MND-P-2374
PHYSICS AND MATHEMATICS

13-WATT CURIUM-FUELED THERMOELECTRIC GENERATOR FOR HARD LUNAR IMPACT MISSION

Final Report - Subtask 5.8

By
J. Bloom

August 1960

Martin Company
Baltimore, Maryland

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FINAL REPORT

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SUBTASK 5.8

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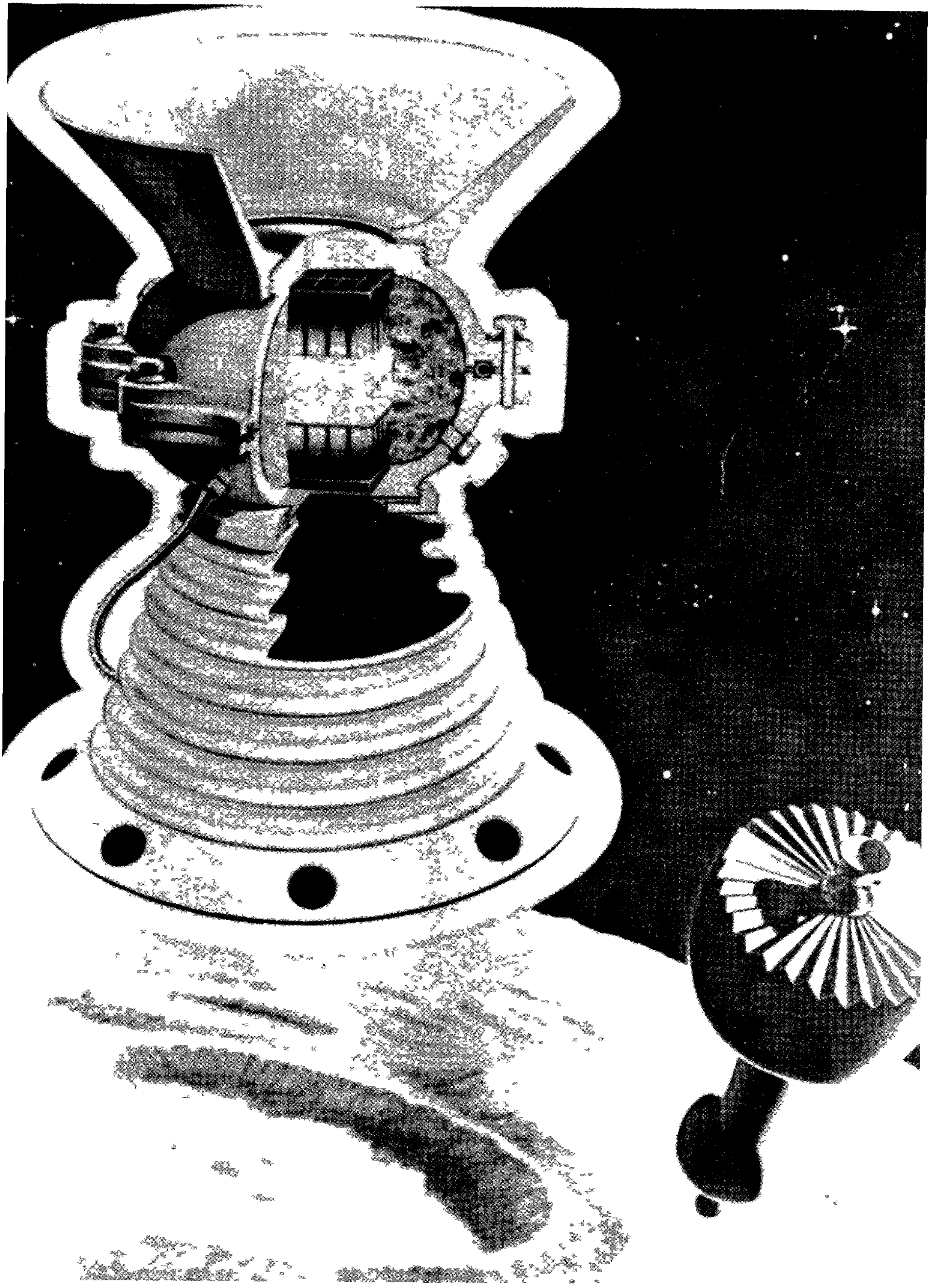

FOREWORD

This is the final report prepared by The Martin Company on the 13-watt Curium-fueled thermoelectric generator for a hard lunar impact mission. It was prepared under AEC contract AT(30-3)-217, Subtask 5.8.

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SUMMARY

Martin Nuclear Division has completed a conceptual design study for a radioisotope-powered thermoelectric generator of minimum size and weight capable of sustaining a hard impact on the moon, followed by continuous generation of electrical power for a period of 60 earth days in the lunar environment.

The generator produces a minimum of 13 watts of DC power at 3 volts and has been designed to withstand uniform decelerations of at least 1000 g. It derives its power from the decay of the radioisotope Curium-242. The generator, complete with mounting structure and radiator (but exclusive of radiation shielding for protection of the payload) weighs 6.2 pounds and occupies a volume of 350 cubic inches.

I. INTRODUCTION

As part of an AEC-sponsored general program for developing radioisotope-fueled auxiliary power units for space applications, The Martin Company, in October, 1959, contracted to design a Curium-242-powered thermoelectric generator suitable for a six-month space mission. In December, the Jet Propulsion Laboratory of the California Institute of Technology--a contractor to the National Aeronautics and Space Administration--asked the Missile Projects Branch, Division of Reactor Development (Aircraft Reactors), AEC Headquarters, to change the specifications for the generator to permit its consideration for a lunar mission in which an instrument capsule containing the generator would impact on the surface of the moon at a velocity of about 500 ft/sec and would survive to produce power for a period of 60 days. This necessitated a radically different approach to the design, and in January, 1960, the new program was begun. In April, the AEC determined that the original approach should be completed also, and the results of the latter study have been published in report MND-P-2373 (Ref. 1).

A preliminary safety analysis of the two concepts has been completed and the results have been published in report MND-P-2366 (Ref. 2).

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II. DISCUSSION OF REQUIREMENTS AND DESIGN ASSUMPTIONS

The conceptual design of the lunar impact generator was to be based on the following specifications:

(1) A maximum weight of 18 pounds, excluding a DC-to-DC voltage converter.

(2) A continuous output of 13 watts at 3 volts DC for a period of 60 days after impact.

(3) Ability to withstand the decelerative forces resulting from lunar impact at 500 ft/sec.

(4) Maximum reliability and efficiency consistent with the other specifications.

(5) Use of Curium-242 as the radioisotope fuel.

(6) Assurance of fuel capsule integrity under adverse conditions of launch vehicle abort.

(7) Reduction of external radiation level to 60 mrem/hr at 1 meter for ground handling purposes, and to the following levels in operation for protection of radiation-sensitive equipment in the instrument capsule: On a circular area of 100 sq cm located 5 meters from the generator, radiation not to exceed

   (a) 1 photon/cm^2-sec in the energy range 0.4-3.0 mev;

   (b) 0.5 photon/cm^2-sec in the energy range 3.0-10.0 mev.

(8) Loading of the fuel capsule occurs 60 days maximum prior to launch.

Since details of the launch vehicle, impact vehicle, and instrument capsule were not available during the course of this study, it was necessary to make some arbitrary assumptions concerning them, so that the generator design would at least reflect representative conditions for launch, abort, or lunar impact.

The Atlas-Agena B rocket system was selected as being typical of the vehicle capable of delivering a 300-pound payload to the moon at the specified 500-ft/sec impact velocity, thereby fixing accelerations at launch, flight trajectories, abort conditions, and missile-induced vibrations during firing of the engines. Flight trajectories and abort conditions were of principal interest in the safety analysis performed concurrently (Ref. 2). Maximum launch acceleration was assumed to be 20 g, and vibrations induced in the generator during flight were taken to be 10-g RMS accelerations in any direction at frequencies in the range of 20 to 2000 cps.

The impact vehicle was assumed to consist of a retrorocket attached to a spherical instrument capsule roughly 3 feet in diameter, consisting primarily of a crushable structure to cushion the impact forces on the instruments contained therein. Since the crushable material might posses thermal insulation properties, the generator was assumed to be mounted on the exterior of the capsule, at a point farthest removed from the retrorocket. In a successful mission, the retrorocket case would probably impact first. Uniform deceleration of the payload was assumed to occur over a distance of one foot, leading to a figure of about 4000 earth g deceleration. To be conservative, the generator design was based on an impact angle varying a maximum of 15° from the axis of the vehicle. The design that evolved was symmetrical about a linear axis which coincided with the axis of the impact vehicle, and maximum stresses were taken to be compressive in nature along the axial direction.

The possibilities of failure of the retrorocket, which could lead to impact velocities on the order of 10,000 ft/sec, or impact of the generator preceding the rest of the payload, were not considered in this study, except that the generator was made as rugged as possible to give some assurance that the fuel capsule would retain its integrity under these extremely adverse conditions. These conditions do not lend themselves to analytical treatment, and an extensive knowledge of the lunar surface--plus a detailed experimental program--would be required before any assurances in these respects could be made.

III. DESIGN CONSIDERATIONS

The resulting generator configuration is shown in Fig. 1. It consists of a fuel block in the shape of a rectangular parallelepiped, containing the curium fuel in four sealed canisters. Two modular arrays of thermoelectric elements are mounted on opposite sides of the fuel block, and the remaining internal volume of the generator is filled with a solid thermal-electrical insulation. The rigid outer shell, in the general shape of an ellipsoid of revolution, conducts rejected heat to radiator surfaces and is attached to a shock-absorbing bellows structure, which in turn is fastened to the impact vehicle. Simplicity of the design and the lack of moving parts are apparent. The following is a more detailed description of the constituent parts.

A. HOUSING

Beryllium metal was selected for the structural casing material for several reasons. It has a relatively high thermal conductivity and an emissivity coefficient of 80 Btu-ft/ft^2-hr-°F and 0.61, respectively, which makes it desirable for heat transfer reasons (Ref. 3).

For impact stress conditions, the yield strength at 500° F is 40,000 psi. For minimum weight, the density is only 0.066 lb/in.^{3} (aluminum = 0.1 lb/in.^{3}).

For minimum deflection or bowing, it is desirable for a material to possess a high modulus of elasticity. The value for beryllium is $E = 40 \times 10^6$ psi (aluminum $E = 10 \times 10^6$ psi).

The geometry of the case was selected to minimize heat losses from the fuel block, i.e., to shape the internal dimensions to the isothermal temperature line of the fuel capsule.

It should be noted that the attachment threads on the casing offer the possibility of numerous applications. The heat sink and mounting structure can be varied to suit the cooling conditions, whether conduction, convection (natural or forced), radiation or any combination of the above in a liquid, air, or vacuum heat sink medium.

B. THERMOELECTRIC ELEMENTS

Sixty-four individual thermoelectric elements of square cross section are bonded into a rigid square module approximately 2 x 2 x 1 inch

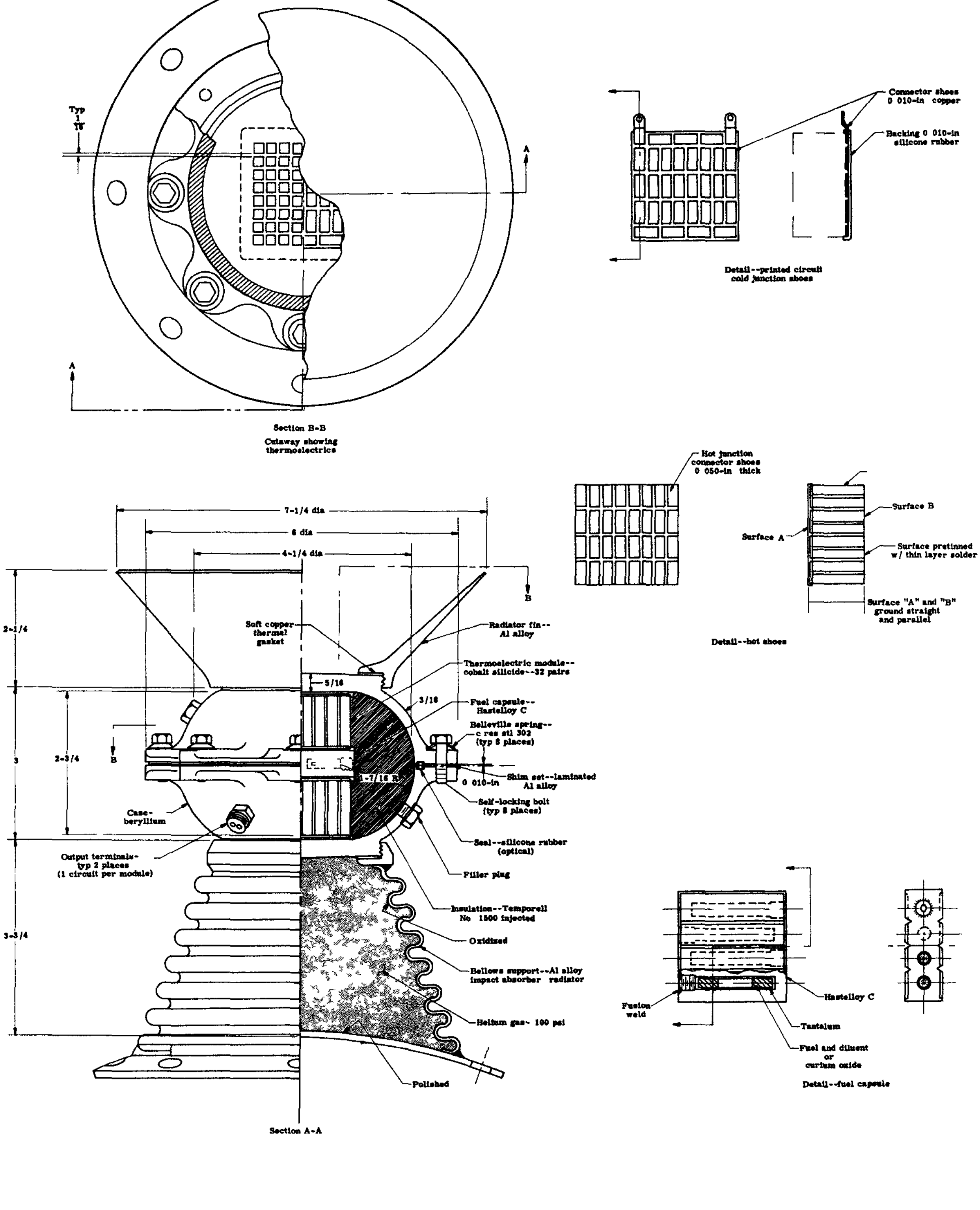


Fig. 1. 13-Watt Thermoelectric Generator for Lunar Impact Mission

high with a thin separator of adhesive insulation cured in place as matrix filler (Fig. 2). The most suitable material to use appears to be Temporell 1500 (Ref. 4). This arrangement will yield several important advantages. Elements of long slender geometry, which contribute to optimum thermal efficiency and higher output voltage, can be used in place of short, squat elements (where previously a short, squat geometry was considered as the best possibility of eliminating columnar failure).

The end surfaces of the modules can be machined parallel to very close tolerances after assembly to obtain uniform length and to insure good thermal and electrical contacts.

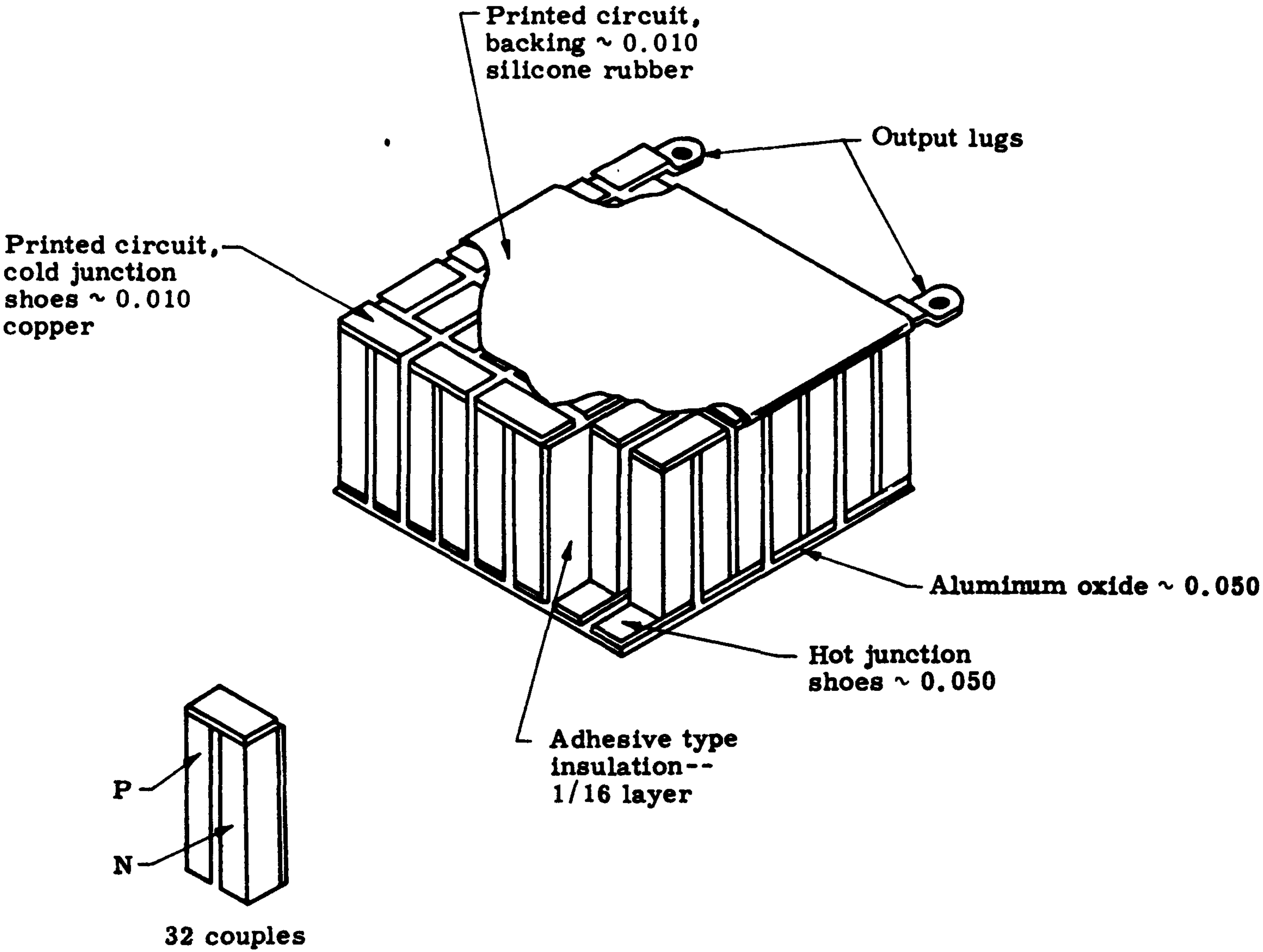
The module arrangement also results in a generator of minimum size and weight. The axes of the elements are oriented coincident with the axis of maximum thrust to provide maximum strength at impact; i.e., direct compression.

To further enhance the impact characteristics, cobalt silicide was selected for use as the thermoelectric semiconductor material because of its superior mechanical properties. Cobalt silicide does not have as high a thermoelectric figure of merit (Z) as lead telluride, but this deficiency is offset by its ability to operate at much higher hot junction temperatures and thus at a higher Carnot efficiency.

C. JUNCTION CONNECTORS

A flexible copper printed circuit (Ref. 5) backed with silicone rubber sheet is used to obtain electrical series contact at the cold junctions. For the radiation level anticipated, the useful life of silicone rubber is about one year (Ref. 6). This arrangement results in simplicity of construction and assembly and minimum weight. Due to the relatively good thermal conductivity of silicone rubber ($60 \frac{\text{Btu-ft}}{\text{hr - ft}^2}$) and high conductivity of copper, the cold junction temperature drop should not exceed 20° F at rated input. The combination of resiliency and ability to yield offers additional benefits in absorbing possible differential expansion between the P and N elements, and in providing maximum reliability on impact.

It was originally planned to use pressure contacts at the aluminum oxide-insulated hot junction electrical shoes. This would have resulted in an electrical contact resistance as high as 40% of the resistance of each element (Ref. 7). Bonding the hot shoes to the element will reduce contact resistance, thus yielding higher efficiency and reduced element cross-sectional area and resulting in lower heat input and overall weight.



Full scale

Fig. 2. Thermoelectric Module Arrangement

Information on techniques for bonding cobalt silicide to electrically conducting materials is extremely limited. Tests at this site indicate that a laboratory program is required to develop techniques and materials for this purpose, but the successful culmination of such a program would result in hot junction contact resistances of about 10% of element resistance.

D. FUEL BLOCK

Hastelloy C was selected as the material for the fuel block because of its impact strength at high temperatures (at 1600° F, Y.S. = 36,000 psi) (Ref. 8).

The thin-walled tantalum fuel canisters provide a corrosion barrier between the Hastelloy and the Plutonium-238 resulting from decay of Curium-242, and permit handling of smaller quantities of the isotope prior to encapsulation.

An attempt was made to optimize the fuel block material and geometry. A minimum thickness is desired to eliminate large lateral heat losses and yet sufficient wall thickness must be available to provide structural strength for abort conditions and helium pressure buildup during radioisotope fuel decay.

The fuel block design is a simple modification (multiple arrangement) of a fuel capsule designed and analyzed previously for another generator concept (Ref. 7).

Linear scoring on the fuel block surfaces, as shown in Fig. 3, is employed to weaken these local areas. In the event of a vehicle abort impact, energy is deliberately absorbed in the fracture at predetermined areas, greatly decreasing the possibility of random or unpredictable fracture of the block or tantalum containers.

E. INSULATION

Temporell 1500 or equivalent insulation is contemplated. After final testing and assembly of the generator, the insulation will be injected through the filler plug to form a rigid, shock-resistant encapsulation of the fuel capsule and thermoelectric modules. An alternative arrangement is to pot the modules in the case with a dummy fuel block and temporary Teflon parting sheet to provide fuel block accessibility and refueling capability. Temporell should not adhere to the Teflon.

It is of interest to note that, due to the minimum thermal insulation cross-section area and the sizing of the case, the heat leakage is reduced to a very attractive 7% of heat input.

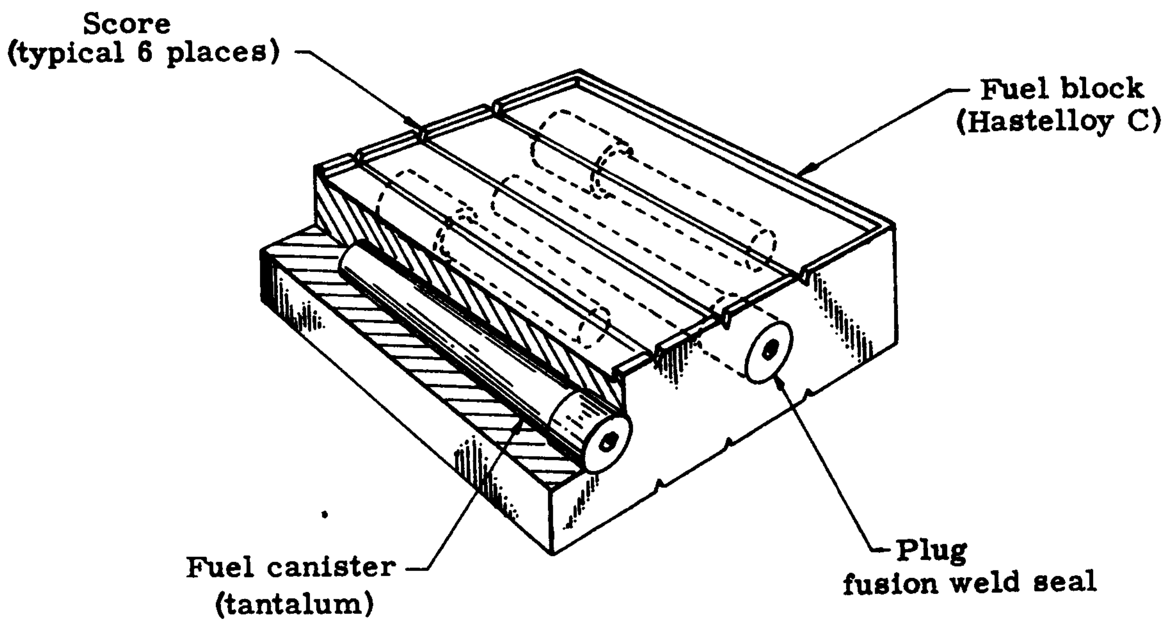


Fig. 3. Fuel Capsule

F. ASSEMBLY FLANGE

Preloading of element modules for good thermal and electrical contacts is essential and is obtained by uniform torquing of the flange bolts. Belleville spring washers (Ref. 9) under the bolt heads provide a simple, lightweight method of maintaining uniform stresses when thermal excursions and expansions occur.

The properties of beryllium at high temperature make it difficult to predict burnup on re-entry to the earth's atmosphere in the event of final stage abort. It is conceivable that intact re-entry may be obtained. Additional experimental studies will be required. Therefore, at this time it has been decided to assemble the housing with aluminum bolts. On re-entry, the bolts should melt preferentially, thus permitting separation of the two halves of the housing and exposing the Hastelloy fuel block to aerodynamic heating and subsequent burnup.

G. SUPPORT STRUCTURE

A conical-shaped aluminum alloy bellows, filled under pressure with helium gas, constitutes the lower radiator and impact absorber. For purposes of maximum heat transfer, it will be desirable to highly polish the bottom surface of the generator assembly and to oxidize the inner wall of the bellows.

The cone configuration is utilized for stability, both under launch and impact conditions.

At impact, the load is carried into the bellows structure and absorbed simultaneously in the compression of the gas and yield of the bellows walls.

H. RADIATOR FIN

The upper radiator fin is mounted on the top attachment threads of the generator casing and is fabricated of soft annealed aluminum alloy. Maximum heat transfer is realized due to the high thermal conductivity and emissivity coefficient.

Under abort conditions, the soft aluminum will readily yield at impact and absorb a large portion of the shock energy.

The fin cross section is of tapered geometry. The root thickness is several times greater than the tip thickness to obtain maximum fin efficiency.

I. INTERNAL CIRCUITRY

For maximum reliability, the generator internal electrical circuitry is subdivided into two individual circuits which feed an external voltage converter. Thus, if one module circuit should fail, half power will remain available.

J. SUMMATION

The mechanical design with reference to selection of materials, general geometry, and stresses and deflections at impact is based upon the weakest link in the system, considered to be the compressive strength of the cobalt silicide thermoelectric elements at their operating hot junction temperature, 1500° F.

Throughout the study, significant factors that affected performance, reliability, safety, weight, operational life, and ease of fabrication were coordinated and optimized to achieve what is hoped to be a practical and realistic concept.

IV. ANALYSIS

A. PARAMETRIC EVALUATION OF THERMOELECTRIC PERFORMANCE

1. Preliminary Hypothetical Design

One of the most important tasks in the early stages of the design study was to determine the feasibility of eliminating a mechanism for flattening the electrical power output over the design life of 60 days by rejecting surplus heat to the environment directly from the heat source. Such a device is ordinarily required when the mission life is equal to or greater than the half life of the isotope employed. Since the electrical power can be expected to decay at a rate roughly proportional to the decay of the isotope, the electrical (and hence the thermal) output at the end of life fixes the design of the generator, and it is then necessary to bypass excess heat during the active life period to avoid overheating the fuel capsule and thermoelectric elements.

By studying the effect of changes in the lunar ambient temperature over a period of 60 days on the operation of a hypothetical generator similar in geometry to the actual design, it was determined that if a design point of 1400° F hot junction temperature at the end of life coincident with the worst ambient condition of lunar daytime were selected, the hot junction temperature would not exceed 1800° F at any time prior to the end of life. Cobalt silicide is limited to a maximum operating temperature of 1800° F.

The study also demonstrated that at least 13 watts of electrical power were produced over the entire period regardless of the external ambient temperature. It should be noted that the lunar surface is estimated to vary in temperature from +250° in the daytime to -250° F in the nighttime.

Details of the parametric study and the earlier results obtained are reported in Ref. 10. A second analysis produced the data shown in Table 1. The later information is more refined in that provision is made for parasitic heat losses through the insulation. For comparison, a similar calculational technique was applied to the actual generator geometry, and the results shown in Table 2 were obtained.

In any generator for which power flattening has not been provided, the electrical output will vary with time. If the demands of the load are such that a constant power output is required, then arrangements must be made to dissipate the excess electrical power through a parasitic resistive load. It is conceivable that the external DC-to-DC converter for transforming the output voltage could be designed with an efficiency

TABLE 1

Results of Second Parametric Study
Based on Mean Thermal Conductivity Curve
for Cobalt Silicide

Original Design at End of Life in 250° F Ambient

Hot junction temperature (°F)	1200	1250	1300	1400
Cold junction temperature (°F)	493	488	485	474
Thermoelectric efficiency (%)	4.64	5.04	5.13	5.54
Overall efficiency (%)	4.2	4.5	4.6	4.9
Output power (watts)	13	13	13	13
Number of elements	156	142	135	121
Current (amps)	4.3	4.3	4.3	4.3
Voltage (volts)	3	3	3	3
Heat loss (watts)	28	31	33	37

Design Characteristics 60 Days Prior to End of Life in 250° F Ambient

Hot junction temperature (°F)	1425	1525	1575	1700
Cold junction temperature (°F)	540	531	529	516
Thermoelectric efficiency (%)	5.13	5.25	5.54	5.71
Overall efficiency (%)	4.72	4.90	4.97	4.90
Output power (watts)	18.4	18.7	18.2	17.36
Number of elements	156	142	135	121
Current (amps)	5.19	5.21	5.2	5.03
Voltage (volts)	3.55	3.59	3.5	3.45
Heat loss (watts)	36	40	42	47

TABLE 1 (continued)

Design Characteristics at End of Life in -250° F Ambient

Hot junction temperature (°F)	1050	1160	1200	1270
Cold junction temperature (°F)	413	403	399	385
Thermoelectric efficiency (%)	5.14	5.55	5.81	6.61
Overall efficiency (%)	4.0	4.85	4.95	5.20
Output power (watts)	12.2	14.0	14.1	13.44
Number of elements	156	142	135	121
Current (amps)	4.21	4.50	4.51	4.42
Voltage (volts)	2.9	3.11	3.13	3.04
Heat loss (watts)	26	30	32	35

Design Characteristics 60 Days Prior to End of Life in -250° F Ambient

Hot junction temperature (°F)	1375	1460	1500	1675
Cold junction temperature (°F)	477	459	454	444
Thermoelectric efficiency (%)	5.59	5.81	5.83	6.30
Overall efficiency (%)	5.19	5.11	4.94	4.72
Output power (watts)	20.2	19	18	17.5
Number of elements	156	142	135	121
Current (amps)	5.4	5.22	5.11	5.00
Voltage (volts)	3.74	3.61	3.6	3.50
Heat loss (watts)	36	40	42	49

Design Characteristics at Launch in 80° F Ambient (60 Days Prior to End of Life)

Hot junction temperature (°F)	1350	1430	1475	1650
Cold junction temperature (°F)	452	443	435	427

TABLE 1 (continued)

Thermoelectric efficiency (%)	5.68	5.97	5.97	6.23
Overall efficiency (%)	5.26	5.39	5.31	5.25
Output power (watts)	20.5	20.11	19.4	18.8
Number of elements	156	142	135	121
Current (amps)	5.44	5.39	5.29	5.23
Voltage (volts)	3.76	3.73	3.67	3.59
Heat loss (watts)	36	40	42	49

TABLE 2

Results of Lunar Generator Design Analysis

End of Life Parameters in 250° F Ambient--Lunar Environment	
Efficiency (%)	
Thermoelectric	5.36
Thermal	95
Overall	5.2
Temperature (°F)	
Hot junction	1400
Cold junction	458
Fuel centerline	1500
Heat Loss (watts)	17
Voltage, End of Life (volts)	3
Power, End of Life (watts)	13
Number of Elements	127
60 Days Prior to End of Life	
Maximum Temperatures (°F)	
Centerline	1800
Hot junction	1660
Cold junction	476

which varies inversely with the input power, so that it would counteract the decrease in power with time of the thermoelectric generator and thus produce a constant output.

2. Actual Design

For the following equations and equations (1) through (16) in the Appendix, the nomenclature employed is:

A	=	Cross-section area (in.^{2})
E_{12}	=	Voltage generation at couple (volts)
f_c	=	Contact factor (ohms/ohm)
I	=	Current flow (amp)
k	=	Thermal conductivity $\frac{\text{Btu-ft}}{\text{°F-hr-ft}^2}$
L	=	Length (in.)
M	=	Matching factor (ohms/ohm)
N	=	Number of couples
P	=	Power to load (watts)
q	=	Heat flow (Btu/hr)
r	=	Internal resistance (ohms)
R_L	=	Load resistance (ohms)
r_c	=	Contact resistance (ohms)
ρ	=	Resistivity (ohm-in.)
S_{12}	=	Seebeck coefficient (couple) (volts/°F)
T	=	Temperature (°F)
V	=	Voltage drop at load (volts)
Z	=	Figure of merit (°F^{-1})

Subscripts

p	=	P element
n	=	N element
2	=	Hot junction
1	=	Cold junction
(-)	=	Average property

The element geometry has a definite effect upon the impact capability of the generator. The following equations were employed to determine the sizing and number of elements:

Matching factor:

$$\frac{R_L}{r} = \sqrt{1 + \frac{S_{12}{}^2}{\rho\, k}\,\frac{(T_2 + T_1)}{2}} = M \qquad (7)*$$

Number elements required:

$$N = \frac{V\,(M + 1)}{S_{12}\,(T_2 - T_1)\,M} \qquad (11)$$

Element area:

$$A_p = \frac{P\,L_p\,f_c\,(M + 1)}{V\,S_{12}\,(T_2 - T_1)}\left(1 + \sqrt{\frac{k_p\,\rho_p}{k_n\,\rho_n}}\right) \qquad (14)$$

See Appendix for origin and derivation of the above.

Notice that sample results which follow are shown for the specific case where contact resistance is 25% of element resistance. The actual element contact resistance, depending upon the effectiveness of bonding, will be between the optimistic 0% contact resistance and the conservative 50%. Thus, a contact resistance of 25% per element was chosen as a realistic representative value.

Table 3 summarizes parameters for various contact resistances.

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*Equation numbers in this section refer to Appendix.

TABLE 3

Parametric Summary for Specific Contact Resistances at End of Life

	0%	25%	50%
Number of elements	128	128	128
Area/element (in.^{2})	0.0187	0.0234	0.0285
Length (in.)	1	1	1
Hot junction (°F)	1400	1400	1400
Cold junction (°F)	500	500	500
Output voltage (volts)	3	3	3
Output power (watts)	13	13	13
Heat input (watts)	215	265	330

Sample Results

For:

$T_H = 1860° R$

$T_c = 960° R$

$L_p = L_n = 1$ in.

$P = 6.5$ watts

$V = 1.5$ volts

$f_c = 1.25$ (25% contact resistance)

$\bar{\rho}_n = \bar{\rho}_p = 7.34 \times 10^{-4}$ ohm-cm

$\bar{k}_n = \bar{k}_p = 6.5 \times 10^{-2}$ watts/cm-°C

$\bar{S}_n = \bar{S}_p = 9 \times 10^{-5}$ volts/°C then

$$Z = 2.8 \times 10^{-4} \,{}^\circ F^{-1}$$

$$M = 1.18$$

$$N = 31.5 \text{ or } 32$$

$$A_n = A_p = 0.0234 \text{ in.}^2$$

$$x = 0.153 \text{ in.}$$

The entire analysis described above is predicated upon the availability of both P- and N-type cobalt silicide elements. At the time of preparation of this report, P-type material was not available commercially, and it was therefore assumed that its thermoelectric properties would be equivalent to the N-type material. The use of all N-type elements, although feasible, would result in a generator of markedly decreased performance because of excessive heat losses through the electrical connections required to complete the series circuit. It is believed that high temperature thermoelectric materials with physical and thermoelectric properties superior to cobalt silicide will be on the market in the near future.

B. HEAT TRANSFER

The following equations were used to determine the preliminary heat transfer parameters and thus the generator geometry:

Heat through element:

$$q_p = \frac{P\, K_p\, \rho_p\, f_c\, (M + 1)}{V\, S_{12}} \left[1 + \sqrt{\frac{k_p\, \rho_p}{k_n\, \rho_n}} \right] \qquad (16)$$

Radiation heat transfer (Ref. 11):

$$q_R = \sigma A_R\, F_e\, F_A\, (T_R^{\,4} - T_o^{\,4})$$

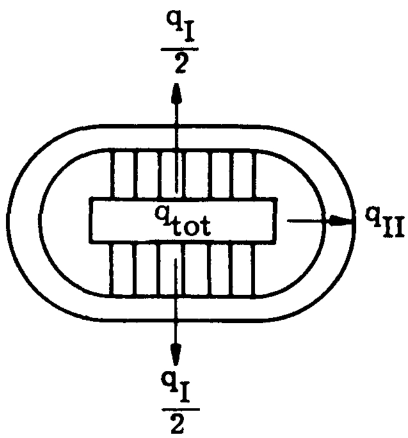
Conduction heat transfer (Ref. 11):

$$q = \frac{A\,(t_1 - t_{n-1})}{\frac{X_1}{k_1} + \frac{X_2}{k_2} + \frac{X_n}{k_n}}$$

Composite cylinder heat transfer (Ref. 11):

$$q = \frac{2\pi k \, l' \, (t_1 - t_n)}{\ell n \frac{r_2}{r_1}}$$

The total heat which must be dissipated from the generator radiator surface is the sum of the heat conducted through the thermoelectric module and the heat leakage from the fuel block. The result is parallel heat flow as shown here:



Heat Flow Paths

The total heat flow is:

$$q_{tot} = q_I + q_{II} = q_{el} + (q_I + q_{II})_{ins}$$

The supplementary nomenclature is:

F_e = emissivity factor

F_A = area factor

r_1 = inner radius (in.)

r_2 = outer radius (in.)

l' = effective length,(in.)

σ = Stefan-Boltzmann const. = 0.173×10^{-8}

Subscripts:

o = ambient

r = radiator

ins = insulation

el = element

Sample Results

For:

q = 261 watts	K_{ins} = 0.025 Btu-ft/hr-°F-ft^2
$F_e = \epsilon$ = 0.6	A_{ins} = 1.5 in.^{2}
F_A = 1	ℓ = 1 in.
T_R = 450° F = 910° R	ℓ' = 2 in.
T_o = 0° F = 460° R	T_H = 1400° F
	T_c = 500° F

then

A = 1.3 ft^2

q_p = 7.1 Btu/hr

$q_I = q_{II}$ = 455 Btu/hr

q_{ins} = 59 Btu/hr

Thermal efficiency = 94%

On the basis of average thermal conductivities listed for the various materials of construction (and their thicknesses when constant) listed in Table 4, the following minor temperature drops have been computed:

Hot connectors (shoes): 12° F

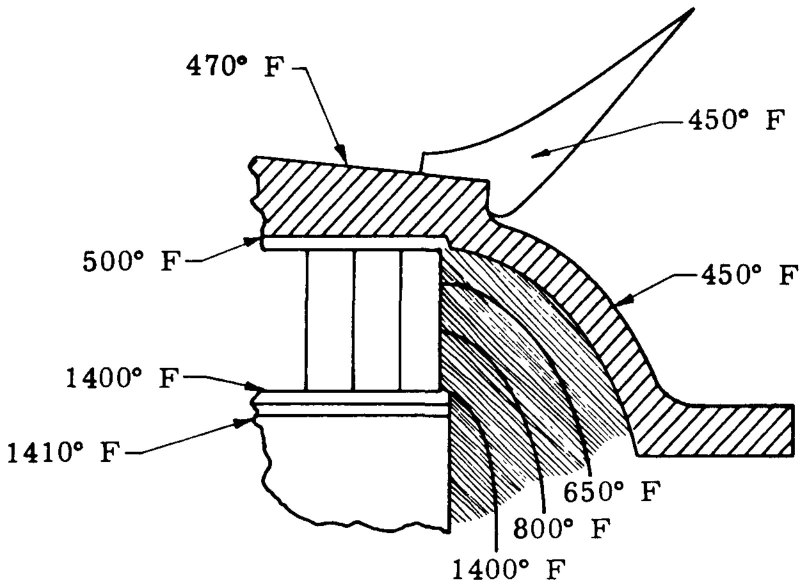
Cold connectors (shoes): 9° F

In summary, the approximate temperature distribution arising from the calculations is shown in the sketch below. Quite obviously, more precise results could be obtained by an iterative process, but this is probably not warranted in view of the uncertainties inherent in many of the physical constants employed.

TABLE 4

Probable Materials of Construction

Material	k (Btu-ft/hr-°F-ft^2)	Thickness (in.)
Al_2O_3	8.5	0.020
Co_2Si	3.7	1
Copper	240	0.010
Beryllium	80	--
Temporell 1500	0.025	--
Silicone Rubber	50	0.010
Molybdenum	84	--
Moly Spray	42	0.050
Melamine	0.5	--
Silicon Carbide	60	--
Temporell 1501	5.2	--



Approximate Temperature Distribution

C. FUEL LOADING AND CAPSULE EVALUATION

The nomenclature to be employed is as follows:

a = Inside radius fuel block (in.)

b = Outer radius fuel block (in.)

f.s. = Factor of safety

n = Number of fuel containers

P_o = Original power loading (watts)

P_t = Power at any time, t (watts)

P_i = Internal pressure (psi)

$p \cdot v$ = From p·v vs t (Fig. 6) (psi · in.^{3})

P = Power density (watts/gm)

ρ = Fuel density (gm/cc)

S_{max} = Maximum stress in fuel block walls (psi)

t = Time (days)

$t_{1/2}$ = Half life of isotope (days)

V_F = Fuel volume per container (cc or in.^{2})

V_v = Void volume per container (in.^{2})

λ = Decay constant (days^{-1})

The following equations were used to establish the fuel block geometry:

$$P_t = P_o \, e^{-\lambda t}$$

Rearranging:

$$P_o = P_t \, e^{\lambda t}$$

Fuel volume:

$$V_F = \frac{P_o}{P_\rho n}$$

Maximum stress (Ref. 12):

$$S_{max} = \frac{b^2 + a^2}{b^2 - a^2} \; P_i \; (\text{f.s.})$$

Void volume:

$$V_v = \frac{pv}{p_i \, n}$$

Enough curium fuel must be provided at encapsulation to give 265 watts of thermal power 120 days later. This is found to be the equivalent of 445 watts at the time of encapsulation. Since the specific power of pure Curium-242 is 122 watts/gram, 3.65 grams of the pure isotope are required at the time of loading into the fuel canisters. The curium actually produced will probably be diluted with Americium-241 and other isotopes of much longer half life, so that the resultant power density of the mixture is about 50 watts/gram. Therefore, containment is provided for 8.88 grams of the mixture in four canisters, or 2.22 grams/canister. At an average density of 10 grams/cc, the fuel volume per canister is 0.222 cc or 0.0137 cu in. For a 2-in. x 2-in. x 5/8-in. Hastelloy C block, the allowable internal pressure due to buildup of helium gas from alpha decay of the curium, is 11,500 psi with a safety factor of 3. The resultant void volume which must be provided in each canister is 0.0120 cu in. on the basis of the equations above and Fig. 4. With allowance for the tantalum liner thickness of 1/64 in. and outside diameter of 3/16 inch, the container is found to be 1.4 in. long. The thin tantalum canisters must be loaded into the fuel block as soon as possible after filling with curium, or they will begin to bulge and will ultimately rupture if they have been welded closed.

D. IMPACT LOAD, STRESSES AND DEFLECTIONS

The nomenclature used is:

A = Area (in.^{2})

a = Acceleration (ft/sec^2)

b = Section width (in.)

d = Mean diameter (in.)

E = Modulus of elasticity (lb/in.^{2})

F = Force (lb)

f = Deflection (in.)

g = Local gravitational acceleration (ft/sec^2)

I = Section modulus (in.^{4})

l = Length (in.)

M = Bending moment (in.-lb)

p = Internal pressure (lb/in.^{2})

r = Radius (in.)

S = Stress (lb/in.^{2})

s = Distance (ft)

t = Section thickness (in.)

U = Energy (ft-lb)

v = Velocity (ft/sec)

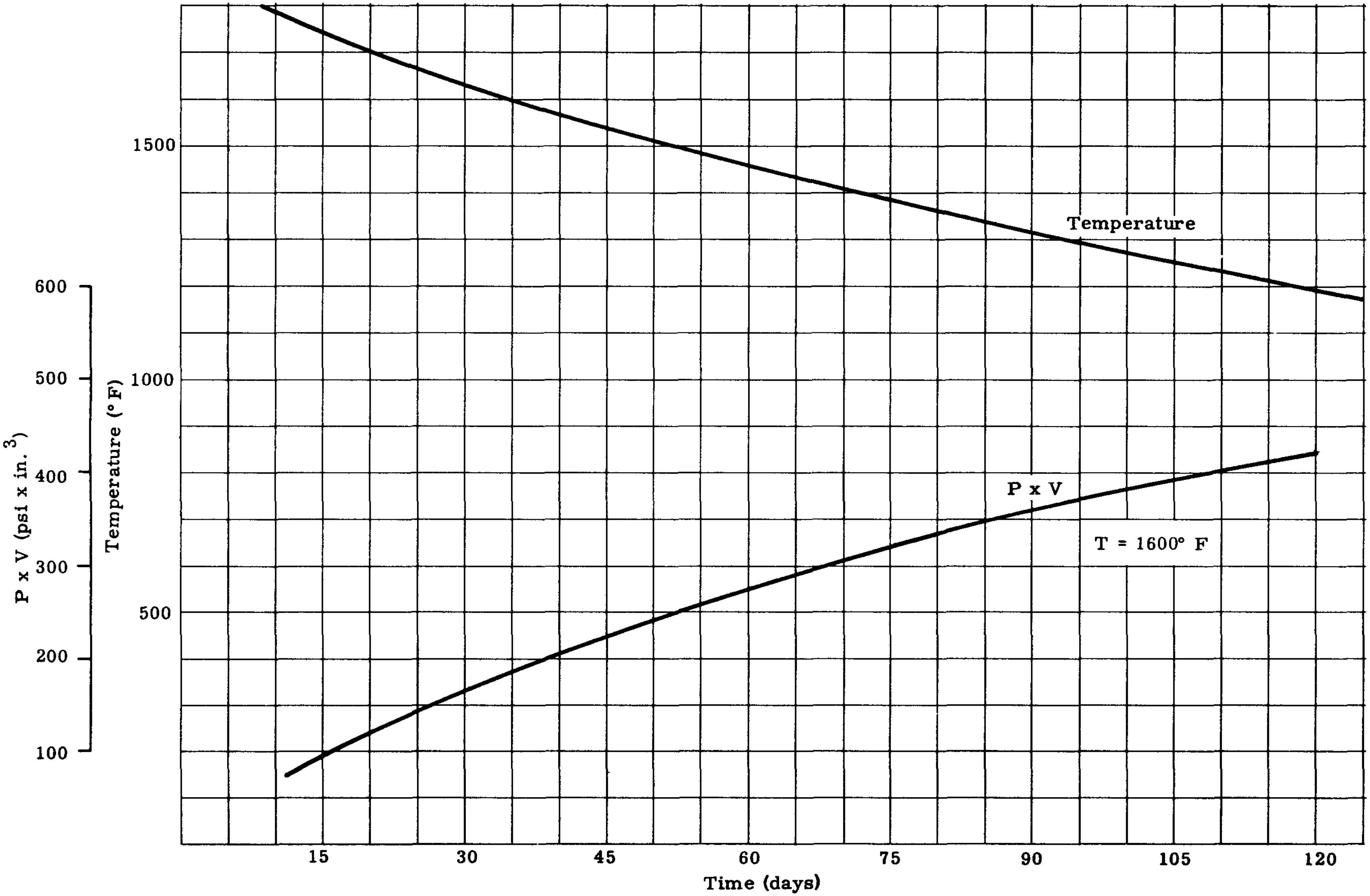


Fig. 4. Helium Gas P x V and T Versus Time--Generator Fuel Block with 3.6 gm Pure Cm-242

Z = Section modulus (in.^{3})

W = Weight (lb)

n = Polytropic constant

Subscripts

I = Impact

b = Bellows

g = Gas

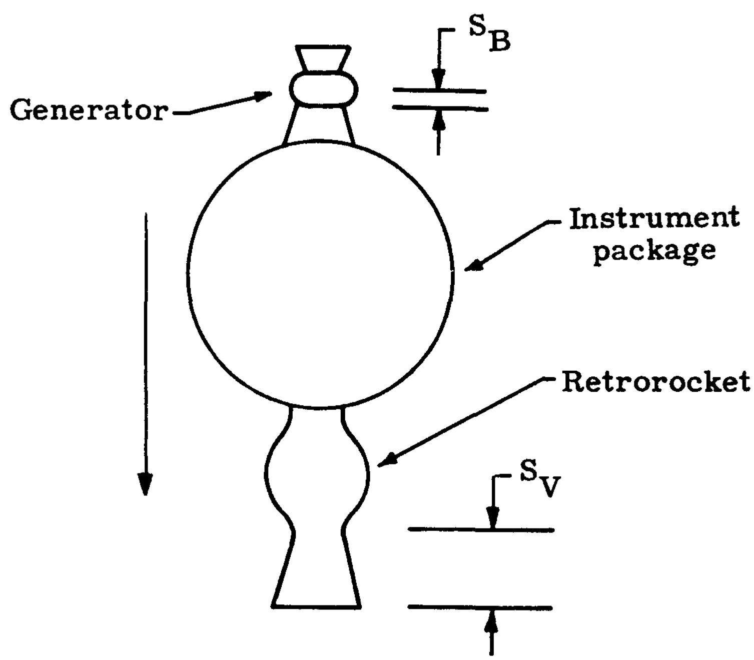
e = Elements

o = Original

f = Final

v = Vehicle

The proposed mounting arrangement is as shown below:



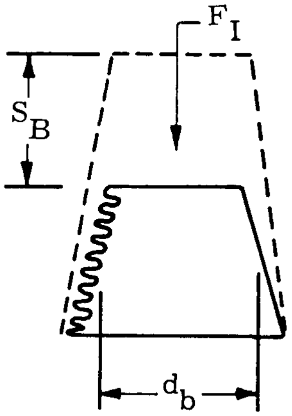
Payload Configuration at Impact

Impact occurs at a given v_o and deceleration takes place through a distance s to final velocity v_f.

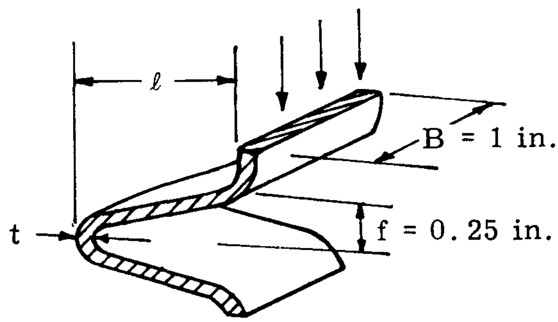
$$v_f^2 = v_o^2 + 2\,a\,s \qquad (1)$$

Thus,

$$a = \frac{v_f^2 - v_o^2}{2s_v}$$



Deflection Diagram



Section Through One Convolute (1-in. segment)

Utilizing a bellows-type impact absorber with deflection s_b as shown above, for the generator support structure, the impact acceleration can be reduced to

$$a = \frac{v_f^2 - v_o^2}{2(s_v + s_b)} \qquad (2)$$

This investigation will be based upon the average acceleration obtained from Eq (2). Thus, the impact force on the bellows support is:

$$F = \frac{W}{g} \cdot a \qquad (3)$$

The scheme considered is a "single shot" type energy absorber. The energy on impact is dissipated in the adiabatic compression of a gas and the permanent yield of a material.

At impact, the, deceleration force is transformed simultaneously into pressurization of the gas and permanent yielding of the bellows walls.

Where x is the fraction of impact force on the gas and y is the fraction on the bellows,

$$F_I = x\,F_I + y\,F_I \tag{4}$$

The governing factors are the hoop stress due to internal pressure and yield deflection of the bellows convolutions.

Approximate deflection as shown in the diagram above is

$$f_b = \frac{Fl^3}{3EI}$$

$$\text{where } F = \frac{y\,F_I}{\pi\,d_b} \qquad \text{and} \qquad I = \frac{bt^3}{12}$$

Combining and solving for y,

$$y = \frac{3\,f\,\pi\,d_b\,E\,bt^3}{4\,F_I\,l^3} \tag{5}$$

Hoop stress due to internal pressure is

$$S = \frac{p_i\,d_b}{2\,t} \qquad \text{where} \quad p_i = \frac{X\,F_I}{\frac{\pi\,d_b^{\,2}}{4}}$$

Combining and solving for x,

$$x = \frac{S\,\pi\,d_b\,2t}{4\,F_I} \tag{6}$$

Substituting Eqs (5) and (6) in Eq (4),

$$\frac{4\,F_I}{\pi\,d_b} = \frac{3E\,bf\,t^3}{\ell^3} + 2\,S\,t \tag{7}$$

The optimum value of t can be determined from Eq (7).

The energy absorbed in compression of the gas is

$$U = \int p\,dv \qquad \text{where } p \cdot v^n = \text{const.}$$

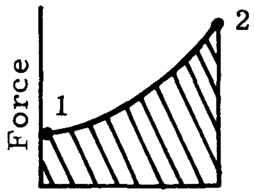
(process occurs in such short time it will be considered adiabatic)

Therefore $$U_{12} = \frac{p_2 v_2 - p_1 v_1}{1 - n} \qquad (8)$$

or

$$U_{12} = \frac{p_1 v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}{1 - n} \qquad (8a)$$

The gas force-deflection diagram is shown below.



Energy Absorbed in Gas

The energy absorbed in the yield of one convolute is (Ref. 12):

$$U = \int \frac{M^2 \, dl}{2\,EI} \qquad \text{where} \quad M = -F \cdot l$$

$$U = \int_0^1 \frac{F^2 l^2 \, dl}{2\,EI} = \frac{F^2 l^3}{6\,EI} \qquad (9)$$

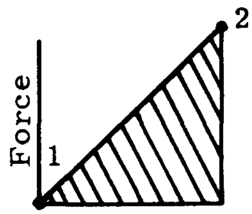
The bellows force-deflection diagram is shown below.

Other equations used in the investigation of element and case loading are

Direct compression loading

$$S_c = \frac{F}{A}$$

$$f = \frac{Fl}{AE}$$



Energy Absorbed in Bellows

Circular plate loaded at center (Ref. 3)

$$S_{max} = \frac{1.365\,F}{\pi t^2} \ln\left(\frac{r}{r_o}\right)$$

$$f = \frac{0.6825\,r^2\,F}{\pi t^3\,E}$$

Bending loads

$$S = \frac{M_c}{I} = \frac{M}{Z}$$

$$f = \frac{F\ell^3}{3\,EI}$$

Sample Results

For v_o = 500 ft/sec

v_f = 0

S_v = 1 ft

S_B = 0.42 ft

Then a = 2750 g

F_I = 16,500 lb.

For s = 10,000 psi

f = 0.2 in.

E = 10×10^6

b = 1 in.

ℓ = 0.5 in.

d_B = 4.5 in.

Then t = 0.050 in.

p = 225 psi.

For p_1 = 100 psi

p_2 = 230 psi

n = 1.66

v_1 = 200 in.^{3}

N = 20

Then U_{12} = 1050 ft-lb (in gas)

U_{12} = 4300 ft-lb (in bellows).

As has been stated before, principal concern is given to compression of the thermoelectric elements on impact. If one conservatively assumes that 50% of the mass of the upper half of the housing and the radiator fin, plus the mass of elements and heat source, contribute to an impact force due to the deceleration of 2750 g, the resulting compressive force on the bottom module of elements is 9100 pounds. By adding to this number the estimated contact force applied by the flange bolts prior to impact, 550 pounds, and dividing by the cross-sectional area of the elements, the compressive stress upon impact is found to be 7000 psi. Cobalt silicide elements should withstand compressive stresses of 5000 to 20,000 psi at operating temperatures. The elements can be expected to compress elastically 0.00023 inch, while the elastic compression of the copper printed circuit and silicone rubber backing is calculated to be 0.007 inch.

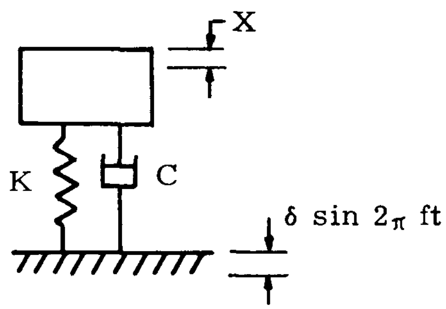
If, for purposes of analysis, the beryllium housing parts are treated as circular plates, supported at the edge and loaded at the center, it is estimated that they will deflect 0.0004 inch upon tightening of the flange bolts, and that the lower housing will deflect 0.0036 inch upon impact. Both types of deformation are elastic in nature. The maximum impact stress induced in the lower housing is 22,000 psi. The assembly bolts do not take up any load on normal impact.

E. ENVIRONMENTAL VIBRATION

Undoubtedly, vibration will be present at launch. For the worst conditions, vibration will span a frequency range of 20 to 2000 cps with a 10-g rms acceleration.

Since the generator internals are held in rigid confinement, no vibration problem is anticipated in this area.

The bellows support is most susceptible to environmental vibrations. A vibration diagram of the mounting arrangement is shown below.



Vibration Diagram

The following equations were employed to investigate vibrations. The equation of motion for a forced vibration in a single degree of freedom is (Ref. 13):

$$X = (A \cos 2\pi f_n t + B \sin f_n t) + \left(\frac{\frac{F}{k} \sin 2\pi f t}{1 - \left(\frac{f}{f_n}\right)^2} \right) \qquad (1)$$

where the first term is transient and is damped out after several cycles.

The effect of steady-state forced vibration with damping may be calculated by retaining $c \frac{dx}{dt}$ in the original differential equation of motion, thus:

$$X_o = \frac{\delta}{\sqrt{\left[1 - \left(\frac{f}{f_n}\right)^2\right]^2 + \left[2 \left(\frac{C}{C_o}\right) \frac{f}{f_n}\right]^2}} \quad (2)$$

where

$$\delta = \frac{F}{k} \quad (3)$$

$$f_n = \frac{1}{2\pi} \quad \frac{kg}{W} \quad (4)$$

$$\frac{C}{C_o} = \text{damping factor.} \quad (5)$$

Essentially, the bellows will be fluid damped due to the internal gas. Also, additional damping will be present, since the bellows structure itself has inherent hysteresis.

Nomenclature

- a = Acceleration (ft/sec^2)
- E = Modulus of elasticity (lb/in.^{2})
- f = Applied frequency (cycle/sec)
- f_n = Natural frequency (cycle/sec)
- g = Gravitational acceleration (ft/sec^2)
- k = Spring constant of system (lb/in.)
- P = Applied force (lb)
- t = Time (sec)
- W = Weight (lb)
- X = Displacement (in.)
- δ = Deflection (in.)

The spring constant of the system is computed to be 1800 pounds per inch, and the resultant resonant frequency is 55 cycles per second. Since the latter value falls within the frequency range of interest, the effect of damping will be critical and must be evaluated experimentally if such a generator is built. The maximum amplitude at the resonant frequency is found to be 0.56 in., giving a magnification factor (X_o / δ) of 17.

F. THERMAL STRESSES AND DEFLECTIONS

To minimize thermal stresses in the assembly, the procedure visualized is to load the element modules and fuel capsule into the case. Flange bolts are not tightened at this time, thus allowing elements, case and other components to approach operating temperature without being stressed. The flange bolts are then torqued down uniformly after steady-state temperature is reached, electrical output is checked, and insulation is injected.

In operation, thermal stresses in the horizontal direction will be negligible, since the temperature distribution throughout the housing will be relatively uniform.

Restraining and thermal stresses in the vertical direction are alleviated by the use of Belleville spring washers.

For thermal stress investigation, the following equations were used with the coefficients of expansion listed in Table 5:

Thermal expansion

$$\Delta L = \alpha L \Delta T \quad (1)$$

Compressive stress due to thermal expansion

$$S_c = \alpha E \Delta T \quad (2)$$

Deflection (direct compression)

$$f = \frac{Fl}{AE} \quad (3)$$

Deflection (bending)

$$f = \frac{Fl^3}{3EI} \quad (4)$$

where

A = Area (in.^{2})

E = Modulus of elasticity (lb/in.^{2})

F = Force (lb)

f = Deflection (in.)

l = Length (in.)

T = Temperature (°F)

α = Expansion coefficient (in./°F)

X = Thickness (in.)

TABLE 5

Thermal Coefficients of Expansion

Material	°F^{-1}
Beryllium	6.9×10^{-6}
Co_2Si--P*	4.7×10^{-6}
Co_2Si--N*	5.4×10^{-6}
Hastelloy C	7×10^{-6}
Aluminum	13×10^{-6}
Temporell 1500	3.5×10^{-6}
Al_2O_3	5×10^{-6}
Molybdenum	8×10^{-6}

* Ref. 14

Preliminary data (Ref. 14) indicate that P- and N-type cobalt silicide elements exhibit different thermal expansion properties, leading to a differential expansion of 0.0006 inch at operating temperatures. To insure good thermal and electrical contact at the cold junction and to avoid the buildup of high compressive stresses in the N-type elements, the use of thin, soft copper connectors and a backing of silicone rubber at the cold junction is recommended. The resiliency of the rubber aids in absorbing shocks, as previously noted. A force of only 5 pounds is required to deform the rubber and shoes by 0.0006 inch, as compared to the design contact pressure of 300 lb/sq in.

Thermal contraction of the generator takes place over its operational life due to the decay of the thermal power of the isotope with time. The differential contraction amounts to 0.0015 inch, which is taken up by the Belleville washers on the flange bolts.

G. WEIGHTS

Calculation of the weights of the individual components shown in Fig. 1 gives a total generator weight of 6.2 pounds. No radiation shielding is provided in the design itself, and the shielding requirement of the payload, if any, would be met by the judicious placement of an external shield between the generator and the radiation-sensitive portion of the payload.

A breakdown of the estimated weight of the generator is given in Table 6.

TABLE 6

Summation of Weights (lb)

Case	1.04
Fin	0.72
Elements	1.10
Fuel Capsule	1.0
Insulation	0.6
Bellows Assembly	1.5
Thermal Gaskets	0.08
Bolts and Springs	0.06
Hot and Cold Shoes	0.10
Total	6.2

V. SHIELDING ANALYSIS

For ground handling purposes, the specifications for external radiation dose rate laid down at the inception of the program can be met without the use of any shielding. This is based on the following contributions to the dose rate from the 3.6 grams of Curium-242 present at the time of encapsulation.

	mrem/hr at 1 meter
Fission neutrons	49.8
Gammas from spontaneous fission	0.39
Gammas from neutron capture	0.04
Gammas from Cm-242 decay	0.009
Gammas from inelastically scattered neutrons	0.002
Total	50.2

To meet the JPL requirement for dose rate to sensitive instruments, 1.2 inches of tungsten or the equivalent would be required between the generator and the payload. Weight of the shield would be determined by the geometry employed.

The limiting photon energy range in determining the shield thickness is 0.4 - 3.0 mev. Only 0.75 inch of tungsten is required to reduce the 3-10 mev dose rate to the required level. At a shield thickness of 1.2 inches, gamma photons from the decay and spontaneous fission processes contribute about equally to the total dose rate.

During the course of this study, it was found that irradiation of the cobalt contained in the cobalt silicide thermoelectric elements by spontaneous fission neutrons resulted in the buildup of gamma-emitting Cobalt-60 at such a slow rate that less than 0.1 photon/cm^2-sec could be expected from this source at a distance of 5 meters.

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VI. GROUND HANDLING AND SHIPPING CONTAINER DESIGN

Since no external shielding is required for ground handling purposes, and since the generator is inherently rugged, the design of a shipping container is alleviated considerably. In addition, no special provisions for removal of heat are required, since any ambient condition anticipated under earthbound restrictions would be less severs than the environment the generator would experience on the moon.

The resulting design, shown in Fig. 5, merely limits direct access to the unit and provides free circulation of air for cooling purposes. It is made of aluminum to reduce weight, since it is rather bulky. The one meter separation distance is determined by the restriction on external ionizing radiation.

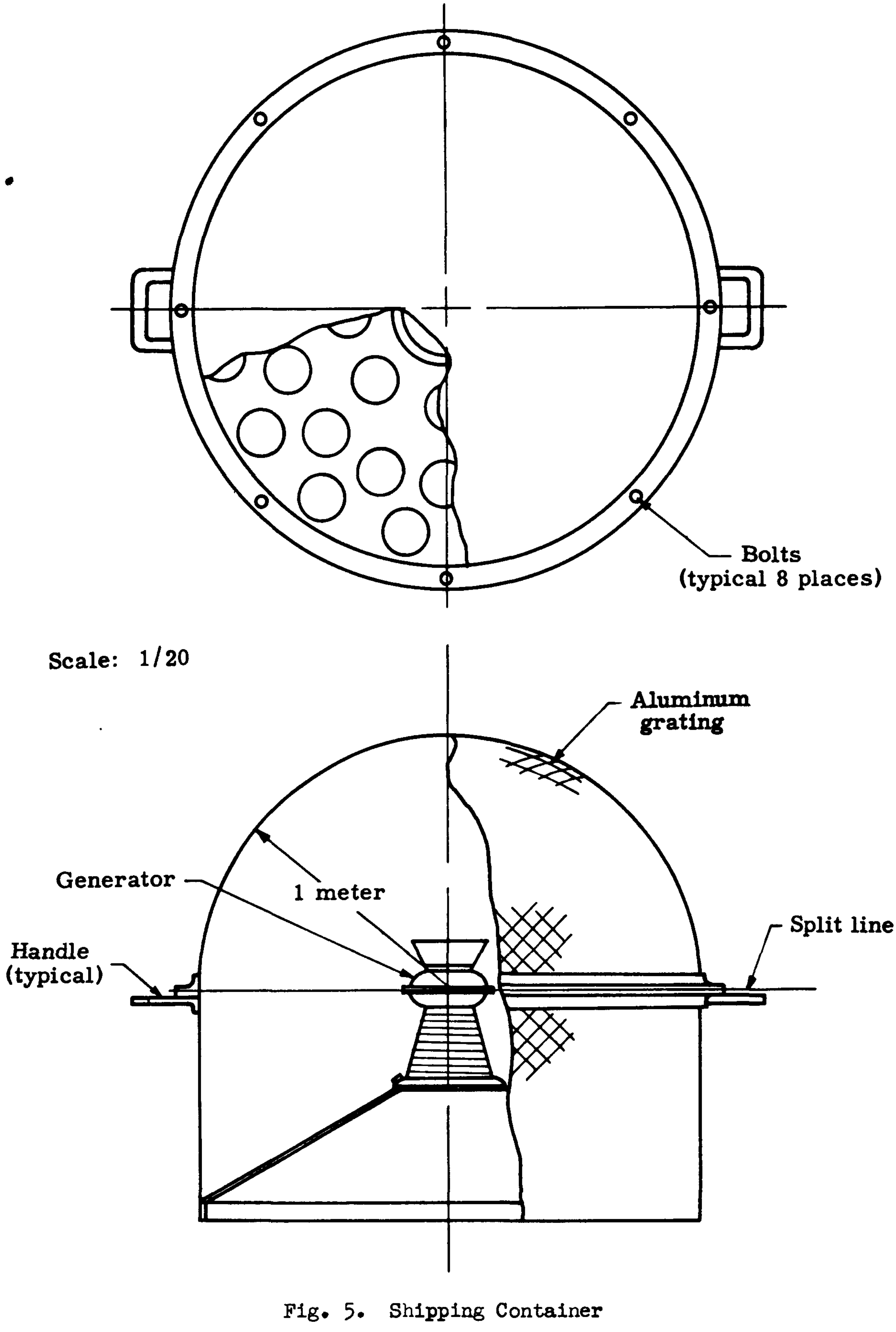

Fig. 5. Shipping Container

VII. RECOMMENDATIONS FOR FURTHER DEVELOPMENT AND CONCLUSIONS

Before a generator of this design can be fabricated and fueled with a high degree of assurance that it will meet the severe operating conditions to be imposed upon it, experimental effort will be required to accurately define parameters in several areas. In particular, the following subjects should be explored:

(1) Mechanical, physical, and thermoelectric properties of cobalt silicide or an equivalent high temperature thermoelectric material.

(2) Life and environmental testing of the thermoelectric module.

(3) Shock and vibration testing of electrically heated generator mockup or prototype.

(4) Process for minimizing electrical contact resistance of thermoelectric elements.

(5) Heat transfer study of generator geometry, under simulated lunar environment.

(6) Simulation of lunar impact conditions on complete generator assembly, with electrically heated power source.

(7) Simulation of retrorocket failure by ultrahigh velocity impact tests, to determine integrity of fuel capsule.

(8) Determination of operational load deflections and thermal stresses for components.

(9) Evaluation of neutron and photon radiation fields, first with simple slab geometries and then with generator prototype.

The results of this study indicate that it is feasible to construct a rugged but lightweight isotope-powered thermoelectric generator capable of surviving the rigors of extraterrestrial flight and a hard oriented landing on another celestial body, and of producing a useful amount of electrical power for a reasonable time.

Verification by experimental means of aspects of the conceptual design which are not amenable to analytical treatment should lead to the development of a new family of auxiliary power supplies.

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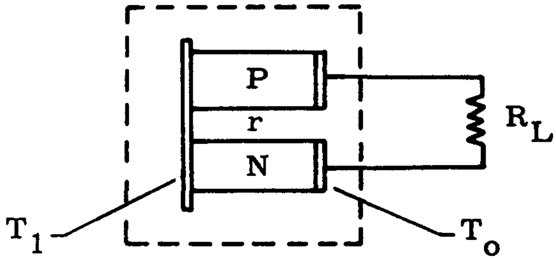
APPENDIX

DERIVATION OF EQUATIONS

Sizing of Thermoelectric Elements



Seebeck EMF (or open circuit voltage) for thermocouple shown in the sketch is (Ref. 15):

$$E_n = S_{12}(T_1 - T_o) \qquad (1)$$

or for any number of couples

$$E = S_{12}(T_1 - T_o) \cdot N \qquad (1a)$$

The terminal voltage is:

$$V = E = Ir \qquad (2)$$

where the current flow is

$$I = \frac{E}{Rl + r} \qquad (3)$$

Power delivered to load

$$P = V \cdot I \qquad (4)$$

or

$$P = EI - rI^2 \qquad (4a)$$

In terms of load resistance

$$P = \frac{V^2}{R_L} = \frac{(E - rI)^2}{R_L} \tag{5}$$

Differentiating (5) with respect to $\frac{R_L}{r}$ and equating to zero yields an optimum value for maximum power of

$$\frac{R_L}{r} = 1 \tag{6}$$

From Ref. 15, maximum thermoelectric efficiency is

$$\frac{R_L}{r} = \sqrt{1 + Z \frac{(T_1 + T_o)}{2}} \tag{7}$$

$$M = \frac{R_L}{r} \tag{7a}$$

and for optimum shape

$$\frac{L_p A_N}{A_p L_N} = \sqrt{\frac{K_p \ \rho_N}{\rho_p \ K_N}} \tag{8}$$

Voltage drop at load is

$$V = I R_L \tag{9}$$

Combining (la) and (3) into (9)

$$V = \frac{S_{12} (T_1 - T_o) \frac{R_L}{r} \cdot N}{\frac{R_L}{r} + 1} \tag{10}$$

Substituting (7a) and solving for number of couples required at optimum:

$$N = \frac{V\,(M + 1)}{S_{12}\,(T_1 - T_o)\,M} \tag{11}$$

Past experience has shown some contact resistance is always present at thermoelectric junctions so that

$$r = \left[\left(\frac{L}{A}\right)_N + \left(\frac{L}{A}\right)_P\right] \cdot f_c \tag{12}$$

where

$$f_c = \frac{r\ \text{element} + r\ \text{contact}}{r\ \text{element}} \tag{13}$$

For determination of optimum element area Eqs (9) and (7a) are substituted into Eq (2):

$$r\,MI = E - rI$$

Rearranging and substituting Eq (4)

$$\frac{1}{r} = \frac{P\,(M + 1)}{V \cdot E_{12}}$$

Substituting for r with Eq (12)

$$\left(\frac{A_P}{\rho\ L}\right)_P + \left(\frac{A}{\rho\ L}\right)_N = \frac{P\,(M + 1)\,f_c}{V\,E_{12}}$$

Substituting for A_N with Eq (8) and E_{12} with Eq (1)

$$\frac{A_P}{L_P} + \frac{A_P}{L_P}\sqrt{\frac{K_P\rho_N}{\rho_P K_N}} = \frac{P\,(M + 1) \cdot f_c}{V \cdot S_{12}\,(T_1 - T_o)}$$

Manipulating of square roots and solving for A_p

$$A_P = \frac{P \cdot L_P \rho_P \ (M + 1) f_c}{V \cdot S_{12} \ (T_1 - T_o)} \left(1 + \sqrt{\frac{K_P \rho_P}{K_N \rho_N}}\right) \tag{14}$$

Likewise for A_N

$$A_N = \frac{P \cdot L_N \rho_N \ (M + 1) f_c}{V \cdot S_{12} \ (T_1 - T_o)} \left(1 + \sqrt{\frac{K_N \rho_N}{K_P \rho_P}}\right) \tag{14a}$$

Heat Flow Through Elements

For conductive heat transfer

$$q = \frac{KA \ (T_1 - T_o)}{L} \tag{15}$$

Substituting Eq (14) for A permits determining with reasonable accuracy the heat flow per element that is required for optimum conditions:

$$q_p = \frac{PK_p f_c \rho_p \ (M + 1)}{V \cdot S_{12}} \left[1 + \sqrt{\frac{k_p \rho_p}{k_N \rho_N}}\right] \tag{16}$$

Thus, for optimum design, the heat transfer through elements can be closely determined without knowing geometry or temperature difference of an element.