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Electric Quadrupole Transitions in Odd-Mass Spherical Nuclei*

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The wave functions derived by Kisslinger and Sorensen from the Pairing plus Quadrupole force model for atomic nuclei are used to compute theoretical E2 electromagnetic transition rates between various low lying states in odd-mass spherical nuclei from Ni to Pb. Comparison is made with experimental data where available. The agreement between theory and experiment is quite good, a large majority of the forty or so cases agreeing within a factor of two while the data covers a range of more than a factor of one thousand.

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DISCLAIMER

I. INTRODUCTION

The occurence of giant quadrupole effects in nuclei has been known for a long time.[1] For the deformed nuclei one observes ground state quadrupole moments which are many times the single particle magnitude, and also E2 transition rates which are many times enhanced above the Weisskopf estimate. For spherical nuclei one observes E2 transitions from the 2+ to 0+ ground state of the even systems which are enhanced from a few times to one hundred times the single particle rate. It has been shown that all these effects as well as "single particle" phenomena can be explained in considerable detail by a nuclear model[2] in which particles moving in a spherical potential well interact with a Pairing plus Quadrupole force. It is the purpose of this note to demonstrate that this model also agrees in considerable detail with the presently available data on E2 transition rates in odd-mass spherical nuclei. In addition, an extensive table of theoretical E2 rates for these nuclei is included to suggest possible interesting cases for future experimental study.

II. THE CALCULATION

With the use of the approximations used by Kisslinger and Sorensen,[3,4] to treat the Pairing plus Quadrupole

Hamiltonian for spherical nuclei, the nuclear states are characterized by two types of excitations, quasi-particles and phonons. For an even-even nucleus the lowest excited state is the one-phonon 2+ state which, because of the energy gap for quasi-particle excitations, is well separated from them and may be treated alone. Good agreement with the 2+ energy and transition rate to the ground state is obtained with the use of a pairing and a quadrupole strength parameter which are smooth functions of mass number, with the exception that the calculated energy is too low, and the B(E2) value too large for nuclei very near a region of deformation. To reduce this difficulty, for the calculation of the properties of odd-mass nuclei, the quadrupole coupling strength was chosen in reference 4 to fit the 2+ energy of the adjacent even-even nuclei.

For odd nuclei, many states of one quasi-particle and zero, one, or two phonons will lie rather close in energy and thus must not be treated as independent excitations. In reference 4, the Pairing plus Quadrupole Hamiltonian is approximately diagonalized in the space of states containing one quasi-particle and up to two phonons. The approximation is to retain in the Hamiltonian only the terms which scatter the quasi-particle while at the same time creating or destroying a phonon. The no-phonon to one-phonon matrix elements are

$$< 0|\alpha_j|H_{INT}|(B^\dagger \alpha^\dagger_{j'})j|0 > = -\bar{X}(\tfrac{5}{4\pi})^{\frac{1}{2}} < j|r^2|j'> C^{2\ j\ j'}_{0\ \frac{1}{2}\ \frac{1}{2}} (-1)^{j-j'} (U_jU_{j'}-V_jV_{j'}), \quad (1)$$

where the effective coupling constant $\bar{X}$, defined in reference 4, depends on the energy of the adjacent even nuclei as described. The creation operators $B^\dagger$ and $\alpha^\dagger_j$ create 2+ phonons and j type quasi-particles respectively, where j represents the angular momentum (and parity) of the shell model state. The state $|0>$ is the vacuum for quasi-particles and phonons and represents the ground state of an even-even nucleus. The quantities U, and V are the usual occupation factors of the Pairing theory. The one-phonon to two-phonon matrix element, given in reference 4, contains the same factor of $(U_jU_{j'}-V_jV_{j'})$.

The wave functions resulting from the diagonalization procedure are of the form

$$\Psi_j = C^j_{j00}\, \alpha^\dagger_j|0> + \sum_{j'} C^j_{j'\ 1\ 2}\, (B^\dagger \alpha^\dagger_{j'})_j|0> + \ldots \quad . \quad (2)$$

The C coefficients (not to be confused with the Clebsch Gordon coefficient of Eq. (1)) for the lowest few states of the spherical nuclei are computed and tabulated in reference 4. The sum on j' is over all single particle states of the same parity as j for which $|j-j'| \leq 2$. The parenthesis indicates that the 2+ photon and j' quasi-particle are coupled to an angular momentum j. Owing to the presence of the U, V factor in Eq. (1), the coupling is often relatively weak for the ground state, for which the factor may be small. For most other low lying states, including some ground states, the coupling is strong enough that $C^{j}_{j', 12}$ is a sizeable fraction of C^{j}_{j00} . It is for this reason that E2 transitions in odd-nuclei are often much more enhanced than the corresponding ground state quadrupole moments.

The E2 transition operator contains two terms[4]

$$\mathcal{M}(E2) = \mathcal{M}(E2)_{S.P.} + \mathcal{M}(E2)_{COL} \quad . \tag{3}$$

The single particle term has matrix elements between quasi-particle states

$$< 0|\alpha_j \mathcal{M}_\mu(E2)_{S.P.}\, \alpha^\dagger_{j'}|0 > = e_{eff} < j|r^2 Y^2_\mu|j' > (U_j U_{j'} - V_j V_{j'}) \quad . \tag{4}$$

It also has matrix elements between quasi-particle states each of which has, in addition, one phonon, but these may be ignored since they will always be overwhelmed by the collective matrix elements discussed below. The effective charge, e_{eff} of Eq. (4), which is to take into account the quadrupole polarization of the core by the particles of the last major shell which are used explicitly in the calculation, is chosen as $e_{eff} = 1$ for neutrons and $e_{eff} = 2$ for protons.

The collective term has matrix elements between states differing by one unit in the number of phonons present. The simplest such matrix element is related to the reduced E2 transition rate for exciting the first excited state of an even nucleus, i.e. the one phonon state:

$$B(E2)_{0+ \rightarrow 2+} = \sum_{\mu m_f} |< 0|B_{m_f} \mathcal{M}_{\mu}(E2)_{COL}|0 >|^2 \quad , \tag{5}$$

where B(E2) is the usual reduced E2 transition probability. The collective matrix elements important for the odd-mass transitions are those between a wave function component containing just a quasi-particle and one containing a quasi-particle and a phonon. Aside from a simple geometrical factor, these matrix elements are just the same as that given by Eq. (5). In order to utilize the information from the even nuclei as much as possible, in evaluating the above matrix element, the average of experimental $B(E2)_{0+ \rightarrow 2+}$ values from neighboring even-even nuclei is used rather than the expression derived in reference 4 for this quantity.

The final form for the reduced transition probability for an E2 transition between two states of the form of Eq. (2) becomes

$$\frac{B(E2)^{TH.}_{j_i \rightarrow j_f}}{2\, j_f+1} = \Bigg| C^{j_i}_{j_i 00}\, C^{j_f}_{j_f 00}\, e_{eff} \frac{< f|r^2|i >}{(4\pi)^{\frac{1}{2}}} (-1)^{j_f - \frac{1}{2}} C^{j_i}_{\frac{1}{2}}{}^{j_f}_{-\frac{1}{2}}{}^{2}_{0} (U_i U_f - V_i V_f)$$

$$+ \left[\frac{B(E2)_{0+ \rightarrow 2+}}{5}\right]^{\frac{1}{2}} \left[(-1)^{j_i - j_f} (2j_f+1)^{-\frac{1}{2}} C^{j_f}_{j_f 00} C^{j_i}_{j_f 12} + (2j_i+1)^{-\frac{1}{2}} C^{j_f}_{j_i 12} C^{j_i}_{j_i 00} \right] \Bigg|^2 . \tag{6}$$

The single particle estimate with which to compare is

$$\frac{B(E2)^{S.P.}_{j_i \to j_f}}{2\, j_f+1} = e^2 \; \frac{< f|r^2|i >^2}{4\pi} \begin{pmatrix} j_i & j_f & 2 \\ C\frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2 \quad . \tag{7}$$

In Eqs. (6)-(7) the factor $2j_f+1$ is included to make the theoretical expression symmetric in j_i and j_f. The theoretical expression, Eq. (6), may then be compared with experimental B(E2) values obtained from Coulomb excitation or lifetime measurements. In the latter case the B(E2) value is related to the partial lifetime by

$$\frac{1}{T_\gamma(E2)} = \frac{4\pi}{75} \; \frac{E^5}{\hbar^6 c^5} \; B(E2) \quad . \tag{8}$$

In Table I, the theoretical values, Eqs. (6)-(7), are given for various possible transitions between low lying states whose wave-functions are computed in reference 4 for spherical nuclei from Ni to Pb. The corresponding experimental value is included when available.

III. DISCUSSION

The theoretical results for E2 transitions in odd nuclei are seen to range from a small fraction of the single particle rate to more than one hundred times single particle. The very large rates occur only for nuclei whose even neighbors have particularly large

$B(E2)_{0+ \rightarrow 2+}$ rates. The small rates are not so common, only about 30 of the 150 calculated cases being less than single particle. These cases occur only if the factor $(U_i U_f - V_i V_f)$ is quite small, corresponding to λ, the Fermi energy of pairing theory, being midway between the two single particle energies in question. The exact isotope for which this occurs depends sensitively on the original choice of the single particle energies. The experimental B(E2) values for odd nuclei in this region also vary over a range of a factor of one thousand. The agreement between theory and experiment is quite good, the large majority of the forty or so cases agreeing to better than a factor of two with the theoretical result.

This agreement shows that there is considerable truth in the picture of wave functions of odd spherical nuclei consisting of linear combinations of quasi-particles and quasi-particles coupled to phonons. Furthermore, the phonons have the same properties as those of the neighboring even nuclei, and the mixing coefficients of Eq. (2) may be computed as in reference 4. Further experimental investigation is desirable, and the calculated rates may serve as a guide to the expected rates. Fast cases might be used to investigate the phonon character of the wave functions in more detail. On the other hand, an observation of the E2 rate in cases for which a large retardation from the single particle rate is predicted might help to determine the validity of the single particle energies used in the calculation and also to indicate possible wave function components not included in Eq. (2).

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TABLE I: Reduced E2 transition probabilities for odd mass spherical nuclei. The first and second column lists the isotope, and the levels between which the transition occurs. Columns 3, 4, and 5, list the single particle, Eq. (7), theoretical, Eq. (6), and experimental B(E2) values divded by $2j_f+1$ where j_f is the final angular momentum, in units of $10^{-50}\ e^2$.

Isotope	Transition	$\frac{B(E2)_{S.P.}}{2j_f+1}$	$\frac{B(E2)_{TH}}{2j_f+1}$	$\frac{B(E2)_{EXP}}{2j_f+1}$
$_{28}Ni^{59}$	$p_{3/2}\ f_{5/2}$	0.010	0.038	
Ni^{61}	$p_{3/2}\ f_{5/2}$	0.010	0.0035	0.012[a]
Ni^{63}	$p_{3/2}\ f_{5/2}$	0.011	0.036	
$_{30}Zn^{63}$	$p_{3/2}\ f_{5/2}$	0.011	0.0002	
Zn^{65}	$p_{3/2}\ f_{5/2}$	0.011	0.078	
Zn^{67}	$p_{3/2}\ f_{5/2}$	0.012	0.105	0.52[a]
	$p_{1/2}\ f_{5/2}$			0.01[a]
$_{37}Rb^{85}$	$p_{3/2}\ f_{5/2}$	0.016	0.148	0.077[b]
Rb^{87}	$p_{3/2}\ f_{5/2}$	0.017	0.122	0.12[b]
$_{48}Cd^{107}$	$s_{1/2}\ d_{5/2}$	0.15	0.023	
	$d_{5/2}\ g_{7/2}$	0.007	0.0065	

TABLE I (cont'd)

$_{48}Cd^{109}$	$s_{1/2}\ d_{5/2}$	0.15	0.81	
	$d_{3/2}\ g_{7/2}$	0.10	0.72	
	$d_{3/2}\ d_{5/2}$	0.022	0.012	
	$s_{1/2}\ d_{3/2}$	0.15	1.98	
	$d_{5/2}\ g_{7/2}$	0.007	0.051	
Cd^{111}	$s_{1/2}\ d_{5/2}$	0.16	1.78	2.4[c]
	$d_{3/2}\ g_{7/2}$	0.10	0.51	
	$d_{3/2}\ d_{5/2}$	0.023	0.0032	
	$s_{1/2}\ d_{3/2}$	0.16	2.09	2.7[c]
	$d_{5/2}\ g_{7/2}$	0.008	0.079	
Cd^{113}	$s_{1/2}\ d_{5/2}$	0.16	2.48	5.0[c]
	$d_{3/2}\ g_{7/2}$	0.10	0.212	
	$d_{3/2}\ d_{5/2}$	0.023	0.0004	
	$s_{1/2}\ d_{3/2}$	0.16	2.02	2.7[c]
	$d_{5/2}\ g_{7/2}$	0.008	0.063	
Cd^{115}	$s_{1/2}\ d_{5/2}$	0.17	2.90	
	$d_{3/2}\ g_{7/2}$	0.11	0.021	
	$d_{3/2}\ d_{5/2}$	0.024	0.0007	
	$s_{1/2}\ d_{3/2}$	0.17	1.43	
	$d_{5/2}\ g_{7/2}$	0.008	0.049	

TABLE I (cont'd)

$_{50}Sn^{117}$	$s_{1/2}\ d_{3/2}$	0.17	0.134	0.02-0.10[d]
Sn^{119}	$s_{1/2}\ d_{3/2}$	0.17	0.0001	< 0.2[d]
Sn^{121}	$s_{1/2}\ d_{3/2}$	0.18	0.111	
Sn^{123}	$s_{1/2}\ d_{3/2}$	0.18	0.393	
Sn^{125}	$s_{1/2}\ d_{3/2}$	0.19	0.61	
$_{51}Sb^{115}$	$s_{1/2}\ d_{5/2}$	0.17	1.34	
	$d_{5/2}\ g_{7/2}$	0.008	0.154	
Sb^{117}	$s_{1/2}\ d_{5/2}$	0.17	1.30	
	$d_{5/2}\ g_{7/2}$	0.008	0.156	
Sb^{119}	$s_{1/2}\ d_{5/2}$	0.18	1.31	
	$d_{5/2}\ g_{7/2}$	0.008	0.155	
Sb^{121}	$s_{1/2}\ d_{5/2}$	0.18	1.42	1.7[e]
	$d_{5/2}\ g_{7/2}$	0.009	0.168	
	$d_{3/2}\ g_{7/2}$	0.11	0.98	1.04[e]
	$d_{3/2}\ d_{5/2}$	0.026	0.141	1.1[e]
	$s_{1/2}\ d_{3/2}$	0.18	0.134	
Sb^{123}	$s_{1/2}\ d_{5/2}$	0.18	1.16	
	$d_{5/2}\ g_{7/2}$	0.009	0.146	0.065[f]
Sb^{125}	$s_{1/2}\ d_{5/2}$	0.19	1.33	
	$d_{5/2}\ g_{7/2}$	0.009	0.168	

TABLE I (cont'd)

$_{52}Te^{121}$	$s_{1/2}$ $d_{3/2}$	0.18	0.174	5.4[d]
Te^{123}	$s_{1/2}$ $d_{3/2}$	0.18	0.86	0.45-0.8[d]
Te^{125}	$s_{1/2}$ $d_{3/2}$	0.19	1.40	
$_{53}I^{125}$	$s_{1/2}$ $d_{5/2}$	0.19	0.96	
	$d_{5/2}$ $g_{7/2}$	0.009	0.134	
I^{127}	$s_{1/2}$ $d_{5/2}$	0.19	1.09	-1.0[n]
	$d_{5/2}$ $g_{7/2}$	0.009	0.156	
I^{129}	$s_{1/2}$ $d_{5/2}$	0.20	1.09	
	$d_{5/2}$ $g_{7/2}$	0.009	0.134	
I^{131}	$s_{1/2}$ $d_{5/2}$	0.20	1.03	
	$d_{5/2}$ $g_{7/2}$	0.009	0.111	
$_{54}Xe^{127}$	$s_{1/2}$ $d_{3/2}$	0.19	1.96	
Xe^{129}	$s_{1/2}$ $d_{3/2}$	0.20	1.87	
Xe^{131}	$s_{1/2}$ $d_{3/2}$	0.20	1.58	
Xe^{133}	$s_{1/2}$ $d_{3/2}$	0.20	1.32	
$_{55}Cs^{129}$	$s_{1/2}$ $d_{5/2}$	0.20	1.25	
	$d_{5/2}$ $g_{7/2}$	0.009	0.229	
Cs^{131}	$s_{1/2}$ $d_{5/2}$	0.20	1.41	
	$d_{5/2}$ $g_{7/2}$	0.009	0.174	
Cs^{133}	$s_{1/2}$ $d_{5/2}$	0.20	1.48	
	$d_{5/2}$ $g_{7/2}$	0.010	0.097	0.20[g]

TABLE I (cont'd)

$_{55}Cs^{135}$	$s_{1/2}$ $d_{5/2}$	0.21	1.30	
	$d_{5/2}$ $g_{7/2}$	0.010	0.043	
Cs^{137}	$s_{1/2}$ $d_{5/2}$	0.21	1.24	
	$d_{5/2}$ $g_{7/2}$	0.010	0.010	
$_{56}Ba^{131}$	$s_{1/2}$ $d_{3/2}$	0.20	2.75	
Ba^{133}	$s_{1/2}$ $d_{3/2}$	0.20	2.20	
Ba^{135}	$s_{1/2}$ $d_{3/2}$	0.21	2.17	0.24[f]
Ba^{137}	$s_{1/2}$ $d_{3/2}$	0.21	1.60	0.65[f]
$_{57}La^{137}$	$s_{1/2}$ $d_{5/2}$	0.21	1.47	
	$d_{5/2}$ $g_{7/2}$	0.010	0.0067	
La^{139}	$s_{1/2}$ $d_{5/2}$	0.22	1.41	
	$d_{5/2}$ $g_{7/2}$	0.010	0.0014	
$_{59}Pr^{141}$	$s_{1/2}$ $d_{5/2}$	0.22	1.50	
	$d_{5/2}$ $g_{7/2}$	0.10	0.0008	< 0.04[a]
Pr^{143}	$d_{5/2}$ $g_{7/2}$	0.011	0.0098	
$_{60}Nd^{145}$	$p_{3/2}$ $f_{7/2}$	0.15	0.78	1.7[h]
Nd^{147}	$p_{3/2}$ $f_{7/2}$	0.15	1.09	

TABLE I (cont'd)

$_{61}Pm^{145}$	$d_{5/2}$ $g_{7/2}$	0.011	0.0088	
Pm^{147}	$d_{5/2}$ $g_{7/2}$	0.011	0.090	0.36[ℓ]
Pm^{149}	$d_{5/2}$ $g_{7/2}$	0.011	0.039	
$_{62}Sm^{147}$	$p_{3/2}$ $f_{7/2}$	0.15	0.89	
Sm^{149}	$p_{3/2}$ $f_{7/2}$	0.15	1.20	
$_{77}Ir^{191}$	$s_{1/2}$ $d_{3/2}$	0.32	11.4	
	$d_{3/2}$ $d_{5/2}$	0.047	7.35	10.5[i]
Ir^{193}	$s_{1/2}$ $d_{3/2}$	0.33	9.15	
	$d_{3/2}$ $d_{5/2}$	0.048	6.40	12.5[c]
$_{78}Pt^{193}$	$p_{1/2}$ $f_{5/2}$	0.33	5.15	
	$p_{1/2}$ $p_{3/2}$	0.33	4.95	
	$p_{3/2}$ $f_{5/2}$	0.048	0.305	
Pt^{195}	$p_{1/2}$ $f_{5/2}$	0.34	4.95	5.0[j]
	$p_{1/2}$ $p_{3/2}$	0.34	2.78	4.5[j]
	$p_{3/2}$ $f_{5/2}$	0.048	0.0005	
Pt^{197}	$p_{1/2}$ $f_{5/2}$	0.34	3.50	
	$p_{1/2}$ $p_{3/2}$	0.34	0.73	
	$p_{3/2}$ $f_{5/2}$	0.049	0.296	
$_{79}Au^{195}$	$d_{3/2}$ $g_{7/2}$	0.22	6.55	
	$s_{1/2}$ $d_{3/2}$	0.33	1.36	3.2[k]
	$d_{3/2}$ $d_{5/2}$	0.048	5.8	

TABLE I (cont'd)

Au^{197}	$d_{3/2}\ g_{7/2}$	0.22	4.40	5.4[i]
	$s_{1/2}\ d_{3/2}$	0.33	0.74	2.7[k]
	$d_{3/2}\ d_{5/2}$	0.048	3.8	5.6[i]
Au^{199}	$d_{3/2}\ g_{7/2}$	0.22	4.20	
	$s_{1/2}\ d_{3/2}$	0.34	0.55	
	$d_{3/2}\ d_{5/2}$	0.049	3.6	
$_{80}Hg^{195}$	$p_{1/2}\ f_{5/2}$	0.34	4.90	11.5[m]
	$p_{1/2}\ p_{3/2}$	0.34	4.35	
	$p_{3/2}\ f_{5/2}$	0.049	0.177	
Hg^{197}	$p_{1/2}\ f_{5/2}$	0.34	3.53	3.5[m]
	$p_{1/2}\ p_{3/2}$	0.34	1.76	
	$p_{3/2}\ f_{5/2}$	0.049	0.0	
Hg^{199}	$p_{1/2}\ f_{5/2}$	0.35	2.64	6.3[m]
	$p_{1/2}\ p_{3/2}$	0.35	0.44	2.5[i]
	$p_{3/2}\ f_{5/2}$	0.050	0.194	
Hg^{201}	$p_{1/2}\ f_{5/2}$	0.35	0.45	
	$p_{1/2}\ p_{3/2}$	0.35	0.042	
	$p_{3/2}\ f_{5/2}$	0.050	0.424	
$_{80}Hg^{203}$	$p_{1/2}\ f_{5/2}$	0.36	0.208	
	$p_{1/2}\ p_{3/2}$	0.36	0.89	
	$p_{3/2}\ f_{5/2}$	0.051	0.407	
$_{81}Tl^{199}$	$s_{1/2}\ d_{3/2}$	0.35	2.74	
	$d_{3/2}\ d_{5/2}$	0.050	0.025	

TABLE I (cont'd)

$_{81}Tl^{201}$	$s_{1/2}$ $d_{3/2}$	0.35	2.87	
	$d_{3/2}$ $d_{5/2}$	0.050	0.014	
Tl^{203}	$s_{1/2}$ $d_{3/2}$	0.36	2.45	3.1[c]
	$d_{3/2}$ $d_{5/2}$	0.051	0.005	< 0.3[c]
	$s_{1/2}$ $d_{5/2}$	0.36	4.08	3.5[c]
Tl^{205}	$s_{1/2}$ $d_{3/2}$	0.36	2.04	2.5[c]
	$d_{3/2}$ $d_{5/2}$	0.052	0.0	< 0.15[c]
	$s_{1/2}$ $d_{5/2}$	0.36	3.73	1.9[c]
$_{82}Pb^{203}$	$p_{1/2}$ $f_{5/2}$	0.36	~ 0.01	0.13[m]
	$p_{3/2}$ $f_{5/2}$	0.051	0.039	
$_{82}Pn^{205}$	$p_{1/2}$ $f_{5/2}$	0.36	0.016	
	$p_{3/2}$ $f_{5/2}$	0.052	0.066	
	$p_{1/2}$ $p_{3/2}$	0.36	0.074	
Pb^{207}	$p_{1/2}$ $f_{5/2}$	0.37	0.37	0.63[m]
	$p_{3/2}$ $f_{5/2}$	0.052	0.052	
	$p_{1/2}$ $p_{3/2}$	0.36	0.36	

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