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THE SIGNIFICANCE OF THE BUCKLING FOR THE SPECTRUM AND THE GROUP CROSS SECTIONS

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Availab[redacted] the
Division o[redacted]ical Information [redacted]n
P. O. Box 10[redacted]
Oak Ridge, Ten[redacted]essee

by

B. MICHEELSEN

[redacted]

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THE SIGNIFICANCE OF THE BUCKLING

FOR THE SPECTRUM AND THE GROUP CROSS SECTIONS

BY

B. Micheelsen

ABSTRACT

The spectrum codes which are used for the calculation of group cross sections assume a leakage term - a buckling, which hitherto has been chosen rather arbitrarily. As the effect of the buckling on the spectrum calculation has been found significant for the Dragon reactor a method is proposed in this paper for the determination of the appropriate bucklings which are to be used in the spectrum code.

The method is iterative as it is using bucklings calculated from preceding diffusion group calculations. The resulting bucklings are dependent on the group energies and the utilisation of these bucklings involve approximations which are least when only small regions are investigated.

The behaviour of the bucklings as functions of energy, the significance for the group cross sections and the multiplication factor is discussed. Finally, a comparison is performed of the spectrum obtained by the aid of the buckling method and by a one-dimensional multi-group diffusion theory code.

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THE SIGNIFICANCE OF THE BUCKLING

FOR THE SPECTRUM AND THE GROUP CROSS SECTIONS

by

B. Micheelsen

1. INTRODUCTION

The procedure used in the Dragon theoretical group for reactivity calculations is to produce spectrum and broad group cross sections by the aid of a spectrum calculation. The spectrum code has many ($\geqslant 43$) fine groups and by the calculation the constants for these are condensed to constants for relatively few ($\sim$4-10) broad groups. These constants are then used in the two-dimensional multi-group diffusion code which gives the reactivity. The number of broad groups in the two-dimensional code is normally strongly limited either by the code or the excessive running times obtained when many groups are used, so it is not possible to avoid the condensation by the spectrum code.

The spectrum calculation by the codes Stewpot, Mupo, or Gam and Gather [1] involve a leakage term, a buckling, and it has been discovered earlier [1] that this buckling has a rather large effect on the calculation for a relatively small reactor like the Dragon Experiment [8].

In this paper a method is outlined in which the bucklings for the spectrum calculation are determined from the space dependence of the flux given by preceding calculations. This leads to bucklings dependent on the group energy. The calculation method and the approximations involved in the use of the bucklings are discussed. Furthermore, the effect of the bucklings on the spectrum, the broad group cross sections, and the multiplication factor is investigated.

2. REMARKS ON THE BUCKLING CALCULATION AND UTILISATION

2.1 Buckling and Spectrum Calculation

The spectrum is, in the following, defined and discussed as the ratio between (a large number of) group fluxes ϕ_i, which are obtained as solution for a system of group equations. An approximation, which is made in the group equations, is that the group energy width ΔE_i is so small that the spectrum within ΔE_i can be assumed constant, i.e. independent of the co-ordinate. This approximation is assumed valid throughout the following.

Spectrum codes as Stewpot [2] and Mupo [3] solve for a homogeneous medium the system of zero-dimensional group equations

$$L_i - \Sigma_i\phi_i + S_i = - D_i B^2\phi_i - \Sigma_i\phi_i + \sum_{j \neq i} C_{j \to i}\phi_j = 0 \qquad (1)$$

where L_i is the leakage in of neutrons, D_i is the diffusion coefficient of group i, B^2 the buckling, Σ_i the removal cross sections, ϕ_i the group flux, and S_i is the source of neutrons transferred into a group i from the groups j ($j \neq i$) - by scattering or by fission.

Equation 1 is obtained from the normal space dependent group equation:

$$- D_i \nabla^2 \phi_i(x) - \Sigma_i \phi_i(x) + \sum_{j \neq i} C_{j \longrightarrow i} \phi_i(x) = 0 \qquad (2)$$

by assuming that all group fluxes have the same shape: $\phi_i(x) = \phi_i f(Bx)$ where ϕ_i is space independent and B^2 is an eigenvalue for the wave equation

$$\nabla^2 f(Bx) + B^2 f(Bx) = 0.$$

This assumption transfers Equation 2 into Equation 1 and the calculated spectrum is independent of x.

This calculation is performed as for a bare homogeneous system with geometrical buckling B^2. The neutron balance (criticality) is, e.g. in Mupo, obtained in Equation 1 either by changing B^2 (changing the dimensions of the bare reactor) or when B^2 is given by changing the fission sources included above in the cross section $C_{j \rightarrow i}$ (source iteration method for a system of given size $\sim B^2$).

When a reflected reactor system is studied the fluxes $\phi_i(x)$ will have different shapes, the spectrum will vary with x, and the buckling $B_i^2(x) = - \frac{\nabla^2 \phi_i(x)}{\phi_i(x)}$ will be a function of x and the group energy (defined by i). If however, the bucklings are given at a point x_o; $B_i^2(x_o) = - \frac{\left(\nabla^2 \phi_i(x)\right)_{x_o}}{\phi_i(x_o)}$, and the fluxes $\phi_i(x)$ (in Equation 2) are approximated with the value ϕ_i^o in a small interval around x_o, then Equation 2 is transferred into

$$- D_i B_i^2 \phi_i^o - \Sigma_i \phi_i^o + \sum_{j \neq i} C_{j \rightarrow i} \phi_j^o = 0. \qquad (3)$$

This equation is equal to Equation 1 and when the bucklings B_i^2 are fed into the spectrum calculation by source iteration, the answer will be the spectrum of group fluxes ϕ_i^o valid only in a small interval around x_o.

In the actual calculations the bucklings B_i^2 are approximated by bucklings obtained from a preceding space dependent diffusion theory calculation and as it is not feasible to calculate B_i^2 and spectra for every point x_o the bucklings are calculated as an average for the region V of interest by:

$$\bar{B}_i^2 = - \frac{\int_V \nabla^2 \phi_i(x) dx}{\int_V \phi_i(x) dx} = - \frac{\int_V \nabla^2 \phi_i(x) dx}{\bar{\phi}_i \cdot V}$$

where $\bar{\phi}_i$ is the average flux in the region V.

The spectrum obtained from Equation 3 is then equal to the spectrum ϕ_i' for a point x' with average bucklings. Another way of describing the spectrum resulting from the use of $\bar{B}_i^2$ is obtained by integrating Equation 2 over the volume of interest [4] and [5]. By the definition of $\bar{B}_i^2$ this transfers Equation 2 into an equation similar to Equation 3 - only with the point fluxes ϕ_i^o substituted with the average fluxes $\bar{\phi}_i$, and this shows that the calculated spectrum is the spectrum of volume averaged fluxes. Whether the spectrum from Equation 3 using $\bar{B}_i^2$ is described as ϕ_i' or $\bar{\phi}_i$, the result can be defined as the "average spectrum", and can be used for the determination of average constants for the region V; but it should be borne in mind that it cannot be expected that a point x' with the average spectrum exists in the region V unless V is infinitely small. The average spectrum defined by the average bucklings is an approximation which is larger, the larger V is.

2.2 Leakage Versus Sources in the Spectrum Code Mupo

The spectrum code Mupo which is used here, and which is able to treat the Equation 3 with B_i^2 as function of group energy assumes a fission process for the source iteration. In the reflector there are no fissions and for the fastest group (group 43) the leakage in L_i is equal to the removal $\Sigma_i \phi_i$; and so $- B_i^2 = \Sigma_i / D_i$. To obtain a spectrum it has been necessary to add a small amount of fissionable material to the reflector graphite. This creates the source but it is then also necessary to reduce L_i so that $- B_i^2 < \frac{\Sigma_i}{D_i}$ to avoid a meaningless answer.

Another factor entering here is that the bucklings B_i^2 are obtained from an earlier calculation and the leakage term = $L_i = - D_i B_i^2 \phi_i$ is only an approximation (even for an infinitely small volume V) to the correct leakage. An error in B_i^2 in groups where $-B_i^2 \sim \frac{\Sigma_i}{D_i}$ as for the fast groups for a reflector can give large errors in ϕ_i (even negative values) and for that reason it is also necessary to make sure that

$$- B_i^2 < \frac{\Sigma_i}{D_i} = 3 \, \Sigma_{tri} \, \Sigma_i,$$

where Σ_{tri} is the transport cross section. It is obvious that these tricks, which normally must be made in the 3-4 fastest groups (above 1.3-2.2 MeV) for a reflector calculation, will lead to a wrong spectrum in these groups, and the spectrum will not be correct before the energy is so low ($E < \sim 1$ MeV) that $S_i \gg L_i$ and S_i is dominated by the slowing down sources.

Problems similar to these for the fast part of the reflector spectrum have not otherwise been encountered by normal calculations, but it must in general be expected that in groups where the leakage L_i is dominating, the uncertainty of the resulting flux ϕ_i is large.

2.3 The Iteration Procedure

The buckling calculation used is principally iterative as the bucklings are calculated from the broad group fluxes given by a preceding (similar) calculation by the multi-group diffusion code (Angie or Gamble [1]). The broad group buckling B_g^2, usually 6-10 groups, are smeared out by crude interpolation to give the bucklings B_i^2 for the fine groups of the spectrum code, and by feeding these bucklings into the spectrum code a set of broad group cross sections is obtained, which is utilised for the multi-group diffusion code. This may lead to a new value of the effective multiplication factor k_{eff} and new broad group bucklings B_g^2, and so the iteration can go on.

However, as the uncertainties of the bucklings are rather large and the full iteration procedure is lengthy, the iteration process is normally stopped after one cycle.

3. CALCULATION METHODS FOR BROAD GROUP BUCKLINGS

The broad group bucklings B_g^2 can be calculated from the results of the multi-group diffusion code in several ways:

(i) The Gamble code gives a neutron balance which directly contains the average number L of neutrons leaking into one cubic centimeter for each group and each region, and the buckling is then calculated as

$$B_g^2 = - \frac{L}{D\phi}$$

where D and ϕ respectively are the diffusion coefficient and the average flux. This average buckling for the region can only be obtained where this region is specified separately in the input for Gamble.

(ii) The Angie code provides no such neutron balance and the buckling has been calculated directly from the fluxes by the formula:

$$B_g^2 = - \frac{\int_V \nabla^2 \phi dv}{\int_V \phi dv} = - \frac{\int_S \nabla \phi ds}{\bar{\phi} \cdot V}$$

where S is the surface of the volume V. $\nabla\phi$ was first calculated from the flux on the boundary and in the adjacent point just inside the boundary, but as this was a too rough approximation the $\nabla\phi$ has in the last version been calculated from a second order polynominal fitted to the fluxes in three consecutive points of which the outermost is on the boundary of the region investigated. By this calculation method it is easy to take a rather large number of fluxes into account in the calculation of both $S\nabla\phi$ and ϕV. A programme No. 5469 has been written for the Ferranti Mercury and this programme allows the fluxes which are used to be read in an arbitrary order from the Angie output.

A programme which reads necessary data for a neutron balance calculation from the Angie output tapes is in progress.

4. BROAD GROUP BUCKLINGS

The result of the calculations for the Dragon reactor with two region core are shown in the Figures 2-7. Core region 1 has $N = \frac{\text{Atoms Th}}{\text{Atoms U-235}} = 20$ while $N = 0$ for region 2. Total number of fuel elements in the core is 37. Calculations of the broad group bucklings have been made for the cases described in Table 1. The old and new energy partition vector (P.V.) in the table indicate the partition of the spectrum groups into the broad groups and Table 2 gives the energies and lethargies for the partitions. (The old partition was used until the results of [1] were found.)

The broad group bucklings are plotted as a function of the average lethargy U of the broad group and curves are drawn through the points to clarify the behaviour in the different cases (these curves are however not identical with the energy dependent bucklings B_i^2 for the fine groups of the spectrum calculations - see below).

Following features have to be observed on the graphs (N.B. Note the irregular negative scales):

(i) The buckling for core region 1 is practically zero at small lethargies (~fast energy region) corresponding to no leakage and flat fast fluxes, while the buckling is negative corresponding to a large leakage in of neutrons at large lethargies (thermal region).

(ii) Region 2 has a rather large positive buckling at low U showing the leakage out to the reflector. Just above thermal energies there is a maximum in buckling which is due to the leakage of neutrons from region 2 to both reflector and region 1 (see the flux plot in Fig. 1). At thermal energies the leakage in from the reflector is very large and the buckling is strongly negative.

(iii) The difference between the 6 group and the 10 group calculations (for Gamble, hot) is not very large (Fig. 2 and 4).

(iv) The change of B_g^2 with the energy partition vector (old and new P.V., hot Angie case, Fig. 3 and 5) is not large. However, it must be noticed that B_g^2 for new P.V. is calculated from an Angie case where another spectrum code than for the old P.V. case has been used with slightly different data and including energy dependent bucklings calculated from the old P.V. case.

(v) There is a decrease in the amplitudes in the thermal range when going from Gamble to Angie (6 groups, hot, 13 elements, Fig. 2 and 4). This is presumably partly due to the differences in scattering model used in the two cases [1], partly it may be due to the differences in the methods used for the calculation of B_g^2.

(vi) The change in buckling from the hot case to the cold case (Fig. 2-5) is not very significant for region 1, but for region 2 the numerical value of the negative bucklings is decreased; - this can be explained by the higher absorption in the reflector. The maximum for region 2 at ~1 eV is decreased in the cold case and the energy of the maximum is also decreased. (It is difficult to say from Fig. 5 whether the cold curve should have a maximum or not.) This is explained by the faster transfer down in energy of the neutrons, and the higher absorption.

(vii) When the diameter of region 1 is decreased by the change from 13 elements to 7 elements, the bucklings of region 1 are again very little sensitive to the changes. The dimensions of region 2 are increased by this change by a factor ~5/4 and as the leakage to and from the reflector only is changed a little the average buckling B_g^2 is decreased, as seen in Fig. 5, by the same factor.

(viii) Bucklings are calculated from the Gamble results for different regions of the reflector, (Fig. 6). Reg. 3 is the radial reflector just outside the core. Reg. 4 is the zone of the radial reflector where the holes for the control rods are. Reg. 5 is the radial reflector outside the control rod zone. Reg. 10 is a reflector region just below the reactor core. The curves for the reflector bucklings give a quickly changing and rather mixed picture, and although the behaviour in several regions can be explained it is not easy to estimate the changes which are going to happen during the iteration for a new system. Furthermore the calculation of bucklings for all the different reflector regions is practically speaking only feasible when the Gamble code is used. As earlier calculations have shown [1] that the k_{eff} for the core changes very slowly with the reflector data, only one buckling curve has been used so far (for all reflector calculations). This curve is constructed as a smooth average curve which shows the general trend of negative bucklings at low U and positive at high U (Fig. 6).

5. BUCKLINGS FOR SPECTRUM CALCULATIONS

The results for the broad group bucklings B_g^2 gave first the impression according to the remarks in Section 4(iii) and 4 (iv) that the smooth curve through the points for B_g^2 would give the curve of the fine group bucklings B_i^2 for the spectrum calculation, and the first set of values C1-H1 and C2-H1 for the two regions of the hot Dragon reactor core with 13 elements in region 1, were obtained from the bucklings B_g^2 for old partition vector as shown at Fig. 7.

When the bucklings B_i^2 are applied in the spectrum code Mupo for a region the multiplication factor k_{eff}^M obtained by Mupo must be equal to the k_{eff} for the reactor if the B_i^2 used are correct. The first sets of bucklings gave a too low value of k_{eff}^M especially for core 2 as seen in Table 3. The reason is that the weighting effect of the fine group fluxes ϕ_i must be taken into account. For example the broad group buckling B_g^2 in group 1 must be assumed to represent the fine group buckling in the fast end of group 1 as the spectrum is increasing rapidly with energy within the broad group, (Angie group 1 is the slowest group). This argument led to new sets of bucklings, labelled C1-H2, and C2-H2 at Fig. 7, and better values of k_{eff}^M were obtained (Table 3).

A more detailed determination of B_i^2 as function of energy has been made for a cylindrical geometry, where the one-dimensional code Zoom can be used [6]. In this study the Dragon core was first represented by a cylindrical geometry in the usual 6 group Angie calculation and the radial bucklings B_g^2 were calculated from the resulting Angie fluxes (Fig. 8 shows B_g^2 for core region 2). From these values of B_g^2 the fine group bucklings called C1-B1 and C2-B1 for the two core regions were estimated. These bucklings were fed into Mupo and as seen in Table 3 this gave a reasonable value for k_{eff}^M for region 1 while the k_{eff}^M for region 2 was a little low. For comparison an 18 group Zoom calculation was made on the cylindrical geometry and 18 group bucklings were calculated for region 2 (Fig. 8). These new bucklings which give a good result for k^M_{eff} of Mupo show that the assumption of the weighting effect within Angie group 1 is correct but that it is exaggerated in the first set: C2-B1. Furthermore, it shows that the maximum around the lethargy $U = 16.6$ ($E \sim 0.6$ eV) is more narrow than it was assumed from the six group bucklings.

Similar procedures of estimating the fine group bucklings and to correct if possible by further iteration must be made for every change of the reactor: temperature change, poisoning, change of geometry etc., but only one more case shall be discussed here.

For the cold Dragon reactor with 7 elements in core region 1, the first buckling sets used in the spectrum code were the C1-H2 and C2-H2 from the earlier hot case with 13 elements in core 1. (It must be remembered that the B^2_g for cold case at Fig. 3 and 5 and the conclusions in Section 4 (vi) and (vii) were first obtained after the following calculation by the two-dimensional diffusion code Angie.) The k^M_{eff} from the Mupo code with these bucklings proved to be far too high (Table 3) for region 2 and as the arguments in 4 (vi) were anticipated a new buckling set C2-C1 was used which had no maximum and numerically smaller negative bucklings. This led to a more reasonable value of k^M_{eff} and the corresponding broad group cross section were used in the diffusion code. The broad group bucklings B^2_g (Fig. 5) justify the changes made in the cold fine group buckling set: C2-C1, but it is seen that two further changes should have been made for region 2 bucklings. The effect of the volume change mentioned in 4 (vii) must be taken into account, and the point at $U \sim 17$ (Fig. 4 and 5), where B^2 is starting to fall rapidly, must be pushed approximately $\Delta U \sim 0.5$ up the lethargy scale when going from hot to cold case. The last effect is to some extent hidden by the six group bucklings B^2_g and is shown most clearly for both core region 1 and 2 by the bucklings from the 10 group Gamble calculations.

In general the buckling for core region 1 is only changing slowly with temperature and size of that region and the bucklings are relatively well defined. For region 2 the uncertainty of the fine group bucklings are rather large around the maximum at $U = 18$ and for high U. The uncertainty for $U > 20$ is however not important as the cross sections of the broad group 1 is almost independent of the buckling in that group - see Section 6.2 and Table 4. The single most important uncertainty is presumably the positioning of the point where B^2 starts to fall rapidly.

For the reflector spectrum calculations the fine group bucklings shown at Fig. 6 have been used - as described in Section 4 (viii) and no detailed study of the behaviour of these bucklings has been made so far.

6. THE DEPENDENCE OF SPECTRUM, GROUP CROSS SECTIONS AND MULTIPLICATION FACTOR ON THE BUCKLINGS

6.1 The Spectrum Dependence

A comparison of the spectrum obtained from the spectrum code with energy dependent bucklings and the spectrum obtained from a space dependent calculation has been possible for the one-dimensional cylindrical geometry mentioned above. The 18 group fluxes from the Zoom calculation on the hot two region Dragon core are shown at Fig. 9 and 10. As a representation of the Zoom flux the 18 group spectrum at the centre of the regions has been chosen as the figures, but as the spectrum is changing rather quickly in region 2, - which in fact indicate that this region should have been divided in two or more regions, - the thermal fluxes in a point r = 45.46 cm a little outside the centre are also shown. The Mupo fluxes condensed into the 18 group system of Zoom are shown at the same figures both for the cases with the energy dependent bucklings: C1-B1 and C2-B2, and the cases with buckling $B^2 = 0$. The last cases correspond to a spectrum calculation for an infinite medium and these spectra would presumably normally have been used if no energy dependent bucklings existed.

Fig. 10 for core region 1 shows only small discrepancies between the three calculations and this is due to the fact that B_i^2 is near 0 for that region. The Mupo case with energy dependent bucklings is, however, slightly better in agreement with the Zoom curve than the $B^2 = 0$ case. For the core region 2 the spectrum calculation with the energy dependent bucklings C2-B2 shows a far better agreement to the Zoom spectrum than the $B^2 = 0$ spectrum. In the epithermal region the infinite medium spectrum is too high, because the leakage out is not taken into account. It is perhaps even more important that the infinite medium spectrum is too hard ($\phi_7 > \phi_6$) in the thermal region, while the C2-B2 Mupo spectrum has a behaviour similar to the Zoom spectrum at r = 45.46 cm. It must be noticed here that the spectrum calculated with the first set of buckling C2-B1 (Fig. 8) also has a shape (is not shown in Fig. 9) which is much more similar to the Zoom spectrum than the infinite medium spectrum.

A similar comparison of spectra for a one-dimensional cylindrical geometry has been made for the cold Zenith reactor, second loading [7]. This reactor has a composition similar to the Dragon reactor but the ratio $N = \frac{\text{Atoms Th}}{\text{Atoms U-235}} = 4.75$ is constant over the core. The core was divided in the spectrum investigation into 3 regions of which regions 1 and 2 are identical in composition while the region 3 is the core edge and has a high voidage. Energy dependent bucklings, Mupo spectra, and Zoom spectra were calculated for each region and the results are shown at Fig. 11 for core region 1 and 3. (The region 2 spectrum is lying just in between the two other spectra.) The agreement between the Zoom spectra and the Mupo spectra is very good and for example the softer thermal spectrum ($\phi_4 > \phi_5$) in region 3 is reproduced very well by the Mupo spectrum calculation. It should be noticed that in the procedure used before energy dependent bucklings were introduced the spectrum and the resulting group cross sections (taking the voidage into account) would have been identical for region 1 and 3.

6.2 Dependence of Broad Group Cross Section

The cross section obtained from the spectrum calculations are normally condensed into 6 broad groups and as a large number of cross sections have been registered only a few general remarks and examples will be mentioned in the following.

The changes in cross section for region 1 when the buckling sets C1-H1, C1-H2, C1-B1 and $B^2 = 0$ are used, are generally small, e.g. changes in absorption cross section are $\lesssim 1\%$. This is of course due to the fact that $B_i^2 \approx 0$ for this region. For core region 2 the changes are larger, and if for example the results are compared for the (Angie) broad group 2, new energy partition vector, which cover the thermal maximum of the spectrum, then the increase in the absorption cross section $\Sigma_{a,2}$ and the cross section for scattering out $\Sigma_{s,\ out,\ 2}$ are 5.5% and 8.5% respectively, when going from the infinite medium spectrum ($B^2 = 0$) to the calculation with the buckling C2-B2 (Fig. 8). The changes for $\Sigma_{a,2}$ and $\Sigma_{s,\ out,\ 2}$ are less than 1 and 2% respectively when the different buckling sets C2-B1, C2-B2 and C2-H2 are used.

A small investigation of the effects of buckling changes has been made by calculations in which the bucklings B_i^2 are kept constant in each broad group, so that a well defined buckling is obtained, but a radical change is made in the bucklings used in two corresponding Mupo calculations. The changes made are artificially high - to create a clear change - and they are larger than the normal uncertainty of the buckling by a factor which is ~5 for core 1 and 2-3 for core 2 - except for group 2, where the factor is ~1. It must be remarked that there is interference between the effects of the buckling changes in the thermal range, and that the effects of the constant bucklings in the broad group can be expected to be different from the effects of changes in the energy dependent bucklings. The changes in per cent of the six group absorption cross sections Σ_a and scattering cross sections $\Sigma_{s,\ out}$ are given in Table 4 for the comparison with new group energy partition. It is seen that rather large changes in cross sections are found especially in $\Sigma_{s,\ out}$. In relation to the uncertainty on the buckling particularly group 2, core 2, is sensitive. The similar comparison [1] with the old six group partition (Table 2) showed that especially the data for group 5, region 1, which covered the thorium resonances, were sensitive to buckling changes. The relative changes for $\Sigma_{a,\ 5}$ and $\Sigma_{s,\ out,\ 5}$ were 14.5% and 26.4% for a change of buckling $\Delta B^2 = 24 \cdot 10^{-4}$ cm^{-2} and this was in fact one of the reasons for the change of group energy partition vector.

6.3 Dependence of the Multiplication Factor

As it can be difficult to see the significance of the changes in group cross sections, a calculation of the infinite multiplication factor k_∞ - as determined by the six group data - has been used for the comparisons. (Mercury programme 5461.) This k_∞ calculation is simple and cheap, but as the leakage is of dominating importance for the neutron balance in the Dragon reactor it shows only some of the buckling effects.

From Table 3 it is seen that the changes in k_∞ for one region when different fine group bucklings B_i^2 are used, are of the order of 0.5%. The effects on k_∞ of the changes of broad group bucklings - mentioned above in Section 6.2 - are shown in Table 5. k_∞ is first calculated for one buckling set, and then the changes in k_∞ are given when the changed cross sections for one group are substituting the cross sections of that group in the first set of data. It can be seen from the table that the normal uncertainty of the buckling - as mentioned above - may in one group change the cross section so much that k_∞ is changed ~0.2-0.3%.

The effects of the buckling changes on the effective multiplication factor k_{eff} obtained by the multi-group diffusion group codes for the reactor have only been determined in a few cases, as the calculations are somewhat lengthy and expensive.

A change of the bucklings used in the Mupo spectrum code from the geometrical buckling of the bare core: $B^2 = + 24.4 \cdot 10^{-4}$ cm^{-2} (fixed in all groups for both regions 1 and 2) to + 0 and + 14.1 . 10^{-4} cm^{-2} for region 1 and region 2 respectively gave a change in k_{eff} obtained from 6 group Angie calculations for the Dragon reactor of 1.8%. Changing from the bare core buckling + 24.4 . 10^{-4} to broad group bucklings calculated from an earlier case gave for the Dragon reactor a change of 1.2% in the k_{eff} from 10 group Gamble calculations [1].

Two other examples have been obtained from control rod calculations on the cold Zenith reactor. The reactivity was calculated without and with control rods using for the core one set of fine group bucklings which was determined from an earlier case without control rods. This gave a $k_{eff} = 1.106$ without control rods, and $k_{eff} = 0.951$ with control rods. The reactor core was then divided in three regions, as described above in Section 6.1 in connection with Fig. 11, and new bucklings were calculated for all three regions with and without control rods. The new bucklings led to $k_{eff} = 1.100$ without control rods and $k_{eff} = 0.937$ with control rods. The changes are 0.5% and 1.5% respectively in k_{eff}.

It appears to be reasonable from these examples to expect changes of 1-2% in k_{eff} when major buckling changes are made. Accordingly it must also be expected that if no effort is made in calculating the bucklings the resulting k_{eff} may be 1-2% wrong, when the present six group diffusion calculations are used.

7. CONCLUDING DISCUSSION

It has been shown that it is feasible to calculate the bucklings as a function of energy. The uncertainty of the calculation and the errors in the application of the bucklings are dependent on the numbers of iteration cycles used, the size of the region investigated, and also on the number and energy partition of the broad group fluxes.

The effects on broad group cross sections when the energy dependent bucklings are introduced, instead of the earlier, rather arbitrarily chosen, fixed buckling, are changes of the order of several per cent. The corresponding change in k_{eff} for the reactor can apparently be up to ~2% and this shows that it is worthwhile to make this extension of the work, even with this comparatively high number (6-10) of broad diffusion groups; for this type of reactor.

The discussion in this paper has mainly dealt with the reactor core and a further study of reflector calculations is necessary. Changes in reflector data are not expected to affect k_{eff} much but the effect for example on reflector temperature coefficient - for which the calculations have been rather poor [7] - might be significant.

At the start of this work it was hoped that it was possible to arrive at one set of fine group buckling for each region and that this buckling set would be almost independent of minor changes, e.g. temperature changes. Such a buckling set should only show the general behaviour of the leakage and the extra work in utilising it would be negligible. However, this has been proved not to be possible - especially for core region 2 - as the changes are too large. As a result somewhat more work than anticipated is needed in the buckling iteration, although it is often possible from earlier experience to guess the change in bucklings created by, e.g. a change in uranium loading.

At this stage it could be questioned whether it would not be better to use the alternative procedure of calculating the spectra in the various regions by a one-dimensional multi-group code (Zoom). The work involved in this is, however, normally much larger than in the buckling procedure and although the one-dimensional calculation in many cases can give well defined and correct answers the influence of the leakage in the direction not taken into account can be very difficult to estimate. It would, for example, be difficult to obtain spectrum variation in axial direction for the relatively high Dragon core with the one-dimensional code, while this gives no extra problems by the buckling procedure. In general the two procedures represent two of the possible methods, which attempt to take the variation of the spectrum with position into account, and the approximations involved depend on the region investigated and the amount of work going into the use. With the buckling procedure as it is used here - by only one iteration and large regions - the uncertainty is rather large and the whole procedure can only be expected to be a first order approximation.

8. ACKNOWLEDGMENT

The author wishes to thank the Dragon theoretical studies group - and especially Mr. J. Blomstrand - for constant interest and valuable discussions during the work.

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Table 1

Dragon and Zenith Cases Investigated by Calculation of Broad Group Bucklings B_g^2

Reactor	No. of elements in region 1 N = 20	Temperature	Group Energy Partition (see Table 2)	Diffusion Code No. of Groups	Spectrum Code	Regions Investigated
Dragon	13	Hot	Old P.V.	Gamble 6 gr	(GAM + GATHER	Core reg. 1 & 2 and reflectors
"	13	"	-	" 10	("	Core reg. 1 & 2
"	13	Cold	-	" 10	("	"
"	13	Hot	Old P.V.	ANGIE 6 gr	Stewpot	"
"	13	"	New	"	Mupo	"
"	7	"	"	"	"	"
"	7	Cold	"	"	"	"
"	13	Hot	"	Angie* 6 gr	"	"
"	13	"	"	Zoom* 18 gr	"	"
Zenith 2nd loading		Cold	New	Angie* 6 gr	"	Core reg. 1,2, and 3

Angie*)
Zoom*) ~one-dimensional calculation

Table 2

Group Energy Partition

Calculation Type	Energies E eV	Lethargies $U = \ln 10^7/E$	Mean U for broad Group	Broad Group No.
Angie 6 gr	10^7	0		
Old P.V.	$0.8208 \cdot 10^6$	2.50	1.25	6
	33.74	12.60	7.55	5
	1.0	16.12	14.38	4
	0.3568	17.15	16.64	3
	0.0225	19.91	18.53	2
	0.001	23.03	21.47	1
Angie 6 gr	10^7	0		
New P.V.	$2.478 \cdot 10^4$	6.00	3.00	6
	$4.54 \cdot 10^2$	10.00	8.00	5
	15.15	13.40	11.70	4
	0.712	16.46	14.93	3
	0.0225	19.91	18.18	2
	0.001	23.03	21.47	1
Gamble 6 gr	10^7	0		
	$0.8208 \cdot 10^6$	2.5	1.25	6
	32.27	12.5	7.50	5
	1.125	16.0	14.25	4
	0.360	17.14	16.57	3
	0.025	19.81	18.47	2
	0.0050	21.42	20.61	1
Gamble 10 gr	10^7	0		
	$6.738 \cdot 10^4$	5	2.50	10
	$1.013 \cdot 10^3$	11.5	8.25	9
	13.71	13.5	12.5	8
	1.855	15.5	14.5	7
	0.400	17.03	16.26	6
	0.200	17.73	17.38	5
	0.100	18.42	18.08	4
	0.0400	19.34	18.78	3
	0.0150	20.32	19.83	2
	0.0050	21.42	20.87	1

Table 3

Mupo Results for Different Sets B_i^2 of Fine Group Bucklings

Case investigated	Core Region	Buckling	Mupo k_{eff}^M	Expected k_{eff}	k_∞^*
HOT DRAGON Region 1 ~N = 20 in 13 elements	1	C1-H1 (Fig. 7)	1.042	1.112	0.995
	1	C1-H2 "	1.100	"	0.990
	1	$B^2 = 0$	0.992	-	0.992
	1	C1-B1	1.108	1.125	
	2	C2-H1 (Fig. 7)	0.697	1.112	1.860
	2	C2-H2 "	1.012	"	1.869
	2	$B^2 = 0$	1.868	-	1.868
	2	C2-B1 (Fig. 8)	0.974	1.125	
	2	C2-B2 "	1.072	"	
Cold Dragon Region 1 ~N = 20 in 7 elements	1	C1-H2	1.253	1.269	1.098
	2	C2-H2	7.41	"	1.889
	2	C2-C1	1.495	"	1.890

*k_∞ calculated from the six group Angie cross sections.

Table 4

The Cross Section Dependence on Bucklings

Change in % of Cross Section for a Buckling Change

Hot Case		New Partition Vector		
Core Region	Group	$\Delta\Sigma_a$ %	$\Delta\Sigma_{s,\,out}$ %	$\Delta B^2 \cdot 10^4$ cm^{-2}
1	1 = slowest	0	0	-200
	2	+4.4	-11.5	-50
	3	-0.65	+3.2	-50
	4	+1.5	+7.8	-30
	5	+3.9	+9	-30
	6	+2.1	+14	-30
2	1	0	0	+250
	2	-2.2	+9.6	+30
	3	-2.1	+0.6	-40
	4	+1.2	+3.4	-13
	5	+1.5	+4.1	-14
	6	+1.7	+10.5	-25

Table 5

Changes in k_∞ with Buckling

	New Partition Vector			
	Core Region 1		Core Region 2	
Buckling	$\Delta B^2 \cdot 10^4$ cm^{-2}	$\frac{k_\infty}{\Delta k_\infty}$ %	$\Delta B^2 \cdot 10^4$ cm^{-2}	$\frac{k_\infty}{\Delta k_\infty}$ %
Fixed in gr	-	0.9922	-	1.869
Gr 6 changed	-30	-0.1	-25	0
" 5 "	-30	+0.3	-14	0
" 4 "	-30	+1.6	-13	+0.06
" 3 "	-50	+1.0	-40	+0.2
" 2 "	-50	+0.4	+30	-0.2
" 1 "	-200	0	+250	0

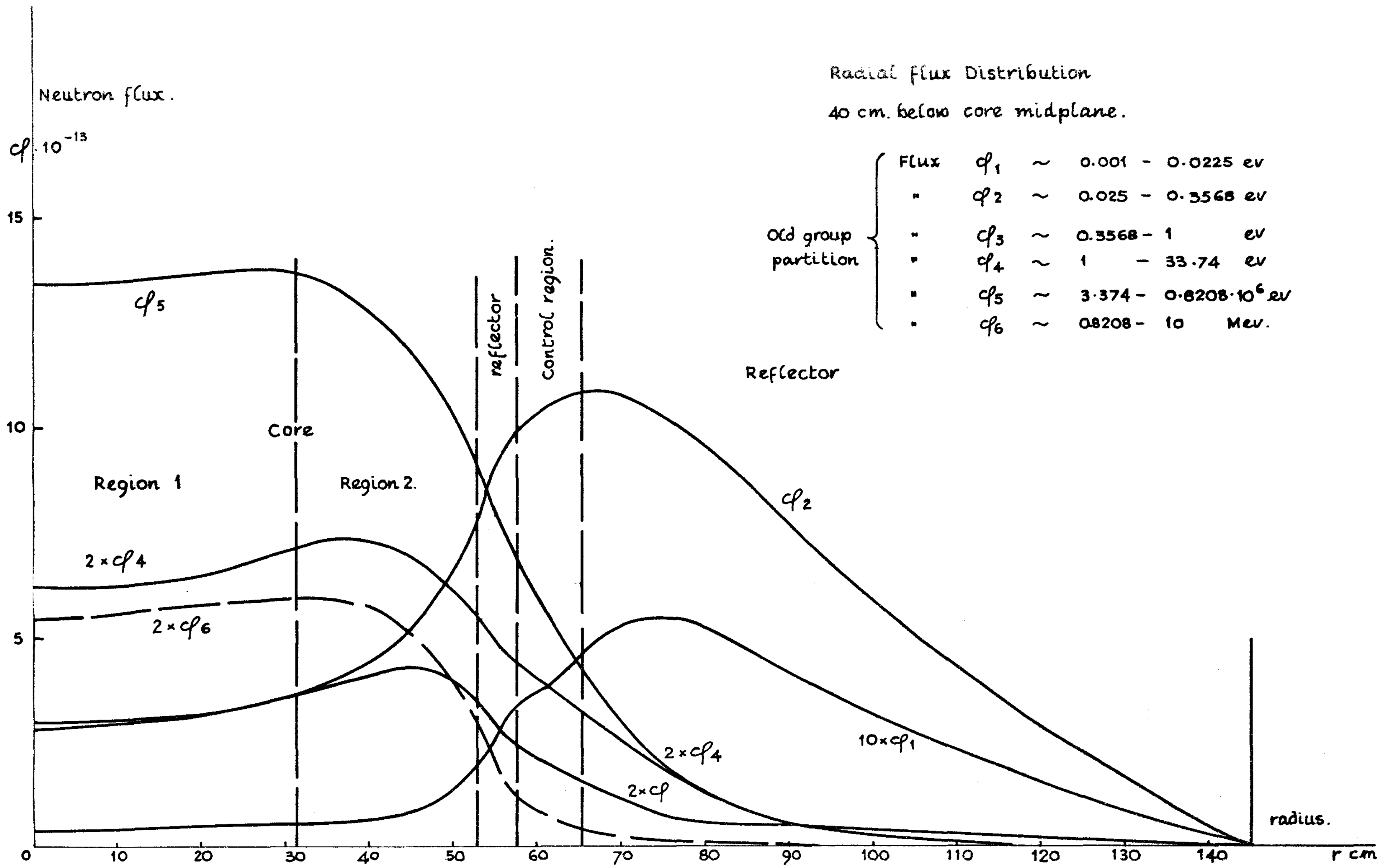

FIG. 1. HOT. DRAGON. TWO REGION CORE. REGION 1: N = 20 IN 13 ELEMENTS.

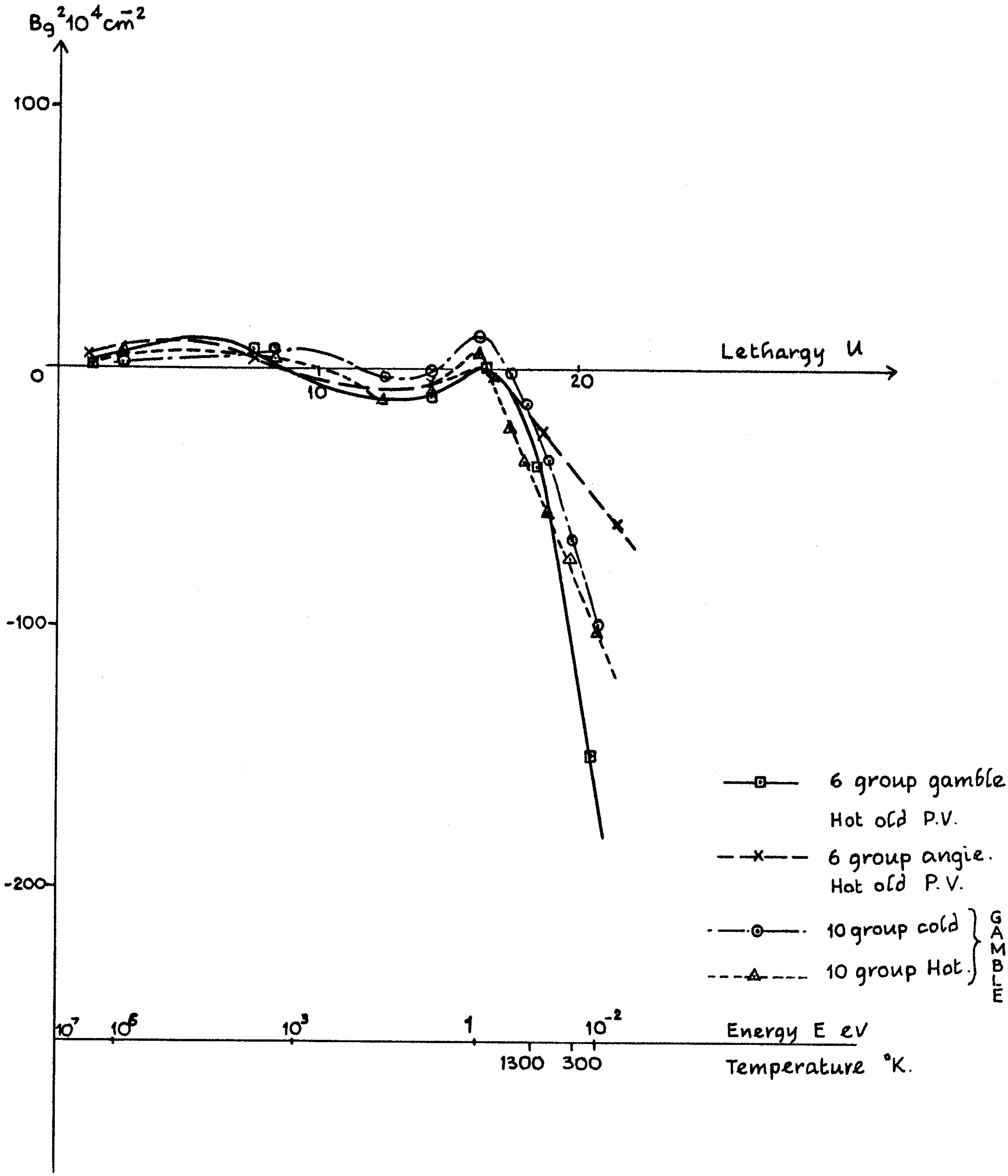

FIG. 2. DRAGON. GROUP BUCKLINGS B_g^2 FOR CORE REGION 1

REGION 1 ~ N = 20 13 ELEMENTS.

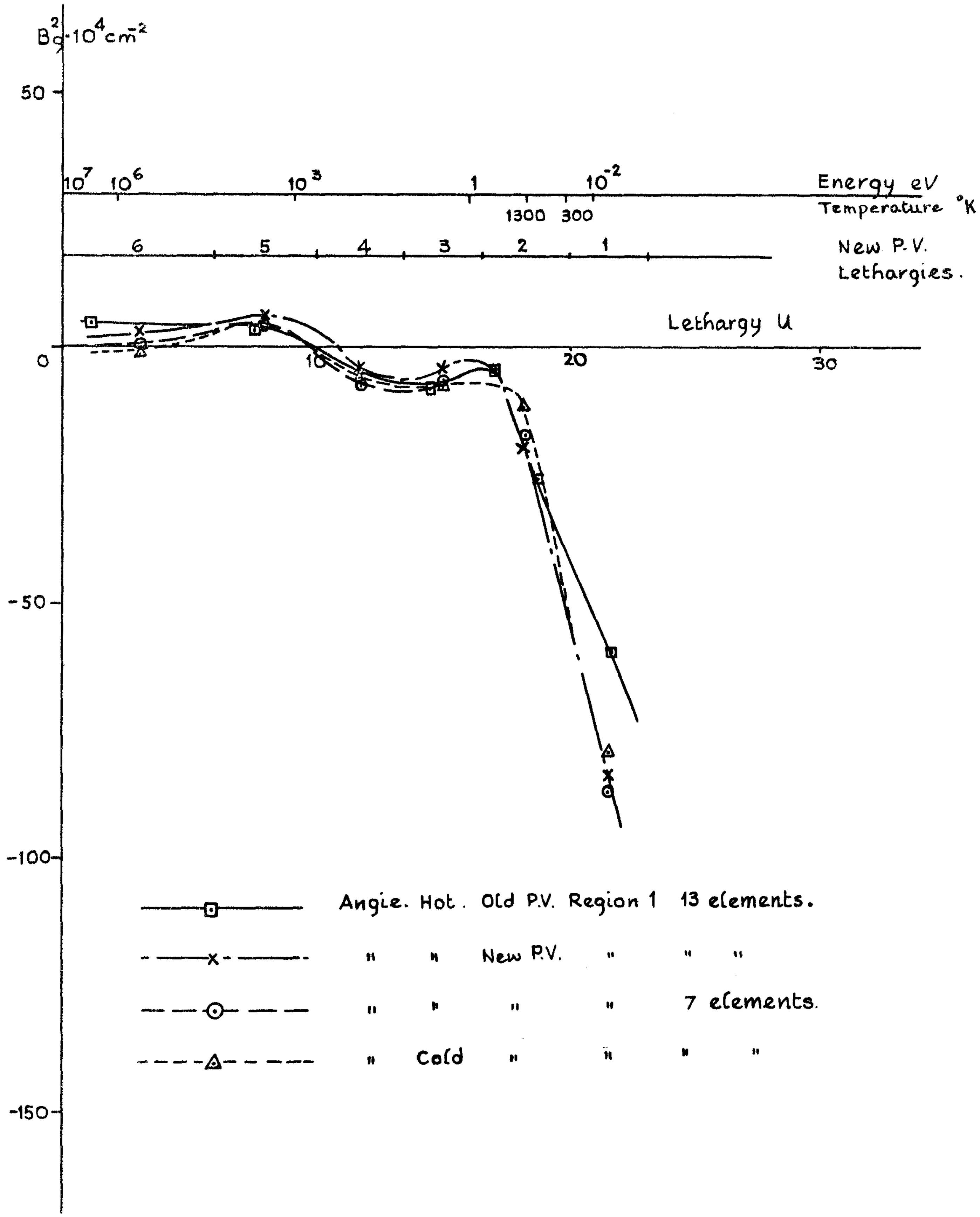

FIG. 3. DRAGON. GROUP BUCKLINGS B_g^2 FOR REGION 1 N=20

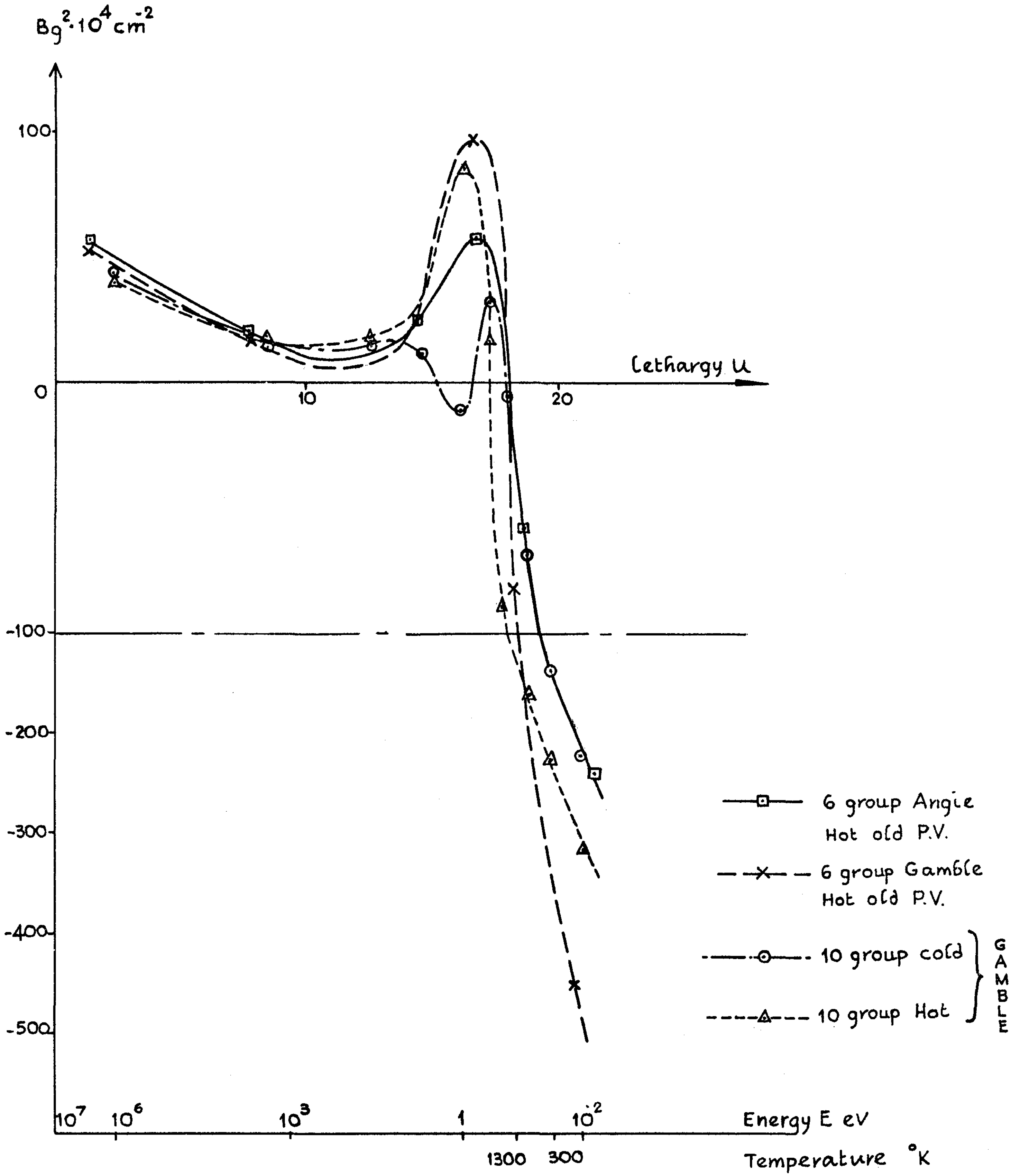

FIG.4. DRAGON. GROUP BUCKLINGS B^2g FOR CORE REGION 2 (N-20 IN INNER CORE WITH 13 FUEL ELEMENTS)

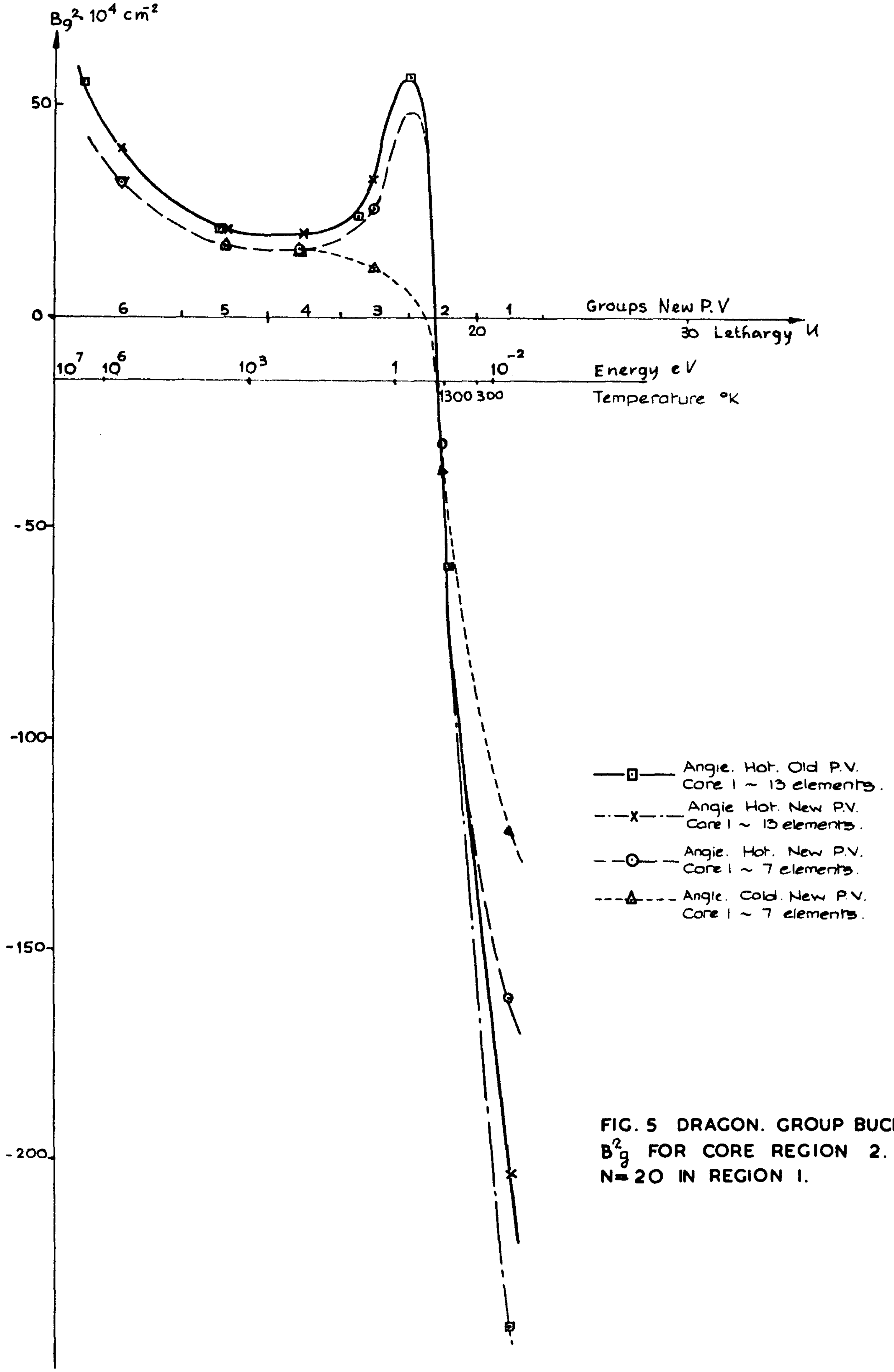

FIG. 5 DRAGON. GROUP BUCKLINGS B_g^2 FOR CORE REGION 2. N=20 IN REGION I.

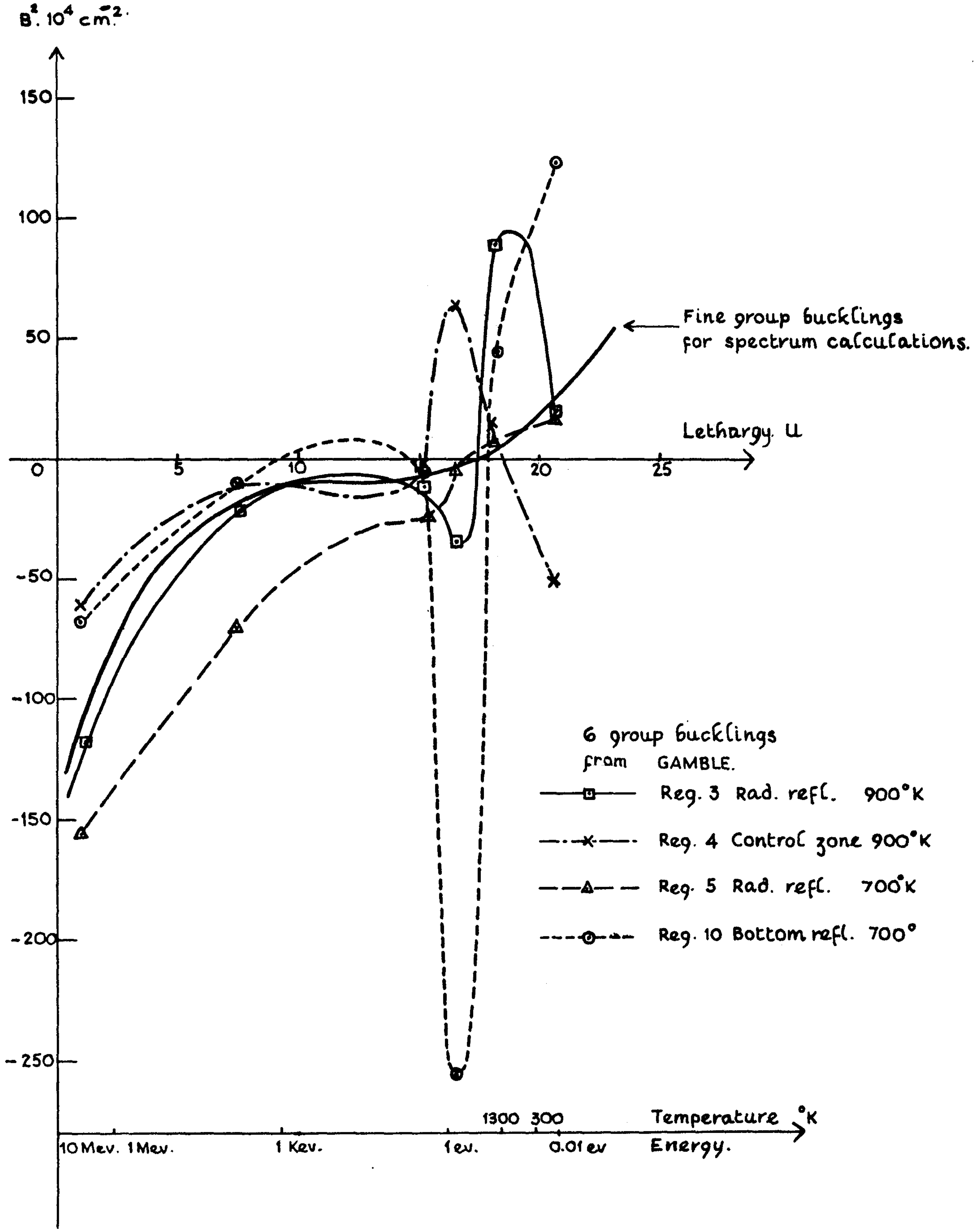

FIG. 6. DRAGON. HOT. REFLECTOR BUCKLINGS.

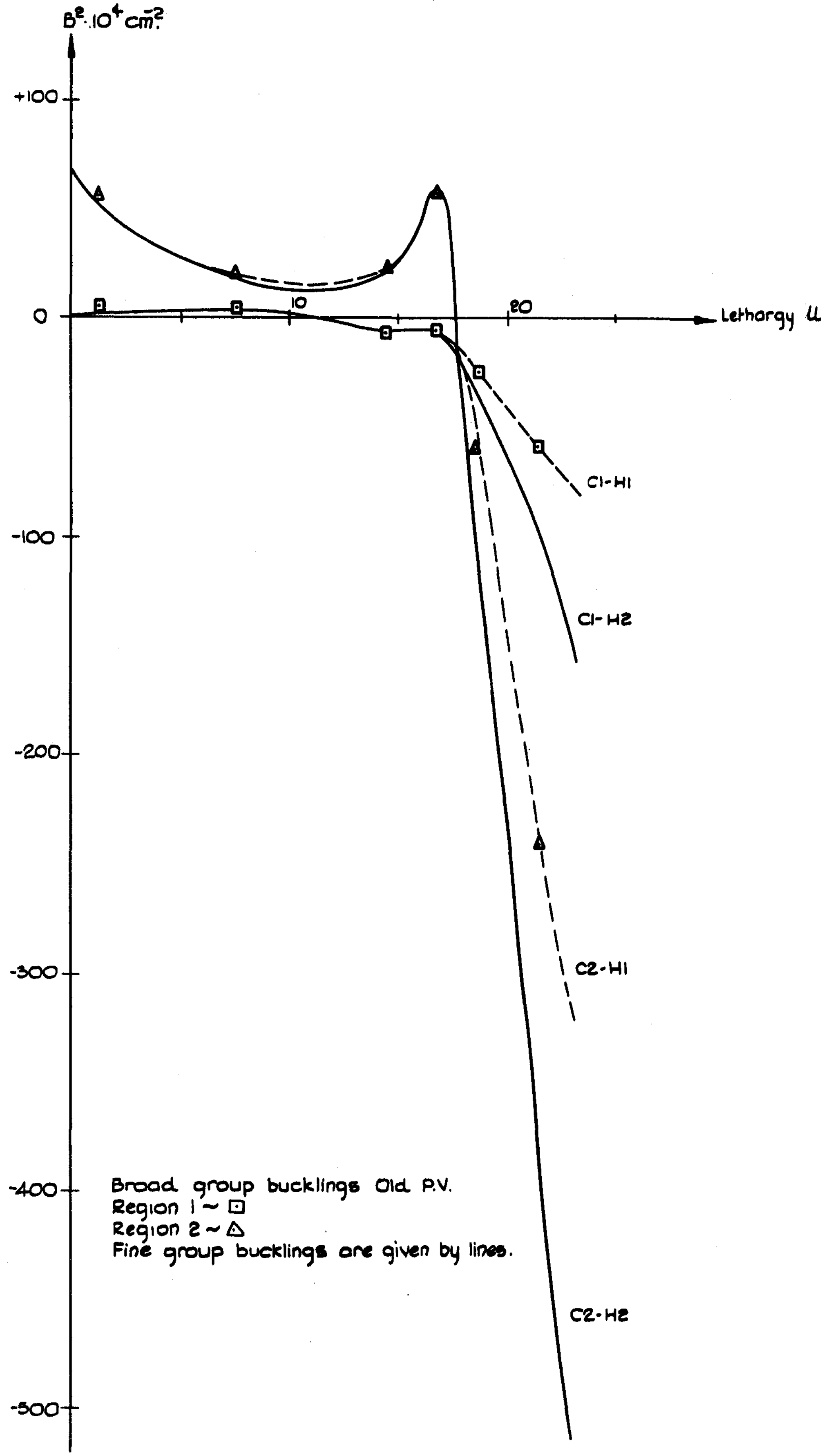

FIG. 7 BUCKLINGS DRAGON TWO REGION CORE.

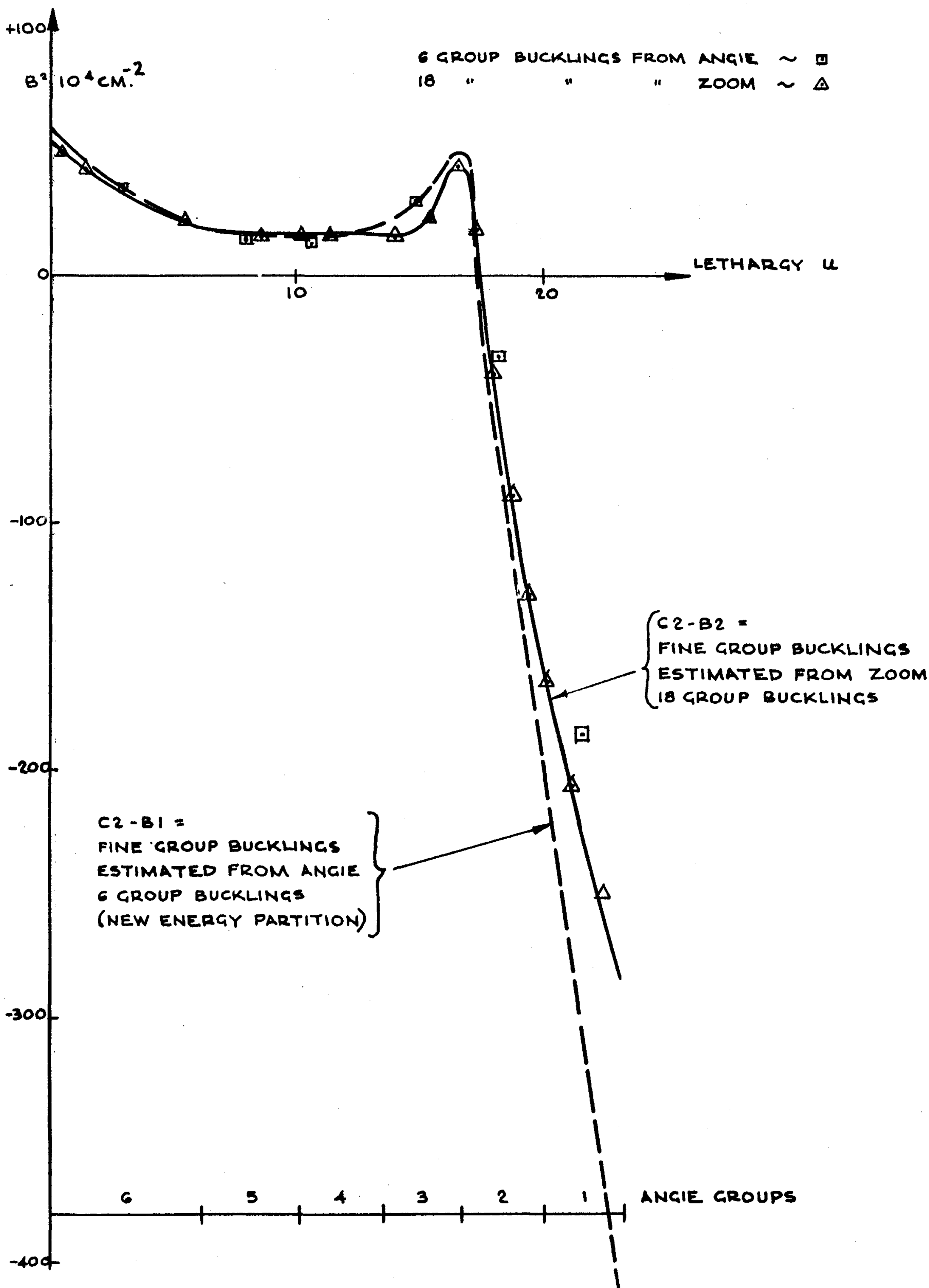

FIG. 8 RADIAL BUCKLINGS FOR CORE REGION 2
ONE DIMENSIONAL GEOMETRY

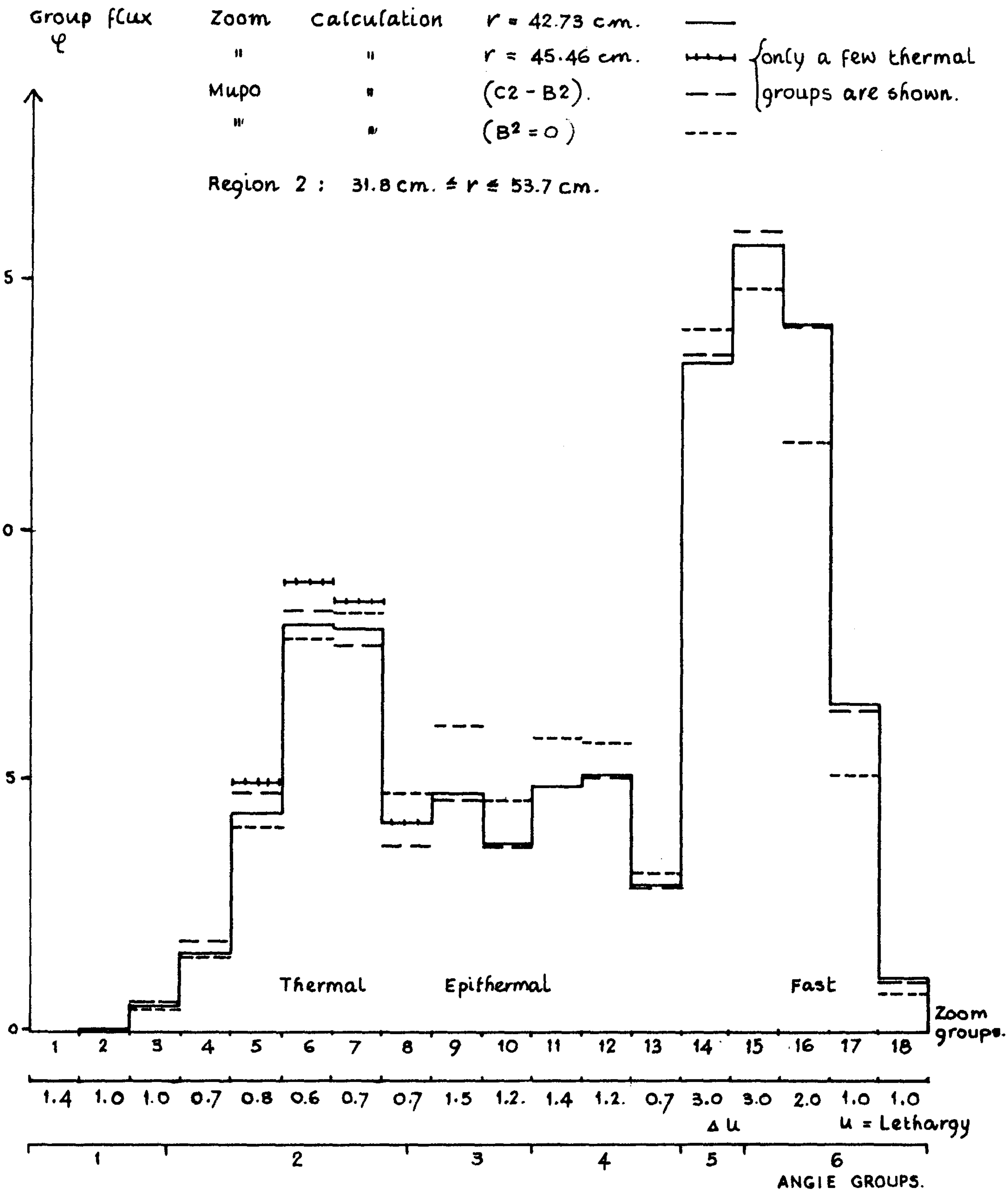

FIG. 9. DRAGON 18 GROUP FLUXES FOR CORE REGION 2 (N=0)

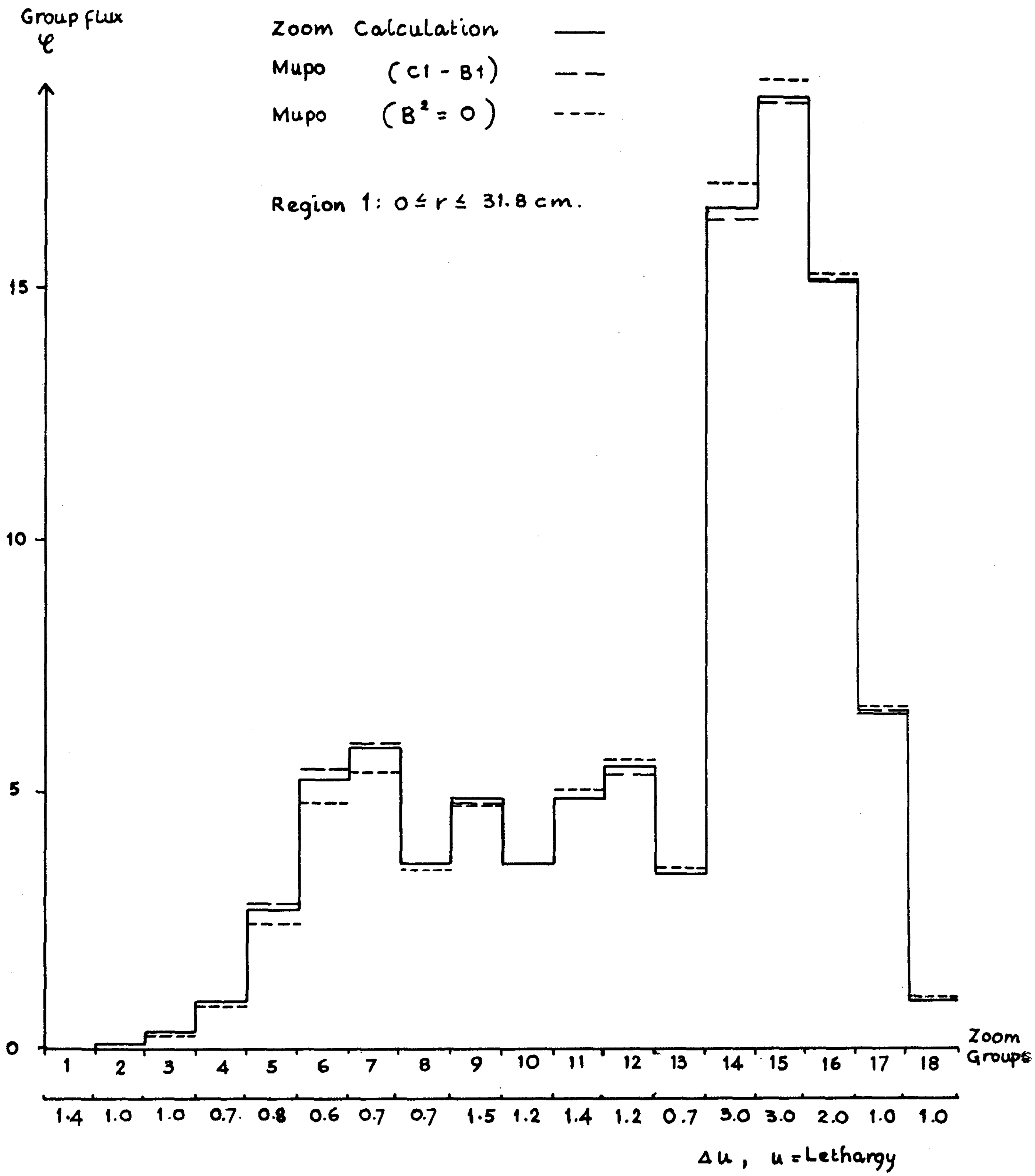

FIG. 10 DRAGON. 18 GROUP FLUXES FOR CORE REGION 1 (N=20)

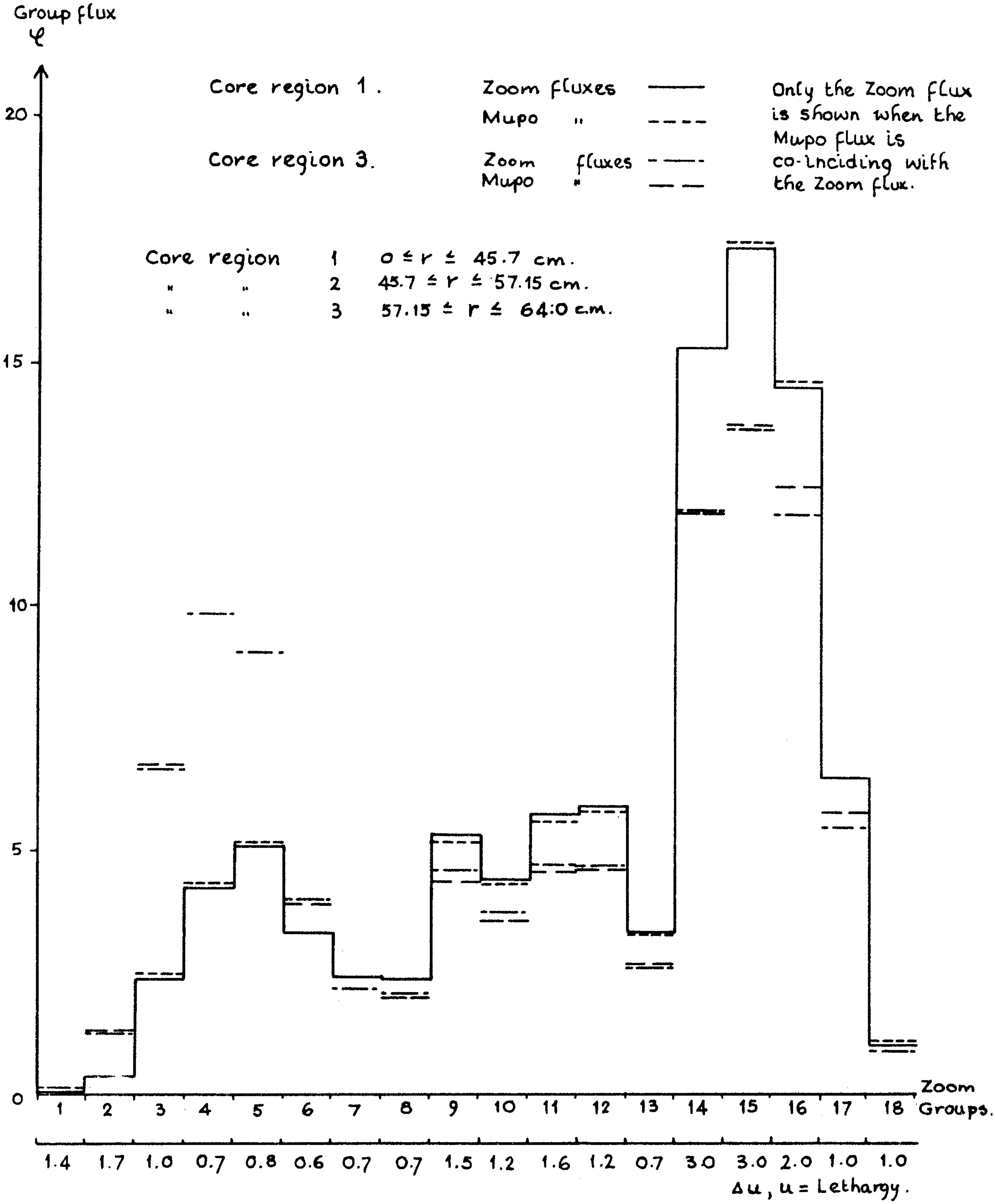

FIG. 11 ZENITH 2L 18 GROUP FLUXES.