AUTOMATIC LENS DESIGN FOR NONEXPERTS*

by

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ABSTRACT

Automatic lens design by nonexperts is practical with the 1962 LASL lens-designing code, a general-purpose program for evaluating and designing optical lens and mirror systems with surfaces generated by conic sections. Using skew ray traces, it analyzes lens performance statistically, finds differences resulting from design-parameter alterations, and computes the linear combination which will improve the prescription. IBM-7090 machine operations, prescription parameters, ray-tracing parameters, weights on lens performance, lens-parameter substitutions, increments to lens parameters, designing instructions, and parameter interactions are described. Sample calculations are given in detail for a Lister-type lens and in summary for a special-purpose zoom lens.

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INTRODUCTION

An instrument designer--whether engineer or scientist--is frequently the only person who can adjust the several requirements of the instrument in order to obtain the best total system because he is the only person with intimate knowledge of the instrument's mechanical, electronic, and optical details. This overall designer, however, is seldom a specialist in optical design. But the modern large computer makes it possible for the nonspecialist to design an optical system, to evaluate it, and to specify the tolerances required for satisfactory performance. The purpose of this paper is to call attention to a new program for the IBM 7090 and to help the engineer or scientist who has no knowledge of lens design or computer operation to use this program. This program gives to the lens design problem a general automated solution which makes full use of the digital computer's capabilities and abandons the classical approach requiring direction by the skilled specialist at every stage.

The code for the 7090 machine was preceded at Los Alamos by a code for the IBM 704. This last, developed in 1959 by J. C. Holladay,[1] was the first program at this laboratory to make possible the design of a complete lens by means of a computer. The Holladay automatic iterative program made statistical analyses of lens performance from skew ray traces. It determined differences in lens performance resulting from design-parameter alterations. A least-squares technique was applied to these differences to compute a linear combination which would give an improved lens prescription. The success of this program created a desire for a code which would be even easier to use and able to solve even more complex problems. This demand was met by C. A. Lehman's 1962 code for the IBM 7090.[2] Improvements are now planned for the Lehman program. There are a number of ways of handling the computations and problems of automatic lens design. These have been reviewed by D. P. Feder,[3] Lehman,[2] Holladay,[1] F. Wachendorf,[4] and others. Investigations are currently going forward into ways of simplifying and speeding up lens optimization by using more precise mathematical analysis. It is the purpose of this paper to describe the use of the Lehman program in its present form.

To study and design lenses, the program uses 3 laws of geometrical optics, that branch of optics which treats the analysis of light rays passing through a lens. The 3 laws are those governing rectilinear propagation, refraction, and reflection. Lens performance is analyzed by tracing bundles of rays through a lens in accordance with these laws. Rays are traced because there is close correlation between the behavior of geometrical rays* and the focusing of light by the lens. The ray-bundle size at the focal plane of interest is the main criterion of lens performance. To obtain the sharpest focus, the design program changes the parameters of the lens system to make the bundles of rays intersect the image surface at the desired positions in patterns of the smallest size. By evaluating the sizes and positions of the image spots, image defects are analyzed precisely; but the traditional Seidel aberrations are not evaluated.

WHAT THE PROGRAM CAN DO

The program has 2 major divisions, the diagnostic section, which evaluates lens performance, and the iterative designing section, which alters specified lens parameters and calculates an improved lens prescription. Both sections use a ray-tracing subroutine, which automatically traces to the focal plane the rays specified by the designer. Each section uses a different method of evaluating lens performance from the ray-trace data. The diagnostic section obtains the size and position of each ray bundle from each object point on the number of planes of interest specified by the designer. The designing section makes a statistical analysis of the weighted deviations of the ray intersections from

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* A ray is the geometrical line representing the propagation of light.

their specified positions, which analysis is presented as a single merit number. These 2 major sections of the program are frequently used alternately so that the designer may observe the details of the designing progress. The diagnostic section is used to determine the performance of an existing lens design in which all the parameters are specified. The designing section is used to optimize lens performance by automatically altering one or more lens parameters. This is the case when one wishes to determine the performance of an existing lens design in which some of the parameters are unspecified, for example, the entrance pupil location and the distance to the best focal plane. Another case might be the adjustment of the lens prescription's air spaces and surface radii to give best performance with a specific field of view and object distance. An extreme case is the optimization of a lens design starting with any desired arrangement of refracting and reflecting surfaces.

The program has 3 very valuable minor divisions which also use the ray-tracing subroutine: the off-axis rim-ray trace, the on-axis paraxial and rim-ray trace, and the spot-diagram coordinate calculation. The off-axis-trace section traces the upper and lower rim rays from the maximum object height and calculates the ray position and direction and the distance to the following surface at each optical surface. The on-axis-trace section traces the paraxial and rim rays from zero object height and calculates the ray position and direction and the distance to the following surface at each optical surface. The spot-diagram section calculates each scaled ray-intersection coordinate relative to the spot's centroid. The coordinates are supplied in tabulated and punched-card form for spot-diagram plotting by the designer.

The program can investigate lens systems with as many as 98 surfaces. The lens or mirror surfaces most used are either spheres or right-circular cylinders, but surfaces generated from any conic section may be used. Any of the surfaces of revolution may be displaced or tilted as desired. There is a wide choice possible in the number and distribution of the rays which may be traced through the lens. The positions and sizes of the entrance and exit pupils are readily controlled. The lens focus may be fitted to a specified focal surface, or the shape of the best focal surface may be determined. Simultaneous design for 7 image points, 6 wave lengths, and 5 conditions of use is possible. Conditions of use may include such things as different object and focal surface positions, different lens-element positions along the optical azis--as in a zoom lens--or different translational and rotational displacements of the lens elements as a result of flexing the lens with external forces. In spite of all this capability, the program is not unwieldy or slow. The reader is referred to the initial reports for details about the program and its use.[2,5,*]

SELECTING THE DESIGN

The first requirement is to decide on the lens type which will best do the job. The decision requires careful thought. Although a simple design with few component elements will be relatively easy to manufacture and will transmit light efficiently, it may not have the desired resolving power. The resolution of a very complex design, on the other hand, may exceed the need. The quality of resolution needed will therefore be a strong factor in the choice of a lens type. For example, if the lens is to be for visual use, the minimum resolution should be that of the eye--an angle of about 0.0003 radian. If it is for photographic use, the minimum resolution should be that of the film--0.02 to 0.002 mm. Other applications will have other requirements. The best lens system to choose is obviously the simplest one which will give the desired performance. In general, more complex lenses are required for the larger relative apertures, the longer focal lengths, and the wider angles of view. The more complex the imaging problem, the greater the number of optical surfaces which will be required.

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* These reports are available at nominal cost from the Office of Technical Services, Department of Commerce, Washington 25, D.C.

One way to determine the lens best suited to a particular imaging problem is to search the literature. The largest selection of specific designs is to be found in patent descriptions, which are abstracted in the Patent Gazette. The U.S. Patent Office will supply, at nominal cost, a list of all Class 88-57 patents, which includes most lens patents. Information about popular lens designs is contained in books describing the characteristic features of photographic lenses.[6,7,8] Articles and books describing specific instruments often have references which give the specifications for optical elements used successfully.[9,10,11] Starting with one of the many popular lens types, it is often possible to modify it to meet the specific needs of a new instrument. Minor alterations of the surface radii and in the spacing of the elements may be sufficient. The more nearly satisfactory the initial lens choice, the less machine time needed to complete the design.

Another procedure is to ignore the literature and let the machine determine the lens design most suitable for the problem. One starts the design by making monochromatic runs on a series of air-spaced glass sheets. To obtain the final prescription with minimum effort, it is useful to keep a few general rules in mind. The more surfaces the lens has, the smaller will be the spot size. Therefore, the design should be started with the estimated maximum permitted number of air-glass surfaces. This number will depend on such factors as cost, time required for fabrication, image quality needed, efficiency in transmitting light, freedom from flare images, space available, and feasibility of construction. Similarly, the higher the refractive index of the glass used, the smaller the spot size. Also, the lens elements should be spread along the axis, as the longest lens usually gives the best performance (smallest spot sizes). If the monochromatic design is satisfactory, it is achromatized by substituting crown (low dispersion) and flint (high dispersion) glasses, respectively, for the glasses in the positive and negative elements. For additional chromatic correction it may be desirable to achromatize some of the elements by making them into cemented doublets containing both crown and flint glasses.

Selecting the optical glass from the bewilderingly long lists of commercially available types is another problem. It is suggested that the beginner use those types which are listed as readily available because this availability is an indication that these glasses are used extensively in successful designs. In this connection, particularly for complex designs, one needs precise values for the refractive indexes of the glasses (melts) used for making the lens elements, since the manufacturing tolerance for glass stock is greater than that allowable if predicted lens performance is to be obtained.

However, catalog values of the indexes of refraction can be used for all designing, as long as the precise values are obtained for final adjustment before the lens is made.

PUTTING THE PROBLEM INTO THE MACHINE

The IBM 7090 loads itself first with the designing code and then with the lens problem by reading instructions and data from punched cards. Each item of data can be located as needed since it has been addressed to a memory cell specifically reserved for it by the 213-card code. The lens data include the lens prescription and the parameters for ray tracing and designing. (The storage nomenclature and the conventions used are detailed in the initial report.[5]) There are 10 individual program-control instructions which the designer uses to order the sequence of machine calculations and information output. A typical 5-minute calculation procedure might include several short design-cycle sequences, with increasing numbers of lens parameters being varied, followed by the machine preparation of a set of punched cards with the latest lens data and a paper print showing the latest lens prescription and the analysis of its performance by multiple ray tracing. After the performance analysis has been studied to determine what progress has been made, the next step in the designing sequence can be planned.

Certain preliminary information must be collected in order to define the problem correctly and to minimize false steps. The performance of an existing lens can be determined if we know or can find the wave length range (in order to use the necessary refractive indexes), the distance from the object, the angle of view, the entrance pupil location, the relative aperture (f/number), the image distance, the shape of the focal surface (plane or curved), and the exit pupil location (for some applications). In the absence of specific conjugates, the object distance is assumed to be infinity, and the angle of view is discovered by tracing a series of ray bundles from object points at increasing angles. If the image distance is specified, the ray-tracing program can set the plane of interest at that distance. If the image field is curved, the additional values of the sagitta for the image points are also used. If the image distance is unknown, the ray-tracing program will set the plane of interest at the back focus calculated from the automatic meridional ray trace with the first refractive index selected (plus the sagitta plus a fine-adjustment increment if these are assigned non-zero values). The entrance pupil distance may be specified or it may be determined by the designing program.

The image spot is evaluated from the RMS radius of the rays from the centroid of the ray-bundle intersection on a plane normal to the optical axis. The designing section of the program uses the image-spot evaluation on only one plane for each image point. However, the printed output can be ordered to show the image-spot evaluation on any number of equally spaced planes, a capability which is useful for determining changes in the ray bundles as the back focal distance is varied. The paraxial focal length and the meridional back focal distance are automatically given for each wave length used.

The choice of input rays and the weighting of ray-intersection deviations are both very important. Since the rays traced through the lens represent the behavior of light, they should be selected to function efficiently as agents for lens performance evaluation. Each ray may be considered as a sample of the focusing ability of the lens region which surrounds the ray path. The larger the surrounding region, the less typical this sampling will be. If each ray samples only a small region of the lens, the sampling will be more typical, but the large number of rays to be traced will require much time for calculation. Thus one may design rapidly (if crudely) with a few rays or design precisely with many rays, a process which is relatively expensive. The solution is to use only a few rays for the early crude design stages and to increase the number as the design is refined.

The pattern of rays to be traced is placed in a unit-radius entrance pupil. The ray-tracing program scales the unit-pupil dimensions to the actual pupil dimensions to obtain the coordinates of each ray to be traced. The rays usually sample the entrance pupil with a grid-pattern array, but as many as 25 rays may be positioned in any pattern desired. In this program, up to 100 rays can be traced from each object point for each color. Since rays traced through a centered optical system are symmetrical, only half the rays of symmetrical ray patterns need be traced to obtain the ray intersections of the other half by a sign change of the x coordinate. We have made a convenient series of ray patterns[5] with 2, 3, 4, 5, 6, 8, 12, 16, 26, 40, 74, and 100 rays which have been used for lens designing and for testing lens performance. The 2-ray pattern is especially useful during the preliminary design stage because it is economical of machine time while still adequately evaluating lens performance for most design problems. The 6-ray pattern is usually adequate for the remainder of the designing. The many-ray patterns are useful for assessing lens performance, expecially when there is vignetting.

To emphasize specific aspects of lens performance, the designer can weight the deviations of the rays from the desired image points. Optimum lens performance requires minimum image-spot sizes on the focal surface. These image spots are made up of a maximum of 6 colors, each from a maximum of 7 object points.

The weights for the sizes of these spots are assigned individually. Weights on the deviations caused by the variation of magnification with color for each of these 7 images are also assigned individually. The 7 desired image heights, the individual weights for each of these image heights, and the sagitta (for a curved field), are assigned by the programmer. The required principal focal length and the weight on the reciprocal focal length may be specified. The exit-pupil position is calculated from a ray traced from an object point at a height selected by the programmer. The desired distance of the exit pupil from the last optical surface and the weight on this distance are also specified by the programmer. The above weights are the total number which control the design program. Another factor of importance to lens performance is the entrance pupil aperture radius or the operating relative aperture of the lens, one of which may be fixed for designing. For further details see the original reports. However, these details are in no sense a full statement of the potentialities of the code. The ingenuity of the user will suggest many other possibilities.

CHOOSING THE DESIGN PROGRAM

The designing program improves lens performance by computing a better number for each of the lens parameters which has been selected for change. Since the code can work with a maximum of only 10 parameters simultaneously, it is usually necessary to work with many different combinations of the parameters so that all possible interactions will be included. Only a small improvement is obtained for each calculation. It is therefore necessary to use an iterative procedure to arrive at the minimum merit number obtainable with a specific set of parameters. It is frequently desirable to use several different sets of parameters because of the tendency of the code to find its best solutions in the impossible-to-build regions which have extremely large thicknesses and negative thicknesses of both the elements and the spaces. There is also a tendency to generate one or more surface curvatures which are so great that some rays are vignetted. The sequence of sets of incremented parameters which has often proved effective is first the curvatures, next the curvatures and some air spaces, then the curvatures and all the air spaces, and finally the curvatures, the air spaces, and the glass thicknesses. Any parameters which tend to create undesirable conditions are fixed at tolerable values, and designing is continued with the remaining parameters. Sometimes these fixed parameters may be made variable again with profit at a later stage in the design.

The program can calculate a better number for a parameter by evaluating the effect of small changes of that parameter on the performance of the lens. These small parameter changes are called parameter increments or simply increments. Each parameter to be incremented and the amount of that increment are specified by the lens designer. The designer has at his disposal an increment instruction storage area for a maximum of 49 independent parameters, each of which may represent as many as 6 dependent parameters. The dependent parameters must be simultaneously given identical alterations in any plus or minus combination specified by the designer. The parameter calling sequence (the order in which the parameters are incremented) is any continuous series which is specified by the designer. With each design cycle each number in the calling sequence is increased by 1 (any becoming larger than the maximum will be reset to the minimum) so that all parameters in a selected series will be incremented in serial order and repeated if enough design cycles are ordered.

This arrangement is used because it is frequently impossible (or undesirable) to alter all parameters at one time, yet all must eventually be allowed to influence the prescription. With crude starting prescriptions, the designing is usually started with only one or two incremented parameters per cycle. As the design is stabilized the number of parameters per cycle is increased until the design progress becomes erratic because of the program's inability to evaluate precisely the too-complex problem.

Incrementing two or more parameters simultaneously, to obtain the interaction between the parameters of each of the pairs present, is another way of increasing the capability of the designing program. Hence it is a good idea to choose a sequence of increments which will give all possible combinations between parameters within the program's capability. A series of m incremented independent parameters will be in the circular form 1, 2, 3, . . . m, 1, 2, etc. The types of parameter interaction found in this series are classified according to the position spacing of the members of each pair. The interacting types present are those spaced 1, 2, 3, . . . $\frac{1}{2}$m apart. A specific sequence of n simultaneously incremented parameters (where n $\leq$ m) from the above series will have part or all of the interacting types present. The number of interactions between n parameters working simultaneously is $\frac{1}{2}n(n-1)$. For example, with this program one obtains 0, 1, 3, 6, 10, 15, 21, 28, 36, or 45 parameter interactions depending on the number of parameters incremented simultaneously. The design-progress rate increases as the number and types of parameter interactions increase. The addresses of the incremented parameters are serially stored, and the calling order is at the command of the designer. For example, with a series of 15 parameters (m) in which 5 are being incremented simultaneously (n), one might select every third parameter; the 3 calling sequences for designing would then be 1, 4, 7, 10, 13; 2, 5, 8, 11, 14; and 3, 6, 9, 12, 15. All parameters are incremented every 3 design cycles, but only 2 types of parameter interaction are present, i.e., those whose positions are spaced 3 and 6 apart. At the other extreme one can select the 15 calling sequences: 1, 2, 3, 4, 5; . . . ; 15, 1, 2, 3, 4. In this case all parameters are incremented every 11 design cycles, with 4 types of parameter interaction present. Or one might use the sequences 1, 2, 3, 5, 14; 2, 3, 4, 6, 15; . . . . Here all parameters are incremented every 9 design cycles, with 5 types of parameter interaction present. Or the reverse sequence 1, 3, 12, 14, 15; . . . would be equally useful. However, the greatest variation in interactions is obtained with the 15 calling sequences 1, 2, 5, 10, 12; 2, 3, 6, 11, 13; . . . , with all parameters incremented every 5 design cycles, and all 7 possible types of parameter interaction (spacings) present. For clarity this information is summarized

Table I in Table I.

Table II Table II gives an extensive series of parameter-increment calling sequences, which have been selected in each case to provide all possible parameter interactions. The number of interactions of each type is distributed as evenly as possible, and each parameter is repeated as often as possible by avoiding sequences with large parameter spacings if there is a choice.

After the designer has scheduled the parameter-increment calling sequence--either one of those described above or one of his own--for the desired number of design cycles, it is only necessary to instruct the program to design.

EVALUATING THE RESULTING LENS

There is no generally accepted method of quantitatively evaluating lens resolution from the sizes of the ray-bundle intersections with the focal surface. This code determines the image spot size by calculating the RMS radius of the rays from their centroid on the focal plane of interest, which is normal to the optical axis. Image evaluation by this method includes the variation of back focal distance with color. The predicted resolution is the RMS radius of the image spot. Variation of the lateral (y) magnification with color is an important lens-performance defect which is measured from the centroid of the bundles which make each image point. The predicted effect of this defect on resolution is the RMS lateral deviation from the centroid.

For measuring the resolving power of the lens and camera-film combination, there is a generally accepted test, the National Bureau of Standards method of determining the resolving power of photographic lenses.[12] The NBS test measures the ability of the lens to produce clear-cut images of an object containing fine details. The object used for this test is a resolution test chart consisting of a series of three-parallel-line patterns arranged in diminishing sizes. The established practice is to consider the resolving power to be the number of lines per millimeter of the finest pattern which is resolved on the negative as separate and distinct lines. For example, the finest pattern might have its lines spaced 40 to the millimeter (the distance between the centers of adjacent lines would be 0.025 mm). The photographic resolution[13] is approximately equal to the optical resolution plus the film resolution. Film resolution is commonly in the range of 0.03 mm for the highest-speed emulsions to 0.002 mm or less for high-resolution plates.

The resolution test charts have also been very useful for visual evaluation of the optical resolution of lens systems. There may be considerable error in the judgment of visual resolution because of the great variation in image contrast among different lens types, because of the spurious resolution which is sometimes associated with diffraction effects, and because of the chromatic errors which observers evaluate over a wide range. However, for high-resolution images there has been close correlation between the visual resolution as determined by the use of the high-contrast parallel-line charts and the predicted RMS image-spot radius of the rays from their centroid, as calculated by this code. Observation is made with a microscope of larger aperture and greater resolution than the image-forming beams. The high correlation between the predicted and the measured chromatic performance is particularly gratifying. It is desirable to keep the lateral chromatic deviation equal to or smaller than the RMS spot radius, so that this defect will not cause loss of resolution in heterochromatic imagery.

EVALUATION OF CONSTRUCTION TOLERANCES

The manufacturing tolerances to be specified for a particular lens design are readily evaluated by the lens-designing program, which allows one to determine quickly the optimum focal position for studying the effect of parameter changes on the image-spot sizes. These parameter changes may be evaluated individually or in any desired simultaneous combination by using the ability of the code to read parameter changes into the memory with ease and to trace rays quickly. The range of allowable tolerance is sometimes very large. In descending order of sensitivity, the following parameter tolerances are suggested for starting a series of tests: 1/2000 for refractive index, 1/1000 for radius of curvature, 1/200 of lens element diameter for decentering, 1/100 for air or glass thicknesses.

If the errors in the manufactured lens elements exceed the assigned tolerances, the designing program can often improve performance by redetermining the air spaces. In this way one can sometimes partially sompensate for resolution lost by improper construction.

SAMPLE CALCULATION

The lens to be designed in the following sample calculation consists of 2 cemented doublets in series which produce an $f/2$ Lister-type lens[14] set to work at a magnification of about one eighth. The objects are at an axial distance of 170 mm from the first lens surface and 1, 3, and 5 mm from the optical axis. The images are on a flat field which is started with the back focal distance held at 10 mm. After the lens has been stabilized, this distance is made a design parameter. The design was started with 2 cemented sheets of 518596 and 621362 glass spaced 18 mm apart. The entrance pupil distance was fixed at -20 mm and the aperture radius at 4 mm. Figure 1 shows the complete series of card images used to design this lens. The data for the starting prescription, the weights, the ray patterns, the parameter alterations, the design instructions, and the machine operations are given. This information is followed by the complete prescription data print, the accomplishments of some of the first 29 design cycles, and the resulting lens prescription.

Figure 1

Designing was started with 3 colors (C, d, F), a 2-ray pattern, and a series of 29 single increments to the 4 glass-air surfaces. As a result the prescription's figure of merit went from 905 to 25. A summary of the complete 15-minute

Table III series of runs used to design this lens is shown in Table III.

These runs produced an $f/2.0$ lens of 21.010-mm (C-light) focal length, which should give 700-lines/mm resolution over most of the 1.2-mm-diameter focal surface at 11.772-mm distance. There is a maximum of 0.0019 mm variation of lateral magnification with color at the edge of the 1.7-degree half field of

Figure 2 view. Figure 2 shows the print of the final prescription and the 12-ray analysis of lens performance. The spot diagrams

Figure 3 of the 3 images are shown in Fig. 3.

A COMPLEX DESIGN

An example of a complex design which can be made with this program is a special-purpose zoom lens. This lens is an objective for a high-speed camera used to photograph explosions through the 4-inch-diameter viewing port of a concrete bunker. Twenty-millimeter images of 150- to 450-mm explosive objects are required, with the objects 9000 mm from the front lens element. To obtain this range of magnification, a zoom ratio of approximately 3.6:1 is needed. Between the viewing port and the objective's image, there is a distance of more than 2000 mm, which can be used for any desired arrangement of lens elements.

In the search for a solution to this difficult imaging problem, the lens designs of US Patents 2,778,272, 3,051,052, and 3,057,259 were tested, and all were found to be inadequate for our purposes because of insufficient correction of off-axis images and excessive shift of the image over the zoom range. It was then decided to let the lens-designing program find the optimum design for lens systems with 3 or 4 or more elements so that we could select the simplest design which would give the required resolution. Using the program's substitution capability, we let the machine design each lens system simultaneously for 5 equally spaced positions of the moving elements. Several monochromatic designs were made with zoom arrangements which previous workers had found to be useful.[15] The so-called 4-lens system, which has 5 elements, was found to be the minimum which would give the desired image quality. The entire available length was used because a long lens nearly always gives the best performance. The design was then restarted with 5 cemented sheets of 517645 and 617366 glass (C, e, and g melt indexes for 7000-feet elevation) in a zoom system consisting of 2 coupled sliding lenses which move between 3 fixed lenses. These paired components quickly assumed shapes to give negative, positive, negative, positive, and positive powers. Before the final design was achieved, several readjustments of the movements and spacings of the elements were necessary.

Figure 4 The prescription data print is shown in Fig. 4.

Table IV Table IV shows selected optical characteristics of this lens for 9 positions. For each of these 9 positions the spot sizes and focal positions for C, d, e, F, g, and h light were determined. There was no discernible shift of the optimum focal plane for any of the positions tested, possibly because the variation of focus with color is the principal defect of this lens. Except for h light, the maximum spot sizes on the focal plane at the 384.709-mm back focus did not go above 0.027 mm, and the average is about 0.013 mm. A visual white-light resolution of about 100 lines/mm is predicted for most of the zoom range.

CONCLUSION

Although mastery of the program requires practice, interested nonexperts have demonstrated that after a short but intense study of the original reports[2,5] and by a few short runs on the machine, they can learn quickly the fundamentals of this program. The first step in the learning process is to rerun one of the completely specified problems in this or the original reports, in order to determine the correct functioning of the program. Next, after the numbers which describe a proposed lens have been selected and assigned, it is necessary to determine if a workable lens has been specified. This decision is most easily made by studying the lens performance evaluation obtained from the multiple ray tracing diagnostic calculation. If an error has been made in specifying the lens data, the output from the off-axis rim-ray trace may aid in locating this error by giving the details of the paths of 2 rays through the lens. After the lens prescription has been shown to function as planned, the designing program can be activated by directing alterations to one or more lens parameters to start improvement of lens performance.

This code has demonstrated its ability to analyze the performance of and to design simple or complex lens systems rapidly and precisely. Correlation between the lens performance predicted by the program's multiple ray-tracing analysis and the measured performance of the constructed lens has been close. When there has been an apparent discrepancy between measured and predicted performance, the cause has always been a manufacturing error. It is therefore necessary to give the manufacturer construction tolerances of sufficient tightness to insure that design specifications will be met. The construction-tolerance specifications can be determined by the program's ability to evaluate the effect of small parameter changes and the displacement and/or tilt of the surfaces.

In conclusion, the code gives to problems of lens design a general automated solution which makes full use of the digital computer's capabilities and abandons the classical approach requiring direction at every stage by the skilled specialist.

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LIST OF CAPTIONS

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L-198 HSLISTER TYPE LENS EXAMPLE    (CONRADY)$
L-192 H$ 04--03--63 $
L-99 X3 X1 X2 X3  L-57 X3(1)-46 +50  L-37 +100 +100 +100  L-17 +10
L-9 +.12 +.36 +.60  L10 -1 +.1 +.  1 +.707 +10+4 +100 +1 +0 +.0001
120 +170 +1 +2 +5.1 +1 +1 +1  130 +0 -20 +0 +0 +1.51544 +1.51807+1.52413
L40 +0 +2.7 +0 +0 +1.6161 +1.62114 +1.63325  L50 +0 +1.5 +0 +0 +1 +1 +1
L60 +0 +18 +0 +0 +1.51544 +1.51807 +1.52413
L70 +0 +1.8 +0 +0 +1.6161 +1.62114 +1.63325  L80 +0 +1 +0 +0 +1 +1 +1
L1010 X-1 X32 C6 +1 +.01  L1020 X-1 X52 C6 +1 +.01 X-1 X62 C6 +1 +.01
L1040 X-1 X82 C6 +1 +.01 X-1 X42 C6 +1 +.01 X-1 X72 C6 +1 +.01
L1070 X1 X61 C6 +1 +.1 X1 X31 C6 +1 +.1 X1 X14 C6 +1 +.01
L-170 X1 X2 +1 +.5 -.5 X2     L-79   X1 X4    X29    X1    X1    C9    X2  +2
L-79    X1 X6    X25    X1    X1    C9    X2      +2
L-79    X1 X6    X25    X2    X1 X3*  C9    X2      +2
L-79    X1 X8    X25    X2    X1 X4     C8    X2      +2
L-49 +2       L-79    X1 X7    X28    X3    X1 X2 X5    C7    X2      +2
L-79    X1 X9    X28    X4    X1 X2 X5 X7    C6    X2      +2
L-170 X1 X6 +.5 +.25 -.75 X4 +.75 -.25 X2    L-79X1X7X28X4X1X2X5X7C7X2 +2
L-46 +19  L-79    X1 X9    X28    X5    X1 X2 X4 X5 X7    C5    X2      +2
L-79    X1 X9    X28    X6    X1 X2 X3 X5 X6 X7    C4    X2      +2
L-170 X1 X12 +.3333 +.1667 -.8333 X6 +.5 -.5 X6 +.8333 -.1667 X2
L-79    X1 X9    X28    X6    X1 X2 X3 X5 X6 X7    C4    X2      +2
+3
+4
+6
+8
+9
```

PRESCRIPTION   LISTER TYPE LENS EXAMPLE (CONRADY)   04--03--63

10	-1.0000000+00	1.0000000-01	9.9999975-05	7.0699999-01	1.0000000+01	4.0000000+00	1.0000000+02
					1.0000000+00	0.0000000+00	9.9999888-06
20	1.7000000+02	1.0000000+00	2.0000000+00	5.1000000+00	1.0000000+00	1.0000000+00	1.0000000+00
30	0.0000000+00	-2.0000000+01	0.0000000+00	0.0000000+00	1.5154400+00	1.5180700+00	1.5241300+00
40	0.0000000+00	2.7000000+00	0.0000000+00	0.0000000+00	1.6161000+00	1.6211400+00	1.6332500+00
50	0.0000000+00	1.5000000+00	0.0000000+00	0.0000000+00	1.0000000+00	1.0000000+00	1.0000000+00
60	0.0000000+00	1.8000000+01	0.0000000+00	0.0000000+00	1.5154400+00	1.5180700+00	1.5241300+00
70	0.0000000+00	1.8000000+00	0.0000000+00	0.0000000+00	1.6161000+00	1.6211400+00	1.6332500+00
80	0.0000000+00	1.0000000+00	0.0000000+00	0.0000000+00	1.0000000+00	1.0000000+00	1.0000000+00

PLANE SETTING MODE ONE.   LATTICE MODE ONE.   SENSE SWITCH MODE ZERO.   NUMBER OF PLANES 3

WEIGHTS

-49	0.0000000+00	0.0000000+00	0.0000000+00	3.0000000+01	0.0000000+00	0.0000000+00	0.0000000+00
-39	1.0000000+02	1.0000000+02	1.0000000+02	0.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00
-29	0.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00
-19	0.0000000+00	0.0000000+00	1.0000000+02	0.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00
-9	1.2000000-01	3.6000000-01	5.9999999-01	0.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00

LATTICE N = 2   DELTA Y = 1.0000000+00
5.0000000-01 -5.0000000-01 2

INCREMENT IDENTIFICATION... 1 4 29 1 1   2

COLOR IDENTIFICATION... 3 1 2 3

ID.	LOC.	NO.	ADDRESSES OF DESIGN PARAMETERS	ACTUAL DET.	REQUIRED DET.	INCREMENT
1	1010	-1	32	0.0000000+00	1.0000000+00	9.9999999-03
2	1020	-1	52	0.0000000+00	1.0000000+00	9.9999999-03
3	1030	-1	62	0.0000000+00	1.0000000+00	9.9999999-03
4	1040	-1	82	0.0000000+00	1.0000000+00	9.9999999-03
5	1050	-1	42	0.0000000+00	1.0000000+00	9.9999999-03
6	1060	-1	72	0.0000000+00	1.0000000+00	9.9999999-03
7	1070	1	61	0.0000000+00	1.0000000+00	1.0000000-01
8	1080	1	31	0.0000000+00	1.0000000+00	1.0000000-01
9	1090	1	14	0.0000000+00	1.0000000+00	9.9999999-03

	1 2 3 4 5 6 7 8 9 10	INCR. FACTOR	DETERMINANT	OLD LENGTH	NEW LENGTH	IMPROVEMENT
MERIT	1	1.0000000+00	1.6496566+04	9.0476220+02	3.7681039+01	9.5835254-01
MERIT	2	1.0000000+00	1.7322608+04	3.7681039+01	3.6238567+01	3.8281109-02
MERIT	3	1.0000000+00	3.0415064+02	3.6238567+01	3.5459544+01	2.1497087-02
MERIT	4	1.0000000+00	2.2931673+02	3.5459544+01	3.5398774+01	1.7137716-03
MERIT	2	1.0000000+00	9.7258874-01	2.6122334+01	2.6121430+01	3.4609589-05
MERIT	3	1.0000000+00	1.0266662+00	2.6121430+01	2.5680417+01	1.6883173-02
MERIT	4	1.0000000+00	1.0311372+00	2.5680417+01	2.5652680+01	1.0800984-03
MERIT	1	1.0000000+00	9.7074433-01	2.5652680+01	2.4925456+01	2.8348864-02

PRESCRIPTION   LISTER TYPE LENS EXAMPLE (CONRADY)   04--03--63

10	-1.0000000+00	1.0000000-01	9.9999975-05	7.0699999-01	1.0000000+01	4.0000000+00	1.0000000+02
					1.0000000+00	0.0000000+00	9.9999888-06
[illegible]	1.7000000+02	1.0000000+00	2.0000000+00	5.1000000+00	1.0000000+00	1.0000000+00	1.0000000+00
[illegible]	0.0000000+00	-2.0000000+01	1.5135896+01	0.0000000+00	1.5154400+00	1.5180700+00	1.5241300+00
[illegible]	0.0000000+00	2.7000000+00	0.0000000+00	0.0000000+00	1.6161000+00	1.6211400+00	1.6332500+00
[illegible]	0.0000000+00	1.5700000+00	1.5129644+03	0.0000000+00	1.0000000+00	1.0000000+00	1.0000000+00
[illegible]	0.0000000+00	1.8000000+01	4.3620544+01	0.0000000+00	1.5154400+00	1.5180700+00	1.5241300+00
[illegible]	0.0000000+00	1.8000000+00	0.0000000+00	0.0000000+00	1.6161000+00	1.6211400+00	1.6332500+00
[illegible]	0.0000000+00	1.0000000+00	-1.4415692+02	0.0000000+00	1.0000000+00	1.0000000+00	1.0000000+00

Figure 1. Machine print showing input data card images, input data from storage, results from first design series, and the resulting Lister lens prescription.

PRESCRIPTION FROM DUMP NUMBER 1 LISTER TYPE LENS EXAMPLE (CONRADY) 04--03--63

10	-1.0000000+00	1.0000000-01	9.9999975-05	7.0692999-01	1.1772135+01	5.6678020+00	1.0000000+02
					1.0000000+00	0.0000000+00	9.9999868-06
20	1.7000000+02	1.0000000+00	2.0000000+00	5.1000000+00	1.0000000+00	1.0000000+00	1.0000000+00
30	0.0000000+00	-2.1429129+01	2.1795673+01	0.0000000+00	1.5154400+00	1.5180700+00	1.5241300+00
40	0.0000000+00	2.7000000+00	-1.4446487+01	0.0000000+00	1.6161000+00	1.6211400+00	1.6332500+00
50	0.0000000+00	1.5000000+00	-1.3046143+02	0.0000000+00	1.0000000+00	1.0000000+00	1.0000000+00
60	0.0000000+00	1.9939361+01	1.2618520+01	0.0000000+00	1.5154400+00	1.5180700+00	1.5241300+00
70	0.0000000+00	1.8000000+00	-7.7183086+00	0.0000000+00	1.6161000+00	1.6211400+00	1.6332500+00
80	0.0000000+00	1.0000000+00	-2.4319753+01	0.0000000+00	1.0000000+00	1.0000000+00	1.0000000+00

PLANE SETTING MODE ONE. LATTICE MODE ONE. SENSE SWITCH MODE ZERO. NUMBER OF PLANES 3

.CLOR	FOCAL LENGTH	FOCAL POINT	H	EXIT PUPIL	F/NUMBER	BACK FOCUS	RAYS
1	2.1009861+01	9.2860284+00	1.0000000+00	-1.6688712+01	1.9999997+00	1.1773229+01	12
	LOC OF PLANE	AVERAGE X	AVERAGE Y	RMS X	RMS Y	SPOT SIZE	
	1.1672135+01	0.0000000+00	1.1980480-01	1.2161439-02	1.2192116-02	1.7220578-02	
	1.1772135+01	0.0000000+00	1.2023697-01	9.6494393-04	9.7981265-04	1.3751908-03	
	1.1872135+01	0.0000000+00	1.2066913-01	1.1010998-02	1.0978221-02	1.5548744-02	
2	2.0986529+01	9.2765597+00	1.0000000+00	-1.6698513+01	1.9976020+00	1.1764187+01	12
	1.1672135+01	0.0000000+00	1.1969845-01	1.1265596-02	1.1298255-02	1.5955069-02	
	1.1772135+01	0.0000000+00	1.2013058-01	6.3235801-04	6.1398833-04	8.8139567-04	
	1.1872135+01	0.0000000+00	1.2056231-01	1.1907588-02	1.1873352-02	1.6815681-02	
3	2.0940019+01	9.2715227+00	1.0000000+00	-1.6693141+01	1.9931200+00	1.1764028+01	12
	1.1672135+01	0.0000000+00	1.1942334-01	1.1673487-02	1.1710967-02	1.6535327-02	
	1.1772135+01	0.0000000+00	1.1985511-01	7.8206419-04	7.9513261-04	1.1152848-03	
	1.1872135+01	0.0000000+00	1.2028687-01	1.1576403-02	1.1538914-02	1.6345019-02	
1			3.0000000+00	-1.6701925+01			12
	1.1672135+01	0.0000000+00	3.5934228-01	1.1917910-02	1.2205725-02	1.7059200-02	
	1.1772135+01	0.0000000+00	3.6063547-01	8.2526085-04	9.4088352-04	1.2515260-03	
	1.1872135+01	0.0000000+00	3.6192866-01	1.1257743-02	1.0968725-02	1.5717815-02	
2			3.0000000+00	-1.6731706+01			12
	1.1672135+01	0.0000000+00	3.5902220-01	1.1028705-02	1.1334417-02	1.5814593-02	
	1.1772135+01	0.0000000+00	3.6031469-01	7.7085200-04	5.7737240-04	9.6310524-04	
	1.1872135+01	0.0000000+00	3.6160719-01	1.2148743-02	1.1846746-02	1.6968716-02	
3			3.0000000+00	-1.6706210+01			12
	1.1672135+01	0.0000000+00	3.5819500-01	1.1454990-02	1.1805767-02	1.6449709-02	
	1.1772135+01	0.0000000+00	3.5948698-01	8.0892972-04	9.0046110-04	1.2104534-03	
	1.1872135+01	0.0000000+00	3.6077896-01	1.1801892-02	1.1469633-02	1.6457130-02	
1			5.0000000+00	-1.6728426+01			12
	1.1672135+01	0.0000000+00	5.9865433-01	1.1432962-02	1.2243044-02	1.6751261-02	
	1.1772135+01	0.0000000+00	6.0080621-01	7.3968949-04	8.9385626-04	1.1602239-03	
	1.1872135+01	0.0000000+00	6.0295810-01	1.1749660-02	1.0944237-02	1.6057112-02	
2			5.0000000+00	-1.6738163+01			12
	1.1672135+01	0.0000000+00	5.9811755-01	1.0557201-02	1.1417627-02	1.5550457-02	
	1.1772135+01	0.0000000+00	6.0026827-01	1.1520503-03	5.5505073-04	1.2787890-03	
	1.1872135+01	0.0000000+00	6.0241900-01	1.2629268-02	1.1787830-02	1.7275744-02	
3			5.0000000+00	-1.6732410+01			12
	1.1672135+01	0.0000000+00	5.9673154-01	1.1020524-02	1.2006707-02	1.6297636-02	
	1.1772135+01	0.0000000+00	5.9888143-01	1.0207015-03	1.1366601-03	1.5276870-03	
	1.1872135+01	0.0000000+00	6.0103132-01	1.2251103-02	1.1326453-02	1.6684666-02	

Figure 2. Machine print showing final prescription and 12-ray analysis of Lister lens performance.

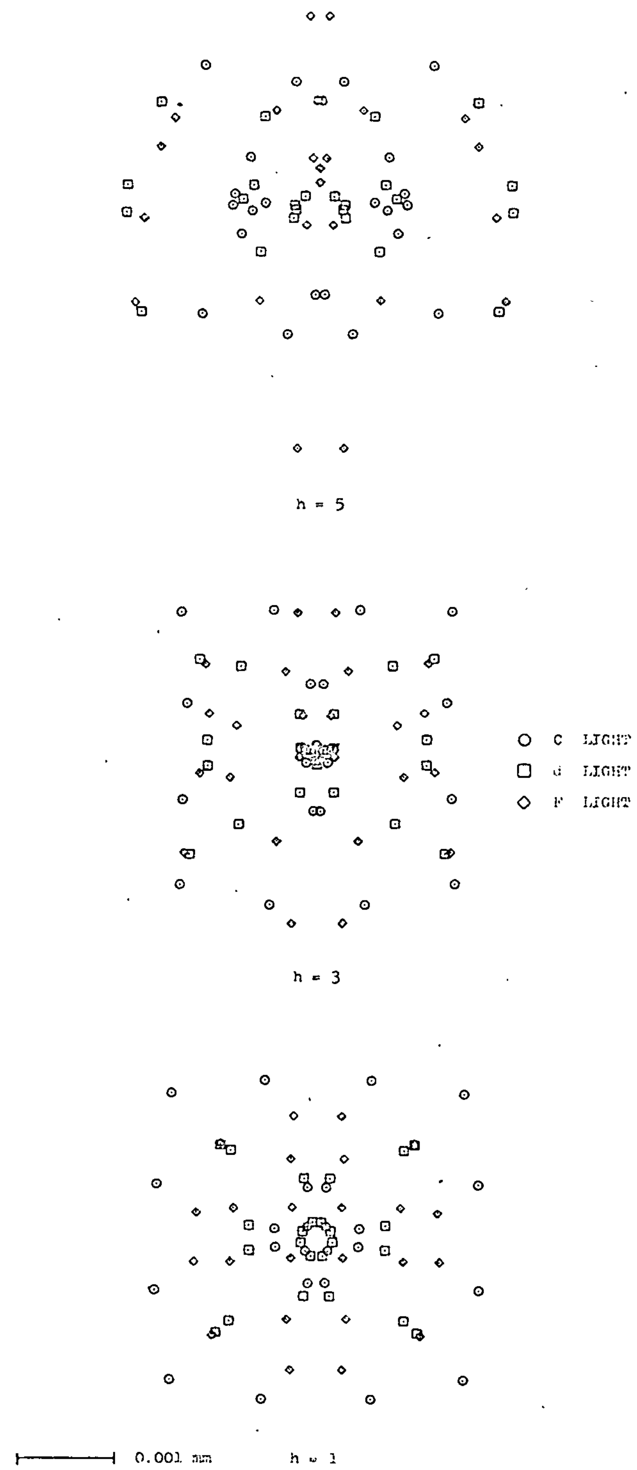


Figure 3. Spot diagrams of 3 images made by Lister lens.

PRESCRIPTION FROM DUMP NUMBER 53 BRIXNER ZOOM FOR 4/1 POSITION ONE. 01--31--63

10	-1.0000000+00	-9.9999999-03	1.0000000-05	7.0699999-01	3.7653234+02	5.4445989+01	1.0000000+00
					1.0000000+00	-2.0000000+01	9.9999999-07
20	9.0000000+03	0.0000000+00	3.7500000+01	7.6000000+01	1.0002060+00	1.0002060+00	1.0002060+00
					1.0002070+00	1.0002080+00	1.0002100+00
30	0.0000000+00	-8.0000000+02	5.0464065+02	0.0000000+00	1.5152100+00	1.5177200+00	1.5196600+00
					1.5233500+00	1.5276900+00	1.5312600+00
40	0.0000000+00	1.0000000+01	2.9565648+02	0.0000000+00	1.6117600+00	1.6167200+00	1.6206900+00
					1.6286000+00	1.6383900+00	1.6469200+00
50	0.0000000+00	1.5000000+01	3.5059407+02	0.0000000+00	1.0002060+00	1.0002060+00	1.0002060+00
					1.0002070+00	1.0002080+00	1.0002100+00
60	0.0000000+00	1.0000000+01	3.5075321+02	0.0000000+00	1.5152100+00	1.5177200+00	1.5196600+00
					1.5233500+00	1.5276900+00	1.5312600+00
70	0.0000000+00	2.0000000+01	-4.6271720+02	0.0000000+00	1.6117600+00	1.6167200+00	1.6206900+00
					1.6286000+00	1.6383900+00	1.6469200+00
80	0.0000000+00	1.0000000+01	5.9126535+03	0.0000000+00	1.0002060+00	1.0002060+00	1.0002060+00
					1.0002070+00	1.0002080+00	1.0002100+00
90	0.0000000+00	5.0000000+02	4.5343400+04	0.0000000+00	1.5152100+00	1.5177200+00	1.5196600+00
					1.5233500+00	1.5276900+00	1.5312600+00
100	0.0000000+00	1.0000000+01	1.1846093+02	0.0000000+00	1.6117600+00	1.6167200+00	1.6206900+00
					1.6286000+00	1.6383900+00	1.6469200+00
110	0.0000000+00	1.5000000+01	2.1867210+02	0.0000000+00	1.0002060+00	1.0002060+00	1.0002060+00
					1.0002070+00	1.0002080+00	1.0002100+00
120	0.0000000+00	1.0000000+01	5.8180050+02	0.0000000+00	1.5152100+00	1.5177200+00	1.5196600+00
					1.5233500+00	1.5276900+00	1.5312600+00
130	0.0000000+00	2.0000000+01	-2.2434079+02	0.0000000+00	1.6117600+00	1.6167200+00	1.6206900+00
					1.6286000+00	1.6383900+00	1.6469200+00
140	0.0000000+00	1.0000000+01	-6.7883659+02	0.0000000+00	1.0002060+00	1.0002060+00	1.0002060+00
					1.0002070+00	1.0002080+00	1.0002100+00
150	0.0000000+00	1.0000000+03	4.4937513+02	0.0000000+00	1.5152100+00	1.5177200+00	1.5196600+00
					1.5233500+00	1.5276900+00	1.5312600+00
160	0.0000000+00	2.0000000+01	-2.2168650+02	0.0000000+00	1.6117600+00	1.6167200+00	1.6206900+00
					1.6286000+00	1.6383900+00	1.6469200+00
170	0.0000000+00	1.0000000+01	-6.4920066+02	0.0000000+00	1.0002060+00	1.0002060+00	1.0002060+00
					1.0002070+00	1.0002080+00	1.0002100+00

PLANE SETTING MODE ONE. LATTICE MODE ONE. SENSE SWITCH MODE ZERO. NUMBER OF PLANES 3

WEIGHTS

-49	1.2000000+01	8.0000000+02	0.0000000+00	1.0000000+02	0.0000000+00	0.0000000+00	0.0000000+00
-39	1.0000000+02	1.0000000+02	1.0000000+02	0.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00
-29	0.0000000+00	5.0000000+01	1.0000000+02	0.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00
-19	0.0000000+00	0.0000000+00	1.0000000+02	0.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00
-9	0.0000000+00	0.0000000+00	1.0000000+01	0.0000000+00	0.0000000+00	0.0000000+00	0.0000000+00

LATTICE N = 6 DELTA Y = 5.0000000-01
2.5000000-01 -7.5000000-01 4
7.5000000-01 -2.5000000-01 2

INCREMENT IDENTIFICATION... 1 18 12 10 8 9 10 11 12 13 14 17 1 4

COLOR IDENTIFICATION... 3 3 5 1

ID.	LOC.	NO.	ADDRESSES OF DESIGN PARAMETERS					ACTUAL DEF.	REQUIRED DEF.	INCREMENT
1	1010	-1	32					1.0001991+02	1.0000000+02	1.1339532-06
2	1020	-1	42					1.0000682+02	1.0000000+02	5.7569209-06
3	1030	-1	52					1.0465263+02	1.0000000+02	9.9999999-07
4	1040	-1	62					1.0000476+02	1.0000000+02	1.0647986-06
5	1050	-1	72					1.0000028+02	1.0000000+02	5.8958878-06
6	1060	-1	82					1.1493105+02	1.0000000+02	9.9999999-07
7	1070	-1	92					1.0000299+02	1.0000000+02	2.6759069-06
8	1080	-1	102					1.0000603+02	1.0000000+02	1.3171679-05
9	1090	-1	112					9.7994932+01	1.0000000+02	1.8061980-06
10	1100	-1	122					1.0000235+02	1.0000000+02	1.5391297-06
11	1110	-1	132					1.0000288+02	1.0000000+02	7.6109598-06
12	1120	-1	142					9.9996854+01	1.0000000+02	1.2817393-06
13	1130	-1	152					9.9997405+01	1.0000000+02	4.3133787-06
14	1140	-1	162					1.0000758+02	1.0000000+02	2.2412374-05
15	1150	-1	172					9.7989856+01	1.0000000+02	3.8367009-06
16	1160	1	1709					1.0000774+00	1.0000000+00	6.4961967-02
17	1170	1	1809					9.9991272-01	1.0000000+00	4.9294689-02
18	1180	1	1909					9.9976323-01	1.0000000+00	8.5637964-02
19	1190	5	14	1602	1702	1802	1902	9.7976886-01	1.0000000+00	8.5473059-02

SUBSTITUTIONS... 4 SUBSTITUTION SETS, OF 11 ITEMS, EACH.

LOC.	1ST SUBST.	2ND SUBST.	3RD SUBST.	4TH SUBST.
-173	-8.8590784+06	-1.0835291+07	3.6607912+15	5.1604435+14
-49	6.5000000+00	8.0000000+00	1.0000000+01	7.0000000+00
14	3.7653234+02	3.7653234+02	3.7653234+02	3.7653234+02
15	3.3457052+01	4.7129263+01	4.9695471+01	4.0865409+01
20	9.0000000+03	9.0000000+03	9.0000000+03	9.0000000+03
61	4.1000000+02	2.1000000+02	1.1000000+02	3.1000000+02
91	1.0000000+02	3.0000000+02	4.0000000+02	2.0000000+02
121	4.1000000+02	2.1000000+02	1.1000000+02	3.1000000+02
151	6.0000000+02	8.0000000+02	9.0000000+02	7.0000000+02
22	1.1250000+02	6.4928912+01	4.9282037+01	8.5562077+01
23	2.2600000+02	1.7500000+02	1.2500000+02	2.0000000+02

Figure 4. Machine print showing final prescription of zoom lens designed for 5 conditions of use.

Table I.

Analysis of Parameter Interaction in Selected Sequences

Design parameter calling sequence	Spaces between parameters					Parameter spacing incidence frequency						
	1st	2nd	3rd	4th	5th	1	2	3	4	5	6	7
1, 4, 7, 10, 13	3	3	3	3	3	0	0	5	0	0	5	0
1, 2, 3, 4, 5	1	1	1	1	11	4	3	2	1	0	0	0
1, 2, 3, 5, 14	1	1	2	9	2	2	3	2	2	0	1	0
1, 3, 12, 14, 15	2	9	2	1	1	2	3	2	2	0	1	0
1, 2, 5, 10, 12	1	3	5	2	4	1	1	1	2	2	1	2

Table II.

EXAMPLES OF EFFICIENT PARAMETER CALLING SEQUENCES

Total No. Parameters	Number of Parameters Used Simultaneously: 4	5	6	7	8	9	10
6	1,2,4,5						
7	1,2,3,5	1,2,3,5,6					
8	1,2,4,6	1,2,3,5,6	1,2,3,5,6,7				
9	1,2,5,7	1,2,4,6,7	1,2,3,5,6,7	1,2,3,4,6,7,8			
10	1,2,4,7	1,2,3,6,8	1,2,3,5,6,8	1,2,3,4,6,7,9	1,2,3,4,6,7,8,9		
11	1,2,4,8	1,2,3,5,8	1,2,3,5,6,8	1,2,4,5,6,7,9	1,2,3,4,5,7,8,10	1,2,3,4,5,6,7,9,10	
12	1,2,4,8	1,2,3,6,9	1,2,3,5,7,10	1,2,4,6,7,8,10	1,2,3,4,6,7,8,10	1,2,3,4,5,7,8,9,11	1,2,3,4,5,7,8,9,10,11
13	1,3,4,8	1,2,5,8,10	1,2,5,7,8,10	1,2,3,5,6,9,11	1,2,3,5,7,8,10,11	1,2,3,5,6,7,8,9,11	1,2,3,4,5,7,8,9,11,12
14	1,3,7,10	1,2,4,7,11	1,2,3,4,8,11	1,2,4,7,8,9,11	1,2,3,5,8,9,10,12	1,2,4,5,6,8,9,11,13	1,2,3,4,6,7,9,10,11,13
15	1,3,6,10	1,2,4,8,12	1,2,3,5,8,12	1,2,3,5,6,9,11	1,2,3,4,6,8,9,12	1,2,5,6,7,8,9,11,13	1,2,5,6,7,8,9,10,11,13
16	1,3,8,11	1,2,4,8,12	1,3,4,5,9,12	1,2,3,5,7,10,13	1,2,3,6,8,10,11,14	1,2,3,4,6,9,10,12,14	1,2,3,4,5,6,9,10,12,14
17	1,3,8,12	1,2,4,8,13	1,2,4,7,10,14	1,2,3,4,7,10,14	1,2,3,4,5,7,10,14	1,2,5,6,7,8,10,12,15	1,2,3,4,5,6,9,10,13,15
18	1,5,8,13	1,3,4,8,13	1,2,3,5,9,14	1,2,6,8,9,11,15	1,2,3,4,6,10,12,15	1,2,5,7,8,9,11,14,16	1,3,4,5,6,9,10,12,14,16
19	1,4,9,13	1,3,4,8,14	1,2,4,7,11,15	1,3,4,5,7,10,15	1,2,3,4,7,9,12,16	1,2,5,8,9,10,11,13,15	1,2,4,5,8,9,10,11,13,15
20	1,4,8,13	1,4,9,11,15	1,3,4,7,11,16	1,2,7,9,11,12,15	1,3,6,7,8,11,14,17	1,3,4,7,9,12,13,14,17	1,3,4,7,9,11,12,13,14,17
21	1,6,9,15	1,3,8,9,12	1,2,4,8,12,17	1,2,3,5,9,12,17	1,4,5,6,10,12,15,18	1,3,7,9,10,12,13,14,17	1,4,5,6,7,10,11,13,15,18
22	1,5,10,16	1,2,5,11,16	1,2,4,8,13,18	1,2,3,5,9,13,18	1,2,3,4,7,10,14,18	1,2,3,5,9,13,14,16,19	1,2,3,5,8,12,13,14,16,19
23	1,5,10,16	1,2,4,9,15	1,2,4,9,13,19	1,2,3,5,9,14,19	1,2,3,4,6,10,14,19	1,2,3,4,6,9,12,16,20	1,2,3,4,5,8,12,14,17,20
24	1,5,10,17	1,2,5,12,17	1,2,4,9,13,19	1,2,3,5,9,14,19	1,2,3,4,8,13,16,20	1,2,3,4,7,12,15,19,21	1,2,3,5,7,12,14,15,17,21
25	1,5,10,16	1,2,5,12,18	1,2,3,6,12,19	1,2,3,5,10,14,20	1,2,3,5,8,13,17,21	1,2,3,5,8,13,17,20,22	1,2,3,4,6,10,14,17,20,22
26	1,6,13,21	1,2,6,12,19	1,2,4,11,15,22	1,2,3,5,10,15,21	1,2,3,4,9,13,17,22	1,2,3,5,9,14,16,19,22	1,2,3,4,6,9,13,15,19,22
27	1,6,13,22	1,2,5,13,19	1,2,4,6,14,22	1,2,4,8,12,17,22	1,2,3,4,7,11,17,22	1,2,3,4,6,10,13,18,24	1,2,3,4,8,10,13,16,20,24
28	1,6,12,20	1,2,5,13,20	1,2,5,16,21,23	1,2,4,9,13,17,23	1,2,3,5,8,12,17,23	1,2,3,5,8,11,15,19,24	1,2,3,4,6,9,15,16,20,24
29	1,6,13,21	1,2,6,13,21	1,2,3,7,14,22	1,2,3,5,11,16,23	1,2,3,5,8,13,17,23	1,2,3,4,6,9,14,18,24	1,2,3,4,6,10,15,19,22,25
30	1,7,14,22	1,2,7,14,22	1,2,3,6,15,25	1,2,3,6,11,17,24	1,2,4,12,13,17,25,27	1,2,3,4,6,10,16,20,25	1,2,3,4,7,11,17,19,22,26

Table III.

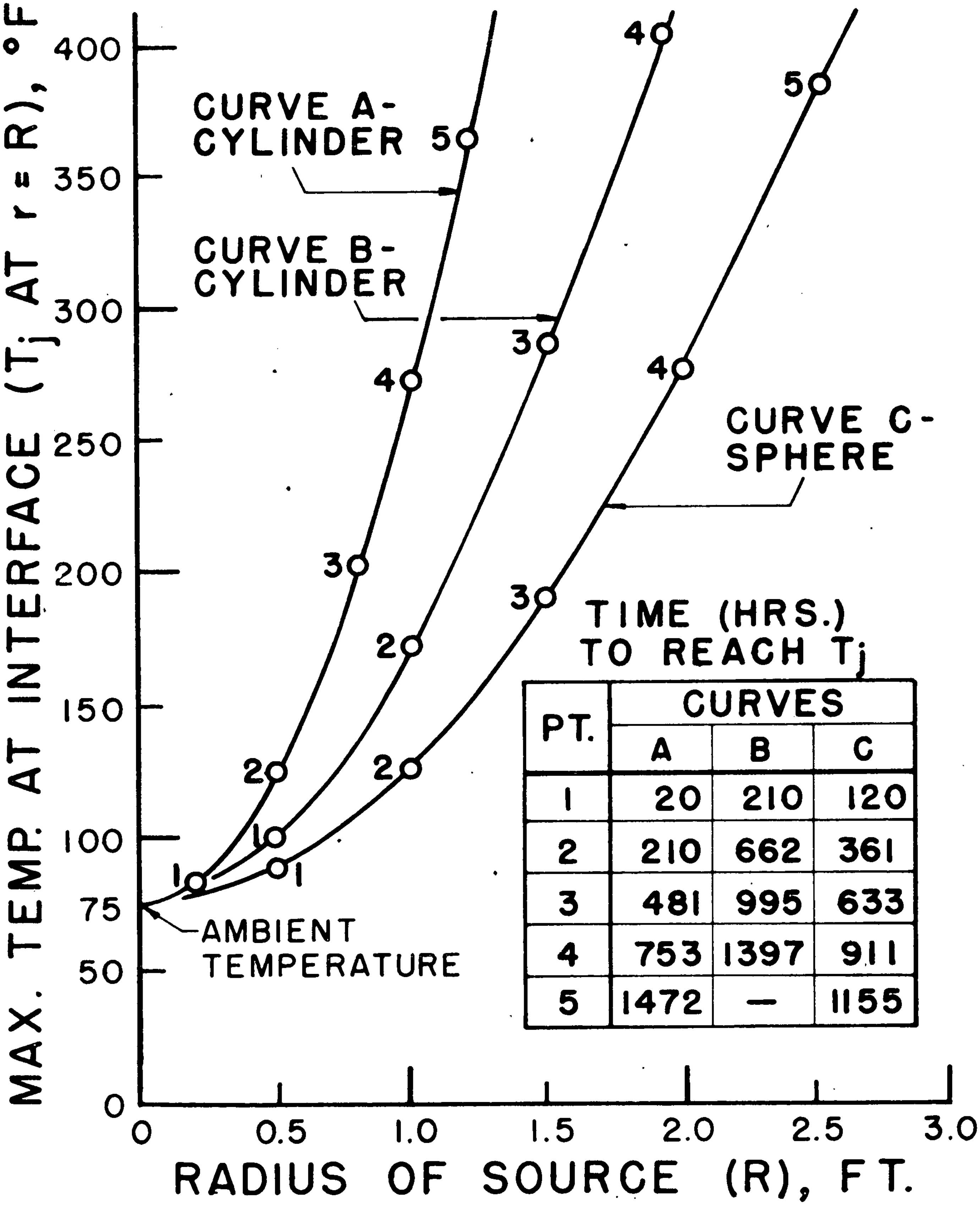
Summary of Lister Lens Designing Sequence and Progress

Operation performed	Ray pattern	Increments used Total	Per cycle	Cycles run	Figure of merit
Start	2	-	-	-	904.7
Designing	2	4	1	29	24.9
Designing	2	6	1	25	18.0
Designing	2	6	2	25	0.26
Designing	2	8	2	25	0.23
f/2.0	2	-	-	-	4.9
Designing	2	9	3	28	0.30
Designing	2	9	4	28	0.22
Ray change	6	-	-	-	4.0
Designing	6	9	4	28	0.47
Designing	6	9	5	28	0.31
Designing	6	9	6	28	0.26
Ray change	12	-	-	-	0.53
Designing	12	9	6	28	0.36

Table IV.

Optical Characteristics of Zoom Lens

Position	Element shift	Object size	Magnification	Relative aperture	Focal length	Back focus
1	0	150.00	0.133	12.0	1590	384.720
2	50	171.8	0.116	10.8	1310	384.565
3	100	197.0	0.102	10.0	1096	384.670
4	150	226.0	0.088	8.8	927	384.755
5	200	259.4	0.077	8.0	790	384.776
6	250	297.9	0.067	7.4	677	384.729
7	300	342.0	0.058	7.0	582	384.658
8	350	392.4	0.051	6.7	503	384.632
9	400	450.0	0.044	6.5	436	384.723



PT.	CURVES A	CURVES B	CURVES C
1	20	210	120
2	210	662	361
3	481	995	633
4	753	1397	911
5	1472	—	1155

Fig. 6. Variation at Interfacial Temperatures