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## BREAKDOWN OF CONVENTIONAL FACTORIZATION FOR ISOLATED PHOTON CROSS SECTIONS

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Using  $e^+e^- \rightarrow \gamma + X$  as an example, we show that the conventional factorization theorem of perturbative QCD breaks down for isolated photon cross sections in a specific part of phase space. Implications are discussed.

Much of the predictive power of perturbative QCD derives from factorization theorems<sup>1</sup>. *Conventional* factorization expresses a physical quantity as the convolution of a partonic part with a nonperturbative matrix element, and it requires that the perturbatively calculated partonic part be infrared safe or order by order in  $\alpha_s$ . Predictions then follow when processes with different hard scattering but the same matrix elements are compared. Using  $e^+e^- \rightarrow \gamma X$  as an example, we demonstrate that the perturbatively calculated partonic part for the isolated photon cross section is not infrared safe in a well-defined phase space.

The essence of isolation is that a cone of half-angle  $\delta$  is drawn about the direction of the photon's momentum, and the isolated cross section is defined for photons accompanied by less than a specified amount of hadronic energy in the cone, *e.g.*,  $E_h^{\text{cone}} \leq E_{\text{max}}$ .

At high energy, photons can result from long-distance fragmentation of quarks and gluons, themselves produced in short-distance hard collisions. In such fragmentation contributions, hadronic energy in the isolation cone has two sources: a) energy from parton fragmentation,  $E_{\text{frag}}$ , and b) energy from non-fragmenting final-state partons,  $E_{\text{partons}}^{\text{cone}}$ , that enter the cone. When the maximum hadronic energy allowed in the isolation cone is saturated by the fragmentation energy,  $E_{\text{max}} = E_{\text{frag}}$ , there is no allowance for energy in the cone from other final-state partons. In particular, if there is a gluon in the final state, the phase space for this gluon becomes restricted. By contrast, isolation does not affect the virtual gluon exchange contribution. Therefore, for isolated photons, there is a possibility that the infrared singularity from the virtual contribution may not be cancelled completely by the restricted real contribution. We showed<sup>2,3</sup> that such incomplete cancellation of infrared singularities appears first at next-to-leading order (NLO) in the quark fragmentation con-



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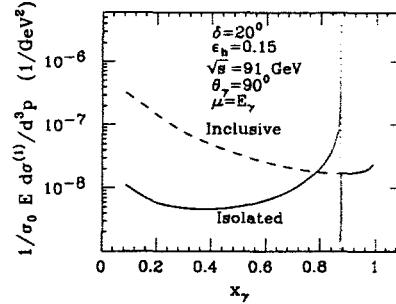


Figure 1: Comparison of the one-loop quark fragmentation contributions to the isolated cross section and the inclusive cross section in  $e^+e^- \rightarrow \gamma X$  as a function of  $x_\gamma = 2E_\gamma/\sqrt{s}$ .

tributions.

If conventional factorization were true, the fragmentation contributions to the cross section for isolated photons would be expressed in the factorized form

$$E_\gamma \frac{d\sigma_{e^+e^- \rightarrow \gamma X}^{iso}}{d^3 \ell} = \sum_c \int_{\max[x_\gamma, \frac{1}{1+\epsilon_h}]}^1 \frac{dz}{z} E_c \frac{d\hat{\sigma}_{e^+e^- \rightarrow cX}^{iso}}{d^3 p_c} \left( x_c = \frac{x_\gamma}{z} \right) \frac{D_{c \rightarrow \gamma}(z, \delta)}{z}; \quad (1)$$

$x_\gamma = 2E_\gamma/\sqrt{s}$ ,  $x_c = 2E_c/\sqrt{s}$ ,  $\epsilon_h = E_{max}/E_\gamma$ , and the sum extends over  $c = q, \bar{q}$  and  $g$ .  $D_{c \rightarrow \gamma}(z, \delta)$  is the nonperturbative function that describes fragmentation of parton “ $c$ ” into a photon. The lower limit of integration results from the isolation requirement with the assumption that all fragmentation energy is in the isolation cone<sup>3</sup>. Because of the isolation condition, the phase space constraints are different in three regions: a)  $x_\gamma < 1/(1+\epsilon_h)$ , b)  $x_\gamma = 1/(1+\epsilon_h)$ , c)  $x_\gamma > 1/(1+\epsilon_h)$ . We show below that the next-to-leading order partonic hard part for quark fragmentation,  $E_q d\hat{\sigma}_{e^+e^- \rightarrow qX}^{iso}/d^3 p_q$ , is infrared sensitive.

When  $x_\gamma < 1/(1+\epsilon_h)$ , subprocesses with two-body final states do not contribute. Therefore, there is no leading-order quark (or antiquark) fragmentation contribution, and one-loop virtual diagrams do not contribute. The well-known  $1/(1-x_q)$  infrared singularity of the real gluon emission diagrams, as  $x_q = x_\gamma/z \rightarrow 1$ , will remain in  $\hat{\sigma}_{e^+e^- \rightarrow qX}^{iso}$ . After convolution with  $D_{q \rightarrow \gamma}(z)$ , this inverse power infrared sensitivity yields a logarithmic divergence proportional to  $\ell \ln(1/x_\gamma - (1+\epsilon_h))$ . As shown in Fig. 1, as  $x_\gamma \rightarrow 1/(1+\epsilon_h)$ , the isolated cross section becomes larger than the inclusive cross section, which is certainly not physical. This infrared sensitivity in  $\hat{\sigma}_{e^+e^- \rightarrow qX}^{iso}$  signals a breakdown of conventional perturbative factorization.

When  $x_\gamma = 1/(1+\epsilon_h)$ ,  $x_q = x_\gamma/z = 1$  is possible. Therefore, the one-loop virtual diagrams, which are proportional to  $\delta(1-x_q)$  will contribute. However,

isolation constraints limit the phase space of real gluon emission in the real subprocess,  $e^+e^- \rightarrow q\bar{q}g$ . Consequently, the infrared divergences in the real and virtual contributions do not cancel completely in the isolated case, unlike the inclusive case. In  $n = 4 - 2\epsilon$  dimensions, we find<sup>3</sup>

$$E_q \frac{d\sigma_{e^+e^- \rightarrow q\bar{q}X}^{(1)iso}}{d^3 p_q} \sim \left\{ \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \left( \frac{3}{2} - \ln \frac{\delta^2}{4} \right) \right\} \delta(1 - x_q) + \text{finite terms}. \quad (2)$$

At  $x_q = 1$ , corresponding to  $x_\gamma = 1/(1 + \epsilon_h)$ , the partonic part for quark fragmentation in Eq. (2) is infrared divergent, and the perturbative calculation is not well-defined. Conventional perturbative factorization again breaks down.

When  $x_\gamma > 1/(1 + \epsilon_h)$ , due to the finite cone size, there is a very small region of phase space where the isolated cross section is not exactly equal to the inclusive cross section. This region of phase space is proportional to  $\epsilon_h \delta^2/4$ . In this narrow region above  $x_\gamma = 1/(1 + \epsilon_h)$ , all one-loop contributions to the isolated cross section have a logarithmic divergence  $\ln(1 + \epsilon_h - 1/x_\gamma)$  as  $x_\gamma \rightarrow 1/(1 + \epsilon_h)$  from above<sup>3</sup>.

In summary, the NLO partonic part for the quark fragmentation contribution to the isolated cross section, calculated in perturbative QCD, is infrared sensitive when  $x_\gamma \leq 1/(1 + \epsilon_h)$ . Conventional perturbative factorization for the cross section of isolated photons in  $e^+e^-$  annihilation breaks down for  $x_\gamma$  in the neighborhood of  $1/(1 + \epsilon_h)$ . Breakdown of factorization means that the isolated cross section cannot be calculated reliably in perturbative QCD near  $x_\gamma = 1/(1 + \epsilon_h)$ . This result challenges theorists to find a modified factorization scheme.

## References

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