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MEASUREMENT OF THE SRE POWER COEFFICIENTS AND REACTOR PARAMETERS UTILIZING THE OSCILLATION TECHNIQUES

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MEASUREMENT OF THE SRE POWER COEFFICIENTS AND REACTOR PARAMETERS UTILIZING THE OSCILLATION TECHNIQUES

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CONTENTS

Page

Appendixes

TABLE

FIGURES

ABSTRACT

An experiment was performed on the SRE, wherein relative stability analysis and the determination of core dynamic parameters were accomplished. The technique was one of producing a sinusoidal change in reactivity through oscillation of an existing control rod and measuring the reactor flux and temperature responses by use of a correlation technique which allows measurement of only the fundamental frequency of reactor response. The ability to isolate the fundamental frequency permitted the analysis of data with distortions or small signal to noise ratios present.

By isolating individual gains and response times, the simplified individual transfer functions of fuel, moderator and coolant were determined. These transfer functions were used in the equation for the power coefficient to separate the isothermal temperature coefficients of reactivity.

The isothermal temperature coefficients of reactivity were determined as

$$\alpha\,(\text{Fuel}) = -1.1 \times 10^{-5}/\,^\circ\text{F}$$

$$\alpha\,(\text{Coolant}) = +0.3 \times 10^{-5}/\,^\circ\text{F}$$

$$\alpha\,(\text{Moderator}) = +1.3 \times 10^{-5}/\,^\circ\text{F (at } 675^\circ\text{F)} \quad .$$

The steady-state power coefficients were determined as

$$H_{ss} = \left(\frac{3}{n_o} - 0.37\right) \times 10^{-4} \pm 50\%/\text{Mwt} \quad \begin{matrix}\text{(1400 gpm}\\ \text{coolant flow)}\end{matrix}$$

or

$$H_{ss} = \left(\frac{-3.5 \times 10^{-4}}{n_o}\right) \pm 50\%/\text{Mwt} \quad (\text{core } \Delta T = 335^\circ\text{F}) \quad .$$

From these individual measurements and one-lumped parameter model calculation, it was possible to approximate very closely the equations for dynamic stability analysis. The work has shown that the SRE has excellent dynamic stability over all frequency ranges and operating conditions of practical importance.

I. INTRODUCTION

A. STABILITY AND REACTOR PARAMETERS

In order to mathematically describe and to improve a specific reactor concept (such as a sodium-cooled, graphite-moderated reactor), it is not only necessary to determine the relative stability of the reactor under study (e.g., the SRE)[1], but also to attempt to separate each of the parameters upon which the stability analysis is dependent.

The relative importance of each major component must be determined from the standpoint of both magnitude and time of response. The power coefficient, in the form of a frequency response curve, denotes the relative stability but is of little assistance in design modifications. For design improvement, a knowledge of the functional dependence of the power coefficient upon each of its parameters is required in order that a specific region of influence may be altered in a direction to improve the response.

In this report the power coefficient curves have been determined to a reasonable degree of accuracy — independent of any approximation. For design usage and extrapolation of the power coefficient curves to the steady state, the separation of parameters required some approximations, assumptions, and the use of data which were obtained by experimental techniques other than oscillation.

B. OSCILLATION OF THE REACTOR

The usefulness of transfer-function or frequency-response measurements of nuclear reactors for the purpose of determining relative stability and parameter magnitudes has become increasingly evident in the last few years as a result of the development of applicable oscillation techniques. The specific application of the oscillation technique for zero-power frequency-response measurement on the SRE has been reported in detail.[2]

For oscillation at-power, the same equipment and procedure were used as for the zero-power measurements. Oscillation of the reactor was accomplished by direct oscillation of a shim rod through a peak-to-peak amplitude of approximately 5/16 in. ($\delta\rho = 1.5 \times 10^{-4}$). This method of oscillation had poor response for frequencies greater than 0.1 cps. As will be explained in Section II, good frequency response to at least 0.5 cps is desirable for analysis of a sodium graphite reactor. This series of tests was conducted for the primary purpose of demonstrating the feasibility of the oscillation method of frequency response measurement at-power, and to determine the general magnitude of the reactor parameters and relative reactor stability.

C. GENERAL THEORY

Stability in a reactor is basically dependent upon the system feedback due to temperature and power changes. There also is a slow response dependence on xenon buildup and decay, but the effect is extremely slow and need not be considered in comparison with the temperature and power dependent feedback effects. The general theory and equations covering the effects of feedback in nuclear reactors, when operating at a sufficient power level to cause temperature changes to become noticeable, have been thoroughly covered in the literature.[3] For the convenience of the reader, however, a block diagram illustration of these effects is shown in Figures 1 and 2 (using control-system-theory notation).

Figure 1 is the basic diagram and illustrates the manner in which the zero-power transfer function, G(s), of a reactor is transformed by the power dependent feedback term, H(s). The zero-power transfer function is dependent only upon the type of nuclear fuel used in the reactor, and upon the ratio of the prompt-neutron lifetime, ℓ, of the reactor to the effective fraction of delayed neutrons, β. H(s), however, is dependent upon nearly all of the components of the reactor core. Figure 2 is a breakdown of some of these effects. A solution of the block diagram of Figure 1 gives for the at-power transfer function:

$$\frac{\delta n(s)}{\delta \rho(s)} = \frac{G(s)_{ZP}}{1 - H(s)\,G(s)_{ZP}} = G(s)_{P} \quad . \qquad \ldots(1)$$

The feedback transference H(s) is then

$$H(s) = \frac{1}{G(s)_{ZP}} - \frac{1}{G(s)_{P}} \quad . \qquad \ldots(2)$$

The nomenclature for this report will be found in Appendix A.

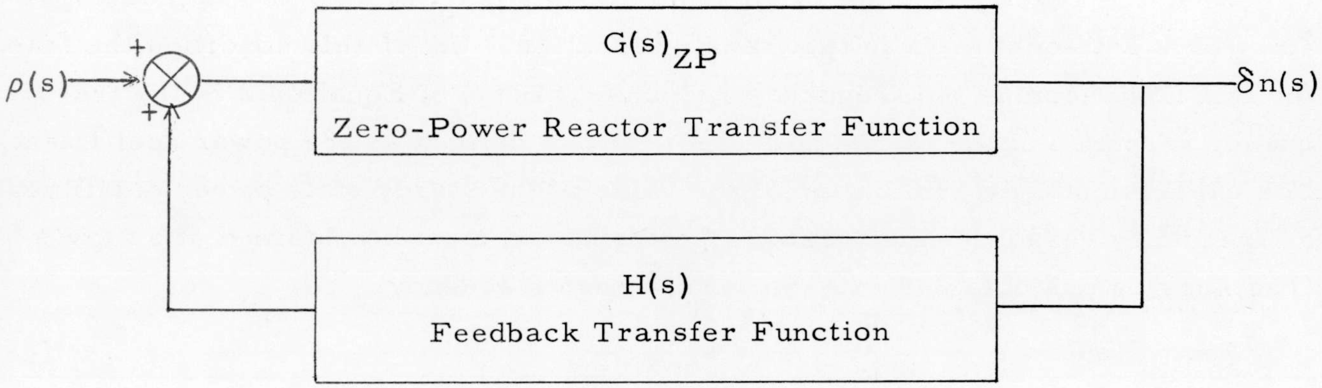

Figure 1. Block Diagram of Basic Reactor Transfer Function When Operating At-Power

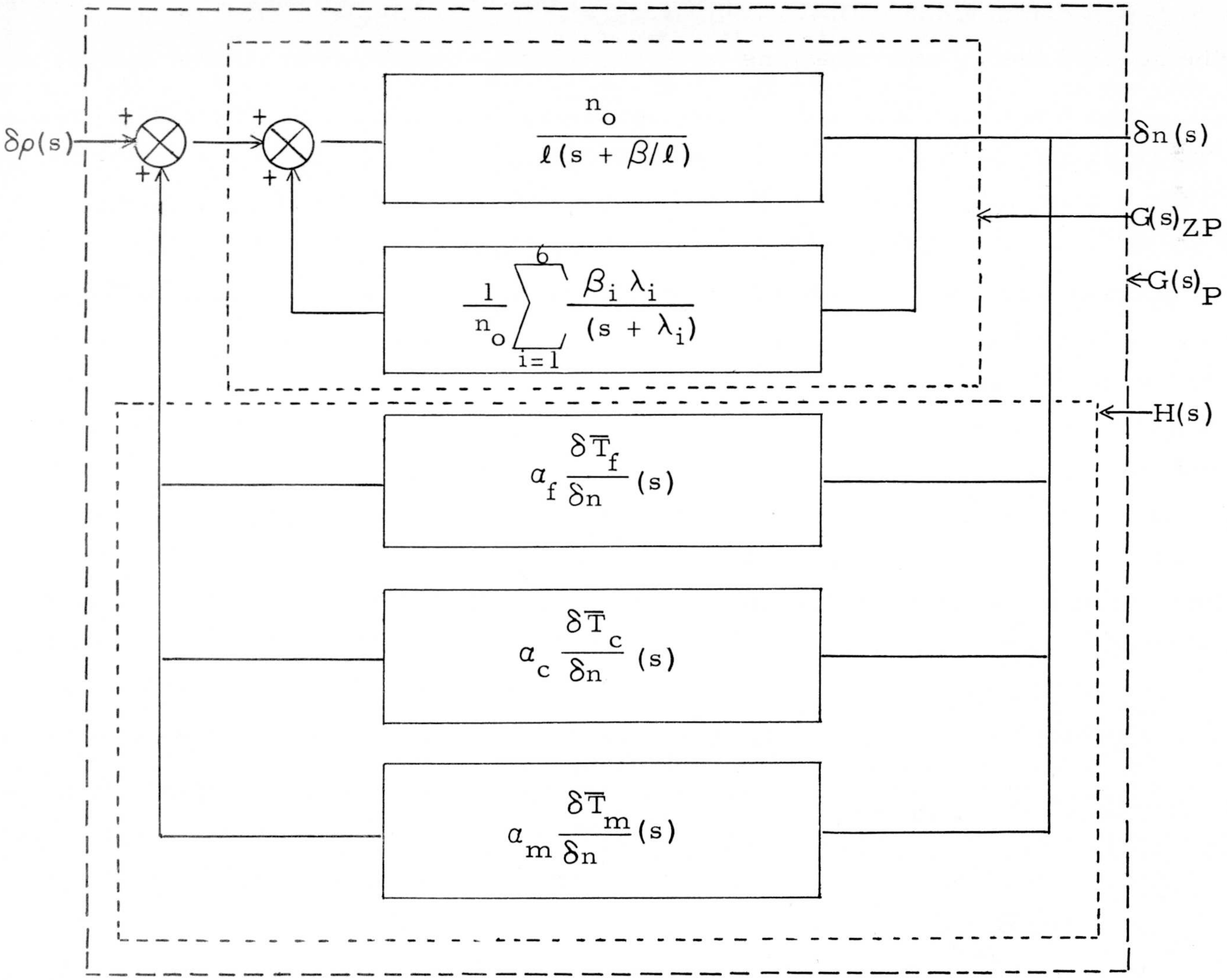

Figure 2. $G(s)_P$ — The Reactor Transfer Function When Operating At-Power

When the reactivity input is sinusoidal ($\delta\rho = \delta\rho_o \sin \omega t$), the Laplace operator (s) is set equal to $j\omega$ in the transfer function. Under this condition the transfer function becomes the frequency response. $H(j\omega)$ of Equation 2 is the frequency response of the feedback system, and is defined as the power coefficient. For experimental determination of the value of the steady state power coefficient by frequency response measurement, information must be obtained at as low a frequency as possible and extrapolated to zero frequency.

From Figure 2

$$H(j\omega) = \alpha_f \frac{\delta \overline{T}_f}{\delta n}(j\omega) + \alpha_c \frac{\delta \overline{T}_c}{\delta n}(j\omega) + \alpha_m \frac{\delta \overline{T}_m}{\delta n}(j\omega) \quad . \qquad \ldots(3)$$

The α's are referred to as the isothermal temperature coefficients of reactivity. They are difficult to calculate accurately and, in a large reactor, difficult to measure. Attempts to measure these at the SRE under semi-isothermal conditions have been only partially satisfactory. By applying various analytical techniques to the measured frequency responses of Equation 3 for several different reactor operating conditions, it is theoretically possible to separate and determine the six unknowns of the equations, providing the individual break frequencies of the three frequency dependent components occur; (1) at least one decade apart; and (2) at a frequency at which the oscillator equipment is capable of functioning. In practice, it is not possible to make this separation, to a high degree of accuracy — in a reactor such as the SRE. The transient behavior of the fuel and sodium coolant is too closely coupled, and the break frequency of the graphite moderator occurs at such a low frequency that it is masked by the reactor power-level drift. However, good approximations of general magnitudes and time responses of the various temperature coefficients of reactivity were obtained.

In order to determine the frequency response of the power coefficient, $H(j\omega)$, directly, it is necessary to take the difference between two complex quantities of the same order of magnitude. In actual experimental measurement, Equation 2 is normalized to

$$\frac{n_o}{\beta} H(j\omega) = \frac{1}{\frac{\beta}{n_o} G(j\omega)_{ZP}} - \frac{1}{\frac{\beta}{n_o} G(j\omega)_P} \quad . \qquad \ldots (4)$$

The normalized zero power transfer function is known from theoretical calculations and experimental measurement.[2] Since the normalized feedback term, or power coefficient, approaches zero at sufficiently high frequencies, the best results are obtained when it is possible to oscillate the reactor at a sufficiently high frequency so that the zero-power and at-power reactor frequency responses functions are equal in both amplitude and phase. Under this condition, any errors due to original equipment setting or torsional strain in the oscillator drive mechanism may be corrected. The required frequency for the SRE would have been about 0.2 or 0.3 cps. The specific equipment used could satisfactorily operate only as high as 0.1 cps with accuracy. This was close enough to the required frequency to afford reasonable extrapolation accuracy.

II. MEASURED AT-POWER FREQUENCY RESPONSE

A. SPECIFIED REACTOR CONDITIONS

At-power measurements for the SRE were conducted by oscillating the reactor at four different power levels and under two different criteria of operation. One criterion called for the different power levels to have approximately the same coolant flow rate; the other called for approximately the same temperature difference across the reactor. These different reactor operating conditions were required to permit the separation of the parameters of Equation 3 and are listed in Table 1. The two criteria were selected because the lumped parameter equations of Appendix B indicated that the gain of the three frequency dependent parameters was flow dependent, whereas nuclear theory and experimental measurements have shown that the isothermal temperature coefficient for the graphite moderator is temperature dependent. Thus, measurements of different flow conditions as well as different average reactor temperatures were required to afford sufficient equations to separate the six unknown variables in Equation 3.

TABLE I

REACTOR CONDITIONS FOR SPECIFIC OSCILLATION RUNS

Run No.	Power (Mw)	Flow (gpm)	T_{in} (°F)	T_{out} (°F)	ΔT (°F)	Operating Conditions
1	9.5	1400	490	670	180	Full Flow
2	14.5	1425	523	795	273	
3	18.5	1400	520	830	330	
4	5.25	357	488	868	380	Variable Flow
5	10.5	746	500	840	340	
6	14.5	1120	510	840	330	

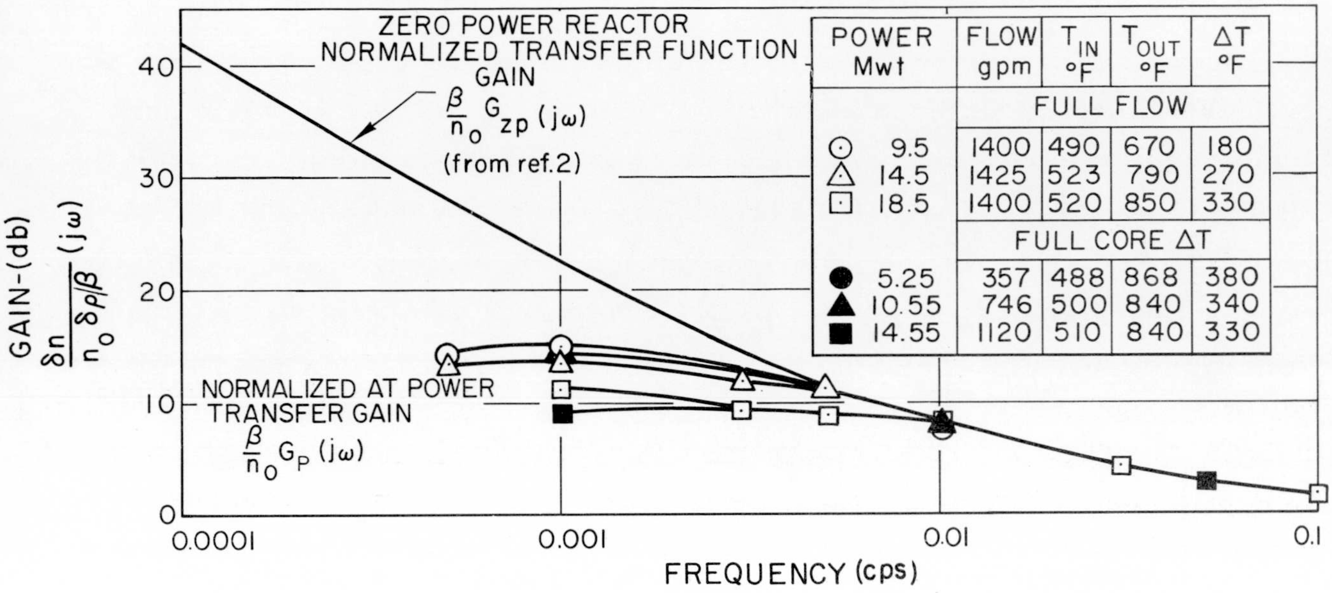

Figure 3. Measured Gain of SRE Transfer Functions

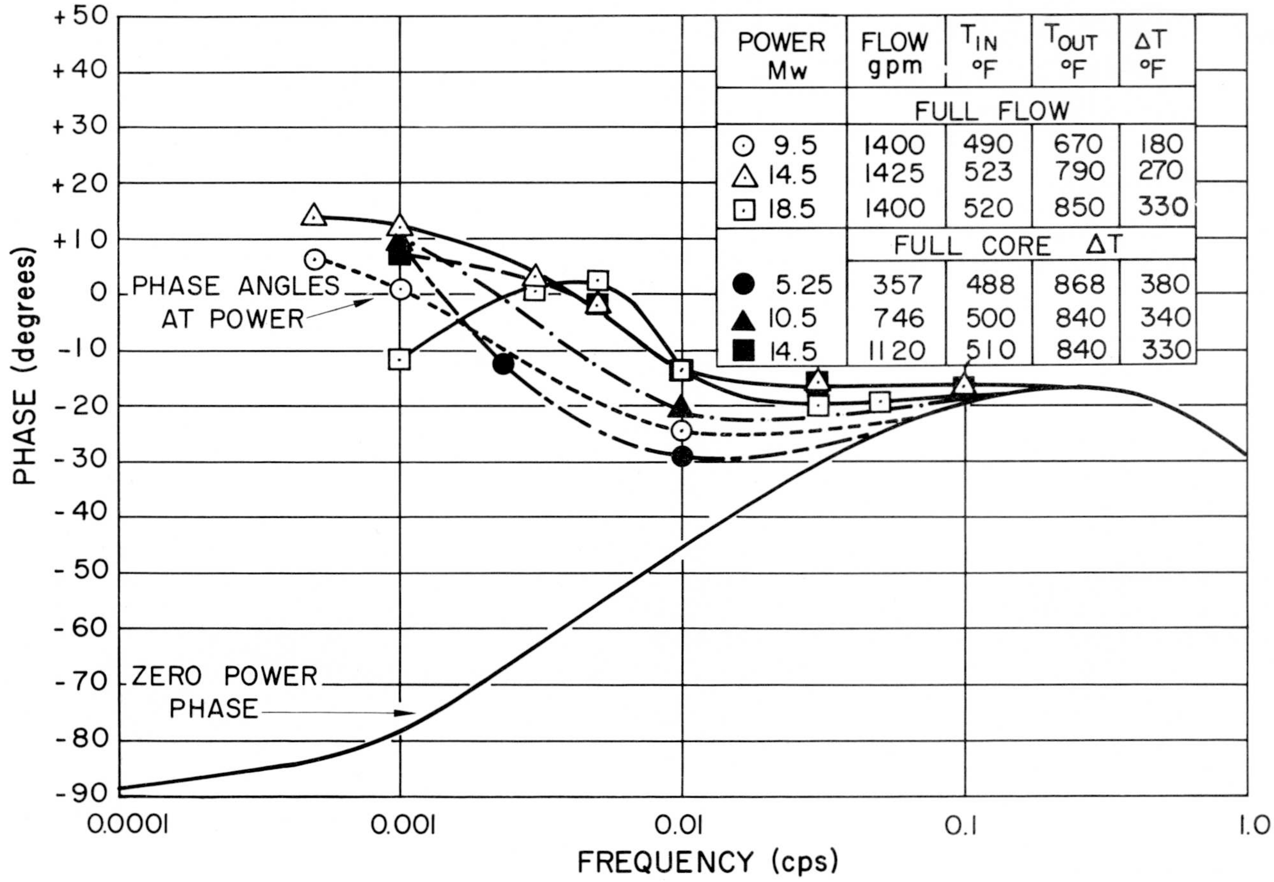

Figure 4. Measured Phase Response of the SRE Transfer Functions

B. REACTOR FREQUENCY RESPONSE MEASUREMENT AT-POWER

Figure 3 shows the measured reactor frequency response amplitudes in decibels. Since the differential worth of the control rod was known to shift slightly with power level, it was necessary to shift the amplitude curve of the at-power measurement to permit the curves to match smoothly into the zero-power curve.

The indicated variation in differential shim-rod worth, for the peak-to-peak oscillation travel of 0.256 inches is shown in Figure 5 for shim rod Number 1 withdrawn 54 inches out. The observed difference is thought to be the result of flux tilting due to xenon buildup and temperature effects on neutron parameters.

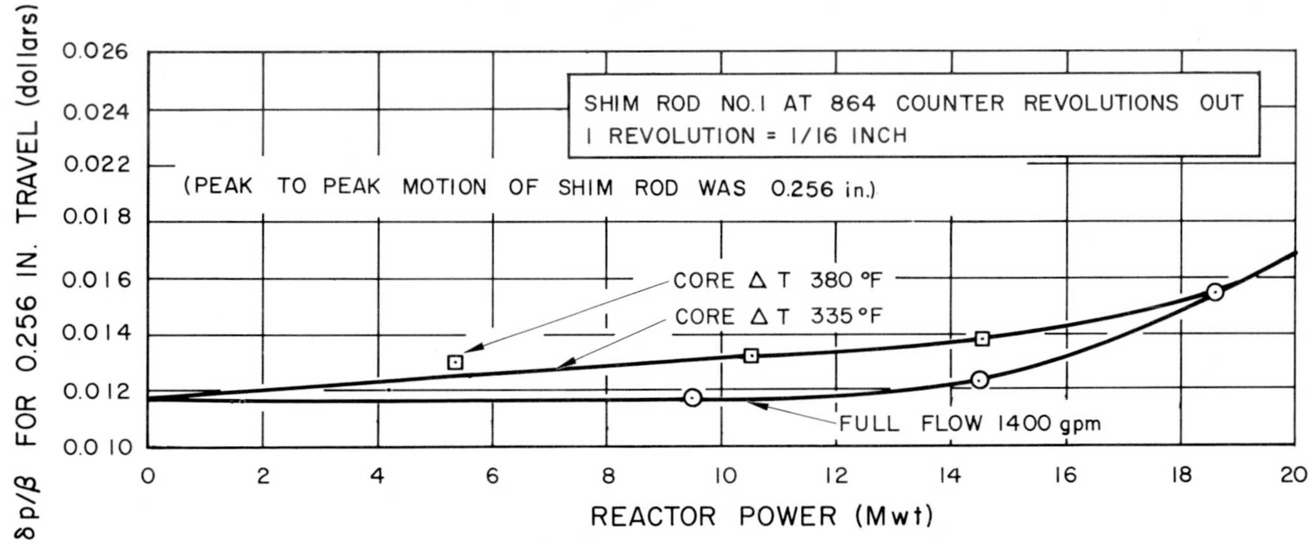

Figure 5. Variation of Differential Shim Rod Worth of the SRE with Respect to Power

The phase curves in Figure 4 had to be elevated 17° to permit smooth transition of the at-power phase curves into the zero-power phase curve. This was not considered unreasonable since 17° was of the order of magnitude of the torsional strain noted on the gearbox position indicator while conducting the experiment. Accuracies of the phase measurements for frequencies of 0.001 to 0.0005 cps are somewhat in doubt since the correlation method of measurement[2] could not be employed, and direct visual analysis of the data was required. The latter method may be in error as much as 25° as compared to a maximum of 3° error when correlation is used.

Before proceeding, there are several observations which should be made in order to note the amount of qualitative information obtainable from Figures 3 and 4. Calculations of isothermal temperature coefficients of reactivity have indicated that α_m and α_c are positive under normal operating conditions. Since $\delta \bar{T}_m/\delta n\ (j\omega)$ and $\delta \bar{T}_c/\delta n\ (j\omega)$ are always positive, these isothermal temperature coefficients of reactivity produce a positive feedback effect which would tend to produce an unstable condition. Likewise, α_f, which is dominated by the Doppler effect, is known to cause a negative feedback which contributes to a stabilizing effect on the reactor. For both a negative and positive power coefficient, the gain will approach a constant level of $1/H(j\omega)$ as ω approaches zero. The phase, however, will approach 0° for a negative power coefficient and -180° for a positive coefficient. Using this information to analyze Figures 3 and 4, it is seen that all amplitudes are, as expected and in confirmation of theory, approaching a constant level in the low frequency region. Similarly, the phase curves indicate that very little positive feedback effect has occurred for any but the 18.5 Mwt run. Even here, long before the positive feedback effect would become noticeable, the curves would project into a region where periods are of the order of hours.

C. POWER COEFFICIENT COMPONENT MEASUREMENT

In order to fully understand the mechanism by which the reactor responds to the stimulus of a change in reactivity, it is not only necessary to know the overall reactor frequency response, it is also necessary to know the responses of the individual components as set forth in Equation 3. This equation is based upon the concept of the lumped-parameter model for the frequency dependent components, and the unrealistic concept of the isothermal reactor for the temperature coefficients.

Derivation of the lumped-parameter model transfer functions for the fuel, coolant and moderator are described in detail in Appendix B. The resultant equations for each are given by equations B-15, B-17, and B-28 respectively. These equations indicate that all three equations may be reduced to a single time-constant system — with an appropriate gain — for simple analysis of the experimentally determined frequency response curves.

Of the three frequency dependent parameters, only one was measured to a satisfactory degree of accuracy. This one was the fuel term $n_o \delta \overline{T}_f / \delta n \, (j\omega)$ which is shown in Figure 6. Solution of Equation B-17 indicated that the coolant

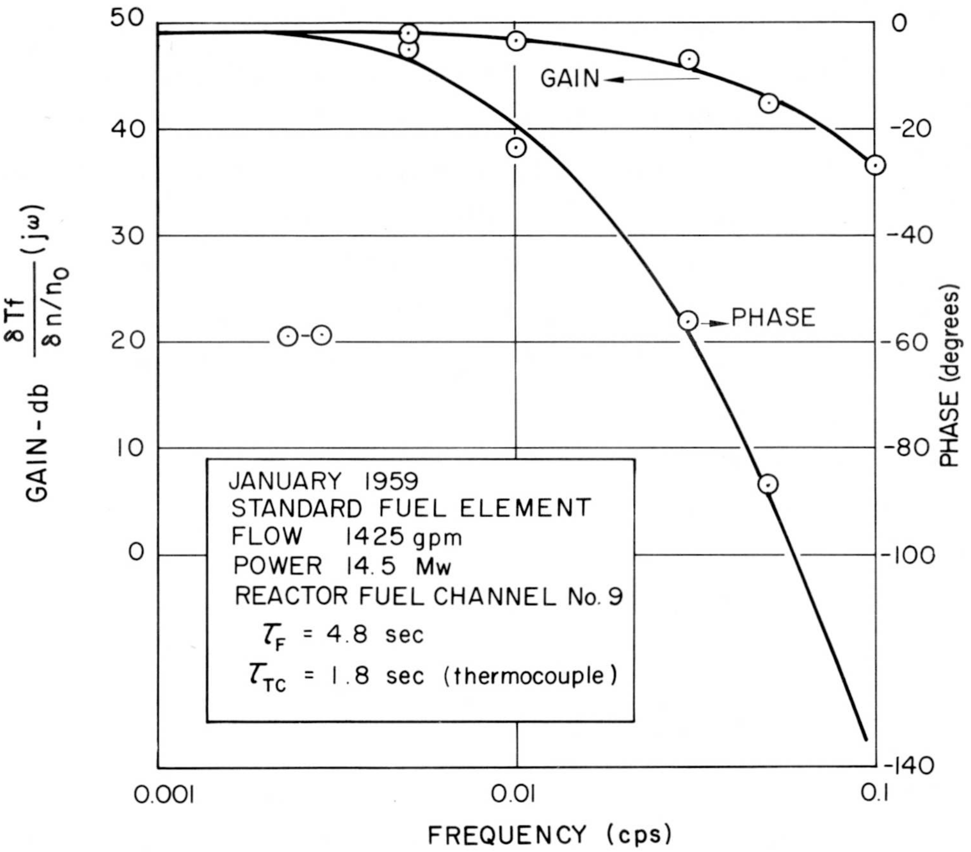

Figure 6. Measured SRE Fuel Transfer Function Including Thermocouple Time-Constant

term $n_o \delta \overline{T}_c / \delta n \, (j\omega)$ is so closely coupled to the fuel as to have essentially the same dominant time-constant with a slight reduction of gain. Since the gain reduction is easily determined, it was deemed unnecessary to measure the complete coolant frequency response. The determination of a few experimental points substantiated this conclusion. Accurate determination of the moderator term $n_o \delta T_m / \delta n (j\omega)$ by direct frequency response measurement was not possible because the break-frequency occurred in too low a region to be measured. Special analytical methods which are explained in Section III were employed to calculate the moderator effects.

Direct measurement of the isothermal temperature coefficients is not possible by reactor oscillation. If all of the oscillation data were of sufficient accuracy, the three frequency dependent terms could be eliminated by obtaining their value through direct oscillation measurement, and the isothermal temperature coefficients determined through solution of simultaneous equations. Since the frequency responses were not all clearly defined by oscillation, the employment of several other experimental and analytical techniques was required to complete the separation of parameters. Since these were quite involved, they are explained in detail in Appendix D.

III. INTERPRETATION OF MEASURED CURVES

A. THE POWER COEFFICIENT

Determination of the normalized power-coefficient $\frac{n_o}{\beta} H(j\omega)$ is accomplished through use of the plots of Figures 3 and 4 as indicated for the solution of Equation 4. The inverse of the complex at-power reactor frequency response is subtracted from the inverse of the complex zero-power reactor frequency response for specific frequencies and power conditions and the resultant values plotted as the smoothed curves of Figures 7 and 8. These measured power coefficients are actual space dependent responses and are not to be confused with the approximate equation for the lumped-parameter model of equation 3. Nevertheless, a comparison between this response and that for the lumped-parameter model is very helpful in the work of separating parameters.

The amplitudes of the curves in these figures are plotted with respect to negative feedback, i.e., $- H(j\omega)$. If only negative feedback were present, the amplitude would approach a constant as the frequency approached zero. If positive feedback was dominant at very low frequencies the amplitude would approach a new and smaller value which would be determined by the difference between the negative and positive feedback gains, while the phase would approach plus 180° as the frequency approached zero. This effect can be seen in the 18.5 Mwt and the 14.5 Mwt full flow curves of Figure 7.

As stated previously, the only negative quantity in the lumped-parameter power-coefficient equations was the isothermal fuel-temperature coefficient of reactivity. It also was noted that the frequency responses of the fuel and coolant were closely coupled and responded to a considerably higher frequency than the moderator. It therefore can be concluded that the positive effect noted can be identified with the moderator-dependent power-coefficient term.

By trial and error fitting of the simplified forms of the derived Equations B-19 and B-28 to the curves, it was observed that the amplitude curves could be fitted to the lumped-parameter model equations, but that the phase shift was too great. This observation, however, is in complete accord with calculated comparisons between lumped and distributed parameter models. From these manipulations the dominant time-constant for the fuel and sodium was determined to

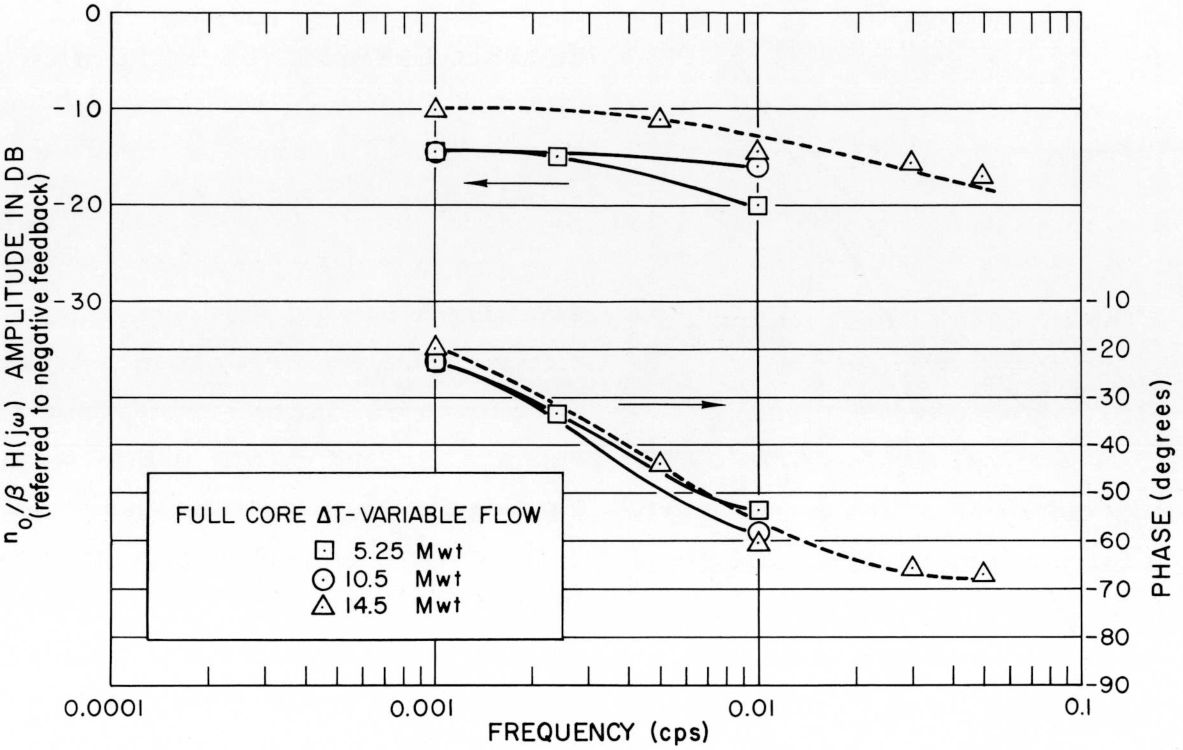

Figure 7. Normalized Measured SRE Power Coefficient for Constant Flow Conditions

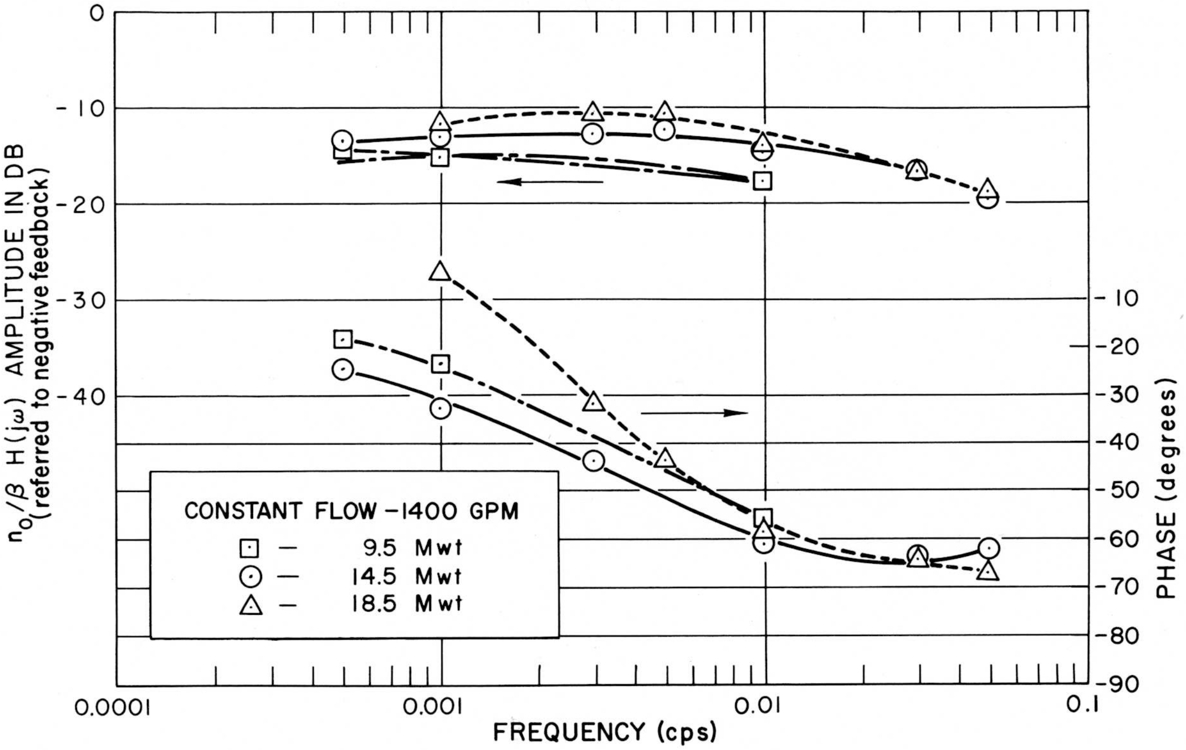

Figure 8. Normalized Measured SRE Power Coefficient for Variable Flow Conditions

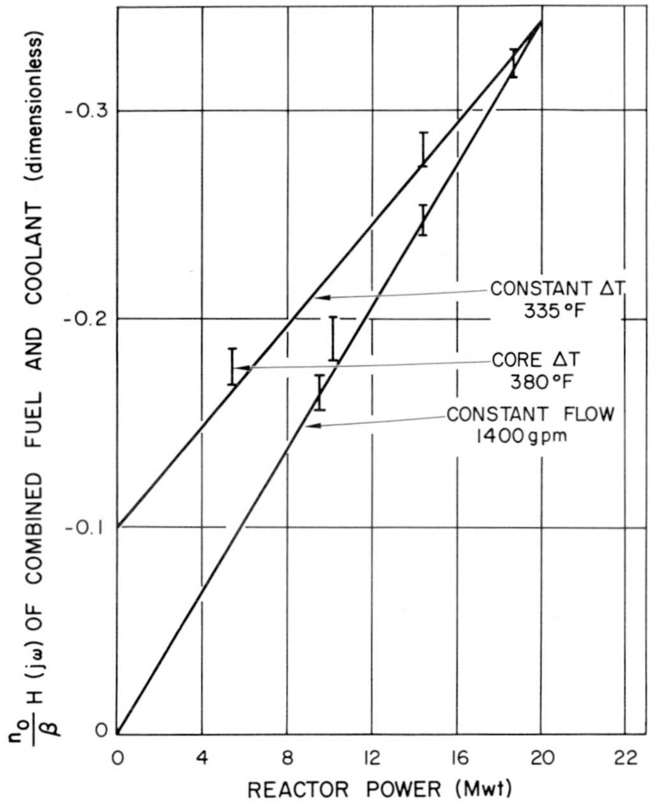

Figure 9. Normalized Power Coefficient Due to SRE Fuel and Coolant

be about 9 seconds for the full flow condition of Figure 7, and to vary from 9 to possibly 28 seconds at the lower flow rates shown in Figure 8.

The indicated dominant time-constant for the moderator, as obtained by similar trial and error manipulation on the curves of Figure 7, was about 600 seconds. Figure 9 was obtained by plotting the zero frequency values of gain, which were obtained during curve fittings of the combined fuel and coolant lumped-parameter power coefficient terms. This figure is described as the contribution of the fuel and coolant terms to the steady-state power coefficient. H(jo) is called the steady-state power coefficient. Now, since this portion of the power-coefficient contributes to negative feedback, and since its maximum time-constant is about 28 seconds, as compared to the minimum time-constant of 600 seconds for the positive feedback contribution of the graphite, the reactor, for all conceivable operating conditions, may be considered quite stable as long as the physical parameters are not changed too drastically.

Figures 7 and 8 do not indicate much of the graphite response. For determining the contribution of graphite to the total power-coefficient, or, in fact, the determination of the total coefficient itself, other information in addition to that obtained by oscillation, and the use of Equation 3 will have to be employed.

B. DETERMINATION OF THE POWER-COEFFICIENT COMPONENT PARAMETERS

In Figure 6, page 10, a plot of the fuel temperature response for a specific reactor condition was shown. If Equation B-15 is solved for this same condition and the best known constants used, the zero frequency gain would be 187 °F and the time constants would be 4.7 and 0.35 seconds. The 0.35-second term would be approximately cancelled by a 0.42-second term in the numerator. The remaining 4.7-second time-constant corresponds very well with the measured time constant of 4.8 seconds of Figure 6. This, however, is a measured one-lumped time constant for actual point of measurement only, and is not the same as that for the distributed reactor since that time constant is affected by the coupled effects of the various distributed parameters and is thus somewhat longer. The distributed-system time constant for the same condition was indicated as about 9 seconds from Figure 7. When, from Figure 6, the observed thermocouple-position node-gain of about 270 °F is transposed to the average gain of the reactor, the resultant gain is 182 °F, which compares very well with the calculated gain of 187 °F.

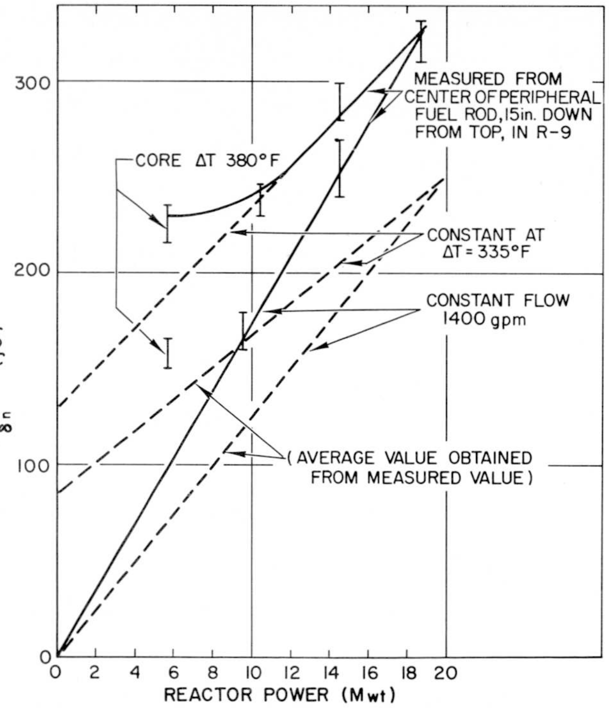

Figure 10. Measured Zero Frequency Fuel Response of the SRE

The zero-frequency gain for the above described run—and for all of the other runs—is plotted as the two upper curves in Figure 10. The approximate degree of accuracy is shown. Since these measurements were made at a specific point in the reactor, and, since Equation 3 requires the volumetric average value of the temperature change, these curves had to be corrected by comparing the temperature distribution in the fuel, as distributed throughout the reactor, with that at the point of measurement. These averages are shown as the two dotted curves in Figure 10, and are used in Equation 3 for the steady-state reactor parameter analysis.

The corresponding average change in the coolant temperature was determined as shown in Appendix C and illustrated in Figure C-1 for use as the gain ratio between fuel and coolant.

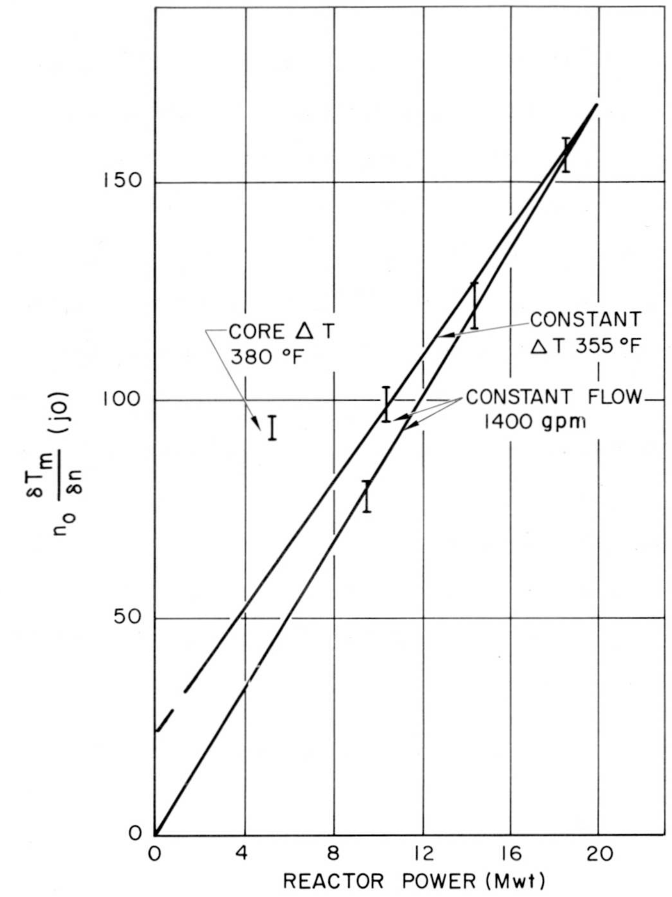

Figure 11. Measured Zero Frequency Moderator Response of the SRE

Determination of the curves for the moderator temperature change, plotted in Figure 11, was not so simple because of the inability to make measurements at frequencies as low as the moderator break frequencies. The break point for the graphite-moderator temperature response occurs at such a low frequency that the visual method of analysis had to be employed for all of the measured points. Also reactor power drift became such a problem at very low frequencies that it was rarely possible to take accurate measurements at less than 0.001 cps.

In Section III-A, it was shown that for two of the runs it was possible to obtain a close approximation of the time-constant, and to separate the combined gain of the two moderator terms from those of the fuel and coolant. With this information, and the additional separate measurements of the isothermal temperature-coefficient described in Appendix D, it was possible to approximate the break frequencies and gains for the frequency-dependent function. The curves were extrapolated through the measured points with the assistance of repeated trial and error solutions of Equation B-28. The trial and error procedure consisted of varying uncertain parameters (primarily thermal resistance) in the equation until the resultant amplitude formed the best approximation to the measured values, and the time-constants approached the region of 600 seconds as indicated by the two measured points. This procedure was considered valid, since, in the case of the fuel, the calculated amplitudes had been very close to those measured.

At this point it can be seen by referring to Equation 3 that all of the components are known except the isothermal temperature-coefficients of reactivity. One point has been measured for the moderator isothermal temperature-coefficient, and, since α_m is known to be temperature dependent (whereas α_f and α_c are approximately independent of temperature), a curve was extrapolated as shown in Appendix D. Now, with only α_f and α_c unknown, these two can be determined by finding the quantities which give the best fit of Equation 3 to the six runs as plotted in Figures 7, 8, and 9. This procedure is given in greater detail in Appendix D, and the isothemal temperature coefficients are plotted in Figure D-1.

Finally, the plotted points in Figures 10, 11, C-1, and D-1 are used in Equation 3 to obtain the plotted approximate steady-state power-coefficient of Figure 12.

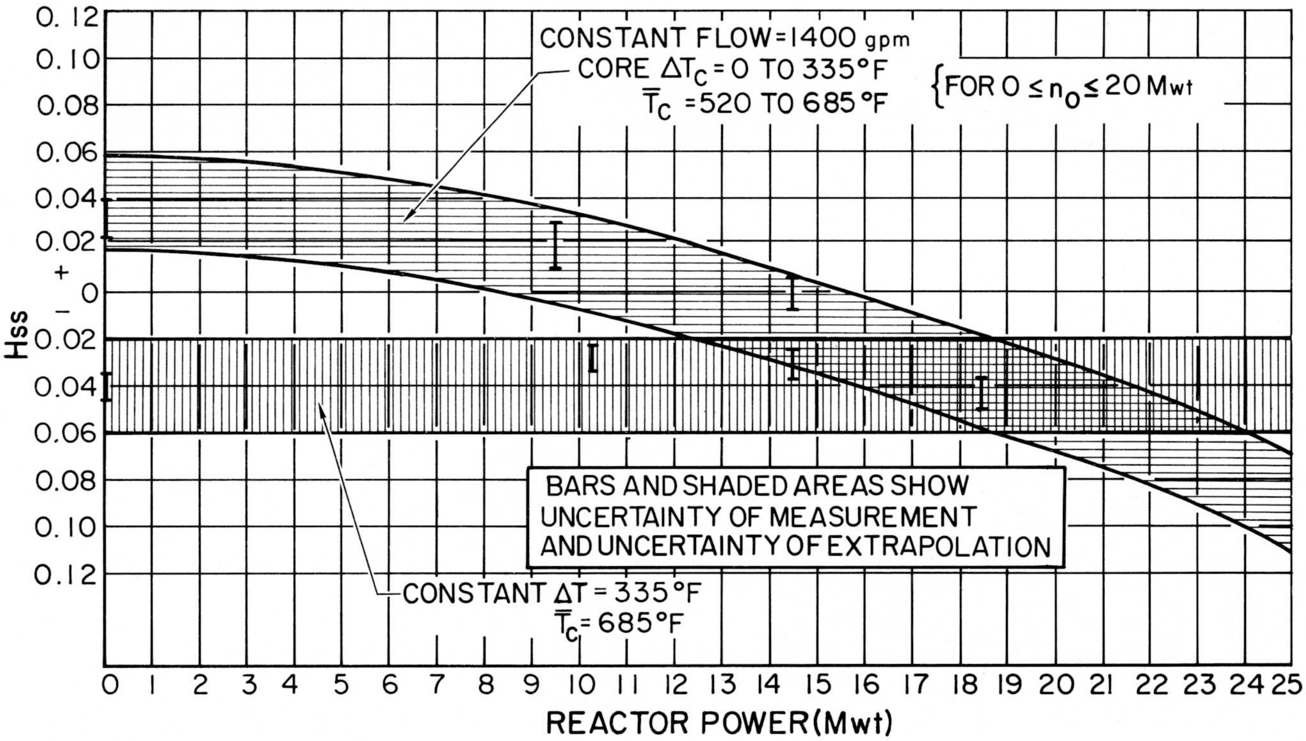

Figure 12. Overall Normalized Power Coefficient

IV. SUMMARY AND CONCLUSIONS

A. SUMMARY

Since many items of information have been derived in various portions of the report, a composite picture is not easily grasped. For this reason the summary will take the form of an itemized grouping of the most important quantities.

1. Steady-State Power Coefficients with Power Expressed in Megawatts ($\Delta\rho = H_{ss}\Delta n$)

   a) The fast-response portion which depends on the response of the fuel and sodium coolant

$$H_{ss} = -1.24 \times 10^{-4} \pm 10\% \text{ per Mwt (for constant flow of 1400 gpm)}$$

and

$$H_{ss} = -\left(\frac{7.3}{n_o} + 0.88\right) \times 10^{-4} \pm 20\% \text{ per Mwt (for constant } \Delta T \text{ of } 335°\text{F)}$$

where

n_o = initial power in Mwt.

   b) Overall-power coefficient (not including xenon effects)

$$H_{ss} = \left(\frac{3}{n_o} -0.37\right) \times 10^{-4} \pm 50\% \text{ per Mwt (for constant flow of 1400 gpm)}$$

$$H_{ss} = \frac{-3.5 \times 10^{-4}}{n_o} \pm 50\% \text{ per Mwt (for constant } \Delta T \text{ of } 335°\text{F)}$$

The first equation is only an approximation of the corresponding curve in Figure 12. The large, 50% uncertanity is a result of the very small difference between the two large component contributions: (1) that due to fuel and sodium, and (2) that due to graphite, with the greater uncertainty lying with the graphite.

2. Isothermal Temperature-Coefficient

a) For fuel

$$\alpha_f = -1.1 \times 10^{-5} \pm 10\%/\,°F$$

b) For coolant

$$\alpha_c = +0.3 \times 10^{-5} \pm 30\%/\,°F$$

c) For moderator

α_m for the moderator varies with average coolant temperature. For values, refer to Figure D-1.

3. Simplified Component Transfer Functions

In general, the transfer functions derived in Appendix A may be reduced to a gain and a single lag term if the dominant time constants and gains are known. The gain factors (K) for the respective responses are obtained from Figures 10, 11, and C-1 (Appendix C) for use in the following general expression:

$$n_o \frac{\delta \overline{T}}{\delta n}(S) = \frac{K}{1 + \tau S}$$

(Where $\overline{T}$, K, and τ are for fuel, moderator or coolant)

a) The distributed-parameter, coolant-coupled, fuel-response time constant at a constant flow of 1400 gpm is 9 ± 2 seconds, and that for a constant ΔT across the core of 335°F is 28 to 9 seconds ± 15% for powers from 5 to 20 Mwt respectively.

b) The moderator at 1400 gpm coolant flow has a dominant time constant of 600 seconds ± 10%, whereas the moderator with a constant core ΔT of 335°F has a dominant time constant of 800 to 600 seconds ± 10% for powers from 5 to 20 Mwt.

c) The coolant has the same coupled response time as the fuel. (See Appendix B) All experimentally determined transient and steady-state equations agreed with the theoretically derived, lumped-parameter equation within 20%, when only the gain and time response was considered. The measured and theoretical phase shifts in the transient equation, as had been predicted by distributed-parameter theory, did not correspond.

4. Overall-Coefficient of Heat Transfer (Btu/hr-ft^2-°F)

a) Moderator to coolant. U = 180 ± 20% for both center and outer channels. Individual center channels may vary as much as 300% due to the wide variation in helium gaps between the process tube and the graphite.

b) Fuel to coolant. U = 1000 ± 10% for the reactor condition of January 1959.

B. CONCLUSIONS

The first conclusion which may be drawn from this report is that the oscillation technique, when used in conjunction with correlation methods of measurement, affords an excellent method of reactor analysis. Specific conclusions from the experiment indicate that:

1) Excellent frequency-response measurements may be obtained even though the oscillation equipment does not generate a perfect sine wave.

2) The amplitude of power deviation is not as critical as has sometimes been reported from theoretical calculations. As much as 6% peak-to-peak deviation in power was used at the low frequencies, for various power levels, without noticeable distortion, and as much as 30% deviation at zero power gave a response which coincided with the theoretical transfer function.

3) The absolute stability of the reactor could be readily determined without dependence upon any theoretical model or any knowledge of system parameters.

4) The specific parameters for the one-lumped model, power-coefficient terms, were separable and could be made available for any desired analytical purpose.

5) From observation of the control rod differential worth changes shown in Figure 5, the oscillation technique could be used for control rod calibrations[4] at power.

The SRE was seen to be very stable as a result of its large, relatively prompt, negative power coefficient. The positive contribution from the graphite is so sluggish in response to changes in reactivity that it takes of the order of three quarters of an hour to reach equilibrium, and thus may be easily controlled. With respect to time response, the control problem compares to that of controlling Xenon buildup and decay, but in actuality is much less critical because the relative magnitude of any net positive coefficient is very much smaller than the xenon effects. Since the positive and negative coefficients are of the same order of magnitude, they tend to almost cancel each other at equilibrium for any power level or mode of operation.

APPENDIX A

NOMENCLATURE

$G(s)_{ZP}$ = zero-power reactor transfer function

$G(s)_P$ = at-power reactor transfer function

$H(s)$ = temperature and power feedback transfer function

$\delta\rho(s)$ = small sinusoidal change in reactivity

$\delta n(s)$ = small sinusoidal change in power about the average power

n_o = average power level in Btu/sec unless noted as megawatts

$\delta\overline{T}_f$ = average sinusoidal change in fuel temperature

$\delta\overline{T}_c$ = average sinusoidal change in coolant temperature

$\delta\overline{T}_m$ = average sinusoidal change in moderator temperature

$\alpha_f = \frac{\delta\rho}{\delta\overline{T}_f}$ = change in reactivity with respect to change in average fuel temperature

$\alpha_c = \frac{\delta\rho}{\delta\overline{T}_c}$

$\alpha_m = \frac{\delta\rho}{\delta\overline{T}_m}$

ℓ = prompt neutron lifetime (sec)

β_i = effective fraction of delayed neutrons in the i^{th} group

β = total effective fraction of delayed neutrons $\beta = \sum_{i=1}^{6} \beta_i$

λ_i = decay constant for the delayed neutrons in the i^{th} group

s = Laplace operator: equals $j\omega$ for frequency response analysis

$j = \sqrt{-1}$

ω = frequency (rad/sec)

APPENDIX B

DERIVATION OF TRANSFER FUNCTIONS FOR CHANGE IN FUEL, COOLANT, AND MODERATOR AVERAGE TEMPERATURES WITH RESPECT TO A CHANGE IN POWER

The time dependent portions of the power coefficient are derived in this section for the linearized and normalized lumped parameter reactor heat transfer model.

The specifically desired transfer functions are

$$\frac{\delta \overline{T}_c}{\delta n}(s) = G_{cn}(s), \quad \frac{\delta \overline{T}_c}{\delta n}(s) = G_{cn}(s) \quad \frac{\delta \overline{T}_m}{\delta n}(s) = G_{mn}(s).$$

$G(s)$ = transfer function (subscripts denote the output and input variables)

$\overline{T}$ = average temperature

Subscripts f = fuel

n = power

c = coolant (sodium)

m = moderator

(mcf) = moderator and fuel channel

(mm) = moderator channel

δ denotes a small change in variable

K = a constant (subscripts denote relation)

τ = time constant (sec)

F = primary sodium flow (lb/sec)

n_o = average power level (Btu/sec)

C_p = heat capacity (Btu/lb - °F)

A = area (ft^2)

U = overall heat transfer coefficient (Btu/hr-ft^2 - °F)

W = mass (pounds)

ρ = reactivity

P = fractional power generation

α = isothermal temperature coefficient of reactivity = $\frac{\delta\rho}{\delta\overline{T}}$

H(s) = power coefficient, feedback transfer function

The basic heat balance equations (after linearizing for small signal changes about a constant level and converting to Laplace notation) are

$$(WC_p)_f\, s\delta\overline{T}_f(s) = P_f\,\delta n_o(s) - (UA)_f\left[\delta\overline{T}_f(s) - \delta\overline{T}_{cf}(s)\right] , \qquad \ldots(B\text{-}1)$$

where

$$\delta\overline{T}_f = \frac{\Sigma\delta\overline{T}_f(V)\Delta V}{\Sigma\Delta V} \qquad (V = \text{volume}) .$$

$$(WC_p)_{cf}\, s\,\delta\overline{T}_{cf}(s) = (UA)_f\left[\delta\overline{T}_f(s) - \delta\overline{T}_{cf}(s)\right] - (FC_p)_c\Big[\delta\overline{T}_{out}(s) - \delta\overline{T}_{in}\Big] + (UA)_{mcf}\left[\delta\overline{T}_{mf}(s) - \delta\overline{T}_{cf}(s)\right] . \qquad \ldots(B\text{-}2)$$

$$\left[1+\left(\frac{WC_p}{AU}\right)_m s\right]\delta\overline{T}_m(s) = \frac{P_m\,\delta n_o(s)}{(AU)_m} + \frac{(AU)_{mm}}{(AU)_m}\delta\overline{T}_{cm} + \frac{(AU)_{mcf}}{(AU)_m}\delta\overline{T}_{cf} . \qquad \ldots(B\text{-}3)$$

$$(WC_p)_{cm}\, s\delta\overline{T}_{cm}(s) = (AU)_{mm}\delta\overline{T}_m(s) - (AU)_{mm}\delta\overline{T}_{cm} - 2(FC_p)_{cm}\delta\overline{T}_{cm}(s) . \qquad \ldots(B\text{-}4)$$

The last term in B-4 was obtained by using the first of two assumptions that:

(1) $\overline{T}_{in}$ is held constant and $\overline{T}_c = \frac{\overline{T}_{out} + \overline{T}_{in}}{2}$

then

$$2\delta\overline{T}_c(s) = \delta\overline{T}_{out}(s) + 0; \qquad \ldots(B\text{-}5)$$

and

(2) $\delta \overline{T}_m(s) = 0$ with respect to $\delta \overline{T}_{cf}(s)$ for the frequency region of interest in this derivation.

Clear Equation B-1 to obtain

$$\left(\frac{WC_p}{UA}\right)_f s\, \delta \overline{T}_f(s) = \left(\frac{P}{UA}\right)_f \delta n_o(s) - \delta \overline{T}_f(s) + \delta \overline{T}_{cf}(s) \qquad \text{. . . (B-6)}$$

$$\left(\frac{WC_p}{AU}\right)_f = \tau_f \qquad \text{. . . (B-7)}$$

$$\left(\frac{P}{UA}\right)_f = K_f \qquad \text{. . . (B-8)}$$

$$(\tau_f s + 1)\, \delta \overline{T}_f(s) = K_f \delta n_o + \delta \overline{T}_{cf}(s) \ . \qquad \text{. . . (B-9)}$$

Clear Equation B-2 using the two assumptions given in B-5: then Equation B-2 becomes

$$\left[(W_c C_p)_c s + (UA)_f + 2(FC_p)_c + UA_{mcf}\right] \delta \overline{T}_{cf}(s) = (UA)_f \delta \overline{T}_f(s) \qquad \text{. . . (B-10)}$$

$$\delta \overline{T}_{cf}(s) = \frac{K_{cf}}{(1 + \tau_{mcf}\, s)} \delta \overline{T}_f(s) \qquad \text{. . . (B-11)}$$

$$K_{cf} = \frac{(UA)_f}{(UA)_f + 2(FC_p)_c + (UA)_{mcf}} \qquad \text{. . . (B-12)}$$

$$\tau_{mcf} = \frac{(WC_p)_{cf}}{(UA)_f + 2(FC_p)_c + (UA)_{mcf}} \ . \qquad \text{. . . (B-13)}$$

$\dfrac{\delta \overline{T}_f(s)}{\delta n(s)}$ is used to determine the feedback effects for a change in reactivity,

where

$$\frac{\delta \rho_f(s)}{\delta n(s)} = \alpha_f \frac{\delta T_f(s)}{\delta n(s)} \quad . \qquad \ldots (B\text{-}14)$$

The contribution of the change in the average temperature to a change in reactivity must be flux weighted[3] in the lumped model by a proportionality factor γ. At any point in the reactor, the change in reactivity is proportional to the product of the temperature coefficient of reactivity and the square of the local flux multiplied by the change in temperature (Refer to Appendix C).

Substituting Equation B-11 into B-8, and using the weighting factor γ:

$$\frac{\gamma \delta \overline{T}_f(s)}{\delta n(s)} = \frac{\dfrac{\gamma K_f}{1 - K_{cf}} (1 + \tau_{mcf} s)}{\left(\dfrac{\tau_f \tau_{mcf}}{(1 - K_{cf})} s^2 + \dfrac{\tau_f + \tau_{mcf}}{(1 - K_{cf})} s + 1 \right)} \quad . \qquad \ldots (B\text{-}15)$$

The effect of fuel and coolant temperature changes upon reactivity are so closely coupled with respect to their response times as to warrant simultaneous consideration. The relatively prompt reactivity effects of the two may be stated as:

$$H(s) = \alpha_f \gamma_f \frac{\delta \overline{T}_f(s)}{\delta n(s)} + \alpha_c \gamma_c \frac{\delta \overline{T}_{cf}(s)}{\delta n(s)} \quad , \qquad \ldots (B\text{-}16)$$

where γ_f weights the fuel temperature and γ_c weights the coolant temperature. (α_f is a negative quantity and α_c is a positive quantity for the SRE.)

$$\frac{\delta \overline{T}_{cf}(s)}{\delta n(s)} = \frac{\delta \overline{T}_{cf}(s)}{\delta \overline{T}_f(s)} \times \frac{\delta \overline{T}_f(s)}{\delta n(s)} = \frac{\dfrac{K_{cf}K_f}{1 - K_{cf}}}{(1 + \tau_1 s)(1 + \tau_2 s)} \qquad \ldots \text{(B-17)}$$

where

$$\tau_1 + \tau_2 = \frac{-(\tau_f + \tau_{mcf})}{2\,(1 - K_{cf})} \pm \sqrt{\frac{(\tau_f + \tau_{mcf})^2}{4\,(1 - K_{cf})^2} - \frac{\tau_f\,\tau_{mcf}}{(1 - K_{cf})}} \quad . \qquad \ldots \text{(B-18)}$$

The contribution to H(s) from fuel and coolant temperature changes is

$$\frac{\dfrac{\gamma \alpha_f K_f}{(1 - K_{cf})}\left(1 + \dfrac{\gamma_c\,\alpha_c\,K_{cf}}{\alpha_f \gamma}\right)\left[1 + \left(\dfrac{\tau_{mcf}}{1 + \dfrac{\gamma_c\,\alpha_c\,K_{cf}}{\alpha_f \gamma}}\right) s\right]}{(1 + \tau_1 s)(1 + \tau_2 s)} \quad . \qquad \ldots \text{(B-19)}$$

The contribution to H(s) from the moderator is given by

$$\alpha_m \gamma_m \frac{\delta \overline{T}_m(s)}{\delta n\,(s)} \quad . \qquad \ldots \text{(B-20)}$$

From Equation B-2,

$$\delta \overline{T}_{cf} = \frac{\dfrac{(AU)_f}{K_{mcf}}\,\delta \overline{T}_f}{(1 + \tau_{mcf} s)} + \frac{\dfrac{(AU)_{mcf}}{K_{mcf}}\,\delta \overline{T}_m}{(1 + \tau_{mcf}\, s)} \quad , \qquad \ldots \text{(B-21)}$$

where

$$K_{mcf} = (AU)_f + (AU)_{mcf} + 2\,(FC_p)_{cf} \quad . \qquad \ldots \text{(B-22)}$$

Similarly,

$$\delta\bar{T}_{cm} = \frac{K_{cm}\delta\bar{T}_{m}}{(1 + \tau_{cm}s)} \quad , \qquad \ldots \text{(B-23)}$$

where

$$\tau_{cm} = \frac{(WC_p)_{cm}}{(AU)_m + 2(FC_p)_{cm}} \qquad \ldots \text{(B-24)}$$

and

$$K_{cm} = \frac{(AU)_{mm}}{(AU)_{mm} + 2(FC_p)_{cm}} \qquad \ldots \text{(B-25)}$$

By substituting in Equation B-3 and by using the following notations

$$K_m = \frac{P_m}{(AU)_m} \qquad \ldots \text{(B-26)}$$

and

$$\tau_m = \frac{(WC_p)_m}{(AU)_m} \quad , \qquad \ldots \text{(B-27)}$$

the resultant equation becomes

$$\frac{\delta \overline{T}_m(s)}{\delta n(s)} = \frac{K_m(1+\tau_{cm}s)(1+\tau_{mcf}s)}{(\tau_m\tau_{cm}\tau_{mcf})s^3+(\tau_m\tau_{cm}+\tau_m\tau_{mcf}+\tau_{cm}\tau_{mcf})s^2+\tau_m s+\left[1-\frac{(AU)_{mm}K_{cm}}{(AU)_m}-\frac{(AU)^2{}_{mcf}}{(AU)_m K_{mcf}}\right]}$$

$$+\frac{(AU)_f\frac{K_f}{1-K_{cf}}(1+\tau_{cf}s)(1+\tau_{cm}s)}{(\tau_m\tau_{cm}\tau_{mcf})s^3+(\tau_m\tau_{cm}+\tau_m\tau_{mcf}+\tau_{cm}\tau_{mcf})s^2+\tau_m s+\left[1-\frac{(AU)_{mm}K_{cm}}{(AU)_m}-\frac{(AU)^2{}_{mcf}}{(AU)_m K_{mcf}}\right](1+\tau_1 s)(1+\tau_2 s)}$$

... (B-28)

The first part of Equation B-28 is the contribution from the moderator coolant channel, and the second part is the contribution from the fuel coolant channel.

For use in Equation B-20, Equation B-28 was solved and reduced to its simplest dominant form,

$$\frac{\delta\rho(s)}{\delta n(s)} = \frac{\alpha_m\,\gamma_m K}{(1+\tau s)} = H(s)_m \quad , \qquad \text{... (B-29)}$$

where α_m, γ_m, K, and τ are dependent upon the specific reactor power level, coolant flow, and average temperature.

APPENDIX C

TEMPERATURE WEIGHTING FACTORS

A. DERIVATION AND COMPUTATION OF FUEL, COOLANT, AND MODERATOR TEMPERATURE WEIGHTING FACTORS

The reactivity effect of a given localized physical change in a reactor core parameter will be proportional to the square of the flux (one-group theory), or to the product of the flux times the adjoint flux (two-group theory).

When the power of the SRE is oscillated, the resulting temperature changes in the fuel and moderator vary in a manner which is dependent upon their location in the core. The temperature change in the SRE coolant varies only with its axial position in the core, since all channels are orificed for the same temperature rise. Furthermore, in the case of the moderator, the effect of a given temperature change on reactivity also depends on the average temperature of the moderator at that location. For these reasons it is necessary to properly integrate over the active core in order to determine the total reactivity change caused by a given change in power and temperature.

The total change in reactivity due to fuel, coolant, or moderator temperature changes — which vary with core position — can be expressed as follows:

$$\delta\rho = \frac{\int_V \alpha(v)\,\Delta T(v)\,\phi^2(v)\,dv}{\int_V \phi^2(v)\,dv} \qquad \ldots C\text{-}1$$

where

$\delta\rho$ = total reactivity change

$\alpha(v)$ = temperature coefficient of reactivity in particular volume element Δv at core location (v)

$\Delta T(v)$ = differential temperature change at core location (v)

$\phi(v)$ = neutron flux at core location (v).

Although Equation C-1 gives the total change in reactivity, it does not explicitly give the desired weighting factor, γ, nor does it contain the average temperature change ($\Delta\overline{T}$). This is obtained by adjusting the temperature change measured during the oscillation experiment at a specific location in the core. Because Equation C-1 does not contain the isothermal temperature coefficient of reactivity α_{iso}, a more appropriate equation will now be derived. It will be expressed in cylindrical coordinates, and the flux will be separated into radial and axial factors. The equation for the fuel weighting factor will be derived first.

The equation for the reactivity change due to a temperature change in fuel may be expressed as follows:

$$\delta\rho_f = \frac{\int_z \int_r \alpha_f(r,z)\, \Delta T_f(r,z)\, \phi_r^2\, \phi_z^2\, 2\pi r\, dr\, dz}{\int_z \int_r \phi_r^2\, \phi_z^2\, 2\pi r\, dr\, dz} \qquad \ldots\text{C-2}$$

The total change in fuel temperature may be expressed as the sum of two terms. The first term is the change due to flux or power generation in the fuel slug and the second term, the change due to the surrounding coolant temperature change. Thus, the change in fuel temperature at any location in the core may be expressed as

$$\Delta T_f(v,z) = \frac{\overline{\Delta T}_{f\phi}\, \phi(r)\phi(z)}{\bar{\phi}_r\, \bar{\phi}_z} + \Delta T_c(v,z)$$

and

$$\Delta T_f(r,z) = \overline{\Delta T}_{f\phi}\, \frac{\phi(r)\phi(z)}{\bar{\phi}_r\, \bar{\phi}_z} + 2\overline{\Delta T}_c(r,z) \ , \qquad \ldots\text{C-3}$$

where

$$\overline{\Delta T}_c = \frac{\Delta T_c}{2}$$

z = axial length of the core

$\Delta T_f(r,z)$ = total temperature change in fuel as a function of (r,z)

$\overline{\Delta T}_{f\phi}$ = average temperature change in fuel measured at average flux location in core and adjusted for localized flux distribution in fuel cluster

$\alpha_f(r,z)$ = fuel temperature coefficient of reactivity

$\phi(r)$ = radial flux

$\phi(z)$ = axial flux

$\overline{\phi}_r$ = average radial flux

$\overline{\phi}_z$ = average axial flux

dr, dz = radial and axial differential elements

$\Delta T_c(r,z)$ = coolant temperature change as a function of (r,z)

$\overline{\Delta T}_c$ = average coolant temperature change $= \dfrac{\Delta T_{out}}{2}$, where $\Delta T_{in} = 0$

$C(r,z) = C_r C_z$ = normalized coolant-channel temperature distribution. Because of coolant channel orificing $C(r,z)$ is a function only of z and thus, $C_r = 1$.

Substituting the expression C-3 for $\Delta T_f(r,z)$ into Equation C-2, and rearranging, gives the following expression:

$$\delta\rho_f = \frac{\alpha_f \displaystyle\int_r\int_z \left[\frac{\phi(r)\phi(z)}{\overline{\phi_r^2}\;\overline{\phi_z^2}}\,\overline{\Delta T}_{f\phi} + 2\overline{\Delta T}_c\, C_r C_z\right]\phi_r^2\,\phi_z^2\; 2\pi r\, dr\, dz}{\overline{\phi_r^2}\;\overline{\phi_z^2}}$$

where

$$\overline{\phi_r^2}\;\overline{\phi_z^2} = \frac{\displaystyle\int_r \phi_r^2\, 2\pi r\, dr \int_z \phi_z^2\, dz}{\displaystyle\int_r 2\pi r\, dr \int_z dz} \quad . \qquad \ldots \text{C-4}$$

Since α_f, the isothermal temperature coefficient of reactivity, is not a function of temperature or composition, it may be taken outside the integral sign.

The next step involves the replacement of the integrals with summation signs and the use of normalized temperature expressions:

$$\delta\rho_f = \frac{\alpha_f \Delta\overline{T}_f \left[\frac{\Delta\overline{T}_{f\phi}}{\Delta\overline{T}_f} \sum_i \frac{\phi_r}{\overline{\phi}_r} \frac{\phi_r^2\, 2\pi r \Delta r_i}{\Sigma 2\pi r \Delta r_i} \sum_j \frac{\phi_z}{\overline{\phi}_z} \frac{\phi_z^2\, \Delta z_j}{\Sigma \Delta z_j} + 2 \frac{\Delta\overline{T}_c}{\Delta\overline{T}_f} \sum_i C_r \phi_r^2 \frac{2\pi r \Delta r_i}{\Sigma 2\pi r \Delta r_i} \sum_j C_z \phi_z^2 \frac{\Delta z_j}{\Sigma \Delta z_j} \right]}{\overline{\phi_r^2}\;\overline{\phi_r^2}}$$

...C-5

Equation C-5 may be written in a more simple form:

$$\delta\rho_f = \alpha_f \overline{\Delta T}_f \left[\frac{\overline{\Delta T}_{f\phi}}{\overline{\Delta T}_f} A_1 + \frac{\overline{\Delta T}_c}{\overline{\Delta T}_f} B_1 \right] = \alpha_f \overline{\Delta T}_f \gamma_f \; , \qquad \text{...C-6}$$

where the expression in the brackets is the desired weighting function for the fuel.

The total expression for the coolant can be shown to be only the last term in the above equation, or

$$\Delta\rho_c = \alpha_c \cdot \overline{\Delta T}_c B_1 = \alpha_c \overline{\Delta T}_c \gamma_c \; , \qquad \text{...C-7}$$

where $\overline{\Delta T}_f$ has been removed.

The expression for the moderator is similar to that for fuel, except α_m must be placed inside the summation signs since its value depends on the temperature of the volume element under consideration. In this case, α_m can be expressed as $\alpha_{mr}\alpha_{mz}(z)$, where $\alpha_{mr} = 1$ and $\alpha_{mz}(z)$ is determined by the value of α_m at the temperature which exists at an axial position Z.

The complete expression for $\delta\rho_m$ is as follows:

$$\delta\rho_m = \frac{\bar{\alpha}_m \Delta\overline{T}_m \left[\frac{\Delta T_{m\phi}}{\Delta\overline{T}_m} \sum_i \frac{\phi_r}{\bar{\bar{\phi}}_r} \phi_r^2 \alpha_{mr} \frac{2\pi r \Delta r_i}{\Sigma 2\pi r \Delta r_i} \sum_j \frac{\phi_z}{\bar{\bar{\phi}}_z} \phi_z^2 \frac{\alpha_{mz}}{\bar{\alpha}_m} \frac{\Delta z_j}{\sum_j \Delta z_j} + 2 \frac{\Delta\overline{T}_{mc}}{\Delta\overline{T}_m} \sum_i C_r \alpha_{mr} \phi_r^2 \frac{2\pi r \Delta r_i}{\Sigma 2\pi r \Delta r_i} \sum_j C_z \frac{\alpha_{mz}}{\bar{\alpha}_m} \phi_z^2 \frac{\Delta z_j}{\sum_j \Delta z_j} \right]}{\overline{\phi_r^2}\,\overline{\phi_z^2}}$$

. . . C-8

or

$$\delta\rho_m = \alpha_m \overline{\Delta T}_m \left[\frac{\overline{\Delta T}_{m\phi}}{\overline{\Delta T}_m} A_2 + \frac{\overline{\Delta T}_{mc}}{\overline{\Delta T}_m} B_2 \right] = \alpha_m \overline{\Delta T}_m \gamma_m \qquad \ldots \text{C-9}$$

Although the expressions appear formidable, the calculations of γ_f, γ_c, and γ_m — the desired weighting functions — may be performed by hand in a few hours. To obtain the values of A_1, B_1, A_2, and B_2, the shape of the radial and axial fluxes must be known. Their actual magnitudes are unimportant since it is only necessary to normalize their values to unity at the center of the core.

Figures C-1 and C-2 are plots of the axial and radial fluxes in the SRE core. Figures C-3 and C-4 give the values for $C_z(z)$ and $\alpha_{mz}(z)$ as needed to calculate the weighting functions.

B. CALCULATION AND MEASUREMENT OF AVERAGE TEMPERATURES

To complete the calculation of the weighting functions for fuel, coolant, and moderator, it is necessary to obtain the proper values for the following temperature terms:

$$\frac{\overline{\Delta T}_{f\phi}}{\overline{\Delta T}_f}, \quad \frac{\overline{\Delta T}_c}{\overline{\Delta T}_f}, \quad \frac{\overline{\Delta T}_{m\phi}}{\overline{\Delta T}_m}, \quad \text{and} \quad \frac{\overline{\Delta T}_c}{\overline{\Delta T}_m} .$$

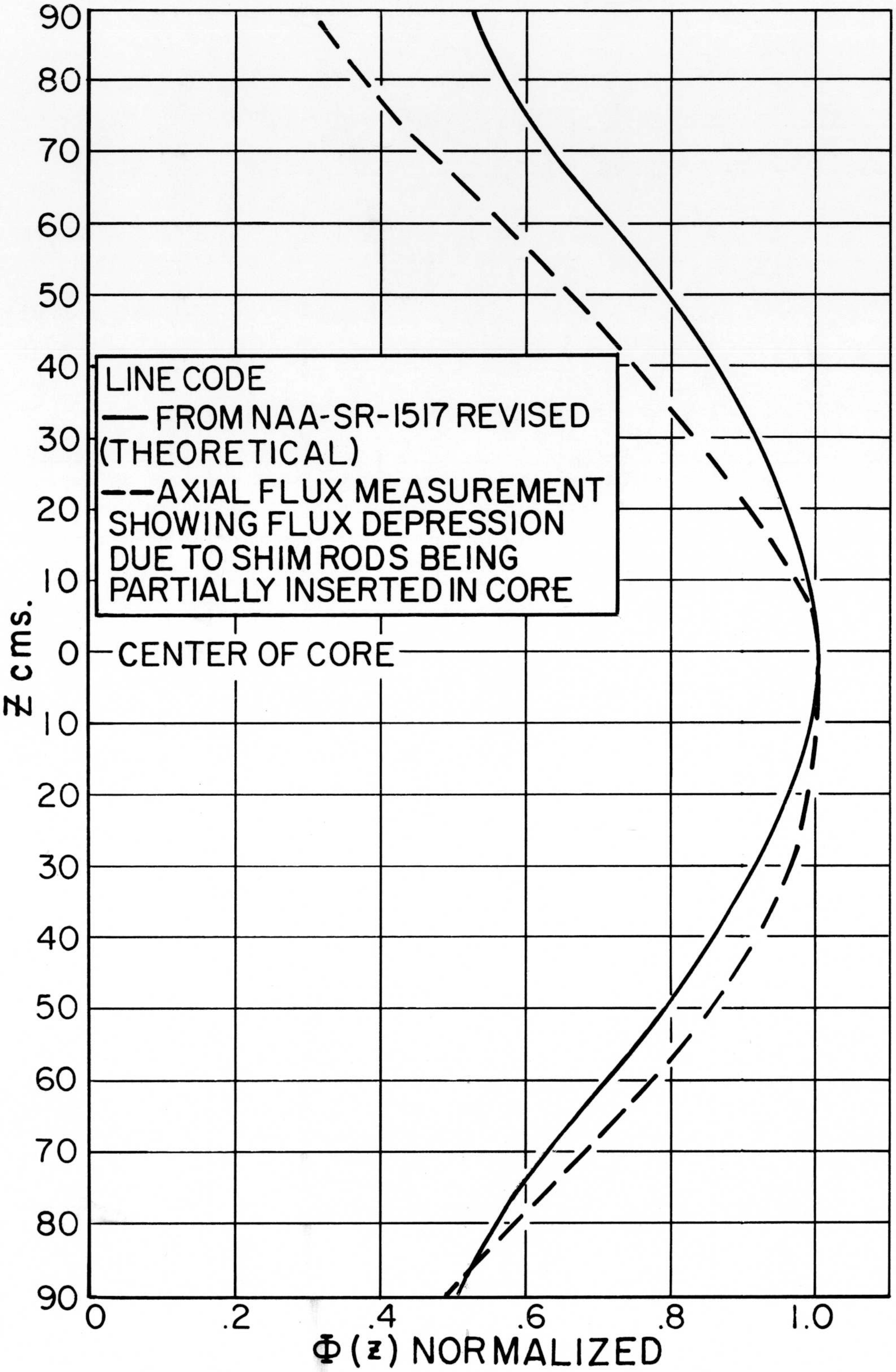

Figure C-1. Normalized Axial Flux Plot for Various SRE Conditions

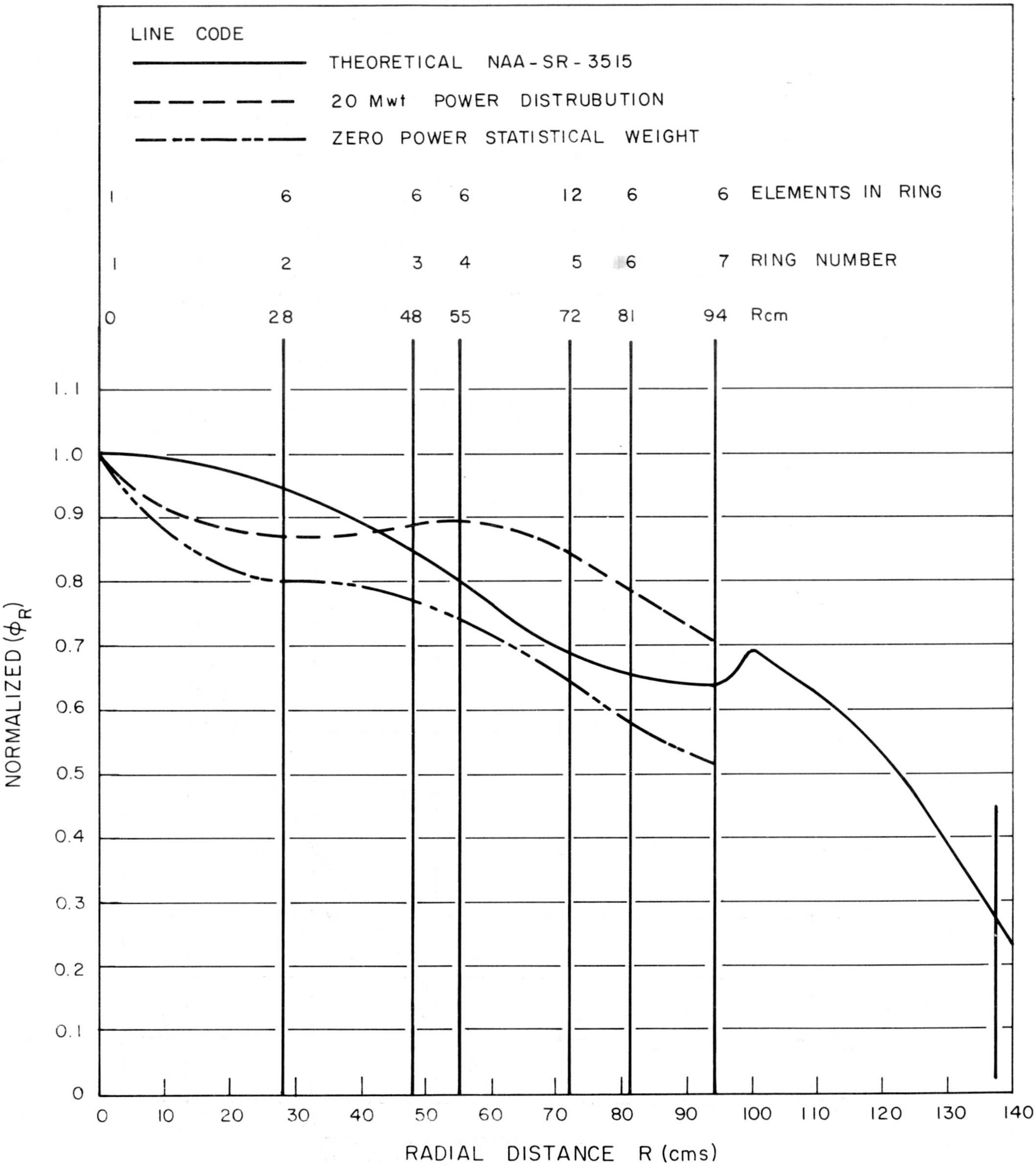

Figure C-2. Normalized Radial Flux Plot for Various SRE Conditions

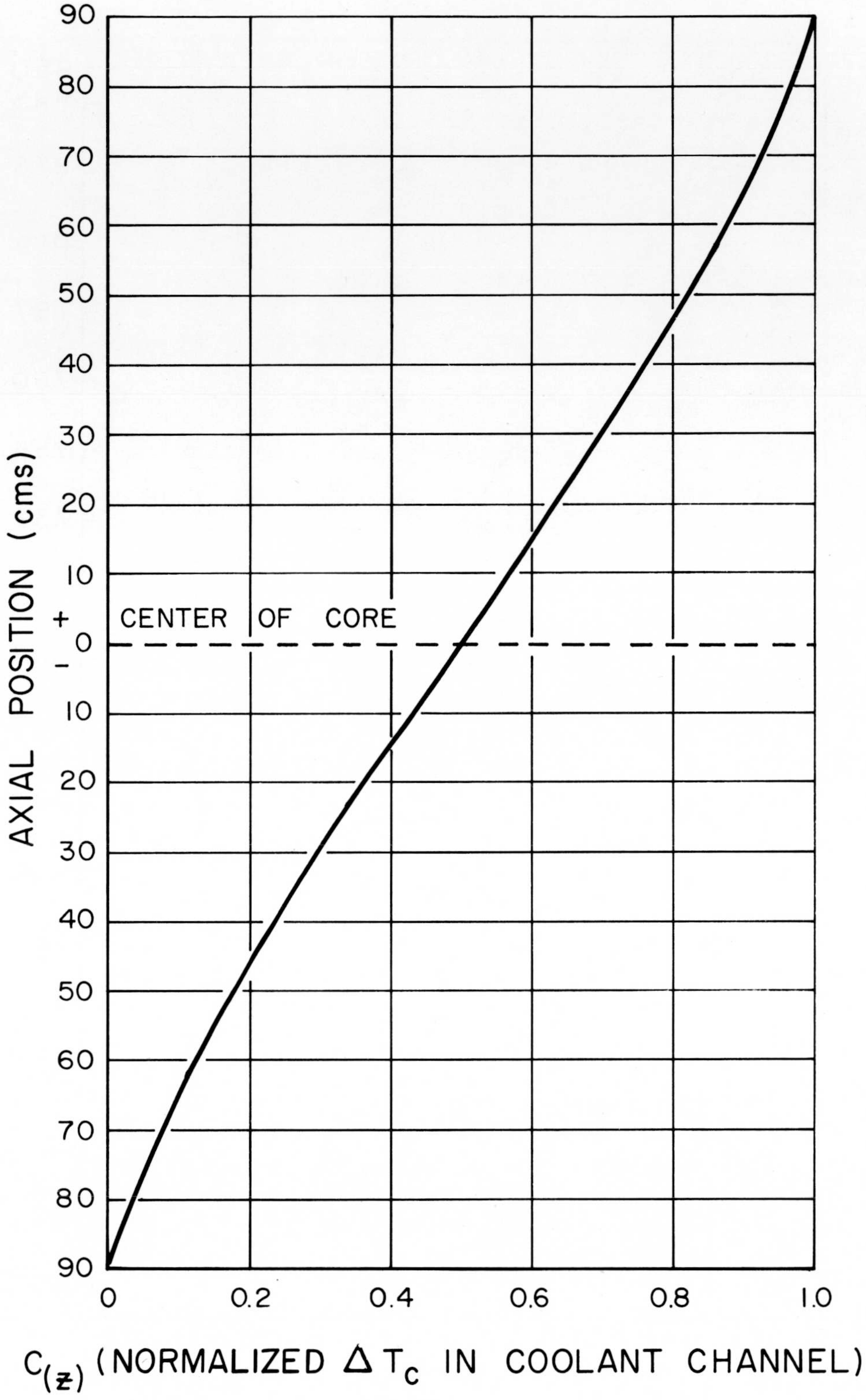

Figure C-3. $C_{(Z)}$, Normalized Temperature Rise in Coolant Channel as Function of Axial Position

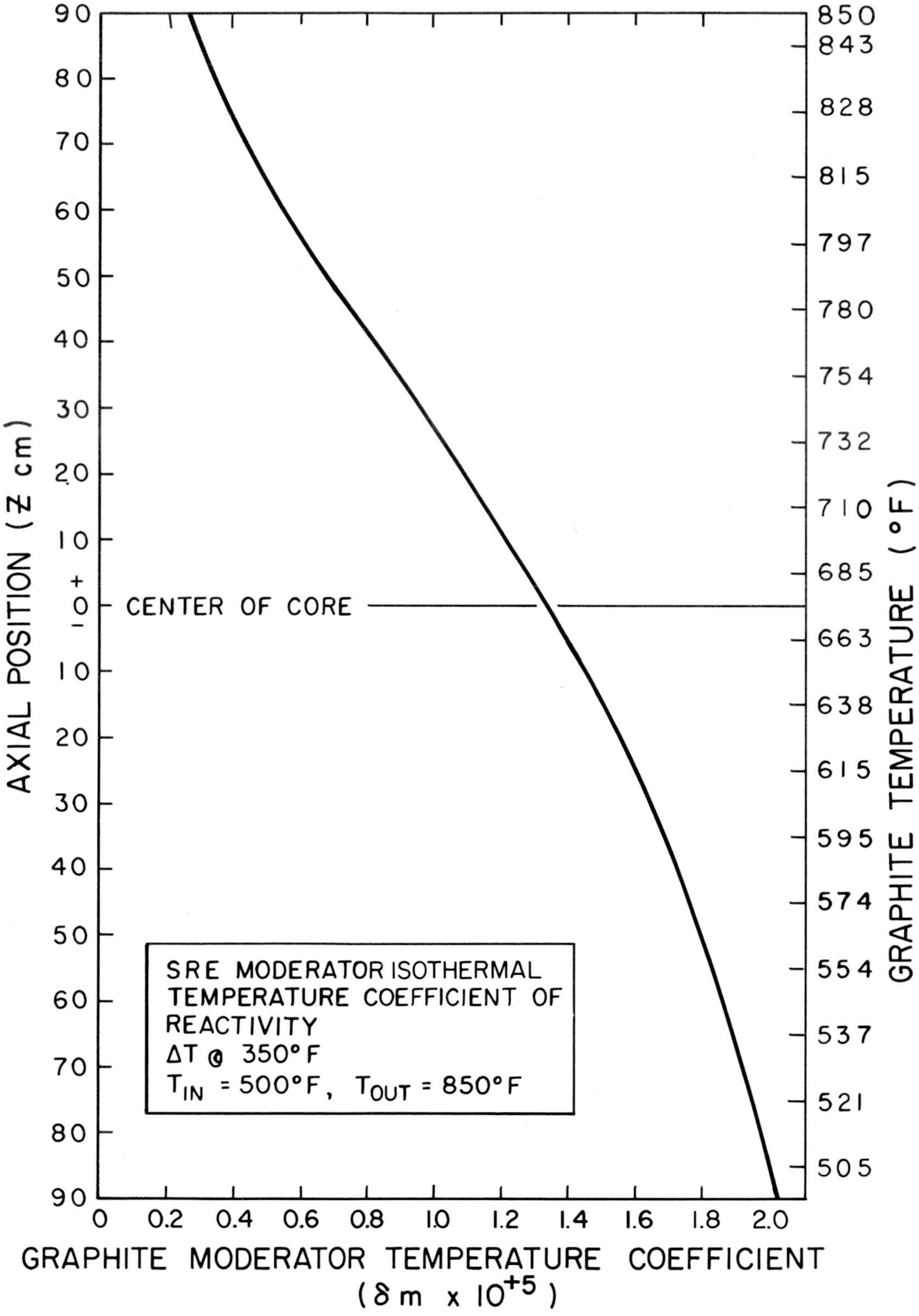

Figure C-4. SRE Moderator Isothermal Temperature Coefficient of Reactivity

The value for $\overline{\Delta T}_{f\phi}$ may be obtained from the equation

$$\text{Power (Btu)} = \overline{U} A (\overline{\Delta T}_{f\phi}) \ ,$$

where

A = total fuel surface area,

and

$\overline{U}$ = average overall heat transfer coefficient; this takes into account: (1) heat transfer coefficients, (2) thickness and conductivity of stainless steel cladding, (3) NaK bond, and (4) appropriate constants for the uranium fuel slug that take into account flux distribution in individual fuel rods.

Power generated in the fuel is 0.91 of total reactor power. Nine percent of the SRE power is considered to be generated in the moderator because of neutron thermalization and gamma absorption.

$$\overline{\Delta T}_c \text{ may be obtained as } \overline{\Delta T}_c = \frac{T_{out} + T_{in}}{2} = \frac{\Delta T_{out}}{2} \ ,$$

where ΔT_{out} is the total change in coolant temperature. It is calculated as follows:

$$P_{Btu} = CF\Delta T_c \ ,$$

where

C = specific heat of sodium coolant,
F = flow rate of coolant,
ΔT_c = total temperature rise of coolant.

Thus we obtain for the sum of the two terms

$$\overline{\Delta T}_f = \overline{\Delta T}_{f\phi} + \overline{\Delta T}_c \quad .$$

It should be noted that $\overline{\Delta T}_f$ is the temperature change measured at a location in the core where $\phi_r \phi_z = \overline{\phi_r \phi_z}$ and where ϕ_r and ϕ_z are taken from the flux plots shown in Figures C-1 and C-2.

Since the thermocouple used in the experiments was at the center of the slug, it was also necessary to adjust the measured temperature change. This allowed for local flux distribution in the fuel element cluster so that it would match the value expected at the point of average local flux in the fuel element cluster. It is also possible to plot normalized curves $\overline{\Delta T}_{f\phi}/\overline{\Delta T}_f$ and $\overline{\Delta T}_c/\overline{\Delta T}_f$ and these are shown in Figures C-5 and C-6.

Figure C-7 gives the calculated values for A and B as obtained from the use of the different axial and radial flux distributions.

The weighting factor for fuel, γ_f, from equation C-6 is calculated as follows:

$$\gamma_f = \frac{\overline{\Delta T}_{f\phi}}{\overline{\Delta T}_f} A_1 + \frac{\overline{\Delta T}_c}{\overline{\Delta T}_f} B_1 \quad .$$

Substitution for the above quantities, using values from Figures C-5, C-6, and C-7, gives values of approximately unity for γ_f for both the 1400 gpm and the 320°ΔT case.

From similar calculations, γ_c is approximately 1.0; γ_m is approximately 0.82, where

$$\frac{\overline{\Delta T}_{m\phi}}{\overline{\Delta T}_m} = 0.09 \quad ,$$

and

$$\frac{\overline{\Delta T}_c}{\overline{\Delta T}_m} = 0.91 \quad ;$$

$$A_2 = 1.25 \text{ and } B_2 = 0.77.$$

Calculated values for A_2 and B_2 are shown in Figure C-8.

Although the values for γ_f, γ_c, and γ_m are dependent on the flux distribution used, the above values are sufficiently accurate for use in calculating the temperature coefficients of reactivity.

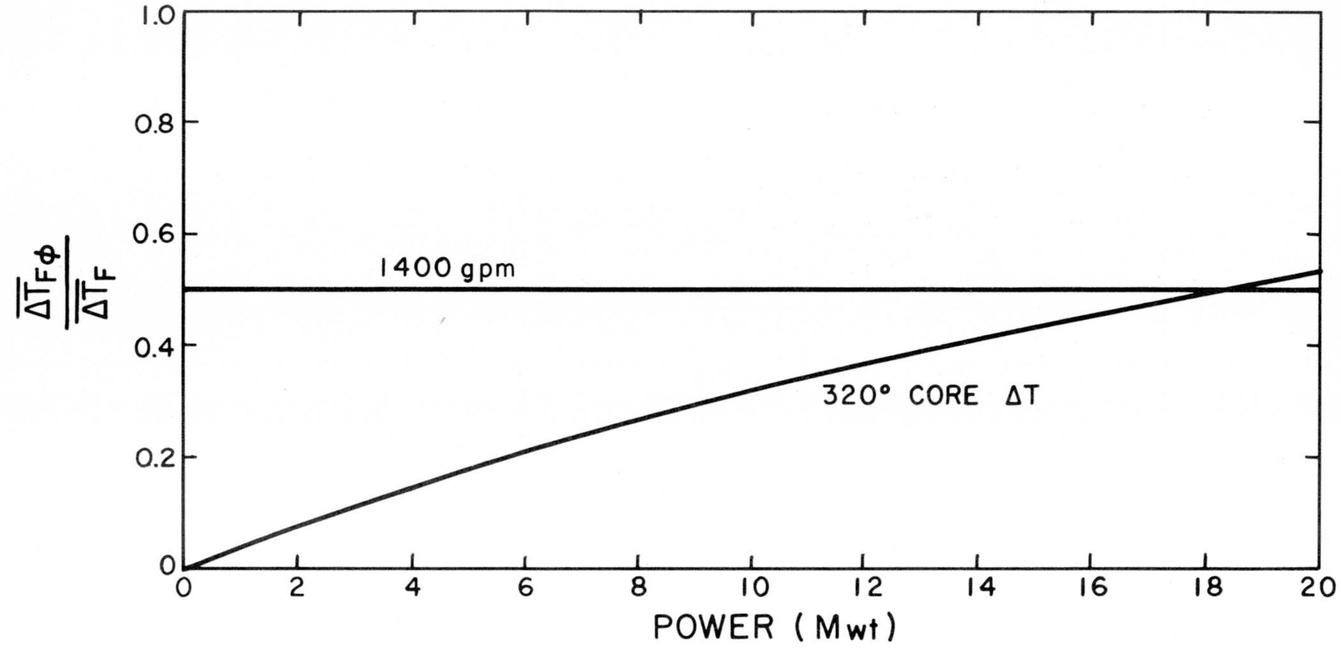

Figure C-5. Calculated Ratio of $\overline{\Delta T}_{f\phi}$ to $\overline{\Delta T}_f$ as a Function of Reactor Power Level

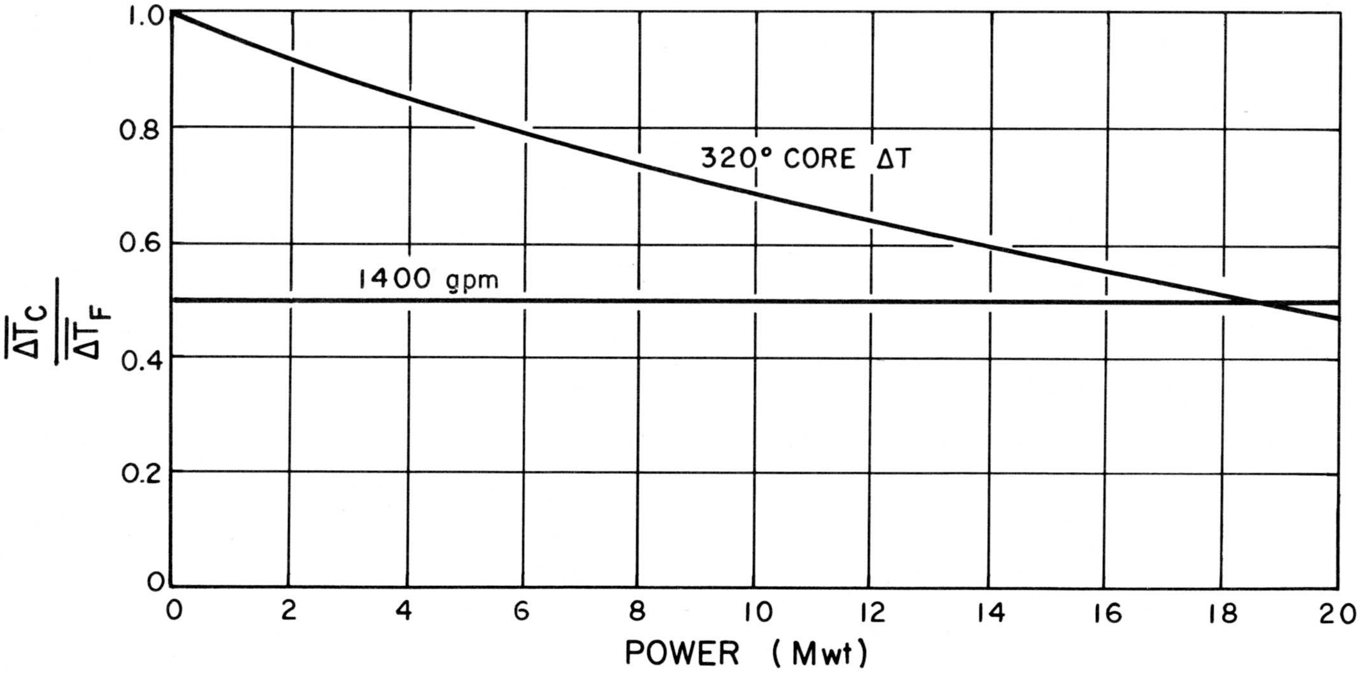

Figure C-6. Calculated Ratio of $\overline{\Delta T}_c$ to $\overline{\Delta T}_f$ as a Function of Reactor Power Level

	ϕ_R (20 Mwt measurement)	ϕ_R (theoretical)	ϕ_R (statistical weight)	
ϕ_z THEORETICAL NAA-SR 1517 REVISED	1.092	1.120	1.162	A_1
	0.994	0.998	0.996	B_1
ϕ_z EXPERIMENTAL	1.169	1.245	1.200	A_1
	0.890	0.890	0.890	B_1

Figure C-7. Calculated Values for A_1 and B_1 Using Various Radial and Axial Flux Distributions

	ϕ_R (20 Mwt measurement)	ϕ_R (theoretical)	ϕ_R (statistical weight)	
ϕ_z THEORETICAL NAA-SR 1517 REVISED	~1.25	~1.25	~1.25	A_2
	0.77	0.77	0.77	B_2
ϕ_z EXPERIMENTAL	~1.25	~1.25	~1.25	A_2
	0.77	0.77	0.77	B_2

Figure C-8. Calculated Values for A_2 and B_2 Using Various Radial and Axial Flux Distributions

APPENDIX D

DETERMINATION OF ISOTHERMAL TEMPERATURE COEFFICIENTS OF REACTIVITY

Determination of the predominant isothermal temperature coefficients of reactivity by oscillation measurements is possible in theory. The desired coefficients could be determined by this technique if the break frequencies of each of the dominant feedback terms listed in Appendix B occurred in a measurable range of frequencies, with each break frequency separated by at least one decade, and if the oscillator equipment was such as to allow very accurate measurements. In the SRE these conditions were not all satisfied. The thermal break frequency of the moderator temperature response occurred below measurement capability. Thus, α_m, the moderator temperature coefficient, could not be determined directly through oscillation. α_f and α_c, the fuel and coolant coefficients, could only be approximated separately since their break frequencies were very close together.

The overall isothermal temperature coefficient of reactivity and some of the individual coefficients were measured during the initial phases of SRE operation. Measurements of the individual coefficients are somewhat in doubt, but the overall coefficient measurements are considered to be fairly good, and are in agreement with the sum of the individual coefficients obtained during the oscillation tests.

The overall temperature coefficient of reactivity measurement for the SRE was reported[6] and has been reproduced in this report as a part of Figure D-1. As a check on the general magnitudes of the individual coefficients, two short experiments were conducted.

The first experiment was designed to measure α_m. A negative step-change of reactivity was made at a power level of 14.5 Mwt, and an average coolant temperature of 675°F. The change in moderator temperature was measured at a point considered to give an average temperature change. After 1400 seconds the moderator temperature had changed 6.2°F. The reactor was then returned to the original power, and the original fuel and coolant temperatures. These responses were fast enough that the moderator temperature did not change

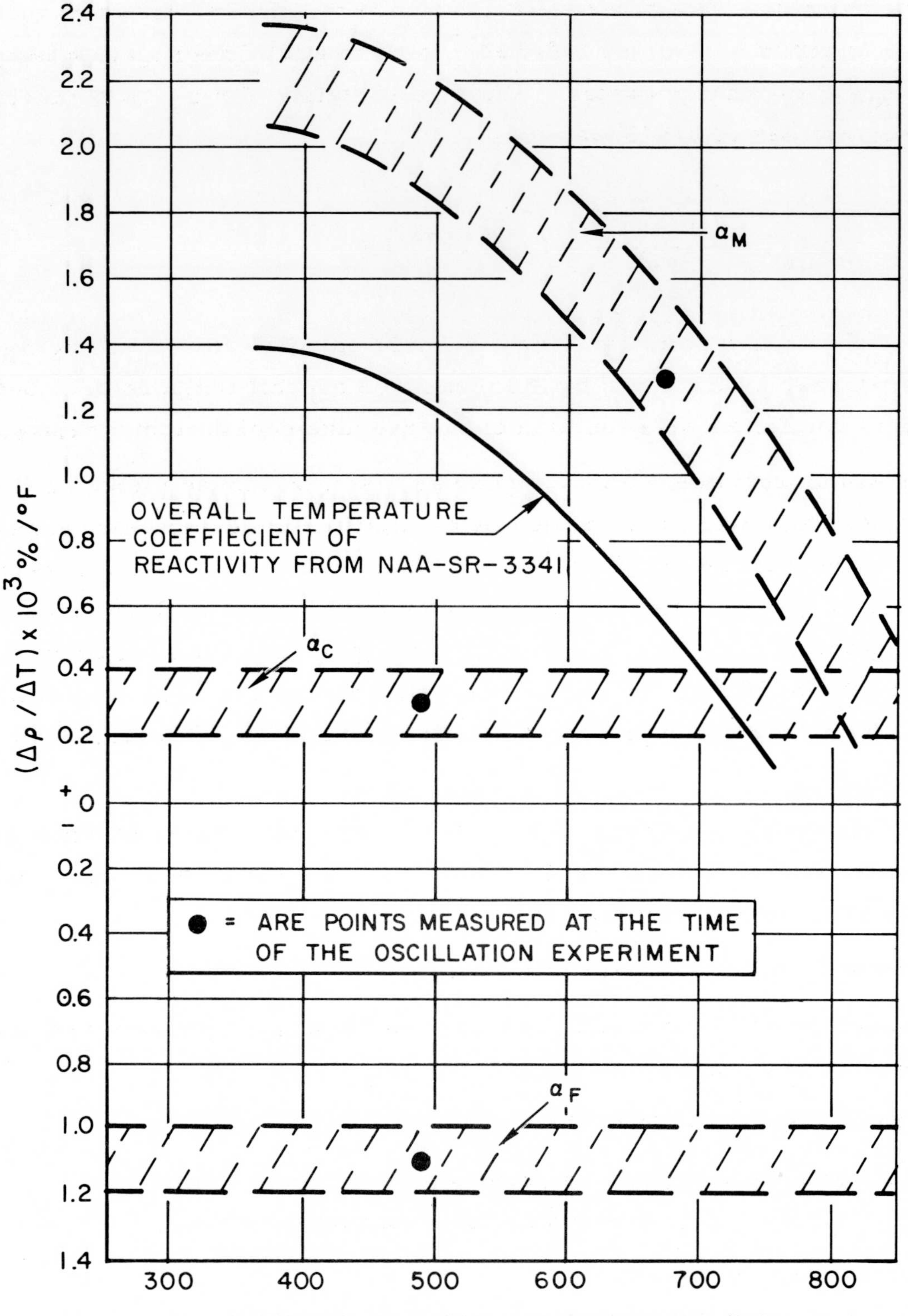

Figure D-1. SRE Isothermal Temperature Coefficient of Reactivity during this interval. The subsequent difference in shim-rod position indicated a change in reactivity of minus 8.0×10^{-5} as a result of the negative temperature change of 6.2°F in the moderator. The value obtained for α_m at an average coolant temperature of 675°F was thus

$$\alpha_m \cong \frac{-8.0 \times 10^{-5}}{-6.2} = 1.3 \times 10^{-5}/°\mathrm{F} \qquad \ldots(\mathrm{D\text{-}1})$$

Figure D-1 shows this point, and the extrapolated curve for other average coolant temperatures, as indicated by the measured overall temperature coefficient curve. This coefficient is a function of the average coolant temperature.

The second experiment was designed as a step response method of checking the α_f and α_c which were determined by oscillation measurements. Since the fuel and coolant time responses were so closely coupled, it was necessary to measure the response of the sum of the two coefficients. With the reactor operating at a power of 5.25 Mwt, the flow at 1330 gpm, and a temperature rise across the core of 100°F, the flow was quickly changed to 1120 gpm while the power was held constant by the automatic flux controller. The observed changes were:

Fuel temperature change ~30°F (as measured by three thermocouples located in the first and second fuel ring from the center, and 6 to 20 inches above the center of the core).

Average coolant temperature change = -19°F

Change in No. 3 shim rod position = -0.48 inches (from 48 inches to 48.48 inches out).

The resultant calculations follow:

Average change in fuel temperature
30°F/1.3 = 23°F (corrected for point of measurement).

Total change in reactivity

$$\Delta\rho = 0.48 \text{ in.} \times 16 \text{ rev/in.} \times 2.46 \times 10^{-5}\ \Delta\rho/\text{rev} = 1.9 \times 10^{-4}\ . \qquad \ldots(\mathrm{D\text{-}2})$$

Isothermal temperature coefficients as determined by the oscillation method are:

$$\alpha_f = -1.1 \times 10^{-5}/°F,$$

$$\alpha_c = +0.3 \times 10^{-5}/°F;$$

where both are approximately independent of average coolant temperature.

Solution for $\Delta\rho$:

$$\Delta\rho = \alpha_f \, \Delta T_f + \alpha_c \, \Delta T_c$$

$$\Delta\rho = (-1.1 \times 23 + 0.3 \times 19)\, 10^{-5} = 1.96 \times 10^{-4}. \quad \ldots(D\text{-}3)$$

The results obtained in Equations D-2 and D-3 show excellent agreement between the observed change in reactivity from shim-rod movement and the calculated effect using the isothermal temperature coefficients of reactivity determined by oscillation measurements. Figure D-1 also shows that the sum of the three measured coefficient values falls very close to the measured overall temperature coefficient value of $0.55 \times 10^{-5}/°F$ for an average reactor temperature of 675°F.

$$\alpha_m + \alpha_f + \alpha_c = \alpha\,\text{total} = (1.3 - 1.1 + 0.3)\, 10^{-5} = 0.5 \times 10^{-5}/°F$$

Because of this agreement, confidence was felt in extrapolating the moderator temperature coefficient curve. Subsequent results of usage in the equation of Appendix B justified the assumed shape of the curve.

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