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THE MEASUREMENT OF PROMPT NEUTRON LIFETIME BY THE PERTURBATION TECHNIQUE THEORY AND APPLICATION TO THE ZENITH REACTOR

by

L. I. TIRÉN.

A.E.E. Winfrith, Dorchester, Dorset, England.

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ABSTRACT

The prompt neutron lifetime of the ZENITH reactor has been determined by a static method which involves measuring the reactivity effect produced by the uniform insertion of a $1/v$ - absorber into the reactor. The theory underlying this experiment is discussed on a basis of multigroup perturbation theory. The theoretical and experimental limitations of the method are considered.

The experimental value of the neutron lifetime obtained with the first loading of ZENITH agrees well with the result of a two group calculation carried out by means of the MERCURY digital computer.

An analytical approach is made on a modified one group basis. The influence of a reflector on the prompt neutron lifetime is considered in particular.

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1. INTRODUCTION

The prompt neutron lifetime of the ZENITH reactor has been determined by a static method, which involves measuring the reactivity effect produced by the uniform insertion of a 1/v - absorber into the reactor. This method has been used previously for example by Ingram, McCulloch and Sanders[1]. In this note some points of the theory underlying this experiment will be discussed and the results of some calculations will be compared with the experimentally determined value of the neutron lifetime in ZENITH.

The reactivity effect produced by an absorber $\delta\sigma_a$ is expressed in the following way, according to one group perturbation theory:

$$|\delta C| = \frac{\int \phi^* \, \delta\sigma_a \, \phi \, dV}{\int \phi^* \, \nu\sigma_f \, \phi \, dV} \tag{1}$$

which is shown in textbooks of reactor physics (see, for example, Weinberg & Wigner, (1958)[2]).

The symbol σ is used for macroscopic cross sections in this note in order to distinguish it from the summation sign Σ.

In the above formula δC is the change in the criticality constant, $\delta\sigma_a \, \phi$ is the neutron destruction rate produced by the inserted absorber, $\nu\sigma_f \, \phi$ is the neutron production rate and ϕ^* is the adjoint flux or neutron importance function. The numerator of (1) is integrated over the region of perturbation and the denominator over the reactor core.

If the absorber follows the 1/v - law, we may write

$$\delta\sigma_a = \frac{\delta\sigma_{ao} \, v_o}{v} \tag{2}$$

and if the 1/v - absorber is inserted uniformly over the entire reactor volume $\delta\sigma_a$ is independent of position and (1) is transformed:

$$\frac{|\delta C|}{v_o \, \delta\sigma_{ao}} = \frac{\int \phi^* \, \phi \, dV}{v \int \phi^* \, \nu\sigma_f \, \phi \, dV} \tag{3}$$

The generation time, $(v\nu\sigma_f)^{-1}$ in a critical reactor is equal to the prompt neutron lifetime, ℓ . So (3) may be rewritten:

$$\frac{|\ \delta C\ |}{v_o\ \delta\sigma_{ao}} = \frac{\int \phi^* \ \phi \ dV}{\int \frac{1}{\ell} \ \phi^* \ \phi \ dV} \tag{4}$$

The measurement of the reactivity change produced by the uniform poisoning, therefore, amounts to a determination of a weighted average lifetime $\bar{\ell}$:

$$\bar{\ell} = \frac{\int \phi^* \ \phi \ dV}{\int \frac{1}{\ell} \ \phi^* \ \phi \ dV} \tag{5}$$

(4) and (5) give together:

$$\bar{\ell} = \frac{|\ \delta C\ |}{v_o\ \delta\sigma_{ao}} \tag{6}$$

which relates the averaged lifetime to the reactivity effect of the poisoning absorber.

This derivation will be examined more closely in the following sections. It will be noted that it holds for a reactor composed of several regions including reflectors, and also in multigroup theory.

Therefore the expression (6) is universally true and can be applied to any reactor system to which diffusion theory is applicable.

In sections 2, 3 and 4 the above expressions are derived in multi-group perturbation theory, following the arguments of Weinberg and Wigner (ibid). The formulae are then applied to a predominantly thermal reactor, where two group theory may be used, and the effect of a reflector on neutron lifetime in such a system is regarded in particular (section 5). Finally some numerical calculations are compared in section 6 with an experimentally determined lifetime value for the first loading of ZENITH, and in section 7 the corresponding experiment with the second loading is discussed.

2. FORMAL EXPRESSION FOR THE EFFECTIVE PROMPT NEUTRON LIFETIME IN N ENERGY GROUPS

On a small reactivity change δC from the critical state the neutron flux level of the reactor will change in time as $e^{\lambda t}$, neglecting the effect of delayed neutrons. The generation time of the system may then be defined as

$$\bar{\ell}_g = \frac{\delta C}{\lambda} \tag{7}$$

which is reasonable from a physical point of view, since the relative change in neutron population per second is λ and per neutron generation is δC.

By using first order perturbation theory in multi-group form it will be shown in this section that the above expression is equivalent with the general definition of prompt neutron lifetime in a critical reactor as given in the Reactor Handbook[3].

In a critical reactor the spatial distribution of neutrons in the n:th energy group satisfies the equation:

$$L_n \phi_n + \gamma_n \sum_{m=1}^{N} \nu\sigma_{fm} \phi_m + \sigma_{s,n-1} \phi_{n-1} = 0 \tag{8}$$

where

$$L_n = \mathrm{div}\,(D_n \,\mathrm{grad}) - \sigma_{an} - \sigma_{sn} \tag{9}$$

and $\phi_n = 0$ on the extrapolated boundary of the reactor.

σ_{an}, σ_{fn} and σ_{sn} are the macroscopic cross sections for absorption, fission, and slowing down, respectively. γ_n is the fraction of fission neutrons born in the n:th group. It is assumed, that the number of neutrons per fission, ν, is independent of energy and also, for simplicity, that a neutron cannot hurdle an energy group during slowing down. Thus only neutrons slowing down from group n - 1 appear as source neutrons in group n.

An adjoint set of equations is formed by transposing the diffusion matrix. The equation for the n:th adjoint is then:

$$L_n \phi_n^* + \nu\sigma_{fn} \sum_{m=1}^{N} \gamma_m \phi_m^* + \sigma_{sn} \phi_{n+1}^* = 0 \tag{10}$$

and $\phi_n^* = 0$ on the extrapolated boundary.

On small perturbations δL_n and $\delta(\nu\sigma_{fn})$ from the steady state the perturbed flux will satisfy the time dependent equation:

$$(L_n + \delta L_n)\, \Phi_n(x,t) + \gamma_n \sum_{m=1}^{N} [\nu\sigma_{fm} + \delta(\nu\sigma_{fm})]\, \Phi_m(x,t) +$$

$$+ \sigma_{s,n-1}\, \Phi_{n-1}(x,t) = \frac{1}{v_n} \frac{\partial \Phi(x,t)}{\partial t} \tag{11}$$

After the decay of transients the flux may be assumed to change in time as:

$$\Phi_n(x,t) = \phi_n'(x) e^{\lambda t} \tag{12}$$

neglecting the influence of delayed neutrons.

Now in first approximation ϕ'_n is replaced by the unperturbed distribution ϕ_n. This approximation will only effect terms in the final equations that are proportional to the second power of the perturbations, which are neglected. (11) is then rewritten

$$(L_n + \delta L_n)\,\phi_n + \gamma_n \sum_{m=1}^{N} [\nu\sigma_{fm} + \delta(\nu\sigma_{fm})]\,\phi_m +$$

$$+ \sigma_{s,n-1}\,\phi_{n-1} = \frac{1}{v_n}\lambda\,\phi_n \qquad (13)$$

It is noted that the above equations are common to the entire reactor volume, although the coefficients may change discontinuously when passing a boundary between different regions, and ν is zero in non-multiplying regions. The equations may still be integrated over such boundaries.

Equation (13) is multiplied by ϕ^*_n and (10) by ϕ_n. On subtracting the equations and integrating over the entire reactor volume, one obtains:

$$\int(\phi^*_n L_n \phi_n - \phi_n L_n \phi^*_n)dV + \int\phi^*_n \delta L_n \phi_n\,dV +$$

$$+ \int[\gamma_n \phi^*_n \sum_{m=1}^{N} \nu\sigma_{fm}\,\phi_m - \nu\sigma_{fn}\,\phi_n \sum_{m=1}^{N} \gamma_m\,\phi^*_m]dV + \int\gamma_n\,\phi^*_n \sum_{m=1}^{N} \delta(\nu\sigma_{fm})\,\phi_m\,dV +$$

$$+ \int[\sigma_{s,n-1}\,\phi_{n-1}\,\phi^*_n - \sigma_{s,n}\,\phi^*_{n+1}\,\phi_n]dV = \frac{\lambda}{v_n}\int\phi^*_n\,\phi_n\,dV \qquad (14)$$

The first term of the left hand side vanishes, since the operator L_n is self adjoint. This may be demonstrated by rewriting the term mentioned using (9):

$$\int[\phi^*_n \,\mathrm{div}\,(D_n \,\mathrm{grad}\,\phi_n) - \phi_n \,\mathrm{div}\,(D_n \,\mathrm{grad}\,\phi^*_n)]dV \qquad (15)$$

The terms $\phi^*_n\,\phi_n\,(\sigma_{an} + \sigma_{sn})$ vanish identically. Now Green's theorem in the form

$$\int \psi \,\mathrm{div}\,\vec{W} dV = \oint \psi\,\vec{W}\cdot\vec{d\delta} - \int\vec{W}\,\mathrm{grad}\,\psi\,dV \qquad (16)$$

is applied to (15), putting $\psi = \phi^*_n$ and $\vec{W} = D_n\,\mathrm{grad}\,\phi_n$ and integrating over the reactor volume. Then the volume integrals on the right hand side of (16) vanish identically. Also the surface integrals are equal to zero, since both ϕ_n and ϕ^*_n vanish on the extrapolated boundary. Thus:-

$$\int(\phi_n^* L_n \phi_n - \phi_n L_n \phi^*)dV = 0 \quad (17)$$

Going back to equation (14) the corresponding equations for all n-values are added together. This will cause the third and the last terms on the left hand side to vanish and one obtains:

$$\lambda \sum_{n=1}^{N} \frac{1}{v_n} \int\phi_n^* \phi_n \, dV = \sum_{n=1}^{N} \int\phi_n^* \delta L_n \phi_n \, dV +$$

$$+ \sum_{n=1}^{N} \int\gamma_n \phi_n^* \sum_{m=1}^{N} (\sigma_{fm} \delta\upsilon + \upsilon\delta\sigma_{fm}) \phi_m \, dV \quad (18)$$

Consider now two hypothetical perturbation experiments. In the first one the cross sections <u>and</u> υ are varied in such a way, that criticality is maintained. Then $\lambda = 0$ and (18) gives the relative change $\frac{\delta\upsilon}{\upsilon}$, or change in reactivity that balances the perturbation of the cross sections:

$$\delta C = -\frac{\delta\upsilon}{\upsilon} = \frac{\sum_{n=1}^{N} \int\phi_n^* (\delta L_n \phi_n + \gamma_n \sum_{m=1}^{N} \upsilon\delta\sigma_{fm} \phi_m) dV}{\int \sum_{n=1}^{N} \sum_{m=1}^{N} \gamma_n \phi_n^* \upsilon\sigma_{fm} \phi_m \, dV} \quad (19)$$

In the second experiment υ is kept constant ($\delta\upsilon = 0$) and (18) now gives the period $1/\lambda$ corresponding to the same perturbation of the cross sections as (19):

$$\lambda \cdot \sum_{n=1}^{N} \frac{1}{v_n} \int\phi_n^* \phi_n \, dV = \sum_{n=1}^{N} \int\phi_n^* (\delta L_n \phi_n + \gamma_n \sum_{m=1}^{N} \upsilon\delta\sigma_{fm} \phi_m) \, dV \quad (20)$$

Combining (19) and (20) yields the period $1/\lambda$ related to the reactivity change δC:

$$\bar{\ell}_g = \frac{\delta C}{\lambda} = \frac{\sum_{n=1}^{N} \frac{1}{v_n} \int\phi_n^* \phi_n \, dV}{\int \sum_{n=1}^{N} \sum_{m=1}^{N} \gamma_n \phi_n^* \upsilon\sigma_{fm} \phi_m \, dV} \quad (21)$$

which is the effective generation time given by (7).

The denominator of (21) is the weighted total production rate of neutrons in the reactor. At the critical state this is equal to the destruction rate, and we may write:

$$\bar{\ell}_g = \bar{\ell}_{crit.} = \frac{\sum_{n=1}^{N} \frac{1}{v_n} \int \phi_n^* \phi_n \, dV}{\int \sum_{n=1}^{N} \phi_n^* [\sigma_{an} \phi_n - \text{div} (D_n \text{ grad } \phi_n)] \, dV} \qquad (22)$$

where $\bar{\ell}_{crit.}$ is the effective prompt neutron lifetime in a critical reactor.

The last expression is in accord with the general formulation of prompt neutron lifetime as given in the Reactor Handbook (ibid), where it is defined as "the ratio of the total neutron importance in the system to the rate in which importance is removed", (cf. R. Avery[4]: "Theory of Coupled Reactors"). However, the physical meaning of this is not very clear. The formula (21) will be further evaluated in Sections 4 and 5.

Since it is found that the equations of perturbation theory produce expressions which can be interpreted in terms of the neutron generation time rather than the neutron lifetime we shall use in the subsequent sections equation (21) rather than (22). The use of neutron lifetime always involves reference to the critical state. All our measurements described below were carried out at critical, however, and therefore the term "lifetime" will appear in the text, being the more commonly used concept.

The advantage of using the concept of generation time in reactor kinetics has been pointed out by Lewins[5].

3. NEUTRON LIFETIME AND POISONING EXPERIMENT

We now go back to equation (19), which expresses the change in reactivity produced by a perturbation of the cross sections, and assume that only the operator L_n has been perturbed.

(19) is then written:

$$\delta C = \frac{\sum_{n=1}^{N} \int \phi_n^* \delta L_n \phi_n \, dV}{\int \sum_{n=1}^{N} \sum_{m=1}^{N} \gamma_n \phi_n^* \nu\sigma_{fm} \phi_m \, dV} \qquad (23)$$

Further assuming that a pure absorber has caused the perturbation we have in view of (9):

$$\delta L_n = - \delta\sigma_{an} \qquad (24)$$

and as in section 1 for a 1/v-absorber:

$$\delta\sigma_{an} = \frac{\delta\sigma_{ao}\, v_o}{v_n} \tag{25}$$

where $\delta\sigma_{ao}$ is the 2200 m/sec. cross section, v_o is 2200 m/sec. and v_n as before is the mean velocity of neutrons in group n. Again assuming that the absorber is uniformly distributed throughout the reactor the factor $\delta\sigma_{an}$ may be brought outside the integration sign:

$$|\ \delta C\ | = \frac{\delta\sigma_{ao}\, v_o \sum_{n=1}^{N} \frac{1}{v_n} \int \phi_n^* \phi_n \, dV}{\int \sum_{n=1}^{N} \sum_{m=1}^{N} \gamma_n \phi_n^* \nu\sigma_{fm} \phi_m \, dV} \tag{26}$$

A comparison with (21) gives:

$$\frac{|\ \delta C\ |}{v_o\, \delta\sigma_{ao}} = \bar{l}_g \tag{27}$$

which is identical (with exception of the index g) with equation (6). <u>The prompt neutron generation time may be determined by measuring the reactivity effect produced by a uniformly distributed poison</u>.

One thus obtains information on a dynamic reactor parameter from a purely static measurement. It is evident from the above derivation, that the theory breaks down in the case that the absorber producing the reactivity change does not obey the 1/v-law. However, if the deviation from 1/v-absorption is small it is still possible to obtain an estimate of generation time (lifetime) from equation (27) under a special condition.

The condition is that all neutrons have the same importance, independent of energy, which implies that one group theory may be used. In a large, well-moderated reactor this is a reasonable assumption. One may then use the effective cross section concept $(\hat{\sigma})$ and a formula corresponding to (27) is obtained by replacing the 2200 meter/sec cross section σ_o by the effective cross section:

$$\frac{|\ \delta C\ |}{v_o\, \delta\, \hat{\sigma}_a} = \bar{l}_g \tag{28}$$

where in Westcott notation[6]:

$$\hat{\sigma}_a = (g + rs)\, \sigma_{ao} \tag{29}$$

4. EVALUATION OF THE LIFETIME EXPRESSION

The expression (21) may be rewritten as a weighted sum of the average generation times (or lifetimes) attributed to the N energy groups.

The local generation time of the n:th group, which we shall denote by $\ell_n(x)$, obeys the equation:

$$\frac{1}{\ell_n(x)} = \frac{v_n}{\phi_n} \left(\gamma_n \sum_{m=1}^{N} \nu\sigma_{fm}\, \phi_m + \sigma_{s,n-1}\, \phi_{n-1}\right) \tag{30}$$

Here the first term is the rate of generation of neutrons due to fission and the second term due to slowing down. By putting (30) into the diffusion equation (8) it is evident that the generation time equals the lifetime in a critical system:

$$\frac{1}{\ell_n(x)} = v_n \left[(\sigma_{an} + \sigma_{sn}) - \frac{\operatorname{div}\,(D_n \operatorname{grad}\, \phi_n)}{\phi_n} \right] \tag{31}$$

By using (30) multiplied by $\phi_n\, \phi^*_n$ and integrated over the reactor we now rewrite the n:th term of (21) in the following manner:

$$P_n\, \bar{\ell}_n = \frac{\int \left(\gamma_n \phi^*_n \sum_{m=1}^{N} \nu\sigma_{fm}\, \phi_m + \sigma_{s,n-1}\, \phi_{n-1}\right) dV}{\sum_{n=1}^{N} \sum_{m=1}^{N} \int \gamma_n\, \phi^*_n\, \nu\sigma_{fm}\, \phi_m\, dV} \times \frac{\int \phi^*_n\, \phi_n\, dV}{\int \frac{1}{\ell_n(x)}\, \phi^*_n\, \phi_n\, dV} \tag{32}$$

Here, the second factor may clearly be defined as a volume weighted average generation time in the n:th energy group:

$$\bar{\ell}_n = \frac{\int \phi^*_n\, \phi_n\, dV}{\int \frac{1}{\ell_n(x)}\, \phi^*_n\, \phi_n\, dV} \tag{33}$$

This volume average is obtained by weighting the inverse local generation time with the local statistical factor $\phi^*_n\, \phi_n$ (cf. Weinberg and Wigner, (ibid)).

The first factor of (32) can be identified as the ratio of the generation rate of neutrons in the n:th group (by slowing down from group n - 1 and by fission) to the total generation rate in all groups (by fission). The generation of neutrons in any group is weighted by the proper importance function ϕ^*_n and averaged over the entire reactor volume. We thus obtain for the relative generation rate in the n:th group, P_n:

$$P_n = \frac{\int (\gamma_n \phi^*_n \sum_{m=1}^{N} \nu\sigma_{fm} \phi_m + \sigma_{s,n-1} \phi_{n-1}) \, dV}{\int \sum_{n=1}^{N} \sum_{m=1}^{N} \gamma_n \phi^*_n \nu\sigma_{fm} \phi_m \, dV} \tag{34}$$

and finally by addition over all groups:

$$\bar{\ell} = \sum_{n=1}^{N} P_n \cdot \bar{\ell}_n \tag{35}$$

5. TWO GROUP CASE

For the purpose of clarity the treatment is now confined to a case when two groups of neutrons are sufficient to give adequate results. This is the case when a predominantly thermal reactor, consisting of a core region (C) and a reflector (R), is regarded. In such a system all fission neutrons are born in the fast group. Thus $\gamma_1 = 1$ and $\gamma_2 = 0$, where index 1 and 2 are used for fast and slow neutrons respectively.

Let us rewrite equation (35)

$$\bar{\ell} = P_1 \bar{\ell}_1 + P_2 \bar{\ell}_2 \tag{36}$$

From (34) $P_1 = 1$, and

$$P_2 = \frac{\int \phi^*_2 \sigma_{s1} \phi_1 \, dV}{\int \phi^*_1 \nu (\sigma_{f1} \phi_1 + \sigma_{f2} \phi_2) \, dV} \tag{37}$$

This expression may be transformed using (14) and (17). In absence of a perturbation the right hand side of (14) is equal to zero. So are also the first, second and fourth integrals on the left hand side and with n = 1 we have

$$\int \phi^*_1 \nu\sigma_{f2} \phi_2 \, dV - \int \phi^*_2 \sigma_{s1} \phi_1 \, dV = 0 \tag{38}$$

which is put into (37):

$$P_2 = \frac{\int \phi_1^* \nu\sigma_{f2} \phi_2 \, dV}{\int \phi_1^* \nu(\sigma_{f1} \phi_1 + \sigma_{f2} \phi_2) dV} \tag{39}$$

P_2 is then the effective ratio of neutron production due to fission in the slow group to total neutron production, weighted over the reactor.

The expressions (33) for $\bar{\ell}_1$ and $\bar{\ell}_2$ may be split into averages over the core and reflector regions. $\bar{\ell}_2$ is written:

$$\frac{1}{\bar{\ell}_2} = \frac{\int \phi_2^* \frac{1}{\ell_2(x)} \phi_2 \, dV_{reactor}}{\int \phi_2^* \phi_2 \, dV_{reactor}} =$$

$$= \frac{\int \phi_2^* \phi_2 \, dV_{core}}{\int \phi_2^* \phi_2 \, dV_{reactor}} \times \frac{1}{\bar{\ell}_{2C}} + \frac{\int \phi_2^* \phi_2 \, dV_{reflector}}{\int \phi_2^* \phi_2 \, dV \text{ reactor}} \times \frac{1}{\bar{\ell}_{2R}} \tag{40}$$

A similar expression holds for $\bar{\ell}_1$. For the fast neutron lifetime in the reflector (30) gives $\frac{1}{\ell_{1R}(x)} = 0$. In two group theory there is no source of fast neutrons in the reflector. Hence the lifetime of those neutrons does not enter the expression.

In the slow region the lifetime is generally much larger in the reflector than in the core. The second term on the right hand side of (40) may therefore be neglected with little loss of accuracy.

Again we define for brevity:

$$W_{core} = \frac{\int \phi^* \phi \, dV_{core}}{\int \phi^* \phi \, dV_{reactor}} \tag{41}$$

W_{core} is usually called the statistical weight of the core region. This is because the reactivity effect produced by the insertion of an absorber in a certain region is proportional to the value of W of that region.

From equations (36), (39), (40) and (41) is finally obtained:

$$\bar{\ell} = \frac{\bar{\ell}_{1c}}{W_{1c}} + \frac{\bar{\ell}_{2c}}{W_{2c}} \cdot P_2 \qquad (42)$$

In a reflected system the statistical weight W_{core} is always less than one. The effective lifetime is therefore larger than that of the core. If $(1 - W_{core})$, the statistical weight of the reflector, increases, this means that an increased number of neutrons in the reflector contribute in maintaining the chain reaction. The very long life of these neutrons increase the effective lifetime of the entire system.

The contribution to reactivity from fission in the fast group reduces the weight of the slow neutron lifetime by a factor P_2.

6. NUMERICAL CALCULATIONS AND COMPARISON WITH EXPERIMENT ON THE FIRST ZENITH LOADING

6.1 Experiment and Computer Calculation

The considerations in the above sections were made in connection with an experimental determination of the prompt neutron lifetime in the ZENITH reactor.

The experiment was carried out by poisoning the reactor with a 1/v-absorber, as mentioned in section 1. On the first loading of Zenith a small sample of copper was employed, the reactivity effect of which was measured with the sample at various positions in the core and reflector. A volume average reactivity change was worked out and equation (27) was used to get a value of the prompt neutron lifetime, the correction for non 1/v-absorption in copper being very small. The value of prompt neutron lifetime so obtained was:

$$\underline{\bar{\ell} = 0.99 \pm 0.06 \text{ m sec.}} \qquad (43)$$

This figure is higher than the lifetime of the equivalent bare core (~ 0.4 m sec.), which is explained by the effect of the very longlived neutrons in the reflector.*

A theoretical prediction of $\bar{\ell}$ can be made using for example equation (21) or (42).

A digital computer programme (MERCURY computer) was available, that computes the integrals $\int \phi_1^* \phi_1 dV$, $\int \phi_2^* \phi_2 dV$ over any region of a reactor and also $\int \phi_1^* \phi_2 dV_c$.

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*However, it may be worth mentioning that in a supercritical transient, where the reactor would be supercritical without the reflector, the reactor period would correspond to the prompt neutron lifetime of the core only.

A calculation with this programme was for the ZENITH first loading. The following value of ℓ was obtained, using (21):

$$\underline{\bar{\ell} = 1.0 \text{ m sec.}} \tag{44}$$

in good agreement with the experimental value[7].

The first loading was chosen so that only a part of the fine control rod was inserted into the reactor at critical. Therefore the effect of control rods on neutron lifetime was negligible.

6.2 <u>Analytical Approach</u>

The effect of the reflector on neutron lifetime will now be determined approximately by carrying out an analytical solution of (42), reduced to one group form:

$$\bar{\ell} = \frac{\bar{\ell}_c}{W_c} \tag{45}$$

where $\bar{\ell}_c$ can be taken as the value for the equivalent <u>bare</u> reactor and W_c is given by (41). Note, that $W_R = 1 - W_c$.

The bare core lifetime $\bar{\ell}_c$ has been calculated for the first loading of ZENITH as:

$$\bar{\ell}_c = 0.37 \text{ m sec.}$$

including a contribution from fast neutrons of 0.02 m sec.

An expression for W_c will be obtained using the one group perturbation formula (1):

$$\delta C = \frac{- \int \phi^* \, \delta\sigma_a \, \phi \, dV}{\int \phi^* \, \upsilon\sigma_f \, \phi \, dV} \tag{1}$$

In the case of a uniform change $\delta\sigma_{aR}$ in the absorption cross section of the reflector (1) may be written, in view of (41):

$$\frac{\delta C}{\delta\sigma_{aR}} = - \frac{1}{\upsilon\sigma_f} \times \frac{\int \phi^* \, \phi \, dV_R}{\int \phi^* \, \phi \, dV_c} = \frac{1}{\upsilon\sigma_f} \times \frac{W_R}{W_c} \tag{46}$$

assuming the fissile material uniformly distributed in the core.

Let us consider the (modified one group) critical equations for a spherical reactor with a reflector of thickness T.

We require the following well known formulae:

$$C = \frac{\nu\sigma_f}{\sigma_{ac}} \cdot \frac{p}{1 + M^2 B^2} \tag{47}$$

$$\operatorname{tg} BR_c = - BL_R \frac{D_c}{D_R} \operatorname{tgh} \frac{T}{L_R} \tag{48}$$

$$L_R^2 = \frac{D_R}{\sigma_{aR}} \tag{49}$$

where the notation is familiar except for macroscopic cross sections, for which the symbol σ is used.

On differentiating each of these formulae in respect of σ_{aR} one obtains, after a straight forward computation:

$$\frac{dC}{d\sigma_{aR}} = - \frac{C}{\sigma_{aR}} \times \frac{M^2 B^2}{1 + M^2 B^2} \times \frac{1 - \frac{2T/L_R}{\sinh 2T/L_R}}{1 + \frac{R_C\, D_R/D_C}{L_R \operatorname{tgh} T/L_R} \left(1 + [BL_R\, D_C/D_R \operatorname{tgh} T/L_R\,]^2\right)} \tag{50}$$

Combining (50) and (46) yields the following expression for the statistical weights, using (47) and putting C = 1:

$$\frac{W_R}{W_c} = \frac{1}{W_c} - 1 = \frac{M^2 B^2}{p} \cdot \frac{L^2_R}{L^2_c} \cdot \frac{D_c}{D_R} \cdot \frac{1 - \frac{2T/L_R}{\sinh 2T/L_R}}{1 + \frac{R_c\, D_R/D_c}{L_R \operatorname{tgh} T/L_R} \left(1 + [BL_R\, D_c/D_R \operatorname{tgh} T/L_R\,]^2\right)} \tag{51}$$

Although only approximately valid, this expression exposes the general characteristics of the usefulness of a reflector. This increases with increasing total leakage ($M^2 B^2$), reflector diffusion area (L^2_R) and ratio of core-to-reflector diffusion coefficient (D_c/D_R). The importance of a reflector also grows with increasing resonance absorption in the core (1 - p).

The factor $(1 - \frac{2T/L_R}{\sinh 2T/L_R})$ contains the dependence on reflector thickness. The variation in the statistical weight of the reflector relative to that of the core at varying reflector thickness is shown on figure 1. The curve is arbitrarily normalised to $W_R/W_c = 1$ for an infinite reflector.

The ZENITH reactor is not a sphere, but a right cylinder. Still equation (51) should give a reasonable estimate of the statistical weight also in this case. For the first loading of ZENITH we put[8]:

$M^2 = L^2 + \tau = 571\ cm^2$

$L_C^2 = 115\ cm^2$

$B_g^2 = 8.7 \times 10^{-4}\ cm^{-2}$

$p = 0.78$ (including $\frac{1}{v}$ - absorption)

$D_c = 1.03$ cm

$R_c = 70$ cm $(R_{sphere} = (\frac{3}{4\pi} V_C)^{\frac{1}{3}})$

$L_R = 52$ cm
$D_R = 0.936$ cm
$T = 90$ cm
} side reflector

$L_R = 58$ cm
$D_R = 1.05$ cm
$T = 60$ cm
} end reflectors

Since the parameters for side - and end relectors differ, we have worked out values using both sets of parameters.

One obtains:

$W_c = 0.31$ (side refl. parameters)

$W_c = 0.38$ (end refl. parameters)

Using (45) and the above values of $\bar{\ell}_c$ and W_c:

$\bar{\ell} = 1.0$ m/sec. (side refl. parameters)

$\bar{\ell} = 1.2$ m/sec. (end refl. parameters)

The good agreement between these figures and the experimental value indicates that the modified one group formulae give useful results in the present application.

7. DISCUSSION OF EXPERIMENTAL RESULTS ON THE SECOND LOADING OF ZENITH

An experiment similar to that described in the previous section was carried out also with the second loading of the reactor. In this case the reactor was loaded with more fissile material, so that criticality was obtained with nine control rods inserted. Since the control rods in ZENITH are placed in the side reflector, close to the core-reflector interface, the effect of the rods is mainly to reduce the efficiency of the side reflector. From the above paragraphs it is clear that the effective prompt neutron lifetime of the system will be reduced accordingly.

The nine control rods inserted were not spaced uniformly in the azimuthal direction, and thus the neutron flux exhibited an azimuthal asymmetry. Accurate calculations on such a system are very complicated and in particular it is difficult to obtain a theoretical assessment of the net influence of the side reflector. Therefore no attempts have been made to calculate a value of the neutron lifetime for the second loading, but it is noted, that the value for the bare core is of the order of

$$\ell_{bare} \sim \underline{0.2 \text{ m sec.}}$$

and for the reflected system with all control rods removed

$$\ell \sim \underline{0.8 \text{ m sec.}}$$

On the experimental side it is noted, that the asymmetry of the distribution of control rods would necessitate measurements in many azimuthal directions in order to obtain a volume average reactivity effect of the copper sample. Such a measurement would be too time consuming and consequently measurements were only carried out in two directions in the reactor. In the first (west) direction the side reflector was partially screened off by several control rods but in the second (east) the nearest control rod was about four diffusion lengths away, so that the effect of the side reflector in this region should be near to that of the rod-free state. (See figure 2).

The data taken in these two directions were analysed as above to give the following values of the prompt neutron lifetime:

$$\bar{\ell} \text{ (west)} = \underline{0.33 \text{ m sec.}}$$

$$\bar{\ell} \text{ (east)} = \underline{0.68 \text{ m sec.}}$$

In the experiments on the second loading of ZENITH a control rod oscillator was in use for determining control rod worths. By operating this oscillator at various frequencies and analysing the transfer function it was possible to obtain a value for the effective prompt neutron lifetime of the reactor. These investigations were made by I. R. Cameron and W. Hage[9] and the value found was:

$$\bar{\ell}_{eff} = \underline{0.53 \text{ m sec.}}$$

As expected the value produced by the perturbation experiment in the region of many rods (west) lies between $\bar{\ell}_{eff}$ and the value for the bare core, while the value in the region far from rods (east) lies between $\bar{\ell}_{eff}$ and the value for the rod free system.

8. SUMMARY AND CONCLUSIONS

The effective neutron generation time in a reactor can be determined by a static method involving the measurement of the reactivity effect produced by poisoning the reactor uniformly with an absorber which obeys the 1/v - law.

The analysis of this experiment neither requires some assumption of the applicability of a low-order approximation such as one group theory, nor is the experiment restricted to reactors of a simple shape or composition. However, the use of diffusion theory imposes a limit on the size and heterogeneity of the system regarded.

By using multi-group perturbation theory the equivalence is demonstrated in the case of a critical reactor between the general definition of prompt neutron lifetime as given by the Reactor Handbook and the neutron generation time defined by the formula (7)

$$\bar{\ell}_g = \frac{\delta C}{\lambda}$$

In connection with perturbation theory the generation time concept is more conveniently used than the neutron lifetime as generally defined. The two quantities are equivalent in a critical reactor, however, and no strict distinction is made in the part of the paper dealing with experiments in the critical state.

The experimental value of neutron lifetime determined with the first loading of ZENITH agrees well with a two group perturbation calculation carried out by means of a digital computer.

In the second loading of the reactor a number of control rods were inserted at critical. The asymmetry caused by these rods makes the poisoning experiment unfavourable in comparison with other methods of determining prompt neutron lifetime and also makes theoretical calculations complicated.

The one group version of the perturbation formula may be expressed in terms of the (modified one group) reactor parameters. This enables a qualitative assessment of the effect of a reflector on the effective prompt neutron lifetime. Numerical calculations with these formulae produce lifetime values in reasonable agreement with the experimental value for the first loading of ZENITH.

9. ACKNOWLEDGMENT

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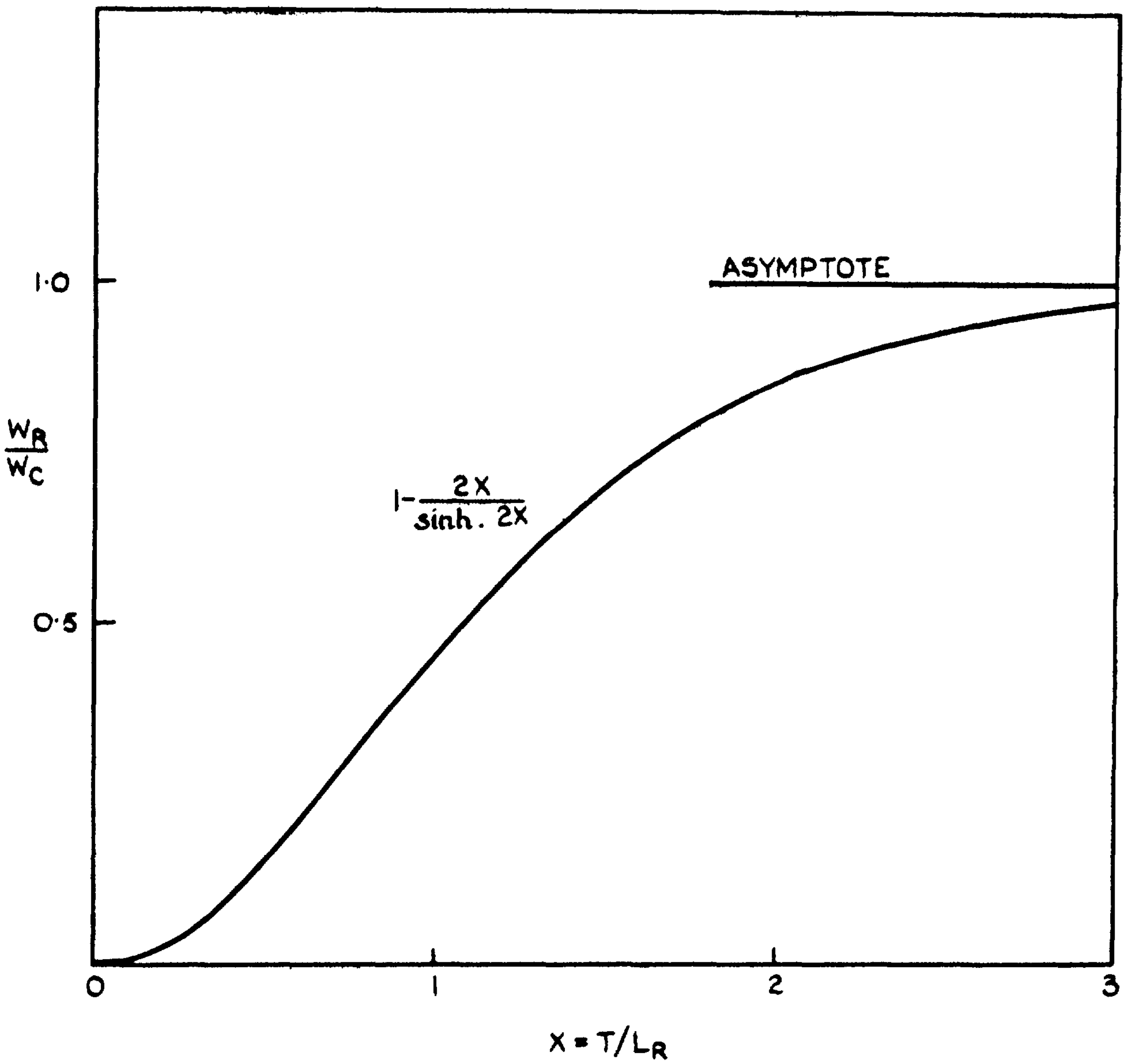
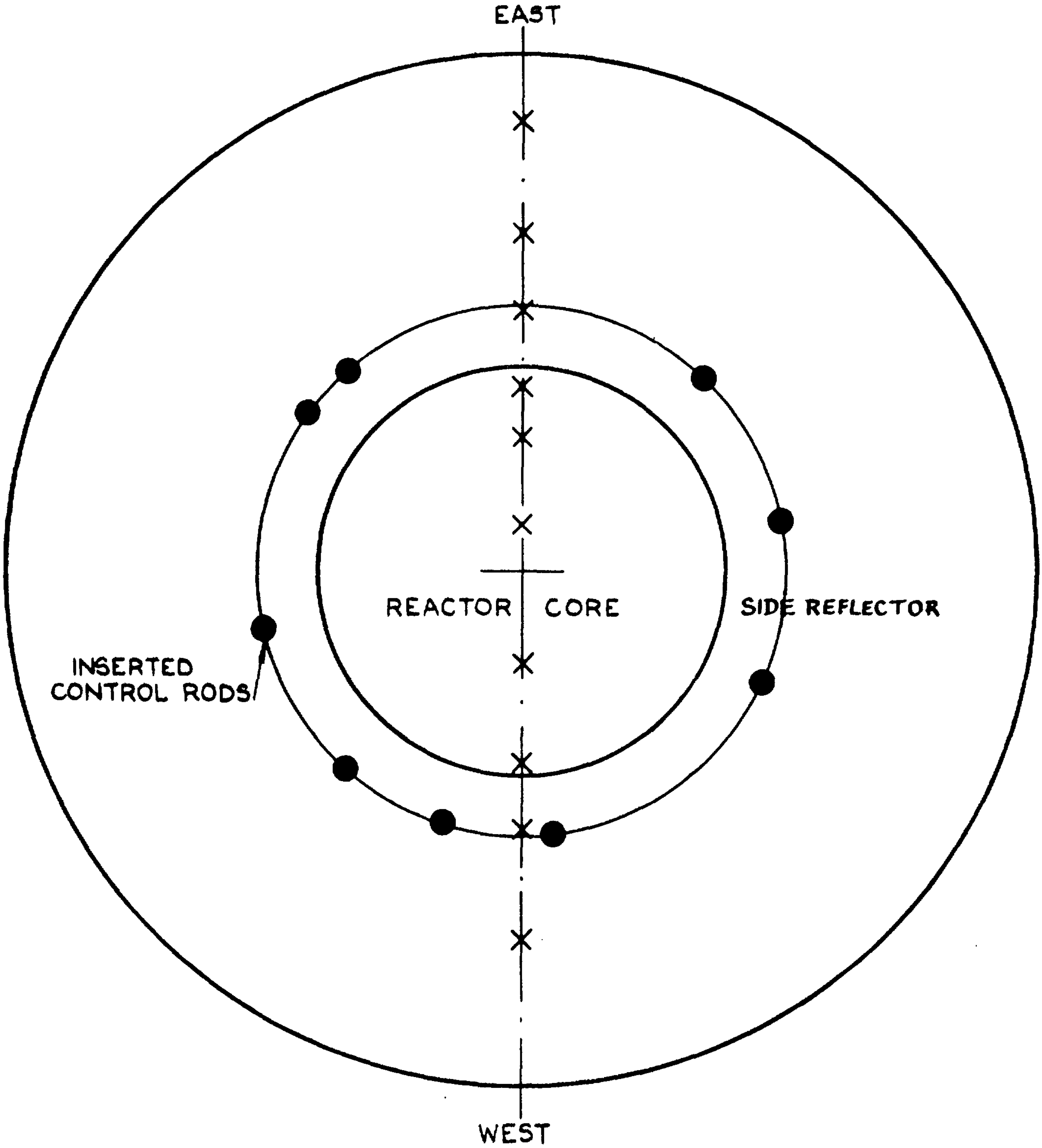


FIG. 1. REFLECTOR-TO-CORE STATISTICAL WEIGHT RATIO VERSUS REFLECTOR THICKNESS.



X DENOTES POSITIONS OF MEASUREMENTS.

FIG. 2. POSITIONS OF MEASUREMENTS WITH COPPER SAMPLE (TUBE)