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CALCULATIONS OF k FOR SOME SMALL U^{233}-, U^{235}-, AND Pu^{239}-FUELED FAST REACTORS

by

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ABSTRACT

Values of the multiplication constant, k, have been calculated for a number of small U^{233}-, U^{235}-, and Pu^{239}-fueled fast reactors through use of a cross-section set previously used by the author for investigation of a number of U^{235}-fueled ZPR-III assemblies.(1-3) The reactors considered in the present report are (a) GODIVA, TOPSY, JEMIMA, JEZEBEL, POPSY, and U^{233} assemblies constructed at Los Alamos; (b) three plutonium-graphite critical assemblies constructed with the reactor VERA at AWRE, Aldermaston, England; (c) the plutonium-fueled Mark-IV loading of the EBR-I reactor at Idaho Division of ANL; and (d) a plutonium- and U^{235}-fueled mockup in ZPR-III of a proposed French reactor, RAPSODIE.

I. INTRODUCTION

A comparison of measured and calculated parameters of U^{235}-fueled ZPR-III fast critical assemblies was previously carried out by the author(1-3) through use of a 16-group cross-section set which is a modification of the set of Yiftah, Okrent, and Moldauer (YOM).(4) In that study, which covered core volumes ranging from 50 to 660 liters and critical masses from 130 to 580 kg of U^{235}, the critical masses could be calculated to an accuracy not worse than $6\frac{1}{2}\%$. In the present study, the same cross-section set and computational methods have been used to calculate the values of k for a number of small, dense, U^{233}- and U^{235}-fueled reactors constructed at Los Alamos and for eight Pu^{239}-fueled systems constructed at Los Alamos, Aldermaston, and the Idaho Division of Argonne National Laboratory.

II. DERIVATION OF THE CROSS-SECTION SET

The derivation of the cross-section set has been described in detail(1) (also Appendix I of reference 3). Briefly, it consists of the YOM Set with: (a) values of α for U^{233}, U^{235}, and Pu^{239} modified to agree with the measurements of Hopkins and Diven;(5) (b) the values of ν for U^{235} modified in accordance with recent measurements;(6-8) and (c) the transport and elastic removal cross sections of steel and aluminum modified in an approximate manner to allow for the effects of strong resonances in these materials.(9,10)

In addition to these data, 16-group cross sections for copper have been added.(11) These are given in Appendix I.

III. REACTOR COMPOSITIONS AND CALCULATIONS

The values of k were calculated by converting (when necessary) the experimental critical configuration to the equivalent, idealized spherical geometry and then finding k for this geometry by means of the DSN neutron transport code. When necessary, corrections to the critical mass were applied for core heterogeneity and some other small effects, thus obtaining critical masses of homogeneous systems of regular geometry. The shape factor used to correct for the core geometry is defined as the ratio of the critical masses of spherical and cylindrical reactors of identical composition, and it is a function of both the core size and the length-to-diameter ratio of the cylindrical core. The method of estimating the shape factors was identical with that used in reference 1 and will not be discussed here.

In the previous study(1) all calculations were made with the DSN code in the S4 approximation. This approximation was accurate for the larger reactors previously considered, but is inaccurate for some of the small reactors examined here. Some S4 and S8 calculations were carried out to estimate the effect of the degree of approximation upon the calculated value of k. All the values of k were then corrected to those that would be obtained by means of S8 calculations.

The reactor compositions and geometries are discussed more fully below.

A. The Los Alamos Assemblies

All these reactors have been considered by Roach(12) and, when necessary, he has estimated the critical masses of homogeneous spherical systems. The idealized compositions and dimensions derived by Roach have been used here. They are given below.

1. GODIVA: A bare highly enriched uranium sphere.

The composition is 93.8% U^{235} and 6.2% U^{238}; the effective density of U^{235} is 17.59 g/cm^3; the critical mass is 48.7 kg U^{235}, and the core radius is 8.71 cm. S4 and S8 calculations were run.

2. TOPSY: A natural uranium reflected highly enriched uranium sphere.

The core was 94.1% U^{235} and 5.9% U^{238}; the effective U^{235} density is 17.60 g/cm^3; the critical mass is 16.28 kg U^{235}, and the core radius is 6.04 cm. The reflector is 9.0 in. of natural uranium at a density of 19.0 g/cm^3.

3. JEMIMA 53.6: A bare 53.6% enriched uranium cylinder.

The idealized composition is a sphere of 53.6% U^{235}, and 46.4% U^{238}; the effective U^{235} density is 10.02 g/cm^3; the critical mass is 75.0 kg U^{235}, and the core radius is 12.14 cm.

4. JEMIMA 37.7: A bare 37.7% enriched uranium cylinder.

The idealized composition is a sphere of 37.7% U^{235} and 62.3% U^{238}; the effective U^{235} density is 7.07 g/cm^3; the critical mass is 92.4 kg U^{235}, and the core radius is 14.61 cm.

5. JEMIMA 29.0: A bare 29.0% enriched uranium cylinder.

The idealized composition is a sphere of 29.0% U^{235} and 71.0% U^{238}; the effective U^{235} density is 5.45 g/cm^3; the critical mass is 109.3 kg U^{235}, and the core radius is 16.85 cm.

6. JEZEBEL: A bare plutonium sphere.

The core is entirely Pu^{239} at a density of 15.56 g/cm^3. The critical mass is 16.22 kg Pu^{239}, and the core radius is 6.29 cm. S4 and S8 calculations were run.

7. POPSY: A natural uranium-reflected plutonium sphere.

The core is Pu^{239} at a density of 15.67 g/cm^3; the critical mass is 5.74 kg Pu^{239}, and the core radius is 4.44 cm. The reflector is 9.5 in. of natural uranium at a density of 18.97 g/cm^3. S4 and S8 calculations were run.

8. Pu/U: A reflected heterogeneous cylinder of plutonium and depleted uranium.

The idealized composition is a homogeneous Pu^{239}/Nickel/U^{238} sphere with $\rho(Pu^{239})$ = 3.81 g/cm^3, ρ(Nickel) = 0.174 g/cm^3 and $\rho(U^{238})$ = 13.87 g/cm^3. The critical mass is 20.09 kg Pu^{239}, and the core radius is 10.80 cm. The reflector is 7.5 in. of U^{238} at a density of 19.0 g/cm^3.

9. U^{233}/U^{238}: A natural uranium-reflected U^{233} sphere.

The core is a 5.04-cm-radius sphere of 9.705 kg of U^{233} and 0.172 kg of U^{238}. The reflector is 0.885 in. of natural uranium at a density of 18.9 g/cm^3.

10. U^{233}/U^{235}: An enriched uranium-reflected U^{233} sphere.

The core is identical with the U^{233}/U^{238} core. The reflector is 0.458 in. of 93.3% enriched uranium at a density of 18.8 g/cm^3.

B. The VERA Assemblies

The compositions and critical dimensions of a number of U^{235}- and Pu^{239}-fueled fast critical assemblies have been reported recently by Weale et al.[13] Of these assemblies, numbers 9A, 10A, and 11A were reflected critical assemblies whose major core constituents were plutonium and carbon. Assembly 8A was fueled with plutonium but did not reach criticality. The core and blanket compositions of VERA 9A, 10A, and 11A are given in Table I. The blanket was the same composition for each assembly.

Table I

CORE AND BLANKET COMPOSITIONS IN VERA ASSEMBLIES 9A 10A, AND 11A

Region	Composition of Region (atoms/cm^3) x 10^{-22}								
	U^{235}	U^{238}	Pu^{239}	Pu^{240}	Cu	Fe	Cr	Ni	C
Core 9A	0.000	0.000	1.071	0.055	1.164	0.607	0.158	0.0665	3.437
Core 10A	0.000	0.000	0.866	0.044	0.941	0.607	0.158	0.0665	4.170
Core 11A	0.000	0.000	0.724	0.037	0.796	0.607	0.158	0.0665	4.606
Blanket	0.025	3.440	0.000	0.000	0.000	0.646	0.168	0.071	0.000

The quoted critical masses of VERA 9A, 10A and 11A are not corrected for core irregularities and edge effects, and these were estimated from the quoted corrections for the VERA U^{235}-fueled assemblies as about $2\frac{1}{2}$%.

The VERA Assemblies were cylindrical. Shape factors to convert the cylindrical masses to equivalent spherical masses were estimated by the method described by Davey.[1] The uncorrected critical masses and corrections are given in Table II.

In all cases the outer radius of the blanket was assumed to be 55.0 cm.

Table II

VERA CRITICAL MASSES AND CORRECTIONS

Assembly Number	Uncorrected Critical Mass (kg)	Critical Mass Corrected for Heterogeneity, etc. (kg)	Shape Factor	Spherical Critical Mass (kg)	Spherical Core Radius (cm)
9A	24.1	24.7	0.98	24.2	11.08
10A	28.4	29.1	0.97	28.2	12.52
11A	33.1	33.9	0.97	32.9	13.98

C. EBR-I Mark IV

EBR-I is a small, fast power reactor whose fourth core loading consists of plutonium fuel rods.[14] The Mark-IV core is a fairly regular pseudocylinder, but there are a number of blanket regions of different composition; the regions are of different composition and thickness in the radial, bottom axial, and top axial directions. Conversion of such a complex real geometry to an equivalent spherical geometry is of dubious validity, but an attempt to estimate k was made by carrying out three spherical DSN calculations. The core composition and dimensions were the same in each of these calculations, but the three reflector compositions corresponded to the blanket compositions in the radial, top axial, and bottom axial directions. No correction to the core critical mass was applied because of heterogeneity effects, but a shape factor of 0.98 was used.[1]

The compositions and dimensions of the various spherical regions used in the calculations are given in Tables III and IV.

Table III

EBR-I MARK-IV CORE COMPOSITION AND DIMENSIONS

Core Radius (cm)	Composition (atoms/cm^3) x 10^{-22}								
	Pu^{239}	Pu^{240}	Fe	Ni	Cr	Al	Zr	Na	K
10.70	1.274	0.069	0.425	0.065	0.107	0.151	0.910	0.178	0.377

The three spherical DSN S4 calculations with the radial, top axial, and bottom axial reflectors gave values of k of 1.044, 1.022 and 0.999, respectively. Since the cylindrical core has a length-to-diameter ratio of approximately unity, the portion of the core surface bounded by each of the above reflectors was in the ratio 4:1:1; the k values were weighted in this proportion. This crude calculation gave a weighted mean value of k of 1.033.

Table IV

EBR-I MARK-IV BLANKET COMPOSITIONS AND DIMENSIONS

Outer Radius of Region (cm)	Composition (atoms/cm^3) x 10^{-22}								
	U^{235}	U^{238}	Fe	Ni	Cr	Al	Zr	Na	K
Radial Blanket									
12.00	0.004	1.686	0.458	0.069	0.116	0.000	0.871	0.184	0.389
18.54	0.017	2.330	0.462	0.070	0.117	0.000	0.774	0.126	0.268
24.59	0.000	0.000	3.457	0.525	0.875	0.328	0.000	0.064	0.136
41.07	0.029	4.056	0.264	0.040	0.067	0.000	0.000	0.000	0.000
Top Axial Blanket									
30.40	0.004	1.695	0.435	0.066	0.110	0.000	0.897	0.181	0.383
44.50	0.000	0.000	4.037	0.613	1.021	0.000	0.214	0.110	0.232
Bottom Axial Blanket									
19.74	0.004	1.695	0.435	0.066	0.110	0.000	0.897	0.181	0.383
32.80	0.000	0.000	1.783	0.270	0.451	0.296	0.147	0.175	0.370
44.13	0.029	4.056	0.264	0.040	0.067	0.000	0.000	0.000	0.000

D. The RAPSODIE Assembly in ZPR-III

This assembly is a simplified mockup of the proposed French oxide-fueled reactor RAPSODIE. The core is fueled with both plutonium and U^{235} and the experimental critical volume is 38.6 liters.(15) Using a 2% correction for heterogeneity and an estimated shape factor of 0.97, we derive a spherical, homogeneous critical volume of 38.2 liters. The core and blanket dimensions used in the DSN calculation are given in Table V.

Table V

RAPSODIE MOCKUP COMPOSITION AND DIMENSIONS

Outer Radius of Region (cm)	Composition (atoms/cm^3) x 10^{-22}								
	U^{235}	U^{238}	Pu^{239}	Pu^{240}	Fe	Cr	Ni	Mo	Al	O
20.9	0.504	0.336	0.156	0.007	1.420	0.181	0.092	0.000	1.029	1.957
32.8	0.005	2.163	0.000	0.000	0.857	0.222	0.113	0.652	0.767	0.000
65.8	0.006	2.977	0.000	0.000	0.446	0.115	0.059	0.000	0.693	0.000

IV. RESULTS

The results of the DSN calculations are given in Table VI. S4 and S8 calculations were performed for GODIVA, JEZEBEL, and POPSY. The difference in the values of k obtained by means of these two approximations decreased with increasing core radius. These data were then used to estimate the values of k that would be obtained for the other reactors if S8 calculations were used.

Table VI

CALCULATED VALUES OF k

Reactor	Fuel	k(S4)[a] - k(S8)	k(S4 calc)	k(S8 calc)[a]
GODIVA	U^{235}	0.003	1.015	1.012
TOPSY	↓	0.007	1.029	1.022
JEMIMA, 53.6	↓	0.001	1.015	1.014
JEMIMA, 37.7	↓	0.000	1.001	1.001
JEMIMA, 29.0	↓	0.000	0.991	0.991
JEZEBEL	Pu	0.007	1.029	1.022
POPSY	↓	0.012	1.062	1.050
Pu/U	↓	0.001	1.014	1.013
VERA 9A	↓	0.001	1.054	1.053
VERA 10A	↓	0.001	1.039	1.038
VERA 11A	↓	0.000	1.032	1.032
EBR-I Mark IV	↓	0.001	1.033	1.032
RAPSODIE	Pu + U^{235}	0.000	1.030	1.030
U^{233}/U^{238}	U^{233}	0.010	1.044	1.034
U^{233}/U^{235}	↓	0.010	1.037	1.027

[a] Calculated for GODIVA, JEZEBEL, and POPSY, and estimated for the other reactors

V. DISCUSSION AND CONCLUSIONS

(a) The values of k for the small assemblies fueled entirely with U^{235} are calculated with about the same accuracy as for the larger, diluted ZPR-III assemblies previously examined.[1,3] This is true for the bare JEMIMA and GODIVA reactors as well as for the reflected TOPSY. In terms of calculation of critical mass, this indicates an accuracy of about 6% or better for all reactors fueled solely with U^{235} up to a mass of 580 kg. This does not imply that the nuclear data are free from significant errors.

(b) The values of k for all the reactors fueled with plutonium are greater than unity, being on average about $3\frac{1}{2}$% greater. Somewhat surprisingly, and perhaps fortuitously, this is true for the RAPSODIE reactor, which is fueled with both U^{235} and plutonium. For the small reactors considered here, an error of $3\frac{1}{2}$% in the calculated value of k means prediction of critical masses that would be about 6% to 10% smaller than would be obtained by experiment. If the same discrepancy of about $3\frac{1}{2}$% in k also occurs in large plutonium-fueled reactors, this could result in errors of critical mass varying from about 20% to 30%.

(c) The values of k for the two U^{233}-fueled reactors are about 3% greater than unity.

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APPENDIX I

GROUP CROSS SECTIONS FOR COPPER (in barns)

Group	σ_{tr}	$\sigma_{n\gamma}$	σ_{er}	$\sigma_{n,n'}$ $j \rightarrow k$						
				k = 0	1	2	3	4	5	6
1	2.3	0.00	0.047	0.124	0.347	0.444	0.356	0.212	0.105	0.076
2	2.4	0.00	0.076	0.172	0.371	0.373	0.137	0.064	0.027	0.018
3	2.45	0.01	0.12	0.144	0.218	0.175	0.104	0.051	0.023	0.015
4	3.0	0.01	0.19	0.054	0.059	0.039	0.021	0.010	0.004	0.003
5	3.5	0.01	0.23							
6	4.15	0.015	0.28							
7	5.0	0.02	0.33							
8	5.75	0.03	0.36							
9	6.6	0.04	0.40							
10	22.0	0.054	3.00							
11	7.8	0.079	0.47							
12	7.9	0.109	0.44							
13	8.9	0.163	0.53							
14	11.5	0.227	0.45							
15	9.05	0.562	0.34							
16	8.1	0.390	0.00							

The group boundaries are those of Yiftah _et al_.[4]