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THE NEUTRONIC VALUES OF U-233 AND U-236 IN THE HTGR

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ABSTRACT

The HTGR utilizes thorium as the fertile material and fully enriched uranium as the makeup feed material. The bred U-233 is recovered and continuously recycled. The discharged feed material is recovered and recycled once more through the reactor before being permanently retired. The bred U-233 is a valuable fuel and its intrinsic neutronic worth, i.e., parity is 1.43 times the value of fully enriched U-235.

Parasitic neutron absorptions in U-236 are significant and are proportional to the product of the in-core U-236 inventory and the effective cross section of U-236. The reference recycle mode of operation limits the U-236 inventory buildup and minimizes the effective cross section of U-236. The U-236 that gradually builds up in the bred uranium is unshielded and its neutronic parity is typically -0.58 the value of fully enriched U-235. The U-236 produced in the feed uranium elements and recycled once more is heavily shielded and has a value of -.20 to -.25 the value of fully enriched U-235.

I. INTRODUCTION

An accurate determination of the neutronic values of bred U-233 and U-236 is a necessity for the proper evaluation of HTGR fuel values and related fuel costs. The results of an extensive evaluation of the U-233 worth in the HTGR was published in 1972.[1] Other reports[2,3] have briefly summarized the results of U-236 value calculations performed at General Atomic Company. This report summarizes in detail the methods and results of more recent calculations of the value of these isotopes with particular emphasis on the U-236 value calculation. A detailed description of the cross section determination of U-236 in the reference HTGR fuel management strategy is included in the Appendix. An understanding of the details of the cross section determination is required to arrive at the proper value of the U-236 parity in the HTGR.

The results of this evaluation are in close agreement with the previously reported values. The neutronic parity values, i.e., the value relative to the value of U-235 in fully enriched uranium, are:

1)	U-233	1.43
2)	U-236 in Bred U	-.58 to -.65
3)	U-236 in discharged feed uranium:	
	First Discharge Segment	-.25
	Equilibrium Discharge Segment	-.20

II. U-236 BUILDUP DURING CORE OPERATION

U-236 In-Core Inventory

The U-236 fuel cycle cost penalty is proportional to the product of the in-core U-236 inventory and the effective cross section of U-236. The latter is a function of the loading per block and the recycle particle characteristics. The U-236 inventory depends on the particular mode of operation being followed, i.e., non-recycle, selective recycle, full recycle, etc. The several possible strategies are described below and illustrated on Figure 1.

Strategy A. No recycle of either the U-233 or U-235 is assumed in Strategy A. However, it is assumed that the bred fuel can be separated from the residual feed (U-235) uranium in the reprocessing plant. The U-233 is assumed to have a value about 40% greater than that of U-235 in 90% enriched uranium, reduced as appropriate by the added cost of fabricating U-233 fuel elements over fresh fuel (U-235) elements. The discharged feed (U-235) value is reduced as appropriate by the negative value of the contained U-236. In about 5 years a steady state condition is achieved with respect to the U-236 inventory of about 350 kg in an 1160 MWe reactor or .30 kg/MW(e), as shown in Figure 2.

Strategy B. In this strategy, recycle of the bred uranium is assumed but the residual feed uranium is recovered and sold as in Strategy A. The reduced U-236 inventory reflects the fact that less feed uranium (U-235) is required with bred U recycle than in the non-recycle mode of operation, and hence less U-236 will be formed. The average inventory is about 270 kg U-236 in an 1160 MW HTGR, or about .23 kg/MW(e).

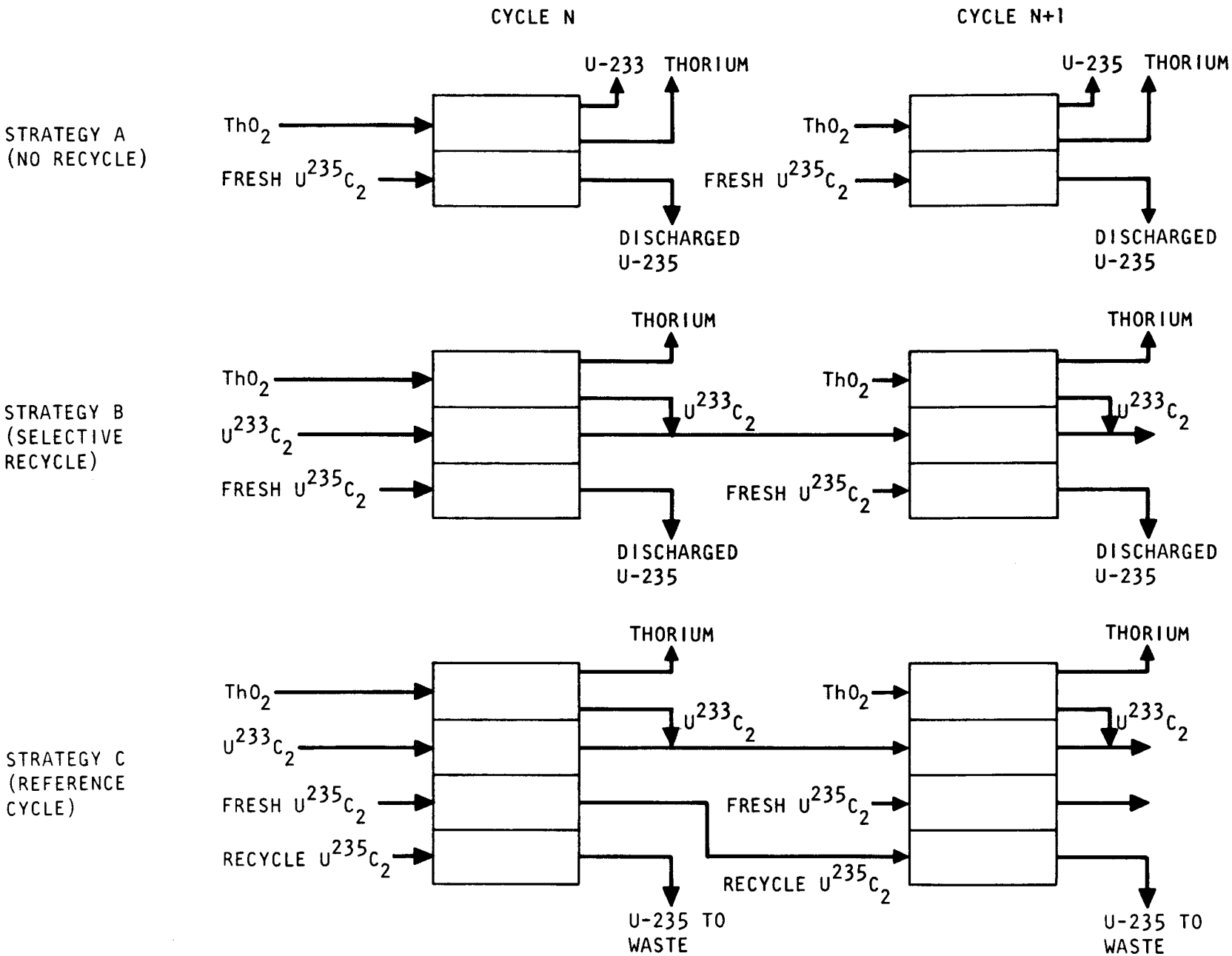


Figure 1 - Alternate fuel cycle strategies.

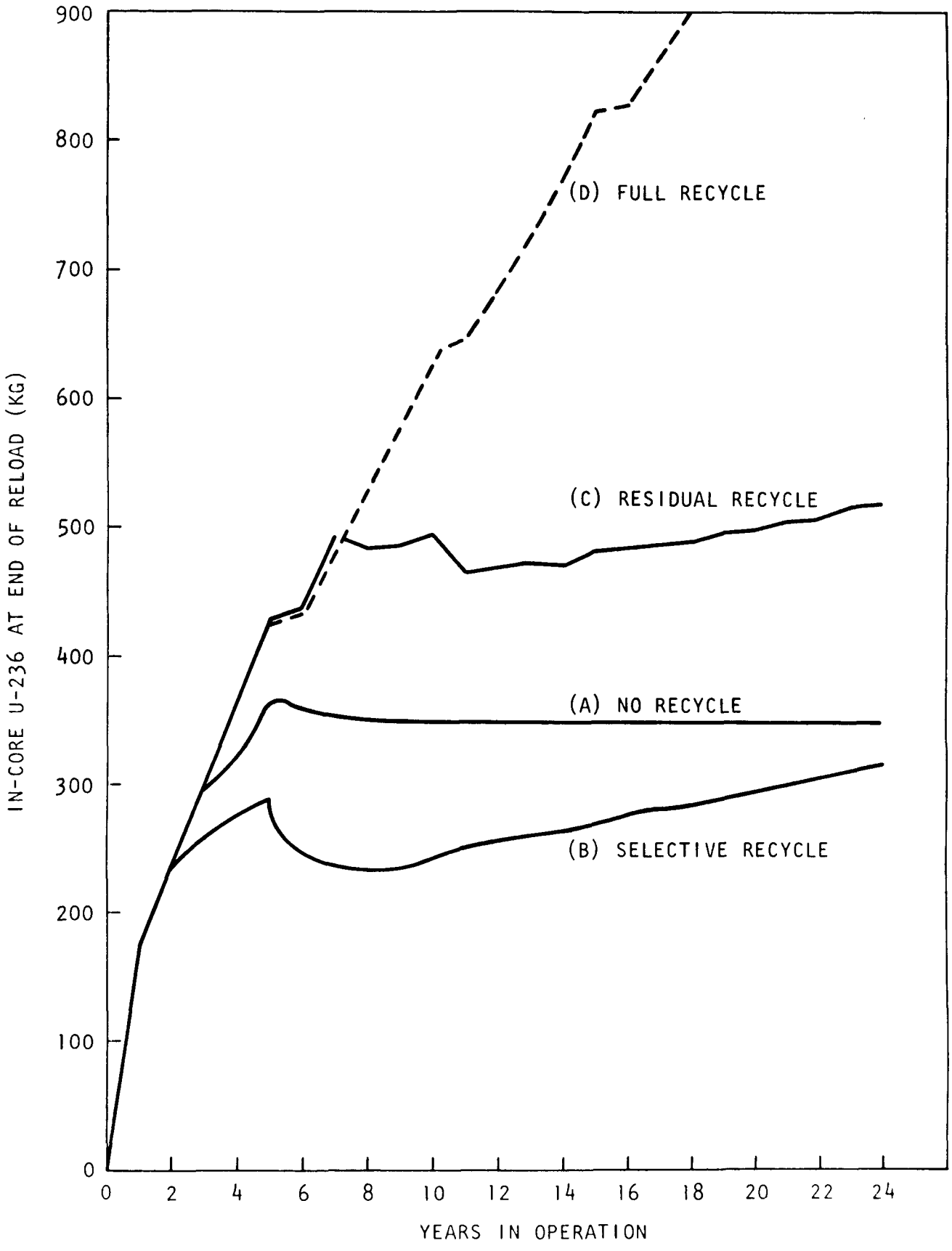


Figure 2 - In-core U-236 inventory (1160 MW HTGR).

Strategy C. This is the current reference strategy for the U.S. HTGR program. The bred uranium is recovered and continuously recycled as in Strategy B. The recovered feed uranium is recycled once more through the reactor, at the end of which it is recovered in the reprocessing plant and buried. It is assumed to have zero value. The average U-236 in-core inventory is about 500 kg for an 1160 MW HTGR or about .43 kg/MW(e).

Strategy D. In this strategy, all of the discharged uranium is continuously recycled. Hence the U-236 steadily builds up, as shown in Figure 2. Strategy D can result whenever a mixed thorium-uranium oxide or carbide is used as the basic fuel, or when the discharged separate fissile and fertile particles are mixed in the reprocessing plant.

The discharged feed uranium has a U-235 enrichment of about 30%, as shown in Table 2. The U-236 enrichment is about 50%. After one more 4 year cycle through the core (i.e., Strategy C), its fissile enrichment is only about 4% and its value is negligible.

U-236 Cross Sections

As mentioned earlier, the poisoning effect of U-236 is proportional to its effective cross section. The majority of parasitic neutron absorptions that occur in U-236 result from neutron captures in the large resonance at about 6 electron volts. The relative average cross section for various recycle strategies is shown on Figure 3. For a self-generated recycle mode the concentration of U-236 increases with time. The composition of recycled fuel also changes with exposure and with the particular mode of recycle assumed. Both of these effects have been included in the U-236 cross section calculations summarized in Figure 3. The estimated number of recycle blocks per reload at equilibrium which contain the recycle U-235 with high U-236 content is also given.

The average cross section in the case of full recycle steadily decreases since the U-236 continues to build up as the uranium is recycled.

Table 2

Isotopic Content of Enriched Uranium Irradiated in HTGR
(Percent)

	U-235	U-236	U-238
Initial feed uranium	93	0	7
Discharged feed uranium	30	50	20
Recycle uranium charged	30	50	20
Discharged uranium (8 years total exposure)	4	70	26

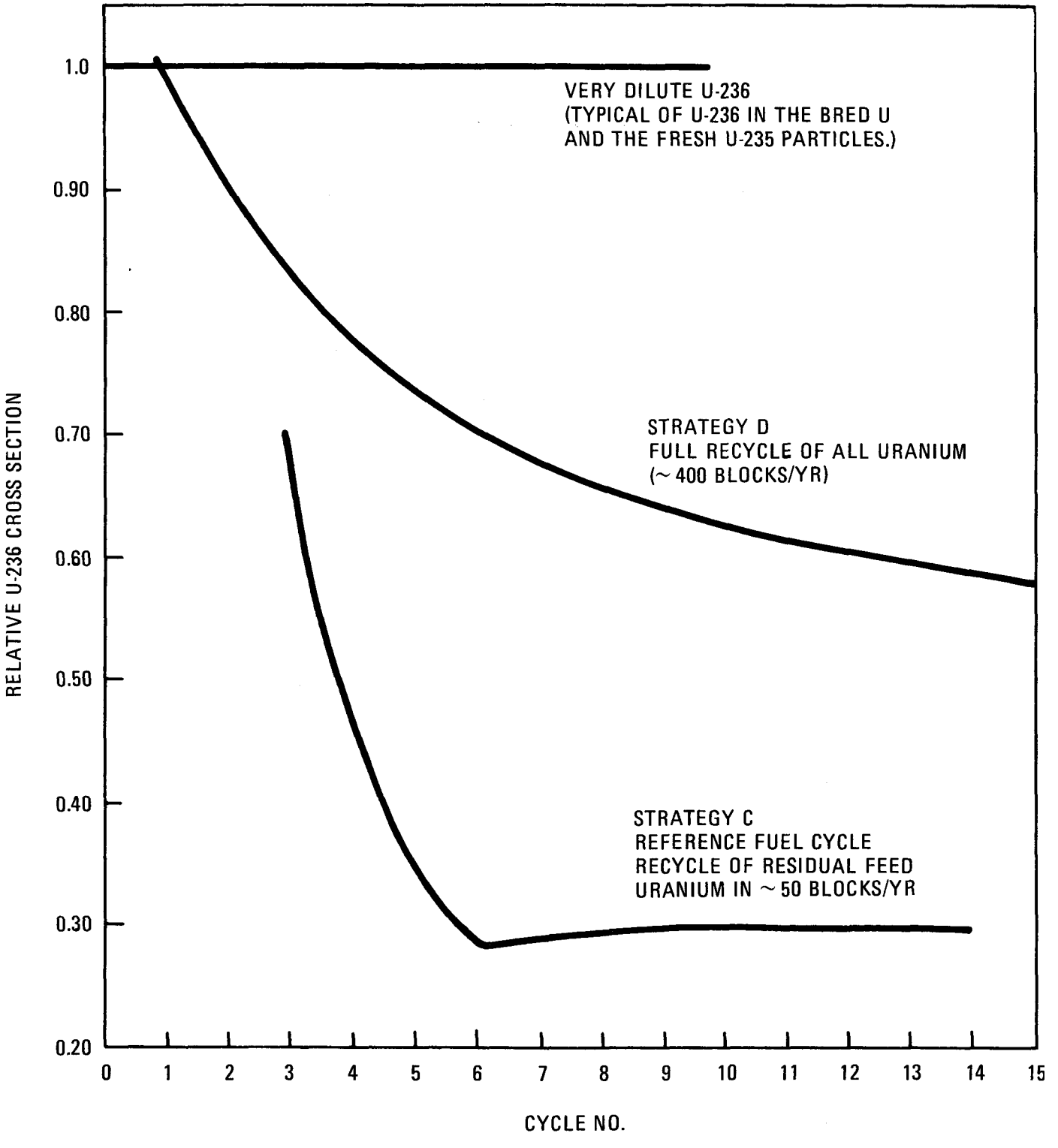


Figure 3 - Relative U-236 Cross Section vs. Cycle For Various Recycle Strategies

For the reference cycle, the U-236 cross section decreases to a constant value since the in-core inventory is limited by the yearly disposal of the residual feed uranium which has been recycled once. The effective cross section is low since the U-236 is concentrated into a few number of blocks. High uranium loading per element is required for these fuel blocks due to the fact that the residual uranium is only about 30% enriched. Such concentration leads to significant self-shielding of the U-236.

In addition to the grain and fuel rod self-shielding effects, the reference residual recycle strategy employs positioning the residual U-235 recycle blocks near the bottom reflector. In these near-reflector locations, the ratio of epithermal to thermal flux is lower than the core average ratio. This further reduces the effective U-236 cross-sections in the residual U-235 recycle elements. The relative cross section shown in Figure 3 takes into account both the rod shielding and the spatial dependence effect on the U-236 cross section.

The details of determining the effective U-236 cross section in the residual U-235 recycle fuel is given in the Appendix of this report.

III. U-236 PENALTY TO FUEL CYCLE COST

The effect of U-236 on the fuel cost has historically been handled in several ways, some incorrect. Generally, fuel cycle studies take into account the conversion of U-235 to U-236, and the effect of neutron capture in the U-236 so formed on fissile material requirements. The presence of U-236 in the system will result in increased U-235 requirements for both inventory and depletion. This neutronic effect is a penalty and is discussed in this report.

The effect of U-236 on the value of the discharged fuel has frequently not been taken into account, however. United States ERDA regulations specify uranium values based on an enrichment computed as the weight fraction of U-235 in total uranium; i.e., any contained U-236 dilutes the enrichment, and hence the value, of the total uranium exactly as if U-236 were U-238. Uranium that contains U-236 is, in fact, less valuable than uranium which contains no U-236. Implicit in this statement is the fact that U-236 is not as valuable a nuclear "fuel" as U-238. If the value of the U-235 discharged from a reactor is properly reduced as a result of the contained U-236, the depletion cost will increase while the in-core inventory cost will decrease. The net result is an increase in fuel cost. This is one component of the U-236 penalty discussed in this report.

Effect of U-236 on Discharged Feed Uranium Value

The majority of the U-236 in-core inventory in the HTGR results from parasitic neutron captures in the U-235 in the feed uranium. The feed uranium is recovered and recycled one more time in the reference

cycle strategy. At equilibrium, the discharged feed uranium is typically 30% enriched in U-235 and contains about 50% U-236.

The recycled feed uranium fissile and fertile loadings per element are adjusted such that those elements have the same power matching characteristics as the fresh makeup elements they replace. (See Appendix.) The low enrichment requires that the uranium loading per block in these elements be ≥3 times the loading of fresh makeup blocks in the same core location. This lumping effect significantly reduces the U-236 epithermal resonance and thus reduces its negative parity.

In addition to the resonance shielding effect, there is a spatial shielding component due to the positioning of the recycled elements near the bottom reflector in a high thermal-to-epithermal flux region. The combination of both effects is to reduce the effective U-236 cross section, and parity, to ~30% of the infinite dilute value characteristic of the U-236 value in the bred uranium fuel.

Detailed "indifference" calculations were performed in which mass flows for non-recycle and recycle of feed uranium of varying compositions were compared. The ERDA "book value" of the discharged feed uranium was varied until equal fuel costs were obtained for the non-recycle and the various recycle cases. These results are shown in Figure 4 where the fractional book value and the equivalent negative U-236 parity are plotted as a function of the ratio of U-236-to-U-235 in the irradiated feed uranium.

From Figure 4 it is seen that the negative U-236 parity value in discharged feed uranium varies from -.25 to -.20 depending on the composition of the discharged uranium. This is about one-third of the value of U-236 in the bred fuel stream.

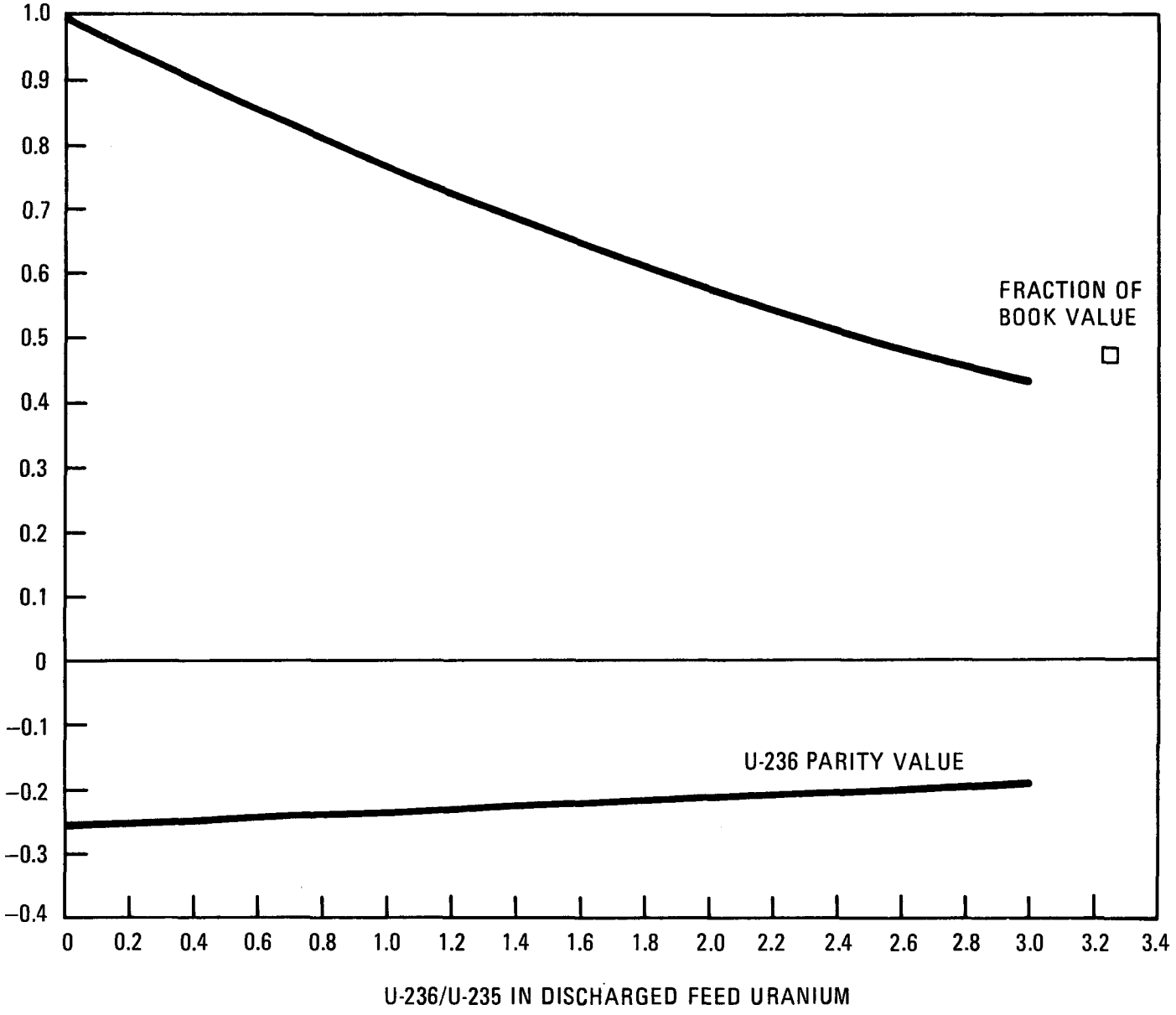


Figure 4 - Relative Book Value and U-236 Parity in Discharged Feed Uranium.

Effect of U-236 on Bred U Value

The value of the bred uranium will change with the number of times the material has been recycled through the reactor since the higher isotopes U-234, U-235 and U-236 will build up. The rate at which this occurs is shown on Figure 5. The U-233 enrichment drops from 92% for 1-year-old fuel (which will be loaded again 2 years after reactor startup) to 60% for 20-year-old recycle fuel. Within that time period, U-234 and U-235 reach an equilibrium enrichment of 25% and 8.5% respectively. The U-236 enrichment continues to increase.

The effect of this changing isotopic content on the relative inherent or neutronic value of the contained U-233 is shown on Figure 6 assuming the value of the U-235 is determined solely by ore and enrichment costs. "Uncontaminated" U-233 has a value relative to U-235 in 90% enriched uranium of about 1.43. As U-234 and U-236 build up, the U-233 value as deduced from indifference calculations drops, so that after 25 years of operation, the effective value of the U-233 in the bred uranium is about 1.35. The U-233 value averaged over 15 years of plant operation is about 1.39, and this is the number frequently used in fuel cycle cost evaluations.

An equivalent method of determining the changing bred uranium value is to assign a value to all uranium isotopes, U-233, U-234, U-235 and U-236. The result is:

	Value Relative to U-235
U-233	1.43
U-234	0
U-235	1
U-236	-.58

That is, U-236 in very dilute concentrations such that its absorption cross section is unshielded has a negative value which is 58% of the value of U-235 in 90% enriched uranium. For example, if U-235 is worth $20/gm, U-236 would be worth -$11.6/gm. The above values assume a working capital rate of 10%. At 15%, the dilute U-236 value is -.65, or -$12.9/gm if U-235 is $20/gm. The next section contains descriptions of the methods for obtaining these values in bred uranium.

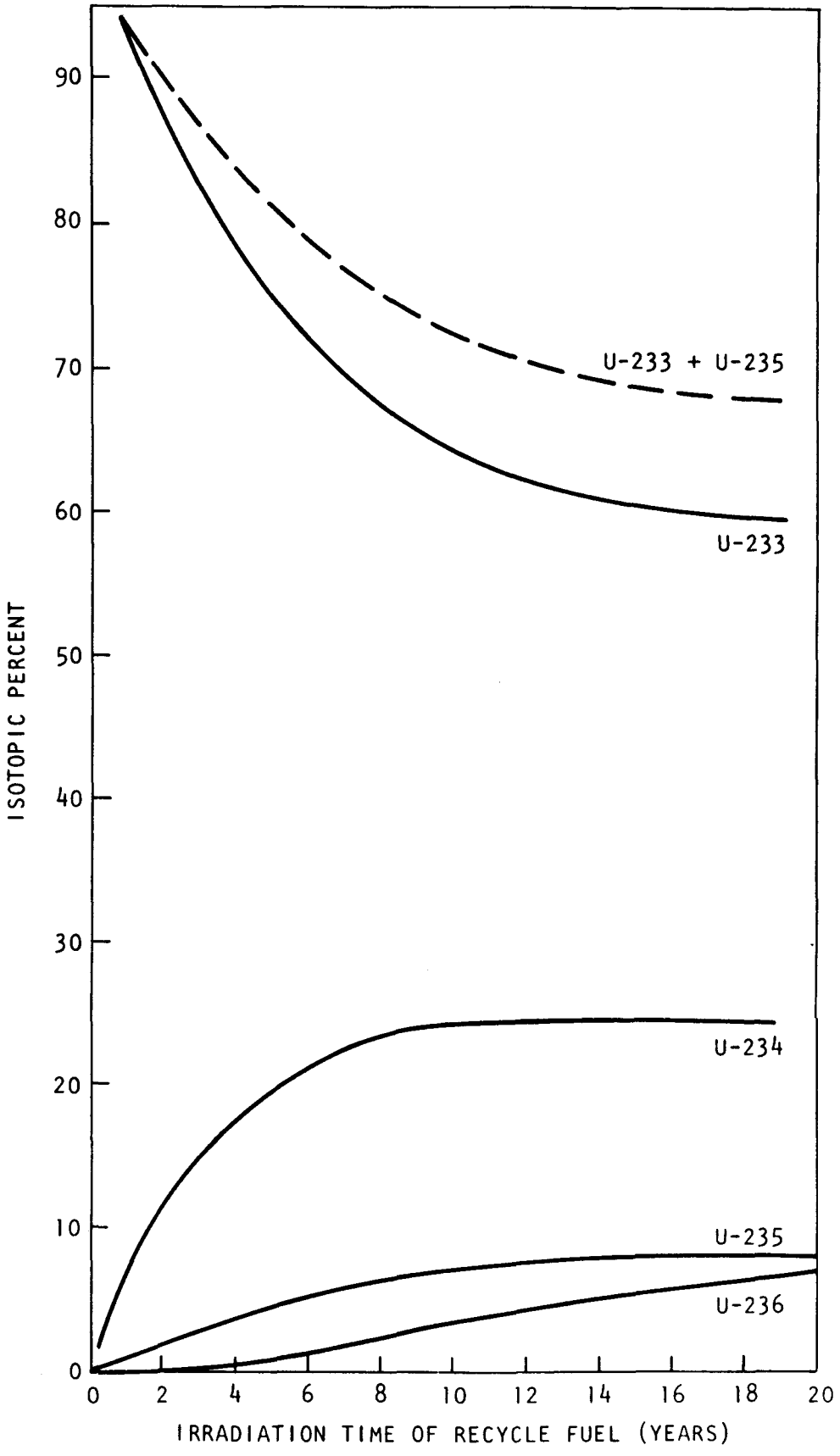


Figure 5 - Isotopic composition of recycle fuel as a function of irradiation time.

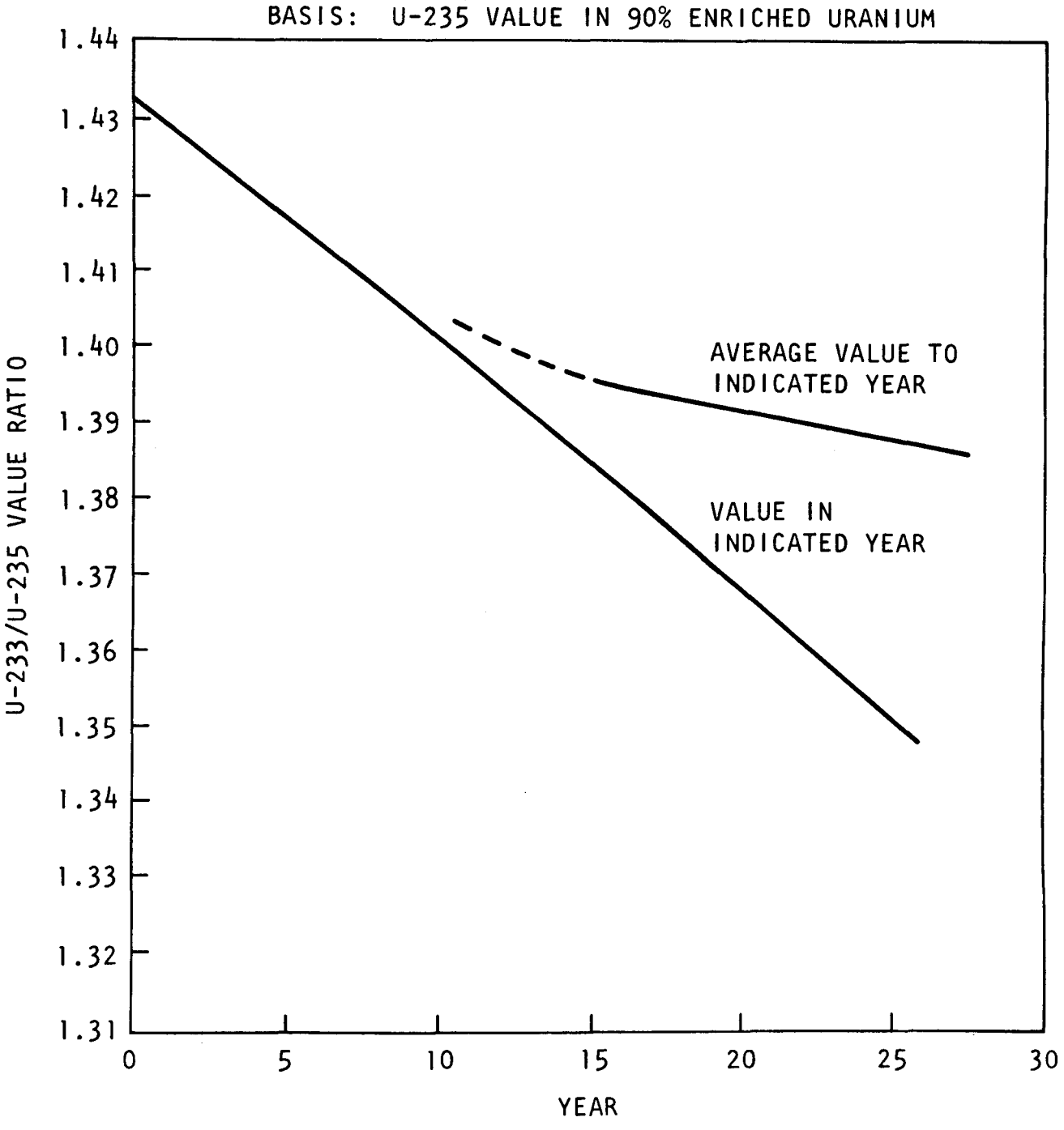


Figure 6 - Relative U-233 value.

IV. SUMMARY OF METHODS

The U-236 resonance cross section is a function of the U-236 concentration and the recycle fuel parameters, i.e., size and composition. Furthermore, for all modes of recycle operation, the U-236 concentration and its cross section change with time. Thus, since the negative value of U-236 is proportional to the neutron absorption rate in U-236, the deduced $/gram value of U-236 will vary with time in proportion to its cross section in the reactor. These facts lead to complications in evaluating in-core as well as out-of-core fuel values since the parity value of U-236 in the fuel mixture is a function of time into recycle, concentration, and its refabricated configuration, i.e., its degree of lumping, particle size, etc.

Three independent methods have been used to calculate the U-236 value based on an infinite dilute U-236 resonance cross section. These three methods all yield a U-236 value in the range of -11.5 to -13$/g of U-236 based on a U-235 value of $20/g. The three methods of determination all yield an upper limit, i.e., unshielded, value of U-236. In practical reactor situations in which the U-236 is shielded the methods can be refined to yield a more accurate value which is consistently lower than for the unshielded conditions.

The methodology used and the results obtained for the three methods are given below. The most accurate method is Method C and the results of that method have been used for determining the final reference U-236 values.

Method A (Perturbation Analysis). In this method a small quantity (1 kg) of U-236 was introduced into a depletion calculation for a typical HTGR reactor at approximately equilibrium conditions. The change in U-235 makeup requirements over the next 3 cycles (12 years) resulting from that perturbation was compared to the unperturbed reference case values. The present worthed sum of the resultant U-235 mass changes was used to obtain the ratio ΔM U-236/ΔM U-235. A 10% discount factor was used to present worth the sum of the U-235 mass flow changes back to the time of the U-236 perturbation.

The result of this calculation was that 1 kg of U-236 introduced into the reactor increased the present worthed U-235 makeup requirements by 0.59 kg. This ratio is equivalent to a -0.59 parity ratio for U-236 or -$11.8/g if the highly enriched U-235 value is $20/g. This result is in close agreement with the Method C (regression analysis) nonshielded result which yielded -$11.5/g with a 10% working capital rate.

Method B (Residual Fissile Indifference Value). This method of determining a U-236 parity value is based on determining the book value of the residual fissile material in the reference design by an indifference technique and then relating the reduction from true book value to the amount of U-236 present in that fuel. This method has the advantage that no bred fuel (U-233) is present in the residual makeup fuel and the complication of properly separating the U-233 effect from the U-236 effect is obviated with this method.

The indifference technique for determining the percent of book value of residual makeup fuel in the HTGR is to:

1. Obtain heavy metal mass flows for both residual recycle and selective recycle modes of operation. Residual recycle refers to recycle of both bred U-233 and residual fissile fuel in separate recycle fuel blocks. Selective recycle refers to recycling the bred U-233 but not the residual makeup fuel, i.e., it is assumed to be sold.

2. Perform fuel cycle cost calculations for both modes of recycle and vary the assumed value of the residual fissile material for both modes of recycle. This is typically done by assuming different values of the reprocessing loss fraction for the residual makeup fuel.

3. Determine the indifference value of the residual makeup fuel for which the fuel cycle costs are the same for the two modes of recycle. This is the true value of that fuel for which the reactor operator is indifferent to whether he sells the fuel or recycles it.

4. Compare the true value of the residual makeup uranium to the book values to determine a percent of book value. Book value is the AEC value, which assumes no explicit penalty for U-236 in the evaluated fuel mixture.

The residual recycle mode involves segregating the recycled residual uranium into as few elements as possible when it is recycled back into the reactor. For the first two or three segments discharged from the initial core, the composition and weight of the discharged residual uranium vary as does its U-236 content. From the fourth and subsequent segments, the discharge fissile material composition is approximately constant. Typical discharges equal 120 kg total uranium of which 25% is U-235 and 53% is U-236.

When recycled back into the reactor, this material is heavily lumped and the U-236 cross section is highly self shielded. Typical values are 1.5 to 2.0 kg uranium per block, which results in a reduction in the U-236 cross section to 30% of its infinite dilute value.

The difference between 100% book value and the true value of this fuel is due to the U-236 poisoning effect when U-236 is heavily shielded,

i.e., for the shielding factor g = 0.28. Dividing the result by the shielding factor yields the infinite dilute U-236 penalty. Thus:

$$\$/g \text{ dilute U-236} = - \frac{V_{100\% \text{ book}} - V_{\text{actual}}}{g \text{ U-236} \times 0.28}$$

Detailed indifference calculations yield a residual uranium value at equilibrium which is ≈60% of the ERDA book value. At 25% U-235 enrichment, the book value of the fuel is given by g U-235 x \$/g U-235 at ε = 25%. With the cost assumptions used in this analysis, the value of U-235 was \$19.08/g at 25% enrichment. Table 3 summarizes the U-236 value calculation.

Table 3
U-236 VALUE BY METHOD B

Average Discharge Composition	100% Book Value (\$)	60% Book Value (\$)	Δ Value (\$)	Shielded (\$/g U-236)	Nonshielded (\$/g U-236)
120 kg total U 30 kg U-235 64 kg U-236 26 kg U-238	572,000	343,000	-229,000	-3.58	-12.79[(a)]

[(a)] U-236 parity ratio $\frac{-12.79}{20.00} = -.64$

It should be pointed out that the indifference method for determining the percent of book value of residual makeup fuel yields an accurate value of the fissile particle U-236 penalty in the reference HTGR design if done carefully and by reload interval. Such a calculation will yield approximately a 95% book value for the first segment discharged (low U-236) down to a value of approximately 60% book value (high U-236) for equilibrium

segments discharged. However, this is generally not done in evaluations in which a constant percent book value is applied to all segments used in levelized cost calculations. In this case an average value of ~70% of book value will result as the value of the discharged feed material. For fuel values yielding $20/gm U-235 in fully enriched uranium, the accurate method will yield a U-236 penalty of typically 0.017 mills/kw-hr. The 70% book figure will yield a 0.033 mill/kw-hr penalty.

Method C (Regression Analysis). The final and most detailed method used for evaluating the U-236 value involved a simultaneous evaluation of the $/gram value of both U-233 and U-236 relative to U-235 in the recycled fuel mixture resulting from assuming continuous recycle of both bred fuel and residual makeup fuel. This method of analysis was used to determine a U-236 parity value both for an assumed infinite dilute U-236 cross section and for a shielded U-236 cross section appropriate to using a 300-μm pure U recycle particle.

A regression analysis technique was used in this method and is explained briefly here and in more detail in the following section. Very briefly, the method simultaneously estimates the value of both U-233 and U-236 relative to the U-235 value in the recycle fuel mixture of bred and residual makeup fuels. It is assumed that the actual value of the fuel mixture, as determined from indifference calculations by reload interval or by segment, is determined solely by the isotopic content of the fuel mixtures; that is,

$$V^{\text{seg } i}_{\text{mixture}} = M^{i}_{233}U_{233} + M^{i}_{235}U_{235} + M^{i}_{236}U_{236} \quad , \tag{1}$$

where $V^{\text{seg } i}_{\text{mixture}}$ = value, in $, of the recycled fuel in segment i as determined from a segment-by-segment indifference calculation

$M^{i}_{233,235,236}$ = mass, in grams, of the U-233, U-235, and U-236 in the recycle fuel in segment i

$U_{233,235,236}$ = value, in \$/g, of the three uranium isotopes in the recycle fuel in segment i

Many segments, corresponding to different times in the cycle, were considered simultaneously in this method. The \$/g value of U-235 was considered known and equal to the standard enrichment table value, i.e., the ERDA book value. Both U-234 and U-238 were assumed to have a zero or negligibly small value and were neglected in the analysis. Independent calculations performed showed that this was a valid assumption.

Equation (1) was rearranged to yield directly the U-233 and U-236 parity ratios. This solution yielded a U-233 parity value of 28.7/20 of the U-235 value in bred fuel. The U-236 value for an infinite dilute cross section was -11.50/20 at 10% working capital and -12.99/20 at 15% working capital rate.

Both a selective-recycle case and a continuous-recycle-all case were used to determine the actual indifference value of recycled fuel by segment, i.e., the $V^{\text{seg i}}_{\text{mixture}}$ values of Eq.(1). In the selective-recycle case, the U-236 content is relatively low in bred fuel since it only appears due to successive neutron captures in uranium starting with U-233. In the continuous-recycle-all case, the U-236 continuously builds up, primarily from the continuous recycle of the residual fissile makeup material, mixed directly with the bred fuel. Mass flow data by segment, along with the resultant recycle fuel indifference value, from both these cases were used simultaneously in the regression, since this yielded data for both low and high U-236 concentrations and allowed a better "fit" to the data.

The continuous-recycle-all case, which is Strategy D described earlier, was used in this evaluation for the reasons given in the previous paragraph. That strategy is not the reference strategy because of the high U-236 penalty that would result from such continuous recycle operation.

A more detailed discussion of this method, as well as result of the value calculations, follows.

V. DETAILS OF REGRESSION ANALYSIS AND RESULTANT FUEL VALUES

Heretofore, the value of bred uranium fuel has been determined solely by the isotopes of U-235 and U-233:

$$V = m_{235}v_{235} + m_{233}v_{235}p_{233} \qquad (2)$$

where V = total value of the uranium fuel

m_i = mass, in grams, of isotope i

v_i = value, in \$/g, of isotope i

p_i = parity ratio for isotope i = $\frac{v_i}{v_{235}}$

Thus, the primary determinant is the per-gram value of U-235, which in turn depends on basic costs such as uranium ore and separative work units of enrichment. The U-233 contributes to fuel value in direct proportion to its parity ratio, p_{233}, which is its value relative to U-235 and thus depends on the reactor design for which fuel is being valued. In HTGR calculations, the parity ratio most generally employed in fuel valuation is the indifference parity, which corresponds to that U-233 value for which the reactor operator is indifferent between recycling the U-233 back into his reactor or selling it on the market and purchasing additional U-235 to make up for the retired U-233. The indifference parity thus represents a market value for U-233 relative to a market value for U-235.

It is known, however, that U-236 contributes a degradation to the uranium value by its presence. This is known from mass flow calculations, which indicate an increased makeup U-235 requirement upon the addition

of U-236 to the system. The goal of the present study was to arrive at a per-gram value, or a parity ratio, for U-236 to be used analogously to that of U-233 in fuel value determinations. The value of the U-236 manifests itself when present in the reactor by means of an enlarged U-235 makeup requirement. Having a parity ratio for U-236 will permit U-236 to manifest its influence on value not only in the reactor but external to the reactor, such as at the time of discharge from a reactor or on the open market where fuel, new and used, will be bought and sold. The total fuel value would then be given by

$$V = m_{235}v_{235} + m_{233}v_{235}p_{233} + m_{236}v_{235}p_{236} \quad , \qquad (3)$$

where the terms are as defined above. Note that p_{236} will have a negative sign since U-236 degrades fuel value. The other isotopes of uranium, such as U-234 and U-238, continue to be neglected since they have very low values compared with the fissile isotopes and with U-236.

One way to try to solve this equation is, quite obviously, to take data for two segments and solve two equations for the two unknowns, p_{233} and p_{236}. This was tried and gave inconsistent results. The reason is that very small changes in V cause very large changes in p_{236}, and V has some amount of randomness in it. This randomness in total value of a segment is a consequence of the six other segments which are present in the core during the residence time of the segment in question. This can have a significant influence on the depletion and working capital costs in the segment being analyzed. Consequently, Eq.(3), instead of being

$$V = m_{235}v_{235} + m_{233}p_{233}v_{235} + m_{236}p_{236}v_{235}$$

should be

$$V = m_{235}v_{235} + m_{233}p_{233}v_{235} + m_{236}p_{236}v_{235} + \varepsilon \quad , \qquad (4)$$

where ε is a random variable. Thus, estimates must be made for p_{233} and p_{236} using statistical techniques rather than simultaneous equation techniques.

Before proceeding, the equation was manipulated somewhat. Dividing

through by total uranium mass in the segment puts the equation in terms of parities and enrichments,

$$\frac{V}{m_{Utotal}} = v' = e_{235}v_{235} + e_{233}p_{233}v_{235} + e_{236}p_{236}v_{235} \quad .$$

Then dividing by v_{235} (a known) gives

$$\frac{v'}{v_{235}} = e_{235} + e_{233}p_{233} + e_{236}p_{236} \quad .$$

Rearranging gives

$$v = \frac{v'}{v_{235}} - e_{235} = e_{233}p_{233} + e_{236}p_{236} \quad ,$$

where v is a computed parameter with no physical meaning. All elements of this equation are dimensionless. Introducing the random term, ε, and using vector notation leads to

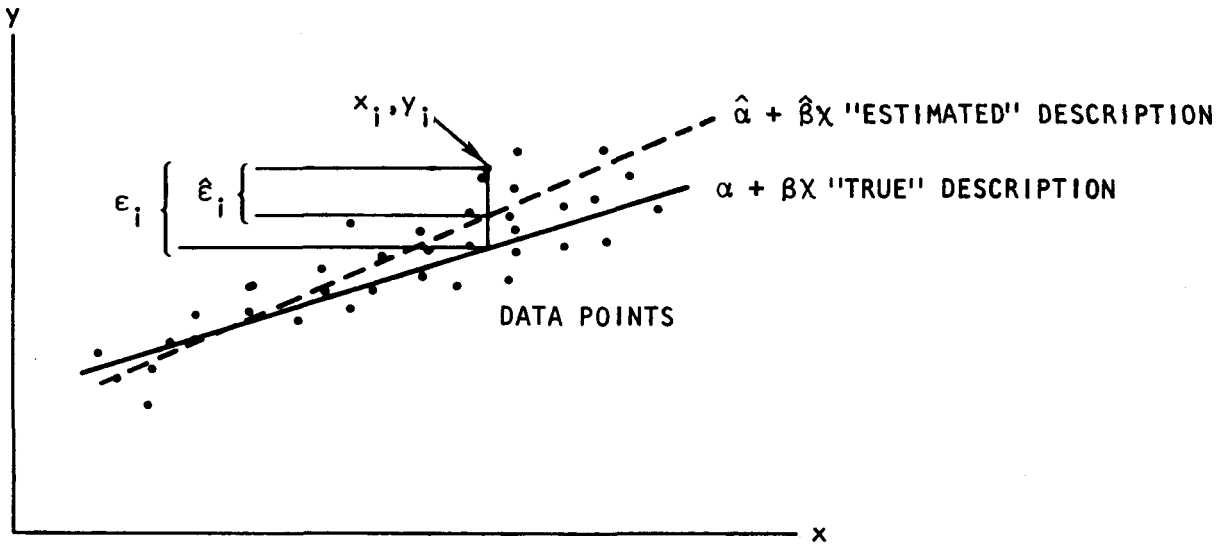
$$\underline{\vec{v} = E\ P + \vec{\varepsilon}} \qquad (5)$$

where

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \cdot \\ \cdot \\ \cdot \\ v_n \end{bmatrix}, \quad E = \begin{bmatrix} e_{233_1} & e_{236_1} \\ e_{233_2} & e_{236_2} \\ \cdot & \\ \cdot & \\ \cdot & \\ e_{233_n} & e_{236_n} \end{bmatrix}, \quad P = \begin{bmatrix} p_{233} \\ p_{236} \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_n \end{bmatrix}$$

and n is the number of segments (i.e., data points) to be considered in the statistical evaluation.

Regression Analysis. The basic concept in regression analysis is best described by the probably more familiar terminology of "least-squares fit." Considering a two-variable case for ease of graphical display, the attempt is to find the best estimate, using available data, of the true relationship that would be obtained with perfect information. The figure below shows the situation.



In the above figure, $\alpha + \beta\chi$ represents the line that would best describe the data with perfect information so that the true relationship for the data is

$$y_i = \alpha + \beta\chi_i + \varepsilon_i \quad i=1,\ldots,n$$

for n data points or observations. Given the less-than-perfect information obtainable in reality, we obtain the line

$$\hat{y}_i = \hat{\alpha} + \hat{\beta}\chi_i$$

such that at any data point the deviation is

$$\hat{\varepsilon}_i = y_i - \hat{y}_i = y_i - \hat{\alpha}_i - \hat{\beta}\chi_i \quad .$$

These residuals are either positive or negative. Squaring them, however, and summing over all data gives the traditional sum of squared residuals

$$\sum_i \hat{\varepsilon}_i^2 = \sum_i (y_i - \hat{\alpha} - \hat{\beta}\chi_i)^2 \quad ,$$

which is non-negative and varies directly with the spread of points from the line. The desired slope and intercept of this line are obtained by minimizing the sum of squares with respect to $\hat{\alpha}$ and $\hat{\beta}$.

This procedure can be easily extended to more than two variables for which

$$y_i = \alpha + \beta_1\chi_{1i} + \beta_2\chi_{2i} + \ldots + \beta_k\chi_{ki} + \varepsilon_i \quad .$$

In vector notation this becomes

$$Y = \chi\beta + \varepsilon$$

where

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad X = \begin{bmatrix} 1 & \chi_{11} & \chi_{21} & \cdots & \chi_{k1} \\ 1 & \chi_{12} & \chi_{22} & & \chi_{k2} \\ \vdots & & & & \\ 1 & \chi_{1n} & \chi_{2n} & & \chi_{kn} \end{bmatrix} \quad \beta = \begin{bmatrix} \alpha \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix} \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} ,$$

If no constant term is present, the column of ones in X and α in β can be eliminated. The minimization as performed above can be applied here in an analogous manner to obtain the best estimate for β (vector of coefficients) such that*

$$\hat{\varepsilon}'\hat{\varepsilon} = (Y - X\hat{\beta})' (Y - X\hat{\beta})$$

(the sum of squared residuals) is minimized, where $\hat{\beta}$ is again the estimated vector of coefficients. The result is

$$\hat{\beta} = (X'X)^{-1} (X'Y) \quad .$$

We now assume the deviation, ε, between the true description of the data, represented by $X\beta$ and the data points, Y, is a random variable for which

$$E(\varepsilon) = 0$$

$$E(\varepsilon\varepsilon') = \sigma^2 I_n \quad .$$

where E( ) indicates "expected value of," σ^2 is the variance in ε, and I_n is the unit matrix of order n.

The first assumption indicates that the expected or average deviation is zero. The second says that the variance in the deviation is constant for all data points, and that the deviation at one point is uncorrelated with that at any other point. Assume further that the matrix X is a fixed set of numbers (nonrandom) and has rank k less than n (i.e., $X_1,\ldots,X_k$ are independent and there are more data points, n, than variables, k). Under the above conditions, the expected value of our estimate $\hat{\beta}$ is β, the true description of the data (with perfect information) and the variance in the estimates is

$$\text{var}(\hat{\beta}) = \sigma^2 (X'X)^{-1}$$

*Note that the "apostrophe" in the equation in this passage connotes the transpose of the matrix to which the apostrophe is appended.

Furthermore, the variance, σ^2, of the true deviation, ε, is estimated without bias by

$$s^2 = \frac{\hat{\varepsilon}'\hat{\varepsilon}}{n-k}$$

U-236 Parity Results for HTGR Bred Fuel. A computer code was written to find the appropriate bred fuel neutronic indifference values, V in Eq.(4) or $\vec{v}$ in Eq.(5), by segment and reload interval. These data together with isotopic masses were punched out to be used in a regression analysis code which made the estimates of parities in the manner described in the preceding paragraphs.

Regression analysis of the continuous-recycle-all (i.e., "full recycle") mode of reactor operation gave estimates with very poor statistics (i.e., large standard deviations), as did anlaysis of selective-recycle mass flows alone. However, combining the data for the two types of operation into a single regression calculation gave estimates with reasonably small deviations. Estimates were made for the case in which the self shielding of the accumulating U-236 in "full recycle" was accounted for, as well as the case in which it was ignored. The regression gives statistical estimates for the U-233 and the U-236 parities simultaneously. Table 4 briefly gives the results.

As the table indicates, the U-233 value is relatively unaffected by working capital rate or by U-236 self shielding. The U-236 parity, on the other hand, becomes almost 15% more negative in going from 10% working capital to 15% working capital. It also becomes less than half as negative when self shielding is accounted for. This of course introduces very strong incentive to maximize the self shielding of the U-236 resonance by such techniques as concentrating the U-236-laden uranium as much as possible. As discussed earlier, this is done in the reference cycle strategy.

Table 4

Statistical Estimates of U-233 and U-236 Parity Ratios in Bred Fuel Stream

	U-236 Nonshielded[a]		U-236 Self Shielded[b] in Full Recycle	
	10% WC	15% WC	10% WC	15% WC
U-233				
Parity estimate	28.62/20 = 1.43	28.62/20 = 1.43	28.21/20 = 1.41	28.21/20 = 1.41
Standard deviation	0.0062	0.0076	0.0038	0.0044
95% confidence interval	1.42 to 1.44	1.42 to 1.45	1.40 to 1.42	1.40 to 1.42
U-236				
Parity estimate	-11.50/20 = -0.575	-12.99/20 = -0.649	-6.63/20 = -0.332	-7.40/20 = -0.370
Standard deviation	0.0157	0.0192	0.0088	0.0104
95% confidence interval	-0.608 to -0.542	-0.689 to -0.609	-0.350 to -0.314	-0.392 to -0.348

[a] Typical values for bred U in reference cycle.

[b] Typical values for full recycle (Strategy D).

VI. CALCULATING FUEL CYCLE COSTS USING U-236 PARITIES

Fuel cycle cost calculations using the results of the U-236 parity determinations were done in two stages. The first stage tested the results against previous fuel cycle cost calculations. The second applied them in some detail to the alternate "full recycle" (Strategy D of Section II) fuel management schemes.

Test Calculations. For the testing stage, the reference selective-recycle fuel cycle was calculated. The U-236 parity for the residual makeup stream was determined in the manner of Method B described earlier in which a U-236 parity value of -0.20 was obtained for the residual makeup particle. For the bred fuel stream in the reference cycle, the U-236 parity of -0.617 was assigned. This corresponds to the regression result in which the U-236 self shielding is ignored, a good assumption for the reference recycle case since very small amounts of U-236 actually build up. This is also the average of the 10% and 15% working capital results, an arbitrary assumption. For U-233, the constant parity of 1.43 was applied.

The previous fuel cycle cost calculations, against which we are testing, used the old techniques for considering U-236 presence, i.e., in this case assuming all fissile fuel discharged at 70% of book value. In particular, for the residual makeup fuel, 30% of the material was assumed "lost" in reprocessing to simulate a reduction in value. For the bred fuel, a levelized, composite U-233 parity of 1.375 was used, composite meaning U-236 was allowed to influence the deduced U-233 parity.

For the test calculations three sets of cases were generated. One used the conventional method of U-236 accounting in both the makeup and bred streams, another used the new parities in both, and the third was an "intermediate" case which used 30% losses in makeup and the new U-236 parity in bred fuel. This permitted isolating comparison of the fuel valuation techniques in the makeup and bred streams. Table 5 presents the fuel cycle cost results for different ore costs and separative work costs.

The basic conclusion from these results is that the indifference and the regression approaches to U-236 accounting agree quite well with respect to a total, levelized (0-15) fuel cycle cost. The yearly results are different, as might be expected, since the indifference techniques utilized levelized average bred fuel values and a 70% book value of discharged fissile material for all segments. Consequently, there is also a difference in the proportion between running cost and working capital cost, especially in the yearly results.

Full-Recycle Fuel Cycle Cost Results. Having demonstrated the validity of the U-236 parity methods, they were applied in cost calculations for the alternate scheme of full recycle.

For these cases, neither U-236 parity ratio used in the reference case is applicable since the U-236 cross section varies by reload interval. A U-236 parity ratio expressed as a function of U-236 enrichment was chosen and set proportional to the one-group U-236 absorption cross section as a function of enrichment. A numerical averaging of individual segment U-236 parity ratios based on this assumption was found to agree well with the shielded, regression analysis U-236 parity value of Table 4.

Table 5

HTGR FUEL CYCLE COST COMPARISON (REFERENCE SELECTIVE RECYCLE)
USING OLD AND NEW TECHNIQUES FOR U-236 PENALTY (15% WORKING CAPITAL)
(In mills/kW-hr)

	"Old" Technique[a]			"Intermediate" Technique[b]			"New" Technique[c]		
	RC	WC	FCC	RC	WC	FCC	RC	WC	FCC
	Running Cost	Working Capital	Fuel Cycle Cost						
$8 ORE, $32 SW									
1-2 yr	0.823	0.689	1.512	0.802	0.695	1.497	0.796	0.696	1.492
14-15 yr	0.980	0.633	1.613	0.988	0.636	1.624	0.993	0.633	1.626
0-15 yr	0.974	0.660	1.634	0.970	0.668	1.638	0.965	0.668	1.633
$8 ORE, $36 SW									
1-2 yr	0.848	0.735	1.583	0.825	0.743	1.568	0.819	0.744	1.563
14-15 yr	1.028	0.681	1.709	1.036	0.683	1.719	1.041	0.681	1.722
0-15 yr	1.016	0.708	1.724	1.012	0.716	1.728	1.007	0.716	1.723
$12 ORE, $32 SW									
1-2 yr	0.873	0.783	1.656	0.848	0.791	1.639	0.841	0.794	1.635
14-15 yr	1.076	0.729	1.805	1.085	0.732	1.817	1.091	0.729	1.820
0-15 yr	1.058	0.758	1.816	1.054	0.765	1.819	1.048	0.765	1.813
[d] $12 ORE, $53 SW									
1-2 yr	1.005	.969	1.974	.971	1.041	2.012	.963	1.048	2.011
14-15 yr	1.329	.986	2.315	1.337	.981	2.318	1.343	.984	2.327
0-15 yr	1.274	1.008	2.282	1.279	1.022	2.301	1.260	1.023	2.283

[a] P-233 = 1.375 in bred fuel; 30% losses in fed fuel.

[b] P-233 = 1.43 and P-236 = -0.617 in bred fuel; 30% losses in fed fuel.

[c] P-233 = 1.43 and P-326 = -0.617 in bred fuel; P-236 = -0.20 in fed fuel.

[d] Typical for $20/gm U-235 value.

Table 6 summarizes fuel cycle cost comparisons for the reference design and for the full recycle modes of operation. All fuel cycle cost results given are based on comparisons of cycles for which only U-236 effects were being evaluated, i.e., all assume essentially the same silicon loading per kilogram of purchased U-235 makeup fuel and all assume the same fabrication cost parameters per kilogram of uranium and thorium. Changes due to TRISO coating, either in fresh fabrication or in refabrication, or other fabrication cost changes have not been included in the results presented.

The fuel cycle cost results presented in both Table 5 and Table 6 are presented for several U_3O_8 and separative work cost assumptions. The results tabulated can be extrapolated to obtain results for higher uranium cost assumptions.

Table 6

HTGR FUEL CYCLE COST COMPARISON USING U-236 PARITY TECHNIQUES

15% WC, TOTAL FCC

(in mills/kw-hr)

	Reference Selective Recycle with Residual Recycle	Full Recycle
(0-15) Levelized FCC		
$8 ORE		
$30 SW	1.589	
$32 SW	1.634	1.659
$36 SW	1.723	1.751
$53 SW	2.098	2.130
$12 ORE		
$30 SW	1.768	
$32 SW	1.813	1.845
$36 SW	1.902	1.935
$53 SW*	2.280	2.320
(14-15) FCC		
$8 ORE		
$30 SW	1.579	
$32 SW	1.626	1.664
$36 SW	1.722	1.763
$53 SW	2.133	2.188

*Typical for $20/gm U-235 value.

APPENDIX

DETERMINATION OF THE EFFECTIVE U-236 CROSS SECTION IN RECYCLE U-235 FUEL

The reference fuel cycle, as now envisioned, consists of recycling once-burned, or residual, U-235 for another four year burn. The difference between the residual makeup uranium (RMU) and makeup uranium (MU) lies in the relative enrichments of the isotopes U-235 and U-236. As can be seen in Table 7, the U-236 to U-235 ratio, a measure of the fuel's isotopic content, is initially zero for MU fuel and takes on a range of values for RMU fuel. The U-236, a parasitic absorber, detracts from the recycle value of RMU in relation to its capture rate, as measured by the effective one-group capture cross section. Therefore, in an effort to enhance the value of RMU, schemes have been devised that reduce these unwanted absorptions, that is, the effective cross section. The following discussion describes such schemes.

Table 7

Isotopic Content of RMU Fuels

Composition	Reload[(a)]	Age[(b)]	Atom Percent				U-236/U-235
			U-234	U-235	U-236	U-238	
A	2	1	1.40	73.06	15.66	9.88	0.214
B	3	2	1.66	52.99	31.55	13.80	0.595
C	4	3	1.77	35.87	45.16	17.21	1.259
D	5	4	1.75	23.13	55.28	19.84	2.390
EQ	18	4	1.67	30.27	50.20	17.86	1.658

(a) Recycle assumed to start at reload 2.
(b) "AGE" means full power years of exposure prior to insertion as RMU fuel.

DISCUSSION

It is well known that U-236 is a strong parasitic absorber due primarily to its resonance behavior at 5.5 ev. The effective resonance group cross section and the overall absorptions can, however, be significantly reduced by lumping the U-236 and taking advantage of the resultant resonance self-shielding. Total self-shielding of this type is composed of grain shielding, which depends upon the size and composition (isotopic content) of the fuel kernel, and rod shielding, which depends upon rod size and the concentration of U-236 in the fuel rod, or equivalently, the number of fuel particles in the rod. In addition, these unwanted absorptions can be reduced by positioning the RMU fuel in the proximity of the axial reflector and taking advantage of a) the "softer" or more thermalized spectrum in these regions, and b) the fact that the axially distributed flux is smallest near the core bottom.

The degree to which RMU may be lumped in order to reduce the U-236 captures is constrained in two ways: 1) a physical limitation imposed by rod size, and 2) the need for RMU and MU fuels to be interchangeable power producers for a given region in the core. The physical limitation is not constraining provided RMU is loaded in regions where the zoning factors are not high. The power-matching need is not easily satisfied since obviously RMU and MU behave differently as nuclear fuel, RMU being to various degrees an inferior fuel. Two measures can be taken to achieve equal power in regions that have fuels with differing characteristics:

a) Increase Σ_f in the region with inferior fuel (RMU) by increasing the loading of this fuel and simultaneously decreasing the loading of the better fuel (MU) in the other region.

b) Reduce Σ_c in the region with inferior fuel (RMU) and increase it in the other region. This can be done by taking LBP and/or thorium from the region with inferior fuel (RMU) and putting it into the other region.

In this study, it was assumed that the MU fissile-fertile loadings were fixed. Furthermore, it was recognized that the LBP loadings of the different regions should be identical lest the power production at the end-of-life diverge. Thus the two measures taken to match RMU and MU power production was to increase Σ_f^{RMU} and decrease Σ_c^{RMU}, the latter done exclusively by reducing, or "diluting", the thorium loading.

Determining the Block Loadings

Methodology

The fissile and fertile loading per block of residual recycle that result in a power match over life with a MU block can be determined with the aid of two criteria. Of the many possible U-235/Th combinations, the proper one is that which

(a) matches K_∞^{RMU} with K_∞^{MU} at BOC.

(b) matches the RMU power scaling factor,[(a)] denoted by S^{RMU} with the MU power scaling factor, S^{MU}.

Condition (a) results in the RMU block having the same reactivity as the MU block and, therefore, approximately the same power. Matching power at BOC, however, does not ensure power matching throughout the exposure history, and, to confuse matters, there are many U-235/Th mixes that result in $K_\infty^{RMU} = K_\infty^{MU}$. Condition (b) determines the "best" U-235/Th combination from the set of such combinations as determined by condition (a). Clearly, the best U-235/Th loading is that which maintains its power level over life, that is, the loading for which $S^{RMU} = S^{MU}$.

The K_∞'s and the parameters needed to calculate the power scaling factors were calculated for several U-236/U-235 values and a variety of fissile-fertile combinations.

(a) This is the GASP[4] scaling factor

$$S = S(r,o) = \sigma_a^{235}\phi \left\{ \frac{\sigma_a Th}{\sigma_a 235} \frac{N^{th}}{N^{235}} - 1.0 \right\}$$

Results

The results of these efforts is the determination of the RMU fissile and fertile block loadings for which the two criteria, matching K_∞ and S, are met (see Fig. 7). However, these are not the final results since the effects of positioning RMU near the bottom reflector have thus far been neglected. To account for this, it was necessary to adjust the loadings predicted from the power scaling calculation.

The change in loadings was deduced by comparing the results of several axial depletion[2] calculations. In one case, axially dependent K_∞ and fluxes were computed for a refueling patch containing makeup fuel only. In the other cases, RMU with compositions A, B, C and D, were placed in the regions adjacent to the bottom reflector. The thorium loading in this region was varied until, again, the RMU regionwise K_∞ and S had the same values as in the case when MU was positioned there. As expected, the thorium content had to be increased to compensate for the increase in reactivity due to the spectrum thermalization. The fissile loading did not require adjustment.

The desired power matching ratios are given in Figure 7. On the basis of these loadings, the resonance (Group 4) self-shielding factors are as shown in Figure 8.

Spatial Flux Distribution Factors

With the knowledge of U-235 and Th loadings per RMU block for a range of U-236/U-235 values, it is possible to calculate correct, heterogeneous reaction rates for RMU nuclides. To this end the FEVER[5] code was used. These reaction rates were then compared with homogeneous (i.e. GARGOYLE[6]) reaction rates and the latter corrected by means of a so-called spatial g-factor which by definition makes the two reaction rates equal. Thus,

$$g^{i,\ell} \cdot [\Sigma^{\ell,i} \cdot \phi^{i}]_{\text{GARGOYLE}} = [\Sigma^{\ell,i} \cdot \phi^{i,R}]_{\text{FEVER}}$$

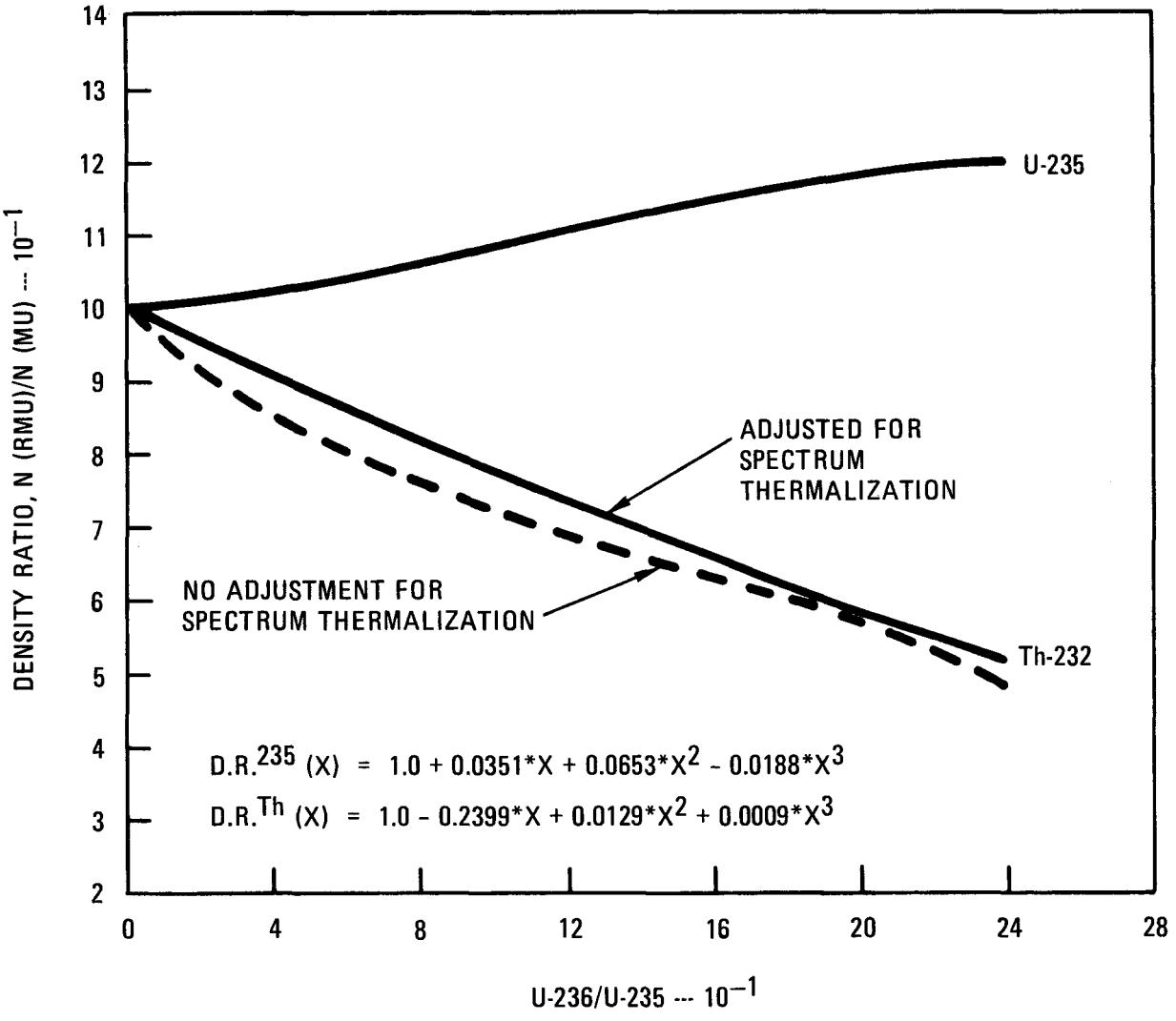


Figure 7 - U-235 and Th Loadings Per Recycle Block

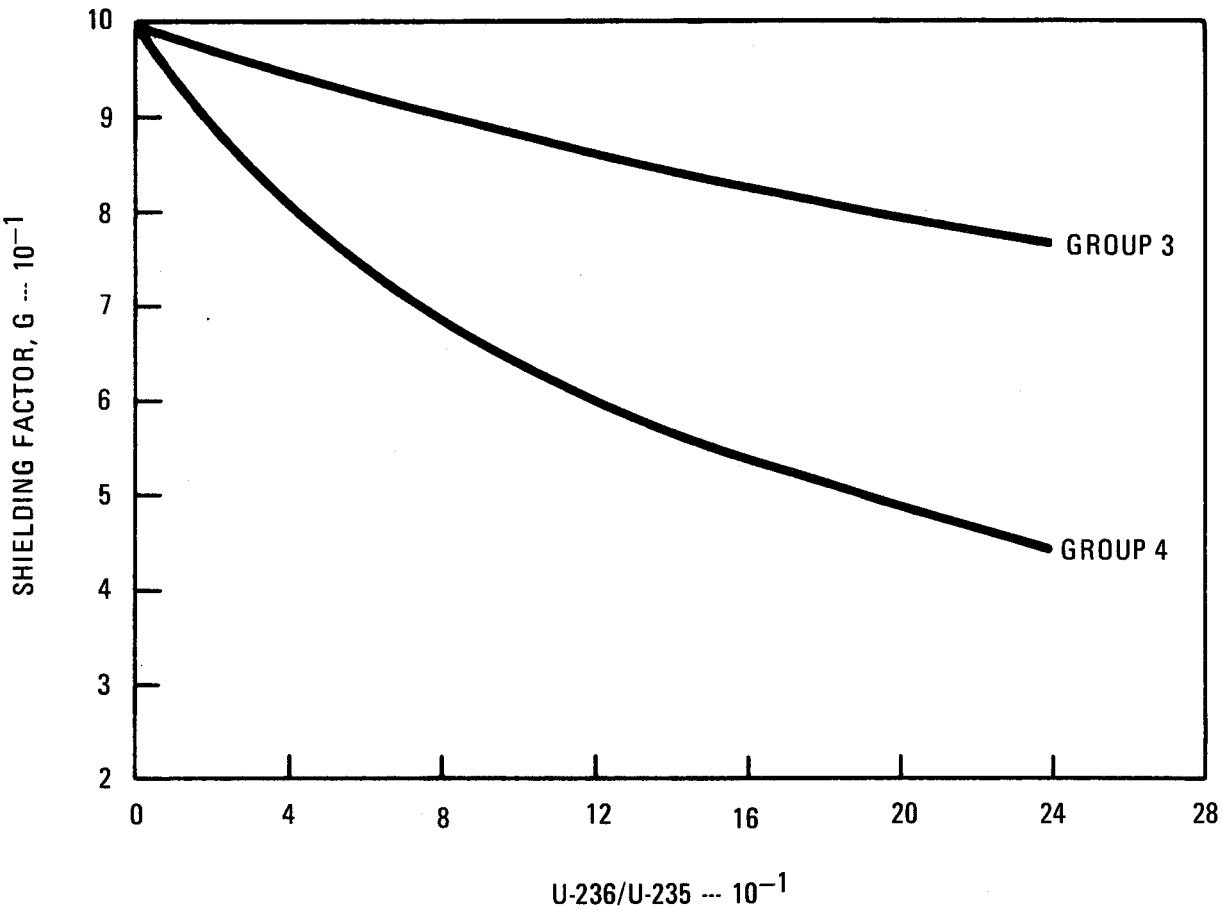


Figure 8 - U-236 Absorption Shielding Factor for Resonance Groups 3 and 4.

where

i = group index

ℓ = nuclide index

R = region index;

= region(s) containing RMU

so that for the same $\Sigma^{\ell,i}$ and fission rate ($\Sigma^F\phi$)

$$g^{i,\ell} = g^i = \frac{\phi^{i,R}\ \text{FEVER}}{\phi^i\ \text{GARGOYLE}}$$

This definition of the flux depression factor should account for a) spectrum thermalization in regions near a reflector, and b) the spatial distribution of the flux, that is, the lower flux amplitude at the core bottom, both of which are important considerations in the calculation of residual recycle reaction rates.

These spatial factors are given in Table 8. The total g-factor is the product of the spatial and energy g-factor. Recall, however, that energy self-shielding is important only for groups 3 and 4, so for all other groups the total g-factor is simply the spatial g-factor. For groups 3 and 4, the product of the spatial and energy g-factor is depicted graphically in Figure 9.

Table 8

Spatial Flux Depression Factors for RMU

	U-236/U-235				
Group	0*	.22	.63	1.63	2.4
1	1.0	.700	.557	.463	.461
2	1.0	.702	.570	.460	.460
3	1.0	.704	.571	.459	.448
4	1.0	.702	.576	.454	.443
5	1.0	.675	.552	.440	.420
6	1.0	.879	.835	.675	.600
7	1.0	.954	.977	.793	.680
8	1.0	.952	.989	.804	.683
9	1.0	.956	1.001	.818	.675

*When U-236/U-235= 0, g is defined to be 1.0.

Using the total U-236 g-factors, and the spectrums calculated by GARGOYLE (incorporating the total g-factor) for reloads in which RMU was recycled, one-group U-236 capture cross sections were computed. This was done for several reloads, that is U-236/U-235 values. The result was shown in Figure 3. It should be noted that at equilibrium (Reload 20) the effective cross section is about 30% of the "infinite dilute" value. Thus the lumping of RMU and the positioning of it near the bottom reflector significantly reduces U-236 parasitic absorptions thereby increasing the recycle value of RMU.

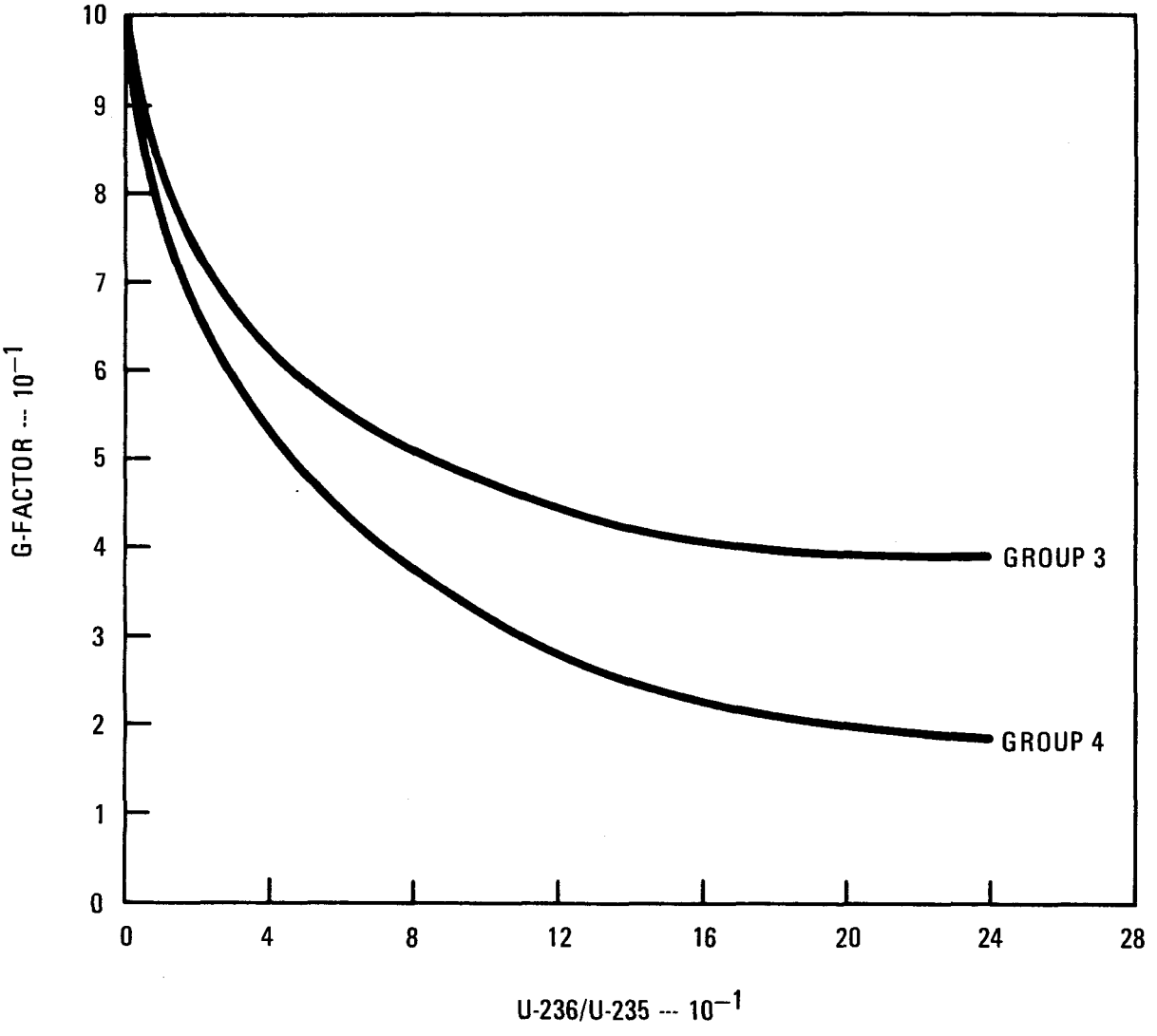


Figure 9 - U-236 Resonance Groups 3 and 4 Total Shielding Factor

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