ANL-HEP-PR-76-03

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January 1976

MASTER

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ARGONNE NATIONAL LABORATORY, ARGONNE, ILLINOIS

operated under contract W-31-109-Eng-38 for the
U. S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

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INTERCEPTS AND RESIDUES OF REGGE POLES IN A STOCHASTIC-FIELD MULTIPARTICLE THEORY

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ABSTRACT

A dynamical theory of multiparticle amplitudes, based on a functional integral representation embodying collective long-range correlations, is applied to the calculation of Regge intercepts and residues. Poles arising in conventional multiperipheral models will characteristically be modified in three ways: promotion, renormalization, and a proliferation of dynamical secondary trajectories, reminiscent of dual models.

We have proposed a theory in I[1] of high energy multiple production phenomena formulated as a stochastic field theory modification of multiperipheral dynamics. In this paper we study the characteristic new features expected for Regge intercepts, compared to conventional multiperipheral models. The intercepts are calculated from a generating functional, as in A-T.[2]

We consider production amplitudes for n secondary hadrons with rapidities y_j, impact-parameters $\vec{b}_j$, and internal quantum-numbers ν_j; the reaction is

$$a + b \to a' + b' + h_1 + \dots h_n .$$

We will simplify the kinematic description as is conventionally done in simple discussions of multiperipheral models, by considering all secondaries to be in the central region; this restriction could be removed at the expense of complex notation.

The A-T "Multi-Eikonal" Ansatz[2] for this production amplitude with a generalization to include quantum numbers, is:

$$\begin{aligned} A_n(\vec{r}_1 \dots \vec{r}_n; \nu_1 \dots \nu_n \mid \vec{R}; \nu_a, \nu_b; \nu_{a'}, \nu_{b'}) = {} & A_o(\vec{R}; \nu_a, \nu_b; \nu_{a'}, \nu_{b'}) \\ & \cdot \prod_{j=1}^{n} G(\vec{r}_j, \nu_j \mid \vec{R}; \nu_a \dots \nu_{b'}) \\ & \cdot \prod_{k} K(\vec{r}_j, \vec{r}_k; \nu_j, \nu_k \mid \vec{R}; \nu_a \dots \nu_{b'}) \end{aligned} \quad (1)$$

where $\vec{r} \equiv (y, \vec{b})$ and $\vec{R} \equiv (Y, \vec{B})$; $Y = \ln S_{ab}$, and $\vec{B}$ is the relative impact parameter between incident particles a and b.

As in AT, we define a generating functional which can be used to express overlap functions and inclusives;

$$F[\xi] \equiv \sum_n \frac{1}{n!} \sum_{\nu_1 \dots \nu_n} \int d^3r_1 \dots d^3r_n\, \xi(r_1, \nu_1) \dots \xi(r_n, \nu_n) \left| A_n\{(\vec{r}_j, \nu_j)\} \right|^2 \Big/ \left| A_o \right|^2 \quad (2)$$

Suppressing notation of dependences on $\vec{R}$ and $(\nu_a \dots \nu_{b'})$, we can write F as:

$$F[\xi] = \sum_n \frac{1}{n!} \sum_{\nu_j} \int \prod_{j=1}^{n} d^3 r_j \; \xi(\vec{r}_j, \nu_j) \left| G(\vec{r}_j, \nu_j) \right|^2 \prod_{k \neq j} \exp\left[V(\vec{r}_j, \vec{r}_k; \nu_j, \nu_k) \right] \tag{3}$$

where we have defined V by

$$\exp(V) \equiv |K|^2 \tag{4}$$

The limits of y integrations in the above expressions are $y = \pm Y/2$; the $\vec{b}$ integrals are cut off by $|G|^2$. Following I, we will assume V can be divided into $V_L + V_S$, where V_S is short range in rapidity, and V_L is small in magnitude compared to unity, but long range in rapidity (dominated by Regge cuts) and of only one sign (presumably positive). Then we can express F as a functional integral over F_S, the short-range-only ensemble generating functional, defined as is F but with no V_L:

$$F_S[\xi] = \sum_n \frac{1}{n!} \sum_{\{\nu_j\}} \prod_{j=1}^{n} d^3 r_j \, \xi(\vec{r}_j, \nu_j) \left| G(\vec{r}_j, \nu_j) \right|^2 \prod_k \exp\left[V_S(\vec{r}_j, \vec{r}_k, \nu_j, \nu_k) \right] \tag{5}$$

We can represent F_S at high energies as a sum over the leading singularities in the Laplace-transform with respect to Y. We may define corresponding functionals $\alpha_m[\xi]$ and $\beta_m[\xi]$ by:

$$F_S[\xi] = \sum_{m=0}^{\infty} \beta_m[\xi] \exp[Y \alpha_m(\xi)] \tag{6}$$

where the summation runs over the leading poles (at a given ξ), with ordering $\alpha_0 > \alpha_1 > \alpha_2 > \ldots$

The one-dimensional parametrization, long-range in rapidity, adopted in I for V_L can be generalized to include $\vec{b}$. If we assume an exponential decrease in $|\vec{b}|$,

$$V_L(\vec{r}) = \frac{1}{(2\pi)^3} \int d^3k \frac{e^{i\vec{k}\cdot\vec{r}}}{\lambda_1^{-\sigma} k_1^{\sigma} + \lambda_2^{-2} k_2^2 + \mu^2} \tag{7}$$

where $\vec{k} = (k_1, \vec{k}_2)$ is Fourier conjugate to $\vec{r} = (y, \vec{b})$. We assume here that $V_L(r_1, r_2)$ depends only on $\vec{r} = \vec{r}_1 - \vec{r}_2$, and also does not depend upon the internal quantum numbers ν_i, as appropriate for a leading-singularity (pomeron) induced cut correction. A cutoff at large k is assumed[(1)] in (7).

Using the approximation[(1)] that short range correlation lengths are much smaller than the characteristic range of V_L, we obtain the A-T generating functional[(2)] in the form:

$$F[\xi_1, \ldots \xi_n] = \frac{\sum_m \int \delta^N\phi \exp\left\{-\int_{-Y/2}^{+Y/2} dy \left(\mathcal{L}_o[\vec{\phi}(y)] + \mathcal{L}_1^{(m)}[\vec{\phi}(y)]\right)\right\}}{\int \delta^N\phi \exp\left\{-\int_{-Y/2}^{+Y/2} dy\, \mathcal{L}_o[\vec{\phi}(y)]\right\}} \tag{8}$$

where, if we ignore the ultraviolet cutoff in V_L for the moment,

$$\mathcal{L}_o[\vec{\phi}] = \int [1/2\, \lambda_1^{-\sigma} (\nabla_y^{\sigma/2} \vec{\phi})^2 + 1/2\, \mu^2 \vec{\phi}^2 + 1/2\, \lambda_2^{-2} (\nabla_{\vec{b}} \vec{\phi})^2]\, d^2b \tag{9}$$

where we will take $\sigma = 1/2$ as in I; and

$$\mathcal{L}_1^{(m)}[\vec{\phi}] = \int \left\{ [\xi_1 e^{\phi_1}, \ldots \xi_N e^{\phi_N}] - 1/Y \ln \beta_m [\xi_1 e^{\phi_1}, \ldots, \xi_N e^{\phi_N}] \right\} d^2b \tag{10}$$

From the formal expression (8) all observables can be calculated, following A& T. The multiplicity generating function, and the Regge-trajectory intercepts, are obtained by specializing to constant $\xi_\nu = Z_\nu$ $(\nu = 1, \ldots N)$.

We wish to investigate here the relation between the intercepts and residues of the short-range-only ensemble, defined by (5), and the final output intercepts and residues calculated from (8).

For this purpose it is sufficient to ignore the impact parameter variables and consider a one-dimensional system. Then we seek to evaluate the behavior, as $Y \to \infty$, of the multiplicity generating function given by (8) with $\mathcal{L}_o$ as in (9) without the $(\nabla_b \phi)^2$ term, and $\mathcal{L}_1$ as in (10) with $(Z_1, \ldots, Z_N)$ replacing $(\xi_1, \ldots, \xi_N)$. We drop the subscript on λ_1 henceforth. The high-lying output intercepts will be determined in general[(1)] by saddle points of the functional integrals in the numerator of (8). We consider each m separately. The saddle-point functions $\vec{\phi}$ will satisfy $\frac{\delta(\mathcal{L}_o + \mathcal{L}_1)}{\delta\phi} = 0$, which gives the set of coupled nonlinear (formally differential) equations:

$$\lambda^{-\sigma} \nabla_y^{\sigma} \phi_j(y) - \mu^2 \phi_j(y) = \rho_j^{(m)}(z_1 e^{\phi_1(y)}, \ldots, z_N e^{\phi_N(y)}) \tag{11}$$

with $j=1, \ldots, N$; $m=0, 1, 2, \ldots$

Here

$$\rho_j^{(m)}(z_1, \ldots, z_N) \equiv z_j \frac{\partial}{\partial z_j} \left[\alpha_m(z_1, \ldots, z_N) + \frac{1}{Y} \ln \beta_m(z_1, \ldots z_N) \right] \tag{12}$$

For each solution $\hat{\phi}^{(m)}$ of (11), the corresponding saddle-point approximation for an associated output intercept and residue is:

$$-\frac{1}{Y} \cdot \int_{-Y/2}^{+Y/2} dy \left\{ \mathcal{L}_0[\hat{\phi}] + \mathcal{L}_1^{(m)}[\hat{\phi}] \right\} = \hat{\alpha}^{(m)} + \frac{1}{Y} \ln \hat{\beta}^{(m)} \tag{13}$$

The leading output intercept (at this level of approximation) will be the largest value of $\hat{\alpha}$ when all possible $\hat{\phi}$'s are used in (13). For large Y, the term involving β_m in (12) plays no role in determining ρ, and we may drop it during calculation of the $\hat{\phi}$'s. The leading $\hat{\alpha}$, for each m, will be expected to arise from a constant $\hat{\phi}$, since contributions from "kinetic" (derivative) terms are negative in $\hat{\alpha}$.

Thus, for the leading intercept, we expect $\hat{\phi}$ to satisfy (11) without the derivative terms. These form N coupled "Mean-Field" equations (for each m):

$$\mu^2 \hat{\phi}_j = \rho_j(z_1 e^{\hat{\phi}_1}, \ldots, z_N e^{\hat{\phi}_N}) \tag{14}$$

Note the residues of the output poles are obtained, in this approximation, by evaluating the short-range β at $(z_1 e^{\hat{\phi}_1}, \ldots, z_N e^{\hat{\phi}_N})$.

When α_o is taken from a "hard-core" (Chew-Snider-Dash-Van der Waals[3]) short-range model, with short-range correlations arising only from kinematic (t_{min}) effects, we obtain from (14) exactly the Van der Waals mixture theory studied (at criticality) by Thomas[4], a very successful description (with essentially no free parameters) of high energy multiplicity distributions.

Corrections to the saddle-point $\hat{\alpha}$'s can be computed by expanding $(\mathcal{L}_o + \mathcal{L}_1)$ in a Taylor series around $\phi = \hat{\phi}$, as explained in I. We can can write the saddle-point expression for (8), keeping only the dominant (constant) $\hat{\phi}$ for each m, as:

$$F(z_1, \ldots, z_N) = D^{-1}\left\{\sum_m \beta_m(z_1 e^{\hat{\phi}_1}, \ldots, z_N e^{\hat{\phi}_N}) \exp\left\{Y\left[\alpha_m(z_1 e^{\hat{\phi}_1}, \ldots z_N e^{\hat{\phi}_N}) - \frac{\mu^2}{2}(\hat{\phi}_1{}^2 + \ldots + \hat{\phi}_N{}^2)\right]\right\} F_m\right\} \tag{15}$$

with

$$F_m = \int \delta\vec{\psi} \, \exp\left\{-\int_{-Y/2}^{Y/2} dy \, \mathcal{L}_2^{(m)}[\vec{\psi}(y)]\right\} \tag{16}$$

We have written $\psi = \phi - \hat{\phi}$ for each m; to 4th order in ψ,

$$\mathcal{L}_2^{(m)}[\psi] = 1/2\lambda^{-\sigma}(\nabla^{\sigma/2}\vec{\psi})^2 + 1/2\mu^2\vec{\psi}^2 - 1/2 \sum_{ij}\psi_i (C_2^m)_{ij}\psi_j - 1/3!\sum_{ijk}(C_3^m)_{ijk}\psi_i\psi_j\psi_k - 1/4!\sum_{ijkl}(C_4^m)_{ijkl}\psi_i\psi_j\psi_k\psi_l \tag{17}$$

where the C_p^m's are defined by:

$$\left[C_p^m(z_1, \ldots, z_N)\right]_{ijk\ldots} \equiv \frac{\partial}{\partial \ln z_i}\frac{\partial}{\partial \ln z_j}\frac{\partial}{\partial \ln z_k}\cdots \alpha_m(z_1 e^{\hat{\phi}_1}, \ldots z_N e^{\hat{\phi}_N}) \tag{18}$$

The factor D is independent of the short-range dynamics and is given by the denominator of (8). Since $\mathcal{L}_o$ is only quadratic in ϕ, we can analytically obtain D if desired, but the expression is not required here.

If the output intercepts are not in the vicinity of a critical point, we may drop terms higher than quadratic in (17); the functional integral in (16) then becomes a quadratic form in ψ, and can be expressed in closed form;

$$F_m = \exp\left\{\frac{-Y}{2\pi}\int dk \, \mathrm{Tr} \ln\left[\lambda^{-\sigma}k^{\sigma} + \mu^2 - \underset{\sim}{C_2^m}\right]\right\} \tag{19}$$

where the trace operation refers to the matrix indices on C_2.

For small λ, the correction to the mean-field result for the intercept from F will be of order λ, unless $\hat{\phi}$ is in the vicinity of criticality, where μ^2 nearly cancels C_2. We will term (19) the "quadratic" approximation to F_m. A cutoff on the k integral in (19) is implied[1], as in (7).

We discuss the shift in trajectory-intercepts, first according to (14), then the role of nonconstant solutions to (11), and finally the influence of fluctuations as in I.

We can distinguish three new characteristic features present in our theory, compared to the simple short-range ensemble.

(1). If the long-range correlations were reduced indefinitely in magnitude, we would find $\mu^2 \to \infty$; thus, from (14), all $\hat{\phi} \to 0$, and the output intercepts $\hat{\alpha}$ would coincide with those of the short-range system alone. As we increase $|V_L|$, μ^2 decreases, and the output intercepts become larger, as determined by $\hat{\phi}$ from (14) and $\hat{\alpha}$ from (13). We may call this a "promotion" of the short-range poles.

(2). For nonzero V_L we encounter also a second phenomenon; (11) will have non-constant solutions which generate secondary (i. e. nonleading) saddle points $\hat{\alpha}$ in (13). There will be usually an infinite number of these, forming a rich spectrum of secondary poles, not present in the short-range ensemble. We will call this a **proliferation** of the high-lying poles. With $\sigma = 2$ and small λ, this is seen explicitly in the similar model calculation of Scalapino and Sugar.[5]

(3). At very large Y, the long range fluctuations will become important, and as in critical phenomena, the true asymptotic behavior associated with the leading intercept will be corrected, compared to the mean-field approximation (14), by a substantial amount. It is only at this stage that the Froissart bound is relevant. We expect generally that this final correction is negative, a downward shift in $\hat{\alpha}$. This can be called a renormalization of the leading intercept. Thus we expect the effective intercept (at FNAL-ISR energies and below) is above unity, but for $Y \to \infty$ only a singularity exactly at unity can appear, with logarithmic corrections.

All three phenomena rely quantitatively on the functional dependence $\alpha_0(z)$ of the short-range ensemble. Thus, the amount of promotion, the characteristics of proliferation, and the final renormalization, will all depend on the quantum numbers of the short-range poles, through the "conventional" MPM dynamics. These dependences may be seen, for example, in the prototype 3-pole MPM, discussed by Pinsky and Thomas[(6)] in the context of inclusive phenomenology, which includes charge symmetry (but not higher symmetries).

The relative magnitudes of the leading-trajectory $\hat{\phi}$'s are dependent, to first order in $\hat{\phi}$, on the magnitudes of the short-range densities ρ_j evaluated at $z \cong 1$. This gives us a systematic way to study the relative shifts in perturbation theory, when $\mu^{-2} \ll 1$, provided the perturbation is not carried out close to a critical point. Unfortunately, this is just the case when the Pomeron is considered. However, we can examine the relative signs of shifts, when $\hat{\phi}$ is unphysically small; the poles then remain well below unity, away from criticality.

Expanding (14) around $\hat{\phi} = 0$ we obtain, to 1st order in $\hat{\phi}$,

$$\sum_j \left[\delta_{ij}\mu^2 - (C_2)_{ij}\right] \hat{\phi}_j = \rho_i \tag{20}$$

where

$$(C_2)_{ij} = z_i \frac{\partial \rho_j}{\partial z_i} \; ; \tag{21}$$

ρ_i and its derivatives are evaluated at $z_1 = \ldots = z_N = 1$ here. The signs of the shifts $\hat{\phi}_j$ are then determined by the matrix

$$M_{ij} = \mu^2 \delta_{ij} - (C_2)_{ij} \, . \tag{22}$$

For $\mu^{-2} \ll 1$, the matrix M is positive definite, so all shifts are positive.

It is instructive to examine the intercept shifts and residue shifts in more detail, and compare our results with those of Chew and Rosenzweig[(7)], and Schmidt and Sorenson[(8)], using the quark-number expansion. For this purpose we need a specific MPM, with quantum number dependent dynamics; the PT model[(6)] would suffice, but an even simpler model first reveals some essential qualitative features, as we now show.

Consider first a single-channel Chew-Pignotti (CP) model, with $\alpha(z) = Gz$. Then we can investigate the dependence of the shift $\hat{\phi}$ upon the value of G. We find $\rho = C_2 = G$ in this case; from (20), $\hat{\phi}$ is given by:

$$\hat{\phi} = \frac{G}{\mu^2 - G} \tag{23}$$

Thus the shift is an increasing function of G, and increases very rapidly as the input intercept (G) is increased toward μ^2. In the critical Van der Waals model(3) we find $\mu^2 = \frac{1}{2a} = \frac{32}{27}$. Thus we expect in realistic models the shift $\hat{\phi}$ will be very sensitive to $\alpha(0)$ for $\alpha(0)$ of order 1/2 or larger. If the short-range pole representing non-quantum-number-exchange is slightly higher than quantum-number-exchange poles, the former will be promoted higher than the latter, even if no further subtleties are involved.

The CP model for the short-range ensemble is inadequate to account for criticality, however, which requires a degeneracy of leading singularities after promotion. It is necessary to include (short range) correlations.

A second feature of short-range dynamics which qualitatively affects the promotion appears when we include N internal degrees of freedom, still within the CP framework;

$$\alpha(z_1, \ldots z_N) = z_1 G_1 + \ldots + z_N G_N \tag{24}$$

with N-fold internal symmetry, we have $G_1 = G_2 = \ldots = G_N$; then we can write

$$\alpha(z_1, \ldots, z_N) = (z_1 + \ldots + z_N)\frac{G}{N} \tag{25}$$

with normalization chosen such that the leading intercept is $\alpha(1, \ldots, 1) = G$ to correspond to the simpler case discussed above.

Now

$$\rho_i = \frac{G}{N} \quad \text{and} \quad (C_2)_{ij} = \delta_{ij}\frac{G}{N}\,; \tag{26}$$

and, from (20),

$$\hat{\phi}_i = \frac{G}{N\mu^2 - G} \qquad (i = 1, 2, \ldots, N). \tag{27}$$

As N increases, for fixed μ^2, the individual shifts $\hat{\phi}_i$ decrease. We find the leading intercept becomes

$$\hat{\alpha} = \alpha(e^{\hat{\phi}_1}, \ldots, e^{\hat{\phi}_N}) - \frac{\mu^2}{2} \cdot \sum_i \hat{\phi}_i^2 = G + \sum_i' (\hat{\phi}_i \rho_i - \frac{\mu^2}{2} \hat{\phi}_i^2)$$

$$= G + \frac{G^2}{N\mu^2 - G} - \frac{\mu^2}{2} \quad \frac{NG^2}{(N\mu^2 - G)^2} \tag{28}$$

In the limit $N \gg G/\mu^2$,

$$\hat{\alpha} \cong G + \frac{G^2}{2N\mu^2} \tag{29}$$

Thus, we see the net promotion of the leading pole decreases as N increases indefinitely. This suggests a reason for limited numbers of internal degrees of freedom in a bootstrap theory; but we will not go into that subject here.

If we wish to now to consider (for comparison of shifts) a quantum-number exchange trajectory, we must introduce dynamics which depend on the quantum numbers; this introduces a third feature, indicating a possibility of a greater shift for vacuum poles than nonvacuum poles.

Consider the relative shifts expected for charge-symmetric vs. charge-antisymmetric exchanges in PT model[6].

I. Leading antisymmetric ρ trajectory (eqn. 3.12 of PT):

$$\alpha^{(3)}(z_{\pm}, z_-, z_0) = z_o C + \ell_2 \tag{30}$$

Associated densities:

$$\rho_+ = \rho_- = 0; \quad \rho_o = C$$

C_2 coefficients: the only nonzero term is $(C_2)_{oo} = C$.

II. Leading symmetric P trajectory (eqn. 3.11 of PT):

$$\alpha^{(1)}(z_+, z_-, z_o) = \frac{1}{2}(z_o A + \ell_1 + z_o C + \ell_2) + \left\{ \left[\frac{1}{2}(z_o A + \ell_1 - z_o C - \ell_2) \right]^2 + 4 z_+ z_- B^2 \right\}^{1/2} \tag{31}$$

Note all three components of $\vec{\phi}$ contribute to the upward shift of $\alpha^{(1)}$, while only one (ϕ_o) contributes to the shift of $\alpha^{(3)}$. Thus, even without a detailed estimate, we expect $\alpha^{(1)}$ will be promoted more than $\alpha^{(3)}$.

The magnitudes of $\hat{\phi}_{\pm}$ are determined by

$$\rho_{\pm} = \frac{2z_+ z_- B^2}{\left\{ \left[\frac{1}{2}(z_o A + \ell_1 - z_o C - \ell_2)\right]^2 + 4z_+ z_- B^2 \right\}^{1/2}}$$

There can be, then, depending on the relative values of A, B, and C, a much larger shift upward in the symmetric channel than in the antisymmetric channel. Thus we see a reasonable mechanism exists for promoting the Pomeron up past unity, while leaving the ρ trajectory well below unity.

Exchange-degeneracy between output poles can only be achieved through the proliferation mechanism. A secondary pole (f) in the Pomeron sector can be degenerate with the leading singularity (ρ) in the antisymmetric sector. However, as the promotion mechanism depends on the same parameters as the proliferation mechanism, such a "conspiracy" need not be accidental.

Further quantitative study of this point will require obtaining nonconstant solutions to equations (11). In the simplest model (CP) we are to study the spectrum of:

$$\lambda^{-\sigma} \nabla_y^{\sigma} \phi(y) + \mu^2 \phi(y) = G e^{\phi(y)}$$

If this equation has an infinite spectrum of nonconstant solutions, we can associate them with dynamically generated secondary poles. The apparent violations of exchange degeneracy in moderate energy 2-body collisions will be studied in this way.

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