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Abstract

A method used in the stress analysis of the PDX toroidal field coil is developed. A multilayer coil design of arbitrary dimensions in the shape of either a circle or an oval is considered. The analytical model of the coil and the supporting coil case with connections to the main support structure is analyzed using the finite element technique. The three dimensional magnetic fields and the non-uniform body forces which are a loading condition on a coil due to toroidal and poloidal fields are calculated. The method of analysis permits rapid and economic evaluations of design changes in coil geometry as well as in coil support structures. Some results pertinent to the design evolution and their comparison are discussed. The results of the detailed stress analysis of the final coil design due to toroidal field, poloidal field and temperature loads are presented.

Introduction

The toroidal field (TF) magnets are subjected to large non-uniform body forces resulting from a magnetic field - current interaction. The effective magnetic pressure of the field produces the tensile and bending stresses in coils. Since it has been found¹ that the mechanical stresses in the coils are important constraints on the performance of toroidal magnets, the stresses in the TF coils for the Poloidal Divertor Experiment (PDX) are calculated.

The design and fabrication of TF coils were previously described². The major parameters are shown in Figure 1 and Table 1. The body forces caused by the interaction of the magnetic field with the current in a toroidal field coil are non-uniform and non-symmetrical. To evaluate the forces, a three-dimensional magnetic field is first calculated by a numerical method. The mechanical stresses in the coil resulting from the effective magnetic pressure are then computed by a finite element technique using the computer model of the PDX TF coil. In this model assumptions are made pertaining to boundary conditions, material characteristics and the location of the magnetic field forces. Since the coil is composed of lamination of copper and epoxy, the material is characterized, as to its homogeneity, isotropy and elasticity, by results of tests on samples which are fabricated using similar

techniques to those anticipated for the actual coil.

Analysis Method

The non-uniform body forces are simulated in a finite element model by concentrated nodal point forces computed by a computer code. These forces are then used as input loads on the finite element stress code model. Two new computer codes were developed: PDXNODE³ which generates the finite element geometry of the toroidal field coil shown in Figure 1, and PDXFORC⁴ which computes the three components of the $\vec{J} \times \vec{B}$ body forces acting on each of the nodal points using the parameters of Table 1. It also generates the boundary conditions of the model. The outputs of these two codes become the input to the ANSYS⁵ code which computes the stresses and deformations of the coil.

PDXNODE has options to generate finite element models of coils of circular, oval or D-shapes in either two-fold or four-fold geometrical symmetry. The code has the ability of modeling coils of arbitrary thickness. In addition the inside leg of the coil can be modeled either with or without a beveled wedging surface. The finite element geometry model used to represent the toroidal field coil is shown in Figure 2. The coil is simulated by finite elements and the computer code PDXNODE generates the node points used to define the finite elements. The three dimensional isoparametric eight node finite element is chosen.

Three layers of elements can be generated. The number of finite elements through the thickness of each layer is respectively N2, N3, and N4. The layers are separated by gaps of arbitrary thickness. The connectivity between the layers can be established with three dimensional interface elements which transmit compression but not tension, or by using a three dimensional solid elements which would then eliminate the gap. There are N1 finite elements spanning the C7 length of an outer leg, an arc section with N6 finite elements spanning the pi radians and an inner leg with N1 finite elements. The number of finite elements through the coil thickness of C2 is N5. The modeling technique also enables the generation of an inner leg of different height from the outer leg. The number of coils is C1, and C6 is

the major radius of the toroidal field coils.

PDXFORC provides a capability to specify boundary conditions and to compute the nodal point forces for two different types of analyses. These are the nodal point force calculations for the TF coil current crossed with the toroidal field or for the TF coil current crossed on a poloidal field such as from the plasma, shaping field coils or ohmic heating coils. The latter nodal point forces are referred to here as lateral forces on the TF coil.

The nodal point information of the computer model of a TF coil is used to generate the corresponding electrical current paths⁶. Two sets of data which simulate the current paths are generated. The first simulates the subject coil and two neighboring coils precisely with many finite line elements of current, the second simulates all other coils of the TF coil system less accurately, using a few line elements.

The magnetic field at every nodal point of a TF coil then computed. The component of the magnetic field from the subject coil and its adjacent coils is computed from the group of current elements which precisely simulate these coils. The component of the magnetic field from all other coils is computed from the group of line current elements which simulate those coils less accurately.

$$\vec{B} = \frac{\mu_0 I}{4\pi |\vec{R}|} (\cos\theta_1 - \cos\theta_2) \vec{n} \quad (1)$$

where

I is the current in amperes

μ_0 is the magnetic permeability constant in henry/m ($4\pi \times 10^{-7}$)

$|\vec{R}|$ is the absolute magnitude of the perpendicular distance from the line to the center of the coordinate system.

Using the coordinate system shown in Figure 3 and substituting in (1) we get

$$\vec{B} = \frac{\mu_0 I}{4\pi} (\vec{B} - \vec{A}) \left[\frac{\vec{B} - \vec{A}}{|\vec{B} - \vec{A}|} \right] \frac{\vec{A} \times \vec{B}}{|\vec{A}| |\vec{B}|} \quad (2)$$

After the magnetic field is computed, the Lorentz force distribution in the coil is determined using

$$\vec{F} = \vec{J} \times \vec{B} \quad (3)$$

where

\vec{J} is the current density in amperes/m².

Due to an uneven current density in the TF coils and the rapid cooling with water after the current pulse, the coils are subjected to non-uniform temperature excursions. It became necessary, therefore, to compute the thermal stresses in the coils. The ohmic heating of the copper is calculated using a previously written computer code⁷.

A heat transfer calculation at each node is performed for discrete time intervals both during the pulse and during the cool down cycle. The temperature distribution throughout the coil is obtained and is used as an input for the temperature stress analysis of the coil with the ANSYS computer code.

The material of the coil was initially assumed to be homogenous and isotropic with a modulus of elasticity listed in Table 2. In the final model the modulus of elasticity in radial and axial directions (Y,Z) was 15.8×10^6 psi. In the direction of lamination (X), the modulus of elasticity of the layered material was determined by testing⁸ to be 8×10^6 psi. The spring constants of the interface elements representing the epoxy layers were computed using a value of 5×10^5 psi for the modulus of elasticity.

Design by Analysis

The stress analysis of the coil has been used to evaluate all design changes. A comparison of different PDX toroidal field coil designs is provided by a matrix of analysis conditions and results (Table 2). Toroidal field coil ampere-turns and the resultant forces for each computation are shown. The differences between the models in assumed material properties and in the type and amount of external support are likewise provided. The results are compared by the outer leg deflections and by the computed maximum shear stresses.

During the evolution of the design there were many changes. Some examples follow. The number of coil was changed from 18 to 20. A single layer design was changed to a two layer design. External supports were added to prevent excessive deformations. They are the outside support ring, the center column support and the coil case. The center column support and the ring are simulated by a series of springs (Figure 4). The spring constants and spring locations were varied in order to minimize the bending at the location of coil joint by minimizing the gradient of Y-direction stress through the coil thickness. The epoxy layers between the coil and the coil case, as well as between the outer and inner layers were modeled by three-dimensional interface elements.

The lateral load analysis was done separately for the inner and outer layers of the coil. Structural constraints against lateral motion were the coil clamp, the spacers between coils, and the coil case. The influence of these constraints has been examined. A detailed comparison of all performed analyses can be found in Reference 11.

The results of stress analyses for various configuration of supports provided the necessary guidance toward the final design.

Results of the Final Design Analysis

The stress analysis of the TF coil for

the final design parameters was performed for the toroidal field, poloidal field and temperature loads.

When a toroidal field load analysis was performed, the TF coil was modeled for one-fourth symmetry. The boundary conditions are shown in Figure 4. The epoxy between the two layers of copper was initially modeled by 3-D interface elements in order to determine the separation of layers. The final model used the 3-D isoparametric solids (Figure 4). The average stress in the epoxy layer was thus determined. There were eight contact pads between the coil case and the coil modeled by interface elements. The outside 6 inch x 6 inch support ring was represented by four springs. The torsional stiffness of the ring was incorporated into the spring constants. The coil case was attached to the clamp in a way not permitting any movement in the radial direction.

The results of the analysis due to the toroidal field load are shown in Figures 5 through 7. The coil displacements show that the coil axial growth is restrained by the center column supports, and that the radial motion is restrained by the outside ring. The Y stress is almost uniform at the top of a coil at 1,500 psi, the Z stress at the outer leg shows the influence of the support ring. The largest principal stresses and maximum shear stress are found in the outer layer of the inner leg with values of 8,551 psi, - 9,278 psi and 7,865 psi, respectively. The stresses throughout the coil case are small except:

- a) in the upper end where the coil case attaches to the clamp
- b) around the opening in side plates and
- c) a stress concentration at outside support ring.

The stresses in the epoxy layer are very small ($\tau_{max} = 980$ psi).

The joint region at the top of the coil is subjected to tension (90,000 lbs.) and shear (25,000 lbs.) forces and a bending moment (18,000 in.-lbs.) about the X axis.

The stress analysis due to the temperature load resulted in small stress levels throughout the coil (Figure 8). The maximum shear stress was 928 psi at the area of center column support.

Poloidal field load analysis required the coil to be modeled for one-half symmetry (Figure 9). The nodes on wedge surfaces are restrained in order to represent the transition of forces into the center column assembly.

The stress analysis of the final design for lateral loads has been done for the critical fault loading condition, when either the equilibrium field coils or the compression field coils are loaded to their full capacity. Only the outer layer, shown to be more critically stressed, was analyzed. The one-half cross-section of the coil at the joint resisting the lateral forces

(Figure 9) was simulated. The lateral forces on the outer leg were attenuated by the coil case subjected to lateral loads was performed and the deformations of the coil case was then prescribed to the corresponding nodal points in ANSYS analysis. The computed stress level throughout the coil was very small ($\tau_{max} = 470$ psi). The joint region was subjected primarily to a bending moment about the Z axis (31,784 in.-lbs.).

The combined effect of toroidal field, poloidal field and temperature loads was determined. The outer layer of the inner leg has a combined maximum shear stress of 8,500 psi. The stress resultants and stress couples were calculated at the joint region for the combined load (Figure 10). The computed stresses at the joint¹⁰ were relatively large ($\tau_{max} = 14,550$ psi) and required re-designing the coil clamp. The changes in design resulted in the more acceptable stress level ($\tau_{max} = 10,000$ psi).

Summary

The finite element analysis of the TF coil subjected to toroidal field, poloidal field and temperature loading indicated that:

- a) the loads are within the elastic limits of the material,
- b) modeling the coil, which is a composite of copper and epoxy, in the model X-direction as an homogenous structure is acceptable for toroidal field loading conditions,
- c) structural supports (rings and center column support) are necessary to keep stresses within acceptable limits, and
- d) the design of the coil, the coil case and the coil joint is possible.

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Table 1:

PDX-TF Coil: Major Parameters

Major Machine radius	1.49 meter
Field at 1.45m	24 kilograms
Total NI	17.5 x 10 ampere turns
Number of coils	20
Number of turns per coil	20
Current per turn	43,700 amperes
Inductance	0.190 henries
Inductive energy	182 megajoules
Resistance	39.59 milliohm at 11 C
Joule heating	207 megajoules
Pulse repetition rate	2 minutes
ESW time	2.7 seconds
Net weight of copper	7,700 pounds
Voltage drop	1,731 volts
Max current density	28 kA/sq in