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The Interaction of Nuclei with Electromagnetic Multipoles*

by

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In connection with the photodisintegration of the deuteron and similar problems, the parts of the interaction energy which are mainly independent of questions concerned with mesonic exchange currents are separated from those affected by the exchange character of nuclear forces. The direct introduction of the intensity of magnetization $\vec{M}$ is shown to be unnecessary, the current density associated with nucleonic magnetic moments accounting for effects often represented by the formal introduction of $\vec{M}$. Transformations of field strengths leading to an equivalent formulation using $\vec{M}$ are devised. The considerations are in terms of the irreducible tensor classification of the multipoles. No explicit use of gauge invariance has to be made. The entrance of limitations caused by electromagnetic form factors is located. The simplifications caused by employing the non-retarded approximation have been studied, although the treatment in its general form does not employ this approximation.

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The Interaction of Nuclei with Electromagnetic Multipoles*

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The differential cross section in the photodisintegration of the deuteron is only reasonably well accounted for by calculations with potential well models up to a γ-ray energy $E_\gamma = 150$ Mev. Definite deviations at $E_\gamma = 177$ Mev indicate pion participation. Together with outstanding discrepancies with models regarding low energy photorecombination and theoretical considerations they confirm the doubt in the correctness of usual forms of interaction energy between nucleons and the electromagnetic field. It is desirable therefore to formulate the interaction in a manner which separates the more certain parts of the interaction energy from those involving exchange currents. A way of accomplishing this systematically is briefly outlined below. It is applicable not only to nucleons but to other particles subject to strong interactions as well.

The classification of electromagnetic multipoles of Franz [1] as presented in the book of Blatt and Weisskopf[2] is employed. In BW use is made of the intensity of magnetization $\vec{M}$. Since there is no evidence for the existence of permanent magnetic dipoles, the direct identification of $\vec{M}$ with effects of nuclear magnetic moments is unjustifiable. It can be shown however that a quantity called $\vec{M}$ below, having its origin in electric currents, can be used to represent effects of nucleon magnetic moments.

II. The Electric and Magnetic Multipole Interaction Energy

A circularly polarized plane wave may be represented by

$$\vec{\mathcal{E}}_s(\vec{r}) = \sum_{\ell} i^{\ell}[2\pi(2\ell+1)]^{\frac{1}{2}} \left\{ \frac{s}{\varkappa} \operatorname{curl} \left[\frac{F_\ell(\rho)}{\rho} \vec{X}_{\ell,s} \right] + \frac{F_\ell(\rho)}{\rho} \vec{X}_{\ell,s} \right\} \qquad (2.1)$$

$$\vec{\mathcal{H}}_s(\vec{r}) = \sum_{\ell} i^{\ell}[2\pi(2\ell+1)]^{\frac{1}{2}}\left\{-i\, s\, \frac{F_\ell(\rho)}{\rho}\vec{X}_{\ell,s} - \frac{i}{\varkappa}\,\mathrm{curl}\left[\frac{F_\ell(\rho)}{\rho}\vec{X}_{\ell,s}\right]\right\} \tag{2.2}$$

where $s = \pm 1$ respectively for right or left circular polarization, the field strengths are

$$\vec{\mathcal{E}}(\vec{r},t) = \vec{\mathcal{E}}(\vec{r})\, e^{-i\omega t} + \vec{\mathcal{E}}^*(\vec{r})\, e^{i\omega t}, \quad \vec{\mathcal{H}}(\vec{r},t) = \vec{\mathcal{H}}(\vec{r})e^{-i\omega t} + \vec{\mathcal{H}}^*(\vec{r})e^{i\omega t} \tag{2.3}$$

and the other symbols are as in BW. Absorption is not influenced by $\vec{\mathcal{E}}^*(\vec{r})$ and $\vec{\mathcal{H}}^*(\vec{r})$.

In the non-retarded approximation in which the dimensions of the nucleus are considered small in comparison with the wave length of the γ ray, the electric multipole part of $\vec{\mathcal{E}}_s(\vec{r})$ may be written as

$$\vec{\mathcal{E}}_{E\ell s}(\vec{r}) \simeq C_\ell \;\mathrm{curl}\,[\vec{L}\, r^{\ell}\, Y_{\ell,s}] \tag{2.4}$$

$$C_\ell = [s\, i^{\ell} \varkappa^{\ell-1}\left\{(2\ell+1)!!\right\}^{-1}]\,[2\pi(2\ell+1)/\ell(\ell+1)]^{\frac{1}{2}} \tag{2.5}$$

which is expressible as the gradient of a potential $\varphi_{E\ell s}$. The relevant potential energy between the charge density $\rho(\vec{r},t) = \rho(\vec{r})e^{-i\omega t} + \rho^*(\vec{r})\, e^{i\omega t}$ and the $E\ell s$ multipole may therefore be written as

$$H'_{E\ell s} = \int \rho^*(\vec{r})\varphi_{E\ell s}\, d\vec{r}\,. \tag{2.6}$$

For a wave polarized with electric vector along the x-axis of a Cartesian co-ordinate system, the z-axis of which is the polar axis,

$$H'_{E\ell x} = (C_\ell/is)(\ell+1)2^{-\frac{1}{2}}\int \rho^*(r)[Y_{\ell,1} - Y_{\ell,-1}]\, d\vec{r}\,. \tag{2.7}$$

This part of the interaction energy does not involve exchange currents, on account of the form of (2.6). Since $\mathrm{curl}^2[F_\ell(\rho)\vec{X}_{\ell s}/\rho] \neq 0$ a modification of the above is needed in the non-retarded approximation. For a Diracian proton in the v^2/c^2 approximation it is readily seen that $\mathrm{curl}\,\vec{\mathcal{H}}_{eff} = \partial\vec{\mathcal{E}}/c\,\partial t + 4\pi\vec{J}_S/c$ where $\vec{J}_S$ is the Schroedinger current density, $\vec{\mathcal{H}}_{eff} = \vec{\mathcal{H}} - 4\pi\vec{M}$, $\vec{M} = (e\hbar/2Mc)\Psi_\alpha^*\,\vec{\sigma}_{\alpha\beta}\Psi_\beta$. Maxwell's equations take the form $\mathrm{curl}\,\vec{\mathcal{E}} = -\partial(\vec{\mathcal{H}}_{eff} + 4\pi\vec{M})/c\,\partial t$, $\mathrm{div}\,(\vec{\mathcal{H}}_{eff} + 4\pi\vec{M}) = 0$, $\mathrm{div}\,\vec{\mathcal{E}} = 4\pi e\Psi_\alpha^*\Psi_\alpha$. It may be shown that if $\vec{M}$ is made to include the effect of the Pauli part of the magnetic moment the equation $\vec{J} = \vec{J}_S + c\,\mathrm{curl}\,\vec{M}$ still holds. Since $\vec{\mathcal{H}} = \vec{\mathcal{H}}_{eff}$ only in the nucleus the field strengths may be calculated in terms of $\vec{\mathcal{H}}_{eff}$ and the equations in BW may thus be justified. Employing the formulation thus outlined one obtains the contribution of $\vec{M}$ to the magnetic multipole interaction by the replacements $\rho(\vec{r}) \to -\operatorname{div} \vec{M}(\vec{r})$, $s \to -i$ in the treatment of the electric multipoles. It is thus seen that

$$H'_{M\ell x} = (C_\ell/s)(\ell+1)2^{-\frac{1}{2}} \int r^\ell (Y_{\ell,1} + Y_{\ell,-1}) \operatorname{div} \vec{M}^* \, d\vec{r}. \tag{2.8}$$

Treating the magnetic field produced by $\vec{J}_S$ as in magnetostatics,

$$\vec{\mathcal{H}}_{eff} \simeq \operatorname{curl} \vec{A}_{mgst} \; ; \; \vec{A}_{mgst} = (-i\, C_\ell /s)\, r^\ell Y_{\ell,s} \tag{2.9}$$

and hence after straightforward manipulation

$$(H'_{\vec{M}} + H'_{J_S})_x = (C_\ell/s)(\ell+1)2^{-\frac{1}{2}} \int r^\ell (Y_{\ell,1} + Y_{\ell,-1}) \left\{ \frac{\operatorname{div}(\vec{r} \times \vec{J}_S^*)}{c\,(\ell+1)} + \operatorname{div} \vec{M}^* \right\} d\vec{r}. \tag{2.10}$$

The connection of emission to absorption is readily seen by means of quantized amplitudes. The exchange current effects are in general present in (2.10).

III. Effect of Retardation and Other Terms

In including retardation and dealing with absorption it is convenient to use a four potential with

$$\vec{A}^{(o)}_{E\ell s}(\vec{r}) = \vec{A}_{E\ell s}(\vec{r}) - \vec{\nabla} \Phi_{E\ell s}(\vec{r})/ i\varkappa \tag{3.1}$$

and 0 for its space and time parts. The field strengths are then

$$\vec{\mathcal{E}}_{E\ell s}(\vec{r}) = -\partial \vec{A}_{E\ell s}(\vec{r})/c\,\partial t - \vec{\nabla}\Phi_{E\ell s}(\vec{r}) \tag{3.2}$$

$$\vec{\mathcal{H}}_{E\ell s}(\vec{r}) = \operatorname{curl} \vec{A}_{E\ell s}(\vec{r}). \tag{3.3}$$

Here $\vec{A}_{E\ell s}$, $\Phi_{E\ell s}$ are proportional to $\hat{r} F_\ell Y_{\ell,m}$ and $-i\,(1+r d/dr)\, F_\ell Y_{\ell,m}/\varkappa r$ respectively.

From Eq. (3.1), the equation of continuity and the assumption $\vec{j}(|\vec{r}| = \infty) = 0$,

$$H'_{E\ell s} = \frac{C'_{\ell s}}{[\ell(\ell+1)]^{\frac{1}{2}}} \int Y_{\ell,s} \left[-i \frac{d\, F_\ell(\varkappa r)}{d\,(\varkappa r)} \rho^*(\vec{r}) - \frac{1}{c} (\hat{r} \cdot \vec{J}^*(\varkappa r)) F_\ell(\varkappa r) \right] d\vec{r}. \tag{3.4}$$

The first term does not contain the current density and therefore the exchange currents do not affect it. This contribution is analogous to the electric multipole contribution in the classification of R.G. Sachs[3] for which there is a proof of the EEA theorem (E - for electric, E - for exchange, A - for absence). The second term in (3.4) contains in $\vec{J}^*$ the effect of $\vec{M}^*$. The part of the vector potential corresponding to magnetic multipole radiation for a 2^ℓ magnetic pole is

$$\vec{A}_{M\ell s} = \frac{C'_{\ell s} F_\ell(\varkappa r)}{i\, s\, (\varkappa r)} \vec{X}_{\ell,s} \tag{3.5}$$

and the corresponding interaction energy is

$$H'_{M\ell s} = \frac{C'_{\ell s}/s}{[\ell(\ell+1)]^{\frac{1}{2}}} \int Y_{\ell,s} \left\{ \frac{d F_\ell(\varkappa r)}{d(\varkappa r)} \operatorname{div} \vec{M}^* - [(\vec{r} \operatorname{curl} \vec{J}_S^*) + c\varkappa^2 (\vec{r}\cdot\vec{M}^*)] \frac{F_\ell(\varkappa r)}{\varkappa r\, c} \right\} d\vec{r}. \tag{3.6}$$

The 2^ℓ-pole contribution to the interaction energy may be therefore put in the form

$$H'_{\ell s} = \frac{i^\ell [2\pi(2\ell+1)]^{\frac{1}{2}}}{\varkappa[\ell(\ell+1)]^{\frac{1}{2}}} \int Y_{\ell,s} \left\{ \frac{s}{c\varkappa} [(\operatorname{div} \vec{J}^*) \frac{d F_\ell}{d(\varkappa r)} - \varkappa(\vec{r}\cdot\vec{J}^*) F_\ell] \right.$$

$$\left. + (\operatorname{div} \vec{M}^*) \frac{d F_\ell}{d(\varkappa r)} - \varkappa(\hat{r}\cdot(\vec{M}^* + \operatorname{curl} \vec{J}_S^*/c\varkappa^2) F_\ell \right\} d\vec{r}. \tag{3.8}$$

With the aid of the boundary condition on the field at infinity, it is found that the calculation of $H'_{E\ell s}$ may be performed employing the following operator in coordinate spin space

$$H^{op}_{E\ell s} = K_\ell \; h^{op}_{E\ell s} \tag{3.9}$$

$$h^{op}_{E\ell s} = \frac{s}{c\varkappa^2} \vec{J}_S^{op} \operatorname{curl} \vec{Z}_{\ell s} + s\vec{\mu}^{op} \vec{Z}_{\ell s}. \tag{3.10}$$

where $K_\ell = i^{\ell+1} \left[\frac{2\pi(2\ell+1)}{\ell(\ell+1)} \right]^{\frac{1}{2}}$; $\vec{Z}_{\ell s} = F_\ell(\varkappa r)\vec{L}\, Y_{\ell,s}/(\varkappa r)$, (3.11)

$$\vec{J}_S^{op} = (\hbar/i) \sum_j (e_j/M_j)\, \delta(\vec{r}_j - \vec{r}) \vec{\nabla}_j \tag{3.12}$$

$$\vec{\mu}^{op} = \sum_j (e_j \hbar/2M_j c)\mu_j \vec{\sigma}_j \delta(\vec{r}_j - \vec{r}). \qquad (3.13)$$

where the summation is taken over all the particles and e_j, M_j, μ_m and $\vec{\sigma}_j$ are respectively the charges, masses, magnetic moments in Bohr magnetons and Pauli spin matrices of the particles. The transition to the operator forms makes use of

$$\int \mathrm{div}_j\, [\varphi^* \psi\, \vec{C}(\vec{r}_j)]\, d\vec{r}_j = 0 \qquad (3.14)$$

which holds provided at least one of the states φ, ψ is a bound state.

Similarly the calculation of $H'_{M\ell s}$ may be performed by means of the operator

$$H^{op}_{M\ell s} = K_\ell\, h^{op}_{M\ell s} \qquad (3.15)$$

where

$$h^{op}_{M\ell s} = \frac{1}{\varkappa c} \vec{J}_S^{\,op}\, \vec{Z}_{\ell s} + \frac{1}{\varkappa} \vec{\mu}^{op}\, \mathrm{curl}\, \vec{Z}_{\ell s}\,. \qquad (3.16)$$

The vectors $\vec{J}_S^{\,op}$ and $\vec{\mu}^{op}$ as written above do not take into account the contributions of the exchange currents which must be present but whose form is still subject to speculation. It is hoped that through an extension of calculations on the $d(\gamma,n)p$ reaction[4] some information regarding this question will be gained on ρ only are still subject to corrections caused by the finite extension of the nucleon.

Footnotes

* Supported by the U.S. Army Research Office-Durham, the Air Force Office of Scientific Research and the U.S. Atomic Energy Commission.

1 W. Franz, Zeits. f. Physik 127, 363 (1950); B. Stech, Z. Naturforsch. 7a, 401 (1952).

2 J.M. Blatt and V.F. Weisskopf, Theoretical Nuclear Physics (John Wiley and Sons, Inc, New York, 1956). This reference will be referred to as BW.

3 R. G. Sachs, Nuclear Theory (Addison-Wesley Publishing Co., Inc. Cambridge, Massachusetts, 1953).

4 For example, M.L. Rustgi, W. Zernik, G.Breit and D.J. Andrews, Phys. Rev. 120, 1881 (1960); W. Zickendraht, D.J. Andrews, M.L. Rustgi, W. Zernik, A.J. Torruella and G. Breit, Phys. Rev. 124, 1538 (1961).

$$\vec{\mu}^{\,op} = \sum_j (e_j \hbar/2M_j c)\mu_j \vec{\sigma}_j \,\delta(\vec{r}_j - \vec{r}). \tag{3.13}$$

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