

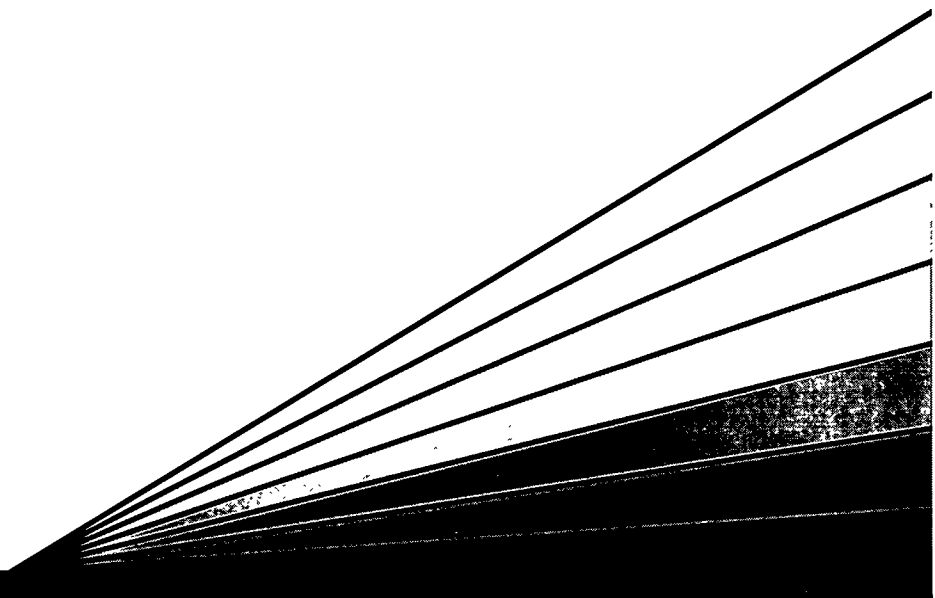
The Integral Equation for a High Gain FEL

by N. A. Vinokurov

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Experimental Facilities Division
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THE INTEGRAL EQUATION FOR A HIGH GAIN FEL

N. A. Vinokurov *

Introduction

The theory of a high gain free electron laser (FEL) is now well developed (e.g., see [1]). In this paper I derive the equation for the electron distribution function, which is valid for FELs with a longitudinally inhomogeneous magnetic system (which may include, in particular, dispersive sections, quadrupole lenses, and simply empty spaces between the undulator sections), magnetic field errors in undulators, and some other options. The integral form of the equation may be useful for numerical calculations.

Calculation of the Radiation Field

Consider the electron beam propagating inside a long planar undulator. The Fourier transform A_x of the transverse horizontal component of vector potential (Lorentz gauge) is given by [2]

$$A_x(\vec{r}) = \frac{1}{c} \int \frac{j_x(\vec{r}') e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} d^3r' + A_{0x}(\vec{r}) , \quad (1)$$

where c is the light velocity, $k = \omega/c$ is the wave number, j_x is the component of a current density, and A_{0x} describes the external electromagnetic wave. In the paraxial approximation

$$|\vec{r}-\vec{r}'| \approx z-z' + \frac{(x-x')^2 + (y-y')^2}{2(z-z')} , \quad (2)$$

and the electric field is

$$E = ikA_x = \frac{ik}{c} \int_0^z \iint \frac{j_x(\vec{r}') e^{ik(z-z')} e^{\frac{ik(x-x')^2 + (y-y')^2}{2(z-z')}}}{z-z'} dx' dy' dz' + E_0 . \quad (3)$$

The fast particle motion along the z axis (i.e., the varying of $j_x(\vec{r}')$ almost like $e^{ikz'}$) was taken into account in Eq. (3). If the equation for the equilibrium trajectory in the undulator is

$$\frac{dx}{dz} = \frac{K(z)}{\gamma} \sin[k_w z + \varphi(z)] , \quad (4)$$

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where γ is the Lorentz factor for the equilibrium particle, k_w and K are the wavenumber and the dimensionless vector potential amplitude of the undulator, then

$$j_x \approx j_z \frac{K(z)}{\gamma} \sin[k_w z + \varphi(z)] \quad (5)$$

and

$$E = \frac{ik}{\gamma c} \int_0^z \iint \frac{j_z(\vec{r}') K \sin(k_w z' + \varphi) e^{\frac{ik(z-z') + ik \frac{(x-x')^2 + (y-y')^2}{2(z-z')}}}{z - z'} dx' dy' dz' + E_0. \quad (6)$$

The Vlasov Equation

To calculate the current density j_z , we may use the unperturbed (by the radiation field) particle trajectories:

$$\begin{aligned} x(z) &= \xi_1(z, x_0, \dot{x}_0, y_0, \dot{y}_0) = x_1(z, x_0, \dot{x}_0, y_0, \dot{y}_0) \\ &+ \int_0^z \frac{K}{\gamma} \left[1 + \frac{\alpha}{2} k_w^2 x_1^2(z', x_0, \dot{x}_0, y_0, \dot{y}_0) + \frac{1-\alpha}{2} k_w^2 y_1^2(z', x_0, \dot{x}_0, y_0, \dot{y}_0) \right] \sin(k_w z' + \varphi) dz' \\ y(z) &= y_1(z, x_0, \dot{x}_0, y_0, \dot{y}_0). \end{aligned} \quad (7)$$

Here and below, point notation (i.e., \dot{x} or \dot{y}) is used for the derivative with respect to z , which will be an independent variable (instead of time), and "0" indicates the initial conditions (at $z = 0$). $\alpha = 0$ for the planar undulator without sextupole focusing. In the more general case, these trajectories depend also on the initial energy, but for the paraxial motion we may neglect this dependence. As the interaction of particles with light changes essentially only the particle energy, the 6-dimensional distribution function may be written in the form:

$$\begin{aligned} F_6(x, \dot{x}, y, \dot{y}, t, \Delta, z) = \\ \int F(\tau, \Delta, z, x_0, \dot{x}_0, y_0, \dot{y}_0) \delta(x - \xi_1) \delta(\dot{x} - \dot{\xi}_1) \delta(y - y_1) \delta(\dot{y} - \dot{y}_1) dx_0 d\dot{x}_0 dy_0 d\dot{y}_0, \end{aligned} \quad (8)$$

where

$$\tau = t - \int_0^z \left[1 + \frac{1}{2\gamma^2} + \frac{\dot{\xi}_1^2(z') + \dot{y}_1^2(z')}{2} \right] \frac{dz'}{c}, \quad (9)$$

$$\int F_6 dx d\dot{x} dy d\dot{y} d\Delta dt = 1 , \quad (10)$$

and $(1 + \Delta) \gamma$ is the Lorentz factor. For the unperturbed motion at the equilibrium energy, τ denotes the moment of time when the particle enters the undulator. Therefore τ is a convenient longitudinal coordinate of a particle. The "mixed" distribution function F obeys the Liouville equation:

$$\frac{\partial F}{\partial z} + \frac{\partial F}{\partial \tau} \left[\frac{1}{v_z} - \frac{1}{c} \left(1 + \frac{1}{2\gamma^2} + \frac{\dot{\xi}_1^2 + \dot{y}_1^2}{2} \right) \right] + \frac{\partial F}{\partial \Delta} \frac{e E_x(\xi_1, y_1, z, t) \dot{\xi}_1}{\gamma m c^2} = 0 , \quad (11)$$

where m is the electron mass, and E_x is the component of the wave electric field. Defining the slow varying amplitude of the electric field as

$$E_x(x, y, z, t) = A(x, y, z, t) e^{ik_0(z-ct)} + c.c. \quad (12)$$

$$k_0 \approx 2 \gamma_{\parallel}^2 k_w \quad (13)$$

$$\gamma_{\parallel} = \frac{\gamma}{\sqrt{1 + K^2/2}} \quad (14)$$

and neglecting all but the slow (by z) part of longitudinal force, one can obtain

$$\begin{aligned} \frac{\partial F}{\partial z} - \frac{\Delta}{\gamma_{\parallel}^2 c} \frac{\partial F}{\partial \tau} = \\ - \frac{\partial F}{\partial \Delta} \frac{e(JJ)K}{\gamma^2 m c^2} \text{Im} \left[A(x_1, y_1, z, \tau + t_1) e^{i\varphi - ik_0 \left(\int_0^z \frac{\dot{x}_1^2 + \dot{y}_1^2 + k_x^2 x_1^2 + k_y^2 y_1^2}{2} dz' + c\tau \right) + ik_w z - ik_0 \int_0^z \frac{dz'}{2\gamma_{\parallel}^2}} \right] , \end{aligned} \quad (15)$$

where

$$(JJ) = J_0 \left(\frac{K^2}{4 + 2K^2} \right) - J_1 \left(\frac{K^2}{4 + 2K^2} \right) \quad (16)$$

is the standard combination of the Bessel functions, which describes the reduction of the particle-wave interaction due to the longitudinal velocity modulation in the planar undulator, and

$$t_1 = \int_0^z \left[1 + \frac{1}{2\gamma_{\parallel}^2} + \frac{\dot{x}_1^2(z') + \dot{y}_1^2(z') + k_x^2 x_1^2 + k_y^2 y_1^2}{2} \right] \frac{dz'}{c} . \quad (17)$$

Wave numbers k_x and k_y describe focusing by the inhomogeneity of the undulator field:

$$k_x = \sqrt{\alpha} \frac{k_w K}{\sqrt{2} \gamma}, \quad k_y = \sqrt{1 - \alpha} \frac{k_w K}{\sqrt{2} \gamma} . \quad (18)$$

The Expression of Radiation Field through the Distribution Function

For an electron bunch having total charge Q , the longitudinal current density is given by

$$j_z = Q \int F_6 d\dot{x} d\dot{y} d\Delta, \quad (19)$$

or, using Eq. (8),

$$j_z = Q \int F(t - \int_0^z \left[1 + \frac{1}{2\gamma^2} + \frac{\dot{\xi}_1^2(z') + \dot{y}_1^2(z')}{2} \right] \frac{dz'}{c}, \Delta, z, x_0, \dot{x}_0, y_0, \dot{y}_0) \times \delta(x - \xi_1) \delta(y - y_1) dx_0 d\dot{x}_0 dy_0 d\dot{y}_0 . \quad (20)$$

In the stationary case, it is more natural to normalize the average value of F_6 :

$$\langle F_6 \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T F_6 dt, \quad (21)$$

$$\int \langle F_6 \rangle dx d\dot{x} dy d\dot{y} d\Delta = 1 . \quad (10')$$

Then the longitudinal current density will be

$$j_z = I_0 \int F_6 d\dot{x} d\dot{y} d\Delta, \quad (19')$$

or

$$j_z = I_0 \int F(t - \int_0^z \left[1 + \frac{1}{2\gamma^2} + \frac{\dot{\xi}_1^2(z') + \dot{y}_1^2(z')}{2} \right] \frac{dz'}{c}, \Delta, z, x_0, \dot{x}_0, y_0, \dot{y}_0) \times \delta(x - \xi_1) \delta(y - y_1) dx_0 d\dot{x}_0 dy_0 d\dot{y}_0 d\Delta \quad (20')$$

where I_0 is the average beam current.

The expression for the field amplitude A , defined in Eq. (12), is coming from Eq. (6):

$$A = A_0 + \frac{ik_0 e^{-ik_0(z-ct)}}{\gamma c} \int_0^z \int \frac{j_z(\bar{r}', t - \frac{z-z'}{c} - \frac{(x-x')^2 + (y-y')^2}{2c(z-z')}) K \sin(k_w z' + \varphi)}{z-z'} dx' dy' dz' \quad (22)$$

Substituting Eq. (20') into Eq. (22) gives:

$$A = \frac{ik_0 I_0 e^{-ik_0(z-ct)}}{\gamma c} \int_0^z \int F(t - \frac{z}{c} - \frac{(x - \xi_1(z'))^2 + (y - y_1(z'))^2}{2c(z-z')}) - \int_0^{z'} \left[\frac{1}{2\gamma^2} + \frac{\dot{\xi}_1^2(z'') + \dot{y}_1^2(z'')}{2} \right] \frac{dz''}{c}, \Delta, z, x_0, \dot{x}_0, y_0, \dot{y}_0) \times \frac{K \sin(k_w z' + \varphi)}{z-z'} dx_0 d\dot{x}_0 dy_0 d\dot{y}_0 d\Delta dz' + A_0 \quad (23)$$

Assuming, that only the slow varying component of the function $F e^{i\omega_0 \tau}$ makes a sufficient contribution to the integral in Eq. (23), we may neglect some rapidly oscillating terms:

$$\sin(k_w z' + \varphi) e^{-i\omega_0 \tau} e^{i\omega_0 \tau} F(t - \frac{z}{c} - \frac{(x - \xi_1(z'))^2 + (y - y_1(z'))^2}{2c(z-z')}) - \int_0^{z'} \left[\frac{1}{2\gamma^2} + \frac{\dot{\xi}_1^2(z'') + \dot{y}_1^2(z'')}{2} \right] \frac{dz''}{c}, \Delta, z, x_0, \dot{x}_0, y_0, \dot{y}_0) \approx - \frac{(JJ)}{2i} e^{ik_0(z-ct)} e^{-i\varphi + ik_0 \int_0^z \frac{\dot{x}_1^2 + \dot{y}_1^2 + k_x^2 x_1^2 + k_y^2 y_1^2}{2} dz'' - ik_w z' + ik_0 \int_0^{z'} \frac{dz''}{2\gamma^2} + ik_0 \int_0^{z'} \frac{(x - \xi_1(z'))^2 + (y - y_1(z'))^2}{2(z-z')} dz''} F e^{i\omega_0 \tau} \quad (24)$$

Actually, we simply change $\sin(k_w z' + \varphi)$ to $ie^{-i(k_w z' + \varphi)}(JJ)/2$. Thus, the expression in Eq. (23) is changed to

$$\begin{aligned}
A = & -\frac{k_0 I_0 e^{-ik_0(z-ct)}}{2\gamma c} \int_0^z \int F\left(t - \frac{z}{c} - \frac{(x - x_1(z'))^2 + (y - y_1(z'))^2}{2c(z - z')}\right. \\
& \left. - \int_0^{z'} \left[\frac{1}{2\gamma_{\parallel}^2} + \frac{\dot{x}_1^2 + \dot{y}_1^2 + k_x^2 x_1^2 + k_y^2 y_1^2}{2} \right] \frac{dz''}{c}, \Delta, z, x_0, \dot{x}_0, y_0, \dot{y}_0 \right) \\
& \times \frac{K(JJ)}{z - z'} e^{-i\varphi - ik_w z'} dx_0 d\dot{x}_0 dy_0 d\dot{y}_0 d\Delta dz' + A_0.
\end{aligned} \tag{25}$$

The Final Equation

Substituting Eq. (25) into Eq. (15), one can easily obtain the final equation:

$$\begin{aligned}
& \frac{\partial F}{\partial z} - \frac{\Delta}{\gamma^2 c} \frac{\partial F}{\partial \tau} = \\
& \frac{\partial F}{\partial \Delta} \text{Im} \left\{ \frac{e(JJ)_0 K_0}{\gamma^2 m c^2} B A_0(x_1, y_1, z, \tau + t_1) e^{ik_0(z - c\tau - ct_1)} e^{ik_w z} \right. \\
& + \frac{k_w D^2 B}{4} \int_0^z \frac{\overline{B} e^{ik_w(z - z')}}{z - z'} \int_{z'}^z F\left(\tau + \int_{z'}^z \frac{dz''}{2c\gamma_{\parallel}^2} - \frac{[x_1(z) - x_1'(z')]^2 + [y_1(z) - y_1'(z')]^2}{2c(z - z')}\right. \\
& \left. + \int_0^z \frac{\dot{x}_1^2 + \dot{y}_1^2 + k_x^2 x_1^2 + k_y^2 y_1^2}{2} \frac{dz''}{c} - \int_0^{z'} \frac{\dot{x}_1'^2 + \dot{y}_1'^2 + k_x^2 x_1'^2 + k_y^2 y_1'^2}{2} \frac{dz''}{c}, \right. \\
& \left. \Delta', z', x'_0, \dot{x}'_0, y'_0, \dot{y}'_0 \right) dx'_0 d\dot{x}'_0 dy'_0 d\dot{y}'_0 d\Delta' dz' \}
\end{aligned} \tag{26}$$

where K_0 and $(JJ)_0$ are constant,

$$1 + \frac{K_0^2}{2} = \frac{2\gamma^2 k_w}{k_0}, \tag{27}$$

$$B = \frac{K(JJ)}{K_0(JJ)_0} e^{i\varphi}, \tag{28}$$

and

$$D = 4 \sqrt{\frac{e I_0}{\gamma m c^3}} \sqrt{\frac{K_0^2}{4 + 2 K_0^2}} (JJ)_0 \quad (29)$$

Equation (26) may be written as an integral equation:

$$F(\tau, \Delta, z, x_0, \dot{x}_0, y_0, \dot{y}_0) = F\left(\tau + \frac{\Delta}{c} \int_0^z \frac{dz'}{\gamma_{\parallel}^2}, \Delta, 0, x_0, \dot{x}_0, y_0, \dot{y}_0\right) + \int_0^z R\left(\tau + \frac{\Delta}{c} \int_{z'}^z \frac{dz''}{\gamma_{\parallel}^2}, \Delta, z'\right) dz' , \quad (30)$$

where $R(\tau, \Delta, z)$ is the right-hand side of Eq. (25). The physical interpretation of Eqs. (26) and (30) is simple. We consider the electron beam as a set of thin beams, each having its own initial conditions $x_0, \dot{x}_0, y_0, \dot{y}_0$ and corresponding trajectory $x_1(z, x_0, \dot{x}_0, y_0, \dot{y}_0)$,

$y_1(z, x_0, \dot{x}_0, y_0, \dot{y}_0)$. Electrons moving along the undulator do not change their trajectories (this is an approximation). Therefore we have, in fact, one-dimensional motion along each trajectory (see the left side of Eq. (26)). The right side of Eq. (26) contains the longitudinal force on the electron, moving along the above-mentioned trajectory. The force is the sum of the contributions of the other electrons, moving along other trajectories $x_1(z, x'_0, \dot{x}'_0, y'_0, \dot{y}'_0)$,

$y_1(z, x'_0, \dot{x}'_0, y'_0, \dot{y}'_0)$.

The Case of a Small Signal

It is enough to put $\partial F_0 / \partial \Delta$ instead of $\partial F / \partial \Delta$ into right side of Eq. (26) to obtain the linear equation for a small signal. We assume that F_0 is the "smooth" part of the distribution function, which does not make a contribution to the radiation field, and keep notation F for the small variable component of the distribution function. Defining the current of the stream

$$J = \int F d\Delta , \quad (31)$$

one can obtain the linear equation for it from Eq. (30):

$$\begin{aligned}
J(\tau, z, x_0, \dot{x}_0, y_0, \dot{y}_0) &= \int F\left(\tau + \frac{\Delta}{c} \int_0^z \frac{dz'}{\gamma_{\parallel}^2}, \Delta, 0, x_0, \dot{x}_0, y_0, \dot{y}_0\right) d\Delta \\
&+ \int_0^z \int \frac{\partial F_0}{\partial \Delta} \text{Im} \left\{ \frac{e(JJ)_0 K_0}{\gamma^2 m c^2} B A_0(x_1(z'), y_1(z'), z', \tau + \frac{\Delta}{c} \int_{z'}^z \frac{dz''}{\gamma_{\parallel}^2} + t_1(z')) \right. \\
&\times e^{\frac{ik_0[z' - c\tau - \Delta \int_{z'}^z \frac{dz''}{\gamma_{\parallel}^2} - ct_1(z')]}{z'} \gamma_{\parallel}^2} e^{ik_w z'} \left. \right\} d\Delta dz' \\
&+ \frac{k_w D^2}{4} \int_0^z \int \frac{\partial F_0}{\partial \Delta} \text{Im} \left\{ B \int_0^{z'} \frac{\overline{B} e^{ik_w(z' - z'')}}{z' - z''} \right. \\
&\times \int J\left(\tau + \frac{\Delta}{c} \int_{z'}^z \frac{dz''}{\gamma_{\parallel}^2} + \int_{z''}^{z'} \frac{dz'''}{2c\gamma_{\parallel}^2} - \frac{[x_1(z') - x_1'(z'')]^2 + [y_1(z') - y_1'(z'')]^2}{2c(z' - z'')} \right. \\
&+ \int_0^{z'} \frac{\dot{x}_1^2 + \dot{y}_1^2 + k_x^2 x_1^2 + k_y^2 y_1^2}{2} \frac{dz'''}{c} - \int_0^{z''} \frac{\dot{x}_1'^2 + \dot{y}_1'^2 + k_x^2 x_1'^2 + k_y^2 y_1'^2}{2} \frac{dz'''}{c}, z'', x'_0, \dot{x}'_0, y'_0, \dot{y}'_0) \\
&\times dx'_0 d\dot{x}'_0 dy'_0 d\dot{y}'_0 dz'' \left. \right\} d\Delta dz'.
\end{aligned} \tag{32}$$

The slow variable may be defined as follows:

$$F = f e^{-i\omega_0 \tau} + c.c. \tag{33}$$

$$J = j e^{-i\omega_0 \tau} + c.c. \tag{34}$$

Then Eq. (32) gives:

$$\begin{aligned}
j(\tau, z, x_0, \dot{x}_0, y_0, \dot{y}_0) &= \int_{-\infty}^{\infty} f(\tau, \Delta, 0, x_0, \dot{x}_0, y_0, \dot{y}_0) e^{-ik_0 \Delta \int_0^z \frac{dz'}{\gamma_{\parallel}^2}} d\Delta + \frac{e(JJ)_0 K_0}{2i \gamma^2 m c^2} \\
&\times \int_0^z \Phi(k_0 \int_{z'}^z \frac{dz''}{\gamma_{\parallel}^2}, \tau, z', x_0, \dot{x}_0, y_0, \dot{y}_0) B(z') A_0(x_1(z'), y_1(z'), z', \tau + \frac{z'}{c} + \int_0^{z'} \frac{dz''}{2c \gamma_{\parallel}^2}) \\
&\times e^{-ik_0 \int_0^{z'} \frac{\dot{x}_1'^2 + \dot{y}_1'^2 + k_x^2 x_1'^2 + k_y^2 y_1'^2}{2} dz''} e^{ik_w z' - ik_0 \int_0^{z'} \frac{dz''}{2 \gamma_{\parallel}^2}} dz' \\
&+ \frac{k_w D^2}{8i} \int_0^z \Phi(k_0 \int_{z'}^z \frac{dz''}{\gamma_{\parallel}^2}, \tau, z', x_0, \dot{x}_0, y_0, \dot{y}_0) B(z') \int_0^{z'} \frac{\overline{B(z'')}}{z' - z''} e^{ik_w(z' - z'') - ik_0 \int_{z''}^{z'} \frac{dz'''}{2 \gamma_{\parallel}^2}} \\
&\times \int e^{ik_0 \frac{[x_1(z') - x_1(z'')]^2 + [y_1(z') - y_1(z'')]^2}{2(z' - z'')}} - ik_0 \int_0^{z'} \frac{\dot{x}_1'^2 + \dot{y}_1'^2 + k_x^2 x_1'^2 + k_y^2 y_1'^2}{2} dz''' + ik_0 \int_0^{z''} \frac{\dot{x}_1'^2 + \dot{y}_1'^2 + k_x^2 x_1'^2 + k_y^2 y_1'^2}{2} dz'''} \\
&\times j(\tau + \int_{z''}^{z'} \frac{dz'''}{2c \gamma_{\parallel}^2}, z'', x'_0, \dot{x}'_0, y'_0, \dot{y}'_0) dx'_0 d\dot{x}'_0 dy'_0 d\dot{y}'_0 dz'' dz'
\end{aligned} \tag{35}$$

where

$$\Phi(\kappa) = \int_{-\infty}^{\infty} \frac{\partial F_0}{\partial \Delta} e^{-i\kappa \Delta} d\Delta . \tag{36}$$

If, for example,

$$F_0 = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\Delta - \varepsilon)^2}{2\sigma^2}} f_0 , \tag{37}$$

where ε , σ , and f_0 are functions of $z, \tau, x_0, \dot{x}_0, y_0, \dot{y}_0$, then

$$\Phi(\kappa) = i\kappa e^{-i\kappa\varepsilon - \frac{\kappa^2 \sigma^2}{2}} f_0 . \tag{38}$$

If F_0 does not depend on τ and z , the Fourier transformation of Eq. (35) gives:

$$\begin{aligned}
j_\omega(z, x_0, \dot{x}_0, y_0, \dot{y}_0) = & \int_{-\infty}^{\infty} f_\omega(\Delta, 0, x_0, \dot{x}_0, y_0, \dot{y}_0) e^{-ik_0 \Delta \int_0^z \frac{dz'}{\gamma_{\parallel}^2}} d\Delta + \frac{e(JJ)_0 K_0}{2i\gamma^2 mc^2} \\
& \times \int_0^z \Phi(k_0 \int_{z'}^z \frac{dz''}{\gamma_{\parallel}^2}, x_0, \dot{x}_0, y_0, \dot{y}_0) B(z') A_{0\omega}(x_1(z'), y_1(z'), z') e^{-i\frac{\omega}{c}(z' + \int_0^{z'} \frac{dz''}{\gamma_{\parallel}^2})} \\
& \times e^{-ik_0 \int_0^{z'} \frac{\dot{x}_1^2 + \dot{y}_1^2 + k_x^2 x_1^2 + k_y^2 y_1^2}{2} dz''} e^{ik_w z' - ik_0 \int_0^{z'} \frac{dz''}{2\gamma_{\parallel}^2}} dz' \\
& + \int_0^z \int K(z, z', x_0, \dot{x}_0, y_0, \dot{y}_0, x'_0, \dot{x}'_0, y'_0, \dot{y}'_0) j_\omega(z', x'_0, \dot{x}'_0, y'_0, \dot{y}'_0) dx'_0 d\dot{x}'_0 dy'_0 d\dot{y}'_0 dz',
\end{aligned} \tag{39}$$

where

$$\begin{aligned}
K(z, z', x_0, \dot{x}_0, y_0, \dot{y}_0, x'_0, \dot{x}'_0, y'_0, \dot{y}'_0) = & \frac{k_w D^2}{8i} \int_{z'}^z \Phi(k_0 \int_{z''}^z \frac{dz'''}{\gamma_{\parallel}^2}, x_0, \dot{x}_0, y_0, \dot{y}_0) \frac{B(z'') \overline{B(z')}}{z'' - z'} e^{ik_w(z'' - z') - i(k_0 + \frac{\omega}{c}) \int_{z'}^{z''} \frac{dz'''}{\gamma_{\parallel}^2}} \\
& \times e^{ik_0 \frac{[x_1(z'') - x'_1(z')]^2 + [y_1(z'') - y'_1(z')]^2}{2(z'' - z')}} e^{-ik_0 \int_0^{z''} \frac{\dot{x}_1^2 + \dot{y}_1^2 + k_x^2 x_1^2 + k_y^2 y_1^2}{2} dz'''} e^{ik_0 \int_0^{z'} \frac{\dot{x}'_1{}^2 + \dot{y}'_1{}^2 + k_x^2 x'^2_1 + k_y^2 y'^2_1}{2} dz'''} dz'' .
\end{aligned} \tag{40}$$

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