ERR-AN-174
Stress

A SPECIALIZED TOROIDAL PRESSURE VESSEL

MASTER

C. E. Edenfield

J. Kinsley Lowry

31 July 1962
(Revised 1 October 1963)

ENGINEERING DEPARTMENT

GENERAL DYNAMICS/ASTRONAUTICS

A DIVISION OF GENERAL DYNAMICS CORPORATION

ERR-AN-174
Stress

A SPECIALIZED TOROIDAL PRESSURE VESSEL

C. E. Edenfield

J. Kinsley Lowry

31 July 1962
(Revised 1 October 1963)

ENGINEERING DEPARTMENT

This work was supported under General Dynamics sponsored research program numbers 111-9372 and 111-9521.

GENERAL DYNAMICS/ASTRONAUTICS

A DIVISION OF GENERAL DYNAMICS CORPORATION

CONTENTS

ILLUSTRATIONS

A SPECIALIZED TOROIDAL PRESSURE VESSEL

C. E. Edenfield[1]

J. Kinsley Lowry[2]

General Dynamics/Astronautics

San Diego, California

ABSTRACT

A method is developed for determining the meridional contours of a toroidal shell which will produce no hoop stress resultants in the curved portion of the shell when subjected to internal pressure. The meridional membrane force will remain tensile. The resulting specialized toroidal pressure vessel design may eliminate the possibility of buckling failures found to be of critical concern in many other types of non-circular tori. Methods for finding the internal volume and surface area of such a pressure vessel are also presented.

INTRODUCTION

The torus is becoming a more prevalent structural configuration for use in certain aerospace vehicles. It is an ideal shape for manned space stations

[1] Senior Structures Engineer, Space Launch Vehicle Stress Group, formerly associated with Structures Research Group.

[2] Structures Engineer, Structures Research Group.

This work was supported under General Dynamics sponsored research program Numbers 111-9372 and 111-9521.

since for stabilization, the spin axis is aligned with the major axis of inertia thereby inducing a fairly constant force of synthetic gravity for living quarters. Further application may be found in small volume propellant tanks for large diameter aerospace vehicles. Most aerospace vehicles at present have ellipsoidal closing bulkheads and conical or cylindrical adapters between stages. A large poorly utilized volume is left between these structural elements.

The information presented in this publication will enable one to design a toroidal pressure vessel of the membrane type which may best utilize the space available. Examples which further illustrate possible applications for the toroidal pressure vessel are shown in Fig. 2.

The stress distribution that results if linear membrane theory is applied to the toroidal shell problem is well known and quite uncomplicated [1]*. As a result of recent developments in the aerospace industry considerable work has been done on toroidal shell problems [2-9]. The problem of designing maximum volume to minimum weight ratio pressure vessels has led to investigations by Turner [2] and Edenfield [3]. Turner has compared the relative merits and design characteristics of the circular torus and elliptical torus. He finds the meridional forces in a circular torus to be significantly greater than the meridional forces in an elliptical torus of equal surface area. Since the shell thickness is dictated by the maximum meridional load, the elliptical torus has a definite weight

*Numbers in brackets indicate references given in the bibliography.

advantage. It is also pointed out that the greater the departure from the design of a circular section the more critical the hoop compressive forces become in the inboard portion of the shell. A recent paper by P. F. Jordan [5] presents a quantitative indication of the degree of confidence with which results of linear membrane theory can be used for practical design applications.

Care must be exercised, when designing certain non-circular tori, to control large compressive hoop forces which might occur in the inboard portions. To eliminate this possibility, we find that a proper relationship between the radii of curvature will render the hoop forces equal to zero in these critical portions of the shell.

The development presented in this publication will consider the axisymmetric-shell problem that is posed by a complete toroid the walls of which are of uniform thickness and are sufficiently thin for membrane theory to seem adequate. Assume the toroid to be pressurized by a weightless gas and the hoop membrane forces to be zero in all portions of the shell except a short cylindrical section about the outer radius. The resulting toroidal configuration is one which may eliminate the possibility of buckling failures, since the remaining meridional force will be tensile.

Solutions for the differential equations derived to obtain the y ordinate and the volume of the specialized toroidal shell are found to be inexpressible in closed form. However, the solutions to these equations are obtained in terms of elliptic integrals of the first and second kinds, convient for handbook computation. Design parameter curves for specialized toroidal pressure vessels are determined for several values of the constant D, (i.e., 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8). This constant allows the presentation of the design contours in non-dimensional form as some ratio between the radius of revolution and x_{max}. These results are tabulated and presented along with the curves in the Appendix.

NOMENCLATURE

A	area
A_s	area projected on horizontal plane, produced by revolving an arc segment about the axis of symmetry.
a	subscript specifying lower limit of integration.
B	radius of revolution of $y - y$ axis about vertical axis of symmetry.
b	subscript specifying upper limit of integration.
C	constant of integration.
D	constant used to define x_{max} as a function of B .
$E(\beta_i, k)$	Legendre's incomplete elliptic integral of the second kind.
$F(\beta_i, k)$	Legendre's incomplete elliptic integral of the first kind.
k	modulus of elliptic functions.
N_θ	hoop force per unit length.
N_φ	meridional force per unit length.
P	total applied load.
p	internal pressure, positive when acting outward.
R	radius of revolution of outboard cylindrical portion of toroid about vertical axis of revolution.
r_1	radius of curvature in meridional plane.
r_2	radius of curvature in circumferential plane.
s	meridian arc length.
V	volume.
x, y	rectangular Cartesian coordinates.
y'	translated ordinate, given by Eq. (25).
x_{max}	maximum value of x at $\varphi = \pi/2$.

α parameter given by Eq. (19) as a function of D.

β transformation variable given by Eq. (22).

θ colatitude angle between r_2 and vertical axis of symmetry.

φ longitude angle in meridional plane.

METHOD OF ANALYSIS

Development of the specialized toroidal pressure vessel is presented in three parts: shell contour, internal volume and shell surface area. By virtue of symmetry, only the portion of the shell above the horizontal center-line will be considered.

3.1 Shell Contour

For any surface of revolution the basic equation of equilibrium is expressed as a relation between the hoop and meridional stress resultants and their radii of curvatures, given as a function of pressure

$$p = \frac{N_\varphi}{r_1} + \frac{N_\theta}{r_2} \, . \tag{1}$$

If N_θ is zero, Eq. (1) becomes

$$p \, r_1 = N_\varphi \, . \tag{2}$$

Since r_1 is always a positive quantity and p is considered positive here, all meridional stress resultants are tensile. It shall be assumed that N_θ is zero in the meridian arc shown in Fig. 1 as KLM. Thus, all remaining stress resultants in this segment will be tensile.

Refer to Fig. 3 and let us consider an arbitrary arc segment s along the inboard portion of the shell surface. Projecting this arc onto a horizontal plane, the expression for the area produced by revolving the arc about the axis of revolution is

$$A_s = \pi \left[B^2 - (B - x)^2\right] = \pi x(2B - x) \quad . \tag{3}$$

The total force from internal pressure applied to this area is

$$P = p\pi x(2B - x) \quad . \tag{4}$$

N_φ must balance this load to maintain equilibrium. The circumference of the shell is

$$2\pi(B - x) \quad .$$

Therefore, the reacting force in the shell will be expressed as follows

$$P = 2\pi(B - x) \; N_\varphi \sin \varphi \quad . \tag{5}$$

Substituting Eq. (4) in Eq. (5) and solving for N_φ we obtain

$$N_\varphi = \frac{px(2B-x)}{2(B-x) \sin \varphi} \quad . \tag{6}$$

Substituting N_φ from Eq. (6) into Eq. (2) and solving for r_1 we find

$$r_1 = \frac{x(2B-x)}{2(B-x) \sin \varphi} \quad . \tag{7}$$

From Eq. (7) a continuous curve of radius r_1 may be developed as a function of x .

From previous equations we find the equation for a differential length of arc to be of the following form

$$ds = r_1 d\varphi = \frac{x(2B-x) \; d\varphi}{2(B-x) \sin \varphi} \quad . \tag{8}$$

Evaluating x as a function of φ we find

$$dx = \cos\varphi \, ds = \frac{x(2B-x)\cos\varphi \, d\varphi}{2(B-x)\sin\varphi}$$

which may be written as

$$dx = \frac{x(2B-x)\cot\varphi \, d\varphi}{2(B-x)} . \tag{9}$$

The differential equation will take the following form as separated variables

$$\frac{2(x-B)\,dx}{x^2 - 2Bx} = \cot\varphi \, d\varphi . \tag{10}$$

Integrating both sides we obtain

$$\ln(x^2 - 2Bx) = \ln C \sin\varphi \tag{11}$$

where C is the constant of integration.

Solving Eq. (11) for x we have

$$x = B \pm \sqrt{B^2 + C\sin\varphi} . \tag{12}$$

It is noted that the positive value of the radical, given here, has no physical significance since the root of x is limited by the radius B. We also find that C cannot be easily evaluated at the apex of the shell where $x = 0$ and $\varphi = 0$ but must be determined at the maximum value of x, where $\varphi = \pi/2$.

Let $x_{max} = DB$ where D is some constant between 0 and 1.0. When DB

is substituted for x in Eq. (12) with $\varphi = \pi/2$ we find

$$DB = B - \sqrt{B^2 + C}\,. \tag{13}$$

Solving Eq. (13) for C we obtain

$$C = -\,B^2\left(2D - D^2\right). \tag{14}$$

Substituting this expression for C into Eq. (12) we obtain the final form of x as a function of φ, given as

$$x = B\left[1 - \sqrt{1 - \left(2D - D^2\right)\sin\varphi}\;\right]. \tag{15}$$

Similarily, y can be expressed in terms of φ as:

$$dy = \sin\varphi\, ds = \frac{x(2B-x)\,\sin\varphi\, d\varphi}{2(B-x)\,\sin\varphi} = \frac{(x^2-2Bx)\, d\varphi}{2(x-B)}\,. \tag{16}$$

Substituting x of Eq. (15) into Eq. (16) we obtain

$$dy = \frac{B(2D-D^2)\,\sin\varphi\, d\varphi}{2\sqrt{1-\left(2D-D^2\right)\sin\varphi}}\,. \tag{17}$$

Integrating both sides of Eq. (17) we find

$$y = \tfrac{1}{2} B\left(2D - D^2\right)\int_{\varphi_a}^{\varphi_b} \frac{\sin\varphi\, d\varphi}{\sqrt{1-\left(2D-D^2\right)\sin\varphi}}\,. \tag{18}$$

A closed form solution for the integral of Eq. (18) is not readily apparent. However, as a result of certain elementary transformations the solution may be expressed in terms of elliptic integrals of the first and second kinds.

To simplify the notation, let

$$\alpha = \frac{1}{\left(2D-D^2\right)}, \quad 1.0 < \alpha < \infty \quad . \tag{19}$$

It follows that Eq. (18) may be expressed in the equivalent form

$$y/B = \frac{1}{2\sqrt{\alpha}} \int_{\varphi_a}^{\varphi_b} \frac{\sin\varphi \, d\varphi}{\sqrt{\alpha - \sin\varphi}} \tag{20}$$

With considerable effort and the use of some elementary transformation substitutions, it may be shown that the general solution to Eq. (20) is of the following form

$$y/B = \sqrt{\frac{2}{\alpha}} \left\{ -\frac{1}{k}\left[E\left(\beta_b, k\right) - E\left(\beta_a, k\right)\right] + \left(\frac{1}{k} - \frac{k}{2}\right)\left[F\left(\beta_b, k\right) - F\left(\beta_a, k\right)\right]\right\}, \tag{21}$$

where

$$\beta_i = \sin^{-1}\sqrt{\frac{1 + \sin\varphi_i}{2}}, \quad i = a, b \; . \tag{22}$$

$$k = \sqrt{\frac{2}{\alpha + 1}}, \quad 0 < k < 1.0 \; . \tag{23}$$

Equation (21) is expressed in terms of Legendre's normal forms, which may easily be evaluated by referring to any comprehensive handbook of elliptic integrals, [10, 11] .

Finally by substituting x of Eq. (15) into Eq. (7), we may express r_1

as a function of φ as follows:

$$r_1 = \frac{B(2D-D^2)}{2\sqrt{1 - (2D-D^2)\sin\varphi}} \quad . \tag{24}$$

By translating the x - x axis by the amount of y evaluated at $\varphi = \pi/2$, a horizontal center line of symmetry common to all specialized toroidal shell meridional contours is obtained. The translated ordinate y' may then be expressed as

$$y' = y_\varphi - y \ , \quad \varphi = \pi/2 \ . \tag{25}$$

Values of x, y and r_1 have been determined for several values of the constant D. The results are presented in Fig. 6 and 7 of the Appendix.

3.2 Internal Volume

The volumes for areas V_1 and V_2 revolved about the axis of revolution (see Fig. 4) are given by the following:

$$dV = \pi\left[B^2 - (B - x)^2\right] dy = \pi\left[2Bx - x^2\right] dy \ . \tag{26}$$

Substituting the expressions for x from Eq. (15) and dy from Eq. (17) into Eq. (26) we obtain

$$dV = \frac{\pi B^3 (2D-D^2)^2}{2} \frac{\sin^2\varphi \, d\varphi}{\sqrt{1 - (2D-D^2)\sin\varphi}} \quad . \tag{27}$$

Integrating both sides of Eq. (27) we find

$$V = \frac{1}{2} \pi B^3 \left(2D - D^2\right)^2 \int_{\varphi_a}^{\varphi_b} \frac{\sin^2 \varphi \, d\varphi}{\sqrt{1 - (2D-D^2) \sin \varphi}} \quad . \tag{28}$$

By making use of certain transformation substitutions, similar to those applied to Eq. (20), we may obtain the closed form solution for Eq. (28) expressed in terms of canonical form elliptic integrals of the first and second kinds.

Using Eq. (19) we may express Eq. (28) in the equivalent form

$$V/\pi B^3 = \frac{\sqrt{\alpha}}{2\alpha^2} \int_{\varphi_a}^{\varphi_b} \frac{\sin^2 \varphi \, d\varphi}{\sqrt{\alpha - \sin \varphi}} \quad . \tag{29}$$

After some elementary transformation substitutions the final expression for the general solution to Eq. (29) is obtained:

$$V/\pi B^3 = \frac{\sqrt{\alpha}}{2\alpha^2} \left\{ - \frac{4(2-k^2)}{3k^3} \left[E\left(\beta_b, k\right) - E\left(\beta_a, k\right) \right] + \left[\frac{8(1-k^2)}{3k^3} + k \right] \left[F\left(\beta_b, k\right) - F\left(\beta_a, k\right) \right] \right.$$

$$\left. + \frac{4}{3k} \left[\sin \beta_b \cos \beta_b \sqrt{1 - k^2 \sin^2 \beta_b} - \sin \beta_a \cos \beta_a \sqrt{1 - k^2 \sin^2 \beta_a} \right] \right\} \tag{30}$$

where

$$\beta_i = \sin^{-1} \sqrt{\frac{1 + \sin \varphi_i}{2}} \, , \quad i = a, b \, . \tag{31}$$

$$k = \sqrt{\frac{2}{\alpha + 1}} \, , \quad 0 < k < 1.0 \, . \tag{32}$$

Equation (30) is expressed in terms of Legendre's normal forms which may easily be evaluated by referring to any comprehensive handbook of elliptic integrals.

It is readily seen that for the area V_3, revolved about the axis of revolution, the volume is given by the following:

$$V_3 = \pi \left[\left(B + | x_\varphi | \right)^2 - B^2 \right] y'_\varphi \; , \; \varphi = - \pi/2 \; . \qquad (33)$$

The total internal volume of the pressure vessel is simply twice the sum of the partial volumes, namely

$$V = 2 \left[V_1 + V_2 + V_3 \right] . \qquad (34)$$

Internal volumes have been determined for each of the meridional contours shown in Fig. 6. The results are presented in Fig. 8 of the Appendix.

3.3 Shell Surface Area

The surface area of arcs KL and LM, revolved about the axis of revolution (see Fig. 5), are given as

$$dA = 2\pi (B - x) \; ds \; . \qquad (35)$$

Substituting the expressions for ds of Eq. (8) and x of Eq. (15) into Eq. (35) we obtain

$$dA = \frac{\pi x (2B - x) \; d\varphi}{\sin \varphi} = \pi B^2 \left(2D - D^2 \right) d\varphi \qquad (36)$$

Integrating Eq. (36) we find the general expression for the surface areas A_1 and A_2 given as follows:

$$A = \pi B^2 \left(2D - D^2\right) \int_{\varphi_a}^{\varphi_b} d\varphi \,. \tag{37}$$

The surface area A_1, is determined by integrating Eq. (37) between the limits $0 \leq \varphi \leq \pi/2$. Similarily, A_2 is found by integrating over the region $-\pi/2 \leq \varphi \leq 0$, and thus A_1 must be the same as A_2.

$$A_1 = A_2 = \frac{\pi^2 B^2}{2} \left(2D - D^2\right) \,. \tag{38}$$

The surface area A_3, produced by revolving the line JK about the axis of revolution, may be expressed as follows:

$$A_3 = 2\pi \left(B + |x_\varphi|\right) y'_\varphi \,, \quad \varphi = -\pi/2 \tag{39}$$

The total surface area of the pressure vessel is simply twice the sum of the partial areas, namely

$$A = 2\left[A_1 + A_2 + A_3\right] \,. \tag{40}$$

Surface areas have been determined for each of the meridional contours shown in Fig. 6. The results are presented in Fig. 9 of the Appendix.

CONCLUSIONS

This publication has presented the development of a method for determining the zero hoop meridional contours of a toroidal shell. The resulting toroidal configuration is one which may eliminate buckling failures caused by hoop compressive forces in the inboard portion of the shell.

Several meridional contours are presented in Fig. 6 to show the relative change in shape with respect to the size parameter D. Figures 8 and 9 are also presented to indicate the relative variation in volume and surface area of the shell with respect to D.

It may be mentioned that while the meridional force is continuous, a large discontinuity of the hoop force may exist at K as a result of the sudden change in the radius of curvature r_1. The consideration of such a possibility, however, is not within the framework of linear membrane theory. As a membrane, it may be pointed out that element JK may not remain straight but may assume some small curvature when subjected to internal pressure. This will tend to minimize the effects of any discontinuity at K.

It may also be shown through a power series expansion that the singularity indicated to exist at the apex of the shell by Eq. (6) is finite in the limit and becomes

$$\lim_{\varphi \to 0} N_\varphi = \frac{(2D-D^2)}{2} pB .$$

This is illustrated by the family of continuous curves for the meridional radii of curvature with respect to the size parameter D presented in Fig. 7.

Referring to Fig. 10, one may compare the relative magnitude of hoop stress in the cylindrical portion of the shell with the maximum meridional stress occurring at $\varphi = \pi/2$.

The authors feel that the development presented in this paper will serve to initiate ideas and subsequent investigations of toroidal shell designs.

The authors extend their sincere thanks to Dr. P. E. Wilson for his stimulating comments and advice during various phases of this investigation. Also we extend appreciation to Dr. A. H. Hausrath for reviewing the manuscript.

BIBLIOGRAPHY

1. Timoshenko, S., and Woinowsky-Krieger, S., "Theory of Plates and Shells," 2nd Ed., McGraw-Hill Book Co., Inc., New York, 1959.

2. Turner, H. M., "Design Parameters for Elliptical Toroidal Pressure Vessels," Aerospace Engineering, November 1962, pp. 33-38.

3. Edenfield, C. E., "Minimum Weight Tank End Closure Systems," Report No. ERR-AN-101 (Structures Research), General Dynamics/Astronautics, 29 December 1961, 62 pp.

4. Dean, W. R., "The Distortion of a Curved Tube Due to Internal Pressure," Philosophical Mag., Vol. 28, 1939, pp. 452-464.

5. Jordan, Peter, F., "Stresses and Deformations of the Thin-Walled Pressurized Torus," Journal of the Aerospace Sciences, February 1962, pp. 213-225.

6. Beskin, L., "Bending of Curved Thin Tubes," Journal of Applied Mechanics, 12, A1-A7, 1945.

7. Clark, R. A., "On the Theory of Thin Elastic Toroidal Shells," Journal of Mathematics and Physics, 29, 1950, pp. 146-178.

8. Reissner, E., "On Bending of Curved Thin-Walled Tubes," Proc. Nat'l. Acad. Sci. U. S. 35, 1949, pp. 204-208.

9. Schuerch, H. U., and Schindler, G. M., "Analysis of Foldability in Expandable Structures," AIAA Journal, Vol. 1, No. 4, April 1963, pp. 873-878.

10. Byrd, P. F., and Friedman, M. D., "Handbook of Elliptic Integrals for Engineers and Physicists (Grundlehren der mathematischen Wissenschaften, Band LXVII)", Springer-Verlag, 1954, (6-place tables).

11. Pearson, K., "Tables of the Complete and Incomplete Elliptic Integrals," (reissued from Tome II of Legendre's "Traité des Fonctions Elliptiques"), London, 1934, (10-place tables).

APPENDIX

6.1 Coordinates and Radii of Curvature of Specialized Toroidal Pressure Vessels*

TABLE I

D = 0.20

φ	x/B	y/B	y′/B	r_I/B
-90.0	-0.16619	0.15917	0.05427	0.15434
-80.0	-0.16384	0.13235	0.08109	0.15466
-70.0	-0.15684	0.10624	0.10720	0.15559
-60.0	-0.14532	0.08155	0.13190	0.15716
-50.0	-0.12950	0.05896	0.15448	0.15936
-40.0	-0.10968	0.03916	0.17429	0.16220
-30.0	-0.08627	0.02278	0.19067	0.16570
-20.0	-0.05977	0.01043	0.20302	0.16954
-10.0	-0.03078	0.00267	0.21077	0.17462
00.0	0.00000	0.00000	0.21345	0.18000
10.0	0.03176	0.00279	0.21066	0.18590
20.0	0.06358	0.01133	0.20211	0.19222
30.0	0.09446	0.02575	0.18770	0.19877
40.0	0.12330	0.04596	0.16748	0.20531
50.0	0.14898	0.07167	0.14178	0.21151
60.0	0.17040	0.10228	0.11117	0.21697
70.0	0.18654	0.13692	0.07653	0.22127
80.0	0.19658	0.17444	0.03901	0.22404
90.0	0.20000	0.21345	0.00000	0.22500

*The value φ is given in degrees.

TABLE II

D = 0.30

φ	x/B	y/B	y′/B	r_1/B
-90.0	-0.22882	0.21505	0.11724	0.20701
-80.0	-0.22566	0.17999	0.13330	0.20801
-70.0	-0.21624	0.14484	0.18645	0.20966
-60.0	-0.20069	0.11152	0.22177	0.21237
-50.0	-0.17927	0.08094	0.25235	0.21522
-40.0	-0.15231	0.05400	0.27529	0.22129
-30.0	-0.12026	0.03158	0.30371	0.22762
-20.0	-0.08371	0.01455	0.31674	0.23530
-10.0	-0.04334	0.00376	0.32903	0.24440
00.0	0.00000	0.00000	0.33329	0.25500
10.0	0.04530	0.00399	0.32920	0.26715
20.0	0.09139	0.01637	0.31692	0.28064
30.0	0.13686	0.03762	0.29567	0.29545
40.0	0.18013	0.06797	0.26532	0.31102
50.0	0.21941	0.10730	0.22599	0.32667
60.0	0.25278	0.15503	0.17823	0.34126
70.0	0.27836	0.20995	0.12534	0.35336
80.0	0.29448	0.27020	0.06304	0.36143
90.0	0.30000	0.33329	0.00000	0.36428

TABLE III

D = 0.40

φ	x/B	y/B	y′/B	r_1/B
-90.0	-0.28062	0.26203	0.20382	0.24927
-80.0	-0.27682	0.21860	0.24726	0.25062
-70.0	-0.26546	0.17624	0.28962	0.25287
-60.0	-0.24669	0.13601	0.32985	0.25607
-50.0	-0.22076	0.09900	0.36686	0.26015
-40.0	-0.18801	0.06628	0.39958	0.26595
-30.0	-0.14891	0.03892	0.42693	0.27802
-20.0	-0.10403	0.01803	0.44783	0.28904
-10.0	-0.05410	0.00469	0.46117	0.30317
00.0	0.00000	0.00000	0.46506	0.32000
10.0	0.05720	0.00505	0.46081	0.33941
20.0	0.11619	0.02092	0.44494	0.36207
30.0	0.17537	0.04859	0.41727	0.38805
40.0	0.23278	0.08887	0.37698	0.41709
50.0	0.28604	0.14224	0.32361	0.44520
60.0	0.33235	0.20852	0.25753	0.47929
70.0	0.36865	0.28653	0.17933	0.50685
80.0	0.39195	0.37366	0.09220	0.52627
90.0	0.40000	0.46586	0.00000	0.53333

TABLE IV

D = 0.50

φ	x/B	y/B	y'/B	r_1/B
-90.0	-0.32287	0.29889	0.31738	0.28347
-80.0	-0.31856	0.24961	0.36665	0.28440
-70.0	-0.30566	0.20152	0.41475	0.28720
-60.0	-0.28433	0.15580	0.46046	0.29197
-50.0	-0.25480	0.11365	0.50261	0.29865
-40.0	-0.21741	0.07629	0.53997	0.30603
-30.0	-0.17260	0.04495	0.57131	0.31980
-20.0	-0.12094	0.02091	0.59536	0.33453
-10.0	-0.06312	0.00546	0.61080	0.35273
00.0	0.00000	0.00000	0.61627	0.37500
10.0	0.06738	0.00596	0.61030	0.40209
20.0	0.13774	0.02490	0.59136	0.43490
30.0	0.20943	0.05844	0.55783	0.47434
40.0	0.28034	0.10823	0.50803	0.52108
50.0	0.34772	0.17581	0.44046	0.57490
60.0	0.40798	0.26213	0.35413	0.63343
70.0	0.45664	0.36683	0.24943	0.69016
80.0	0.48873	0.48700	0.12926	0.73347
90.0	0.50000	0.61627	0.00000	0.75000

TABLE V

D = 0.60

φ	x/B	y/B	y'/B	r_1/B
-90.0	-0.35646	0.32780	0.46536	0.30962
-80.0	-0.35175	0.27397	0.51919	0.31070
-70.0	-0.33766	0.22142	0.57175	0.31398
-60.0	-0.31432	0.17141	0.62175	0.31955
-50.0	-0.28198	0.12525	0.66791	0.32761
-40.0	-0.24094	0.08425	0.70891	0.33845
-30.0	-0.19163	0.04977	0.74339	0.35245
-20.0	-0.13459	0.02322	0.76994	0.37017
-10.0	-0.07045	0.00609	0.78707	0.39235
00.0	0.00000	0.00000	0.79317	0.42000
10.0	0.07580	0.00671	0.78645	0.45444
20.0	0.15578	0.02826	0.76490	0.49750
30.0	0.23842	0.06694	0.72622	0.55148
40.0	0.32172	0.12548	0.66768	0.61921
50.0	0.40290	0.20696	0.58620	0.70340
60.0	0.47794	0.31458	0.47858	0.80451
70.0	0.54102	0.45052	0.34264	0.91508
80.0	0.58435	0.61317	0.17999	1.01047
90.0	0.60000	0.79317	0.00000	1.05000

TABLE VI

D = 0.70

φ	x/B	y/B	y'/B	r_1/B
-90.0	-0.38202	0.34959	0.66337	0.32922
-80.0	-0.37701	0.29235	0.72061	0.33042
-70.0	-0.36202	0.23645	0.77652	0.33406
-60.0	-0.33719	0.18322	0.82974	0.34026
-50.0	-0.30272	0.13404	0.87892	0.34926
-40.0	-0.25894	0.09030	0.92266	0.36141
-30.0	-0.20623	0.05344	0.95952	0.37720
-20.0	-0.14509	0.02499	0.98797	0.39734
-10.0	-0.07611	0.00657	1.00639	0.42281
00.0	0.00000	0.00000	1.01297	0.45500
10.0	0.08240	0.00731	1.00565	0.49585
20.0	0.17008	0.03094	0.98202	0.54824
30.0	0.26175	0.07388	0.93908	0.61632
40.0	0.35574	0.13997	0.87300	0.70624
50.0	0.44963	0.23429	0.77867	0.82672
60.0	0.53965	0.36356	0.64940	0.98839
70.0	0.61936	0.53576	0.47721	1.19538
80.0	0.67778	0.75590	0.25707	1.41203
90.0	0.70000	1.01297	0.00000	1.51666

TABLE VII

D = 0.80

φ	x/B	y/B	y'/B	r_1/B
-90.0	-0.40000	0.36480	0.94802	0.34285
-80.0	-0.39478	0.30519	1.00763	0.34413
-70.0	-0.37916	0.24696	1.06587	0.34803
-60.0	-0.35328	0.19149	1.12133	0.35469
-50.0	-0.31734	0.14021	1.17261	0.36436
-40.0	-0.27164	0.09455	1.21827	0.37746
-30.0	-0.21655	0.05603	1.25679	0.39455
-20.0	-0.15253	0.02624	1.28658	0.41647
-10.0	-0.08013	0.00692	1.30591	0.44438
00.0	0.00000	0.00000	1.31283	0.46000
10.0	0.08714	0.00774	1.30509	0.52582
20.0	0.18045	0.03289	1.27993	0.58568
30.0	0.27888	0.07902	1.23380	0.66564
40.0	0.38119	0.15099	1.16183	0.77568
50.0	0.48560	0.25597	1.05685	0.93314
60.0	0.58937	0.40520	0.90762	1.16894
70.0	0.68711	0.61700	0.69582	1.53412
80.0	0.76636	0.91790	0.39492	2.05450
90.0	0.80000	1.31283	0.00000	2.40000

TABLE VIII

D = 0.90

φ	x/B	y/B	y'/B	r_1/B
-90.0	-0.41067	0.37379	1.43837	0.35089
-80.0	-0.40533	0.31278	1.49938	0.35222
-70.0	-0.38935	0.25318	1.55898	0.35628
-60.0	-0.36285	0.19639	1.61577	0.36320
-50.0	-0.32604	0.14387	1.66830	0.37329
-40.0	-0.27920	0.09708	1.71508	0.38695
-30.0	-0.22270	0.05757	1.75459	0.40484
-20.0	-0.15697	0.02699	1.78517	0.42783
-10.0	-0.08254	0.00712	1.80504	0.45725
00.0	0.00000	0.00000	1.81217	0.49500
10.0	0.09000	0.00799	1.80417	0.54395
20.0	0.18673	0.03408	1.77808	0.60665
30.0	0.28936	0.08219	1.72998	0.69656
40.0	0.39697	0.15792	1.65425	0.82066
50.0	0.50845	0.27006	1.54210	1.00702
60.0	0.62232	0.43403	1.37814	1.31066
70.0	0.73598	0.68111	1.13105	1.87488
80.0	0.84175	1.08798	0.72418	3.12813
90.0	0.90000	1.81217	0.00000	4.95000

TABLE IX

D = 0.95

φ	x/B	y/B	y'/B	r_1/B
-90.0	-0.41332	0.37603	1.92903	0.35289
-80.0	-0.40795	0.31467	1.99038	0.35423
-70.0	-0.39188	0.25472	2.05033	0.35832
-60.0	-0.36523	0.19761	2.10744	0.36532
-50.0	-0.32820	0.14478	2.16028	0.37550
-40.0	-0.28108	0.09771	2.20735	0.38931
-30.0	-0.22423	0.05795	2.24710	0.40739
-20.0	-0.15808	0.02717	2.27788	0.43066
-10.0	-0.08315	0.00717	2.29788	0.46046
00.0	0.00000	0.00000	2.30506	0.49875
10.0	0.09072	0.00806	2.29699	0.54851
20.0	0.18831	0.03438	2.27067	0.61446
30.0	0.29200	0.08299	2.22207	0.70445
40.0	0.40098	0.15969	2.14537	0.83261
50.0	0.51433	0.27373	2.03132	1.02694
60.0	0.63102	0.44181	1.86324	1.35173
70.0	0.74968	0.70006	1.60499	1.99250
80.0	0.86713	1.15503	1.15002	3.75368
90.0	0.95000	2.30506	0.00000	9.97500

6.2 Internal Volumes of Specialized Toroidal Pressure Vessels

TABLE X

D	$V_1/\pi B^3$	$V_2/\pi B^3$	$V_3/\pi B^3$	$V/2\pi B^3$	$V/\pi B^3$
0.20	0.06123	0.04458	0.01953	0.12536	0.25072
0.30	0.13668	0.08545	0.05979	0.28193	0.56387
0.40	0.24228	0.12975	0.13045	0.50249	1.00498
0.50	0.38027	0.17311	0.23803	0.79143	1.58286
0.60	0.55634	0.21234	0.39090	1.15960	2.31921
0.70	0.78376	0.24508	0.60367	1.63252	3.26504
0.80	1.09623	0.26960	0.91010	2.27594	4.55189
0.90	1.60903	0.28476	1.42398	3.31778	6.63557
0.95	2.10721	0.28860	1.92420	4.32002	8.64005

6.3 Surface Areas of Specialized Toroidal Pressure Vessels

TABLE XI

D	A_1/B^2	A_2/B^2	A_3/B^2	$A/2B^2$	A/B^2
0.20	1.77652	1.77652	0.39770	3.95076	7.90153
0.30	2.51674	2.51674	0.90520	5.93870	11.87740
0.40	3.15827	3.15827	1.64008	7.95663	15.91327
0.50	3.70110	3.70110	2.63802	10.04022	20.08045
0.60	4.14523	4.14523	3.96628	12.25674	24.51349
0.70	4.49067	4.49067	5.76047	14.74181	29.48363
0.80	4.73741	4.73741	8.33929	17.81411	35.62822
0.90	4.88545	4.88545	12.74905	22.51996	45.03992
0.95	4.92246	4.92246	17.13020	26.97513	53.95026

6.4 Hoop Stress on Cylindrical Portion of Shell and Maximum Meridional Stress at $\varphi = \frac{\pi}{2}$

TABLE XII

D	$\frac{N_\varphi}{pB}$	$\frac{N_\theta}{pB}$
0.20	0.22500	7.25686
0.30	0.36428	7.54309
0.40	0.53333	7.74516
0.50	0.75000	7.83399
0.60	1.05000	7.82833
0.70	1.51667	7.73130
0.80	2.40000	7.54211
0.90	4.95000	7.24647
0.95	9.97500	7.03553

SPECIALIZED TOROIDAL SHELL

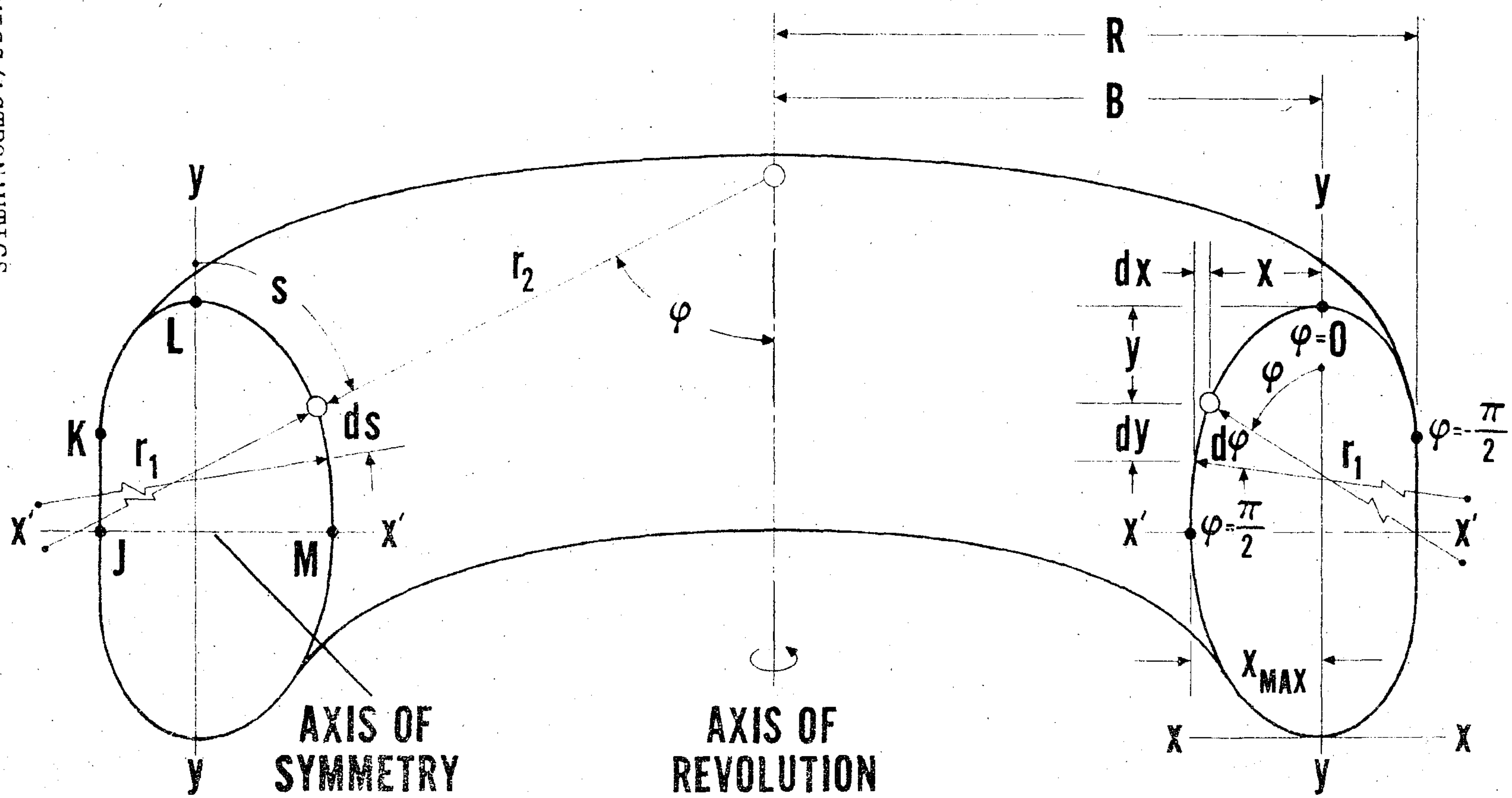

FIGURE 1

POSSIBLE APPLICATIONS

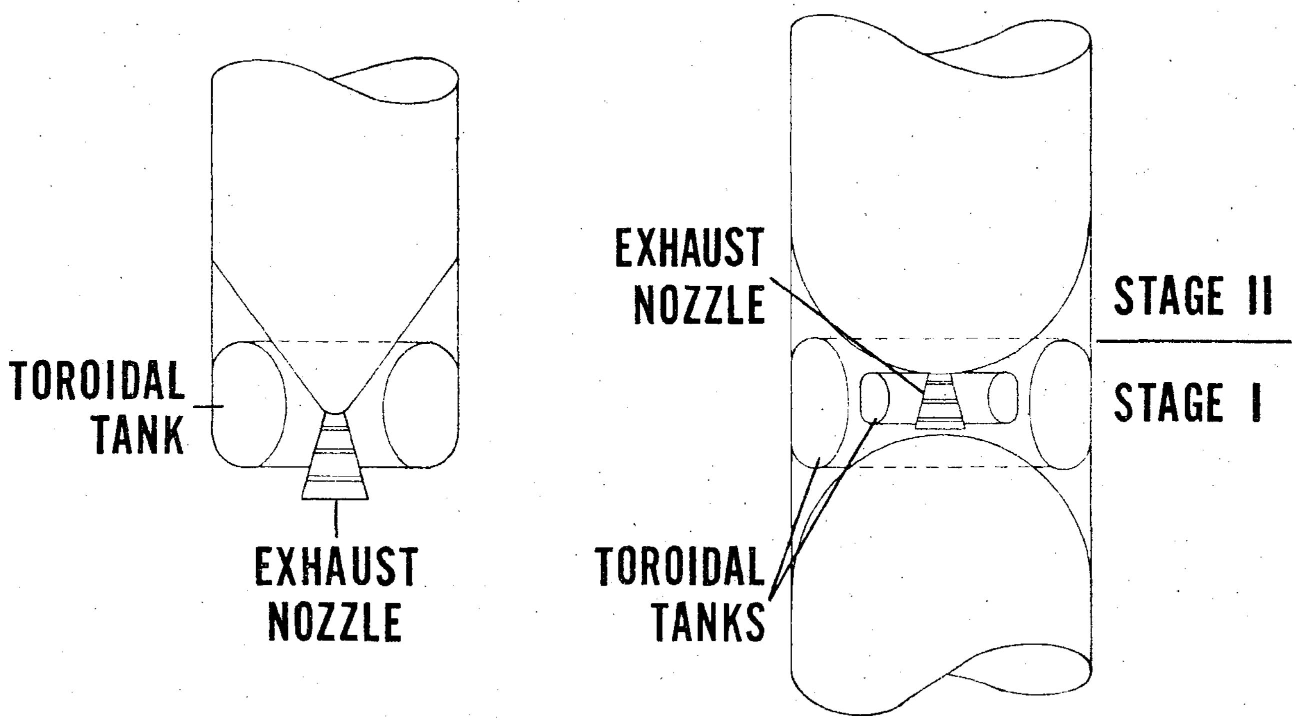

FIGURE 2

PRESSURE LOADS & MERIDIONAL STRESS RESULTANT

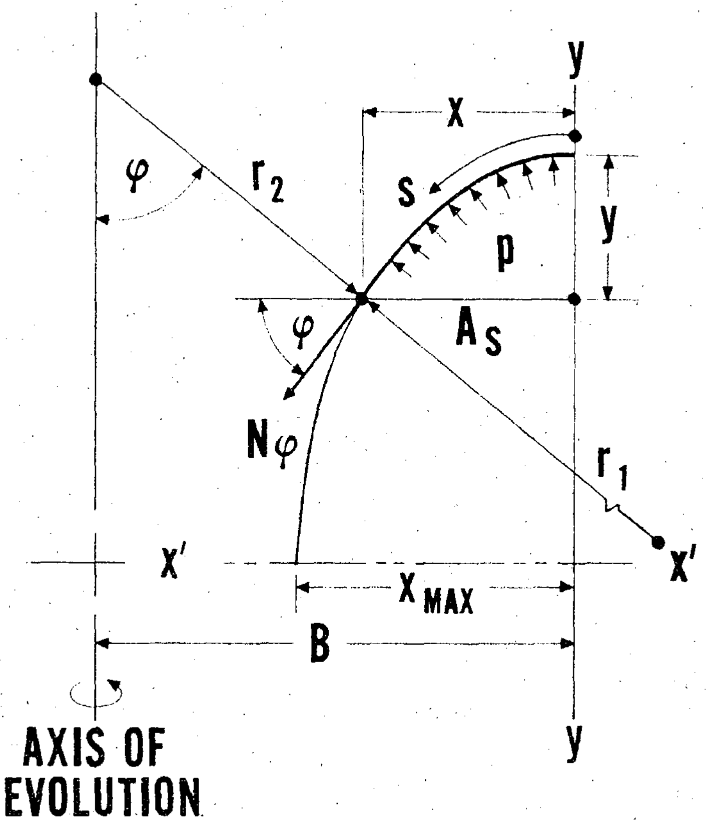

FIGURE 3

INTERNAL VOLUME CALCULATIONS

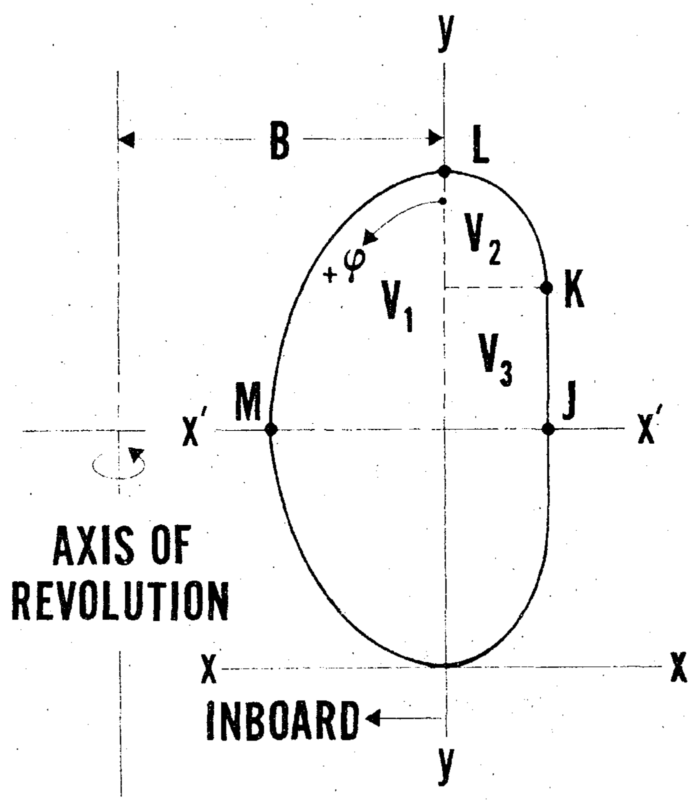

FIGURE 4

SURFACE AREA CALCULATIONS

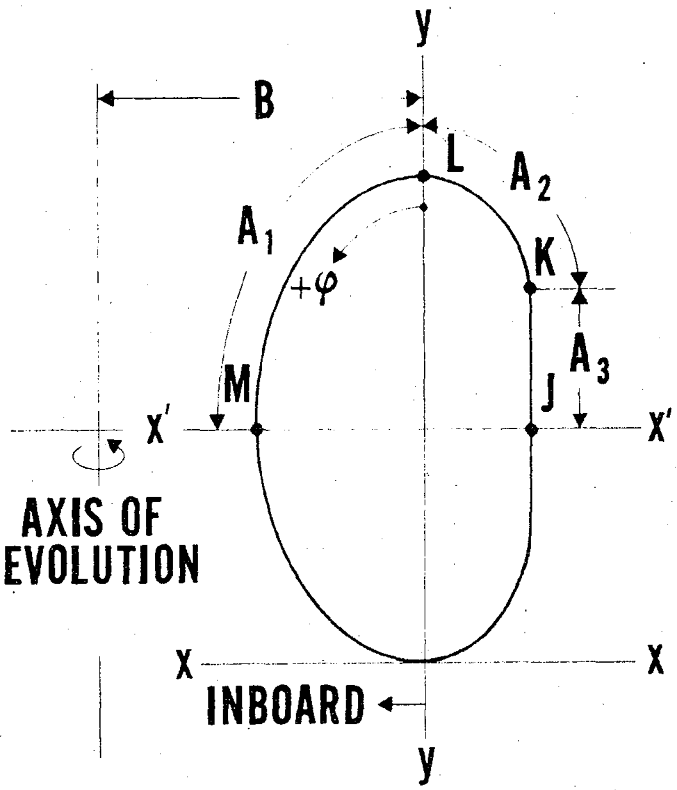

FIGURE 5

MERIDIONAL CONTOURS

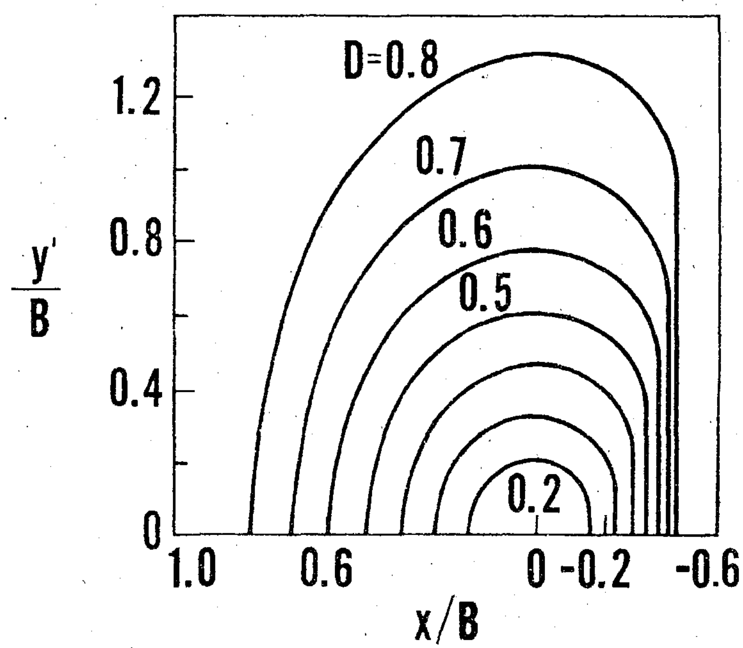

FIGURE 6

MERIDIONAL RADII OF CURVATURE

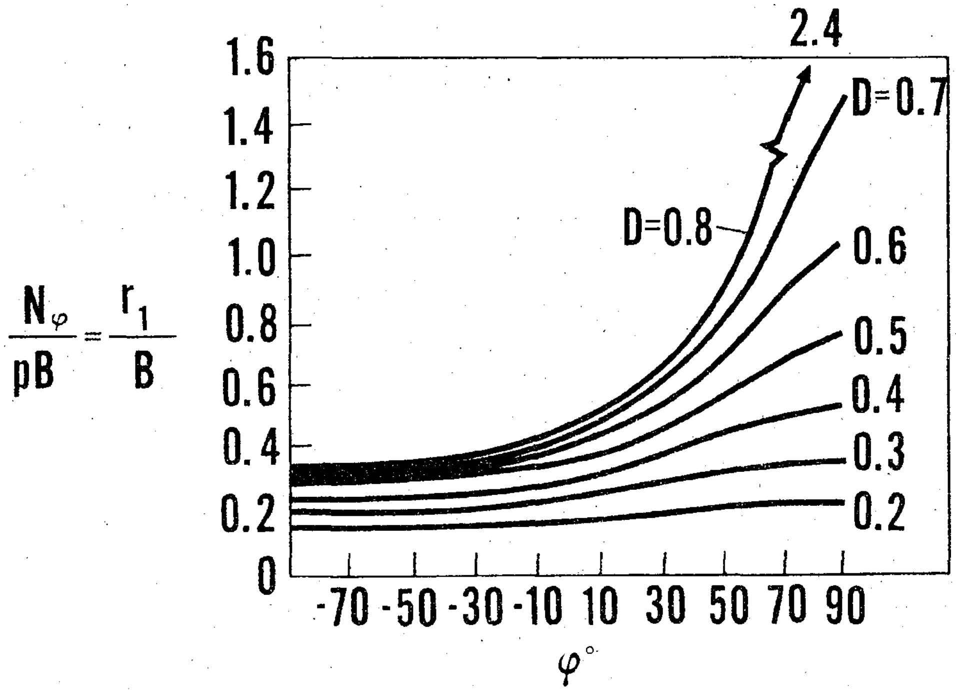

FIGURE 7

RELATIVE VARIATION OF INTERNAL VOLUME

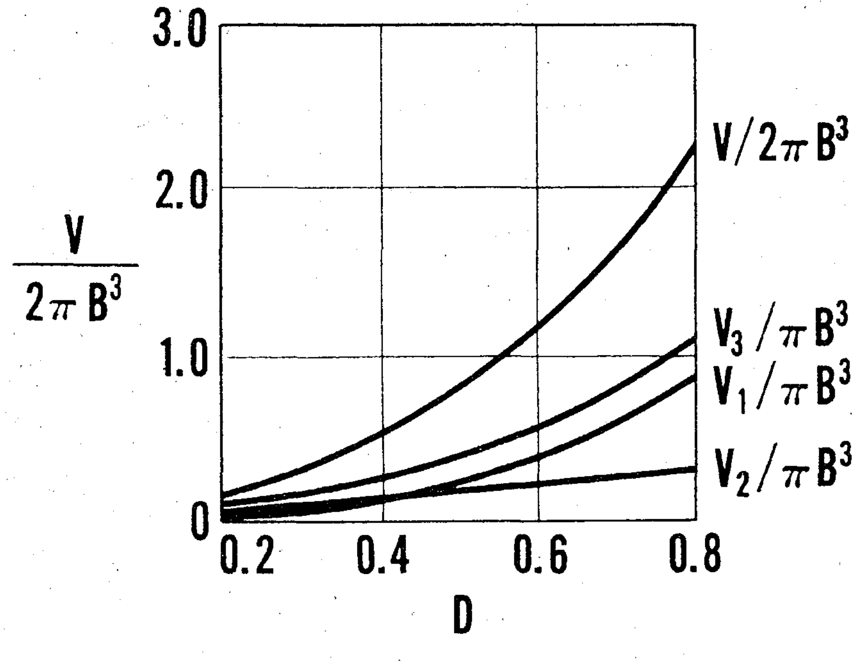

FIGURE 8

RELATIVE VARIATION OF SURFACE AREA

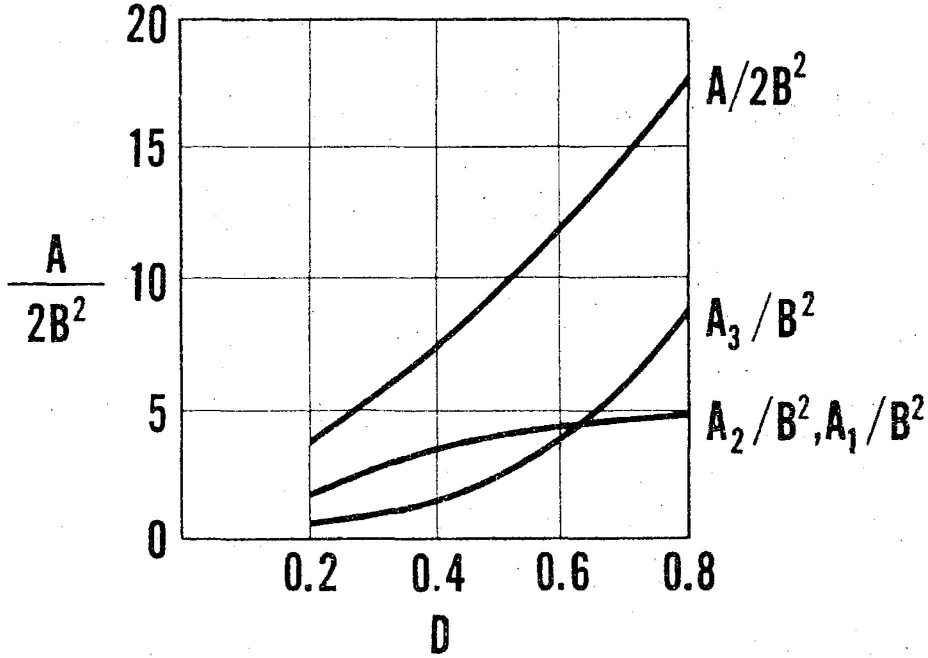

FIGURE 9

HOOP STRESS ON CYLINDRICAL PORTION OF SHELL & MAXIMUM MERIDIONAL STRESS AT $\varphi = \frac{\pi}{2}$

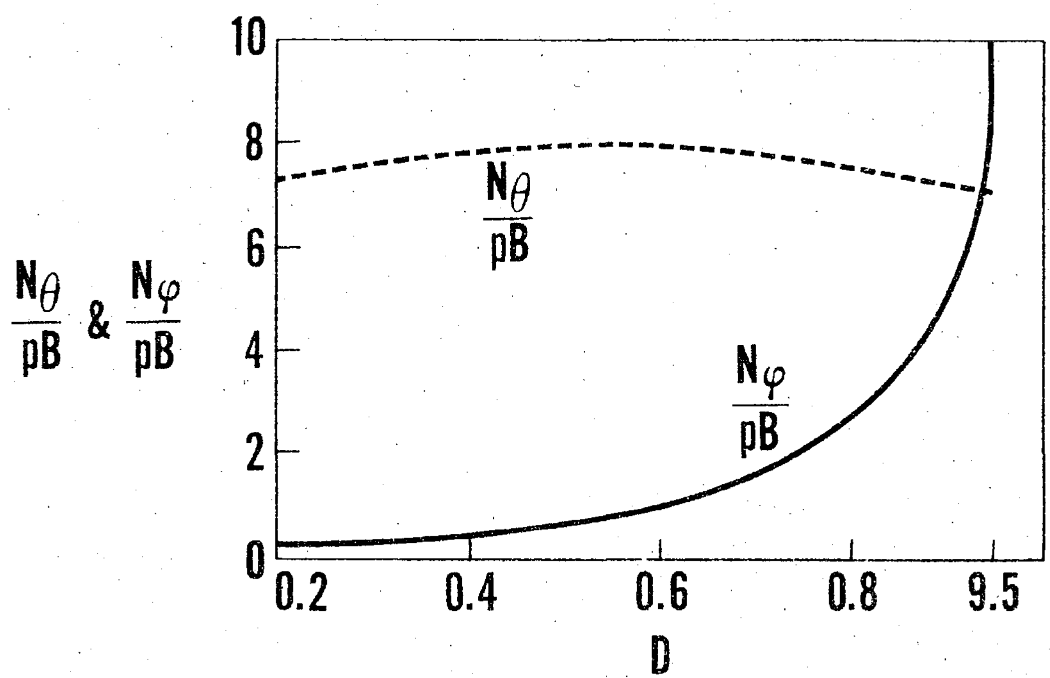

FIGURE 10

DISTRIBUTION

Division	Recipient	Mail Zone	No. of Copies
G. O.	W. J. Eding	Rockefeller Plaza N. Y. 20, N. Y.	1
Convair	G. G. Green	6-143	2
	Division Library	6-153	1
Fort Worth	E. L. Secrest	E-69	3
	E. H. Watts	C-67	2
	Division Library	Y-10	1
Pomona	G. E. Burkheimer	6-18	3
	M. F. Stoughton	6-95	1
	J. J. Marcinek	6-95	1
	H. A. Swift	6-56	2
	Division Library	6-20	1
Astronautics	H. F. Dunholter	590-00	1
	F. J. Dore	580-00	1
	V. A. Babits	592-00	1
	A. H. Hausrath	557-10	1
	Division Library	128-00	2
	Appropriate Research Personnel		