NYO-9687

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QUANTUM FIELD THEORY

by

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ROCHESTER , NEW YORK

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ABSTRACT

The formulation of field theories by means of Wightman functions is studied. It is shown that, given two field theories that satisfy all the axioms, one can construct a family of Wightman fields with the same properties by a process of superposition of Wightman functions. The condition of unitarity is formulated without reference to asymptotic conditions, and it is proved that the Wightman fields constructed by the above superposition process (starting with "unitary" fields) fail to preserve unitarity; and a forteori, the standard asymptotic condition.

1. Introduction

In the search for a dynamical scheme for describing elementary particle phenomena consistent with relativistic invariance and quantum mechanical principles, the theory of quantized fields has been favoured with more study and has provided more insight than any other scheme. The use of manifestly covariant local Lagrangians as a starting point and the use of perturbation expansions lead to questionable mathematical operations with infinite quantities. In view of this, during the last few years the study of general field theories without starting with any specific Lagrangian has received much attention. [1] The more fundamental part of such a program concerns the study of an abstract axiom system more or less suggested by earlier Lagrangian theories. In such a study it is worthwhile to know if the axioms are independent and whether they are compatible; while the axioms are "related" to general physical requirements their truth is neither "self evident" nor can one trust intuitive "physical" justifications for the compatibility of these axioms.

Among the set of axioms usually taken as characterizing as quantized fields, these comments apply particularly to the so-called "asymptotic condition" [2] which enables one to relate the field operators to particle scattering amplitudes. The somewhat provisional nature of this axiom has been noted before; and perhaps not unconnected with this is the fact that the other "field axioms" have been the subject of a structure analysis by Wightman. [3] Making use of the tools developed in this brilliant study we show in this paper that the "asymptotic condition" is an independent axiom and that one can construct systems satisfying all other axioms but not this axiom provided that at least one quantum field theory yielding a non-trivial scattering matrix exists. In the course of this study we have been able to construct several examples of fields with a trivial scattering matrix.

In sec. 2 we review Wightman's theory and construct certain elementary families of Wightman fields using the technique of vacuum expectation values. Sec. 3 discusses the weak axiom of asymptotic particle interpretation and the

normalization of the field. The main result of the present paper is to show that almost all members of the families of fields constructed in sec. 2 do not satisfy the (weak) axiom of asymptotic particle interpretation; this result is stated and proved in sec. 4. Certain related comments are made in the concluding section.

2. Families of Wightman Fields

According to Wightman[3], a quantum field theory is defined in terms of a Hilbert space $\mathcal{H}$ and a set of hermitian linear operators $\phi(x)$ labelled by a four-vector x provided the following conditions are satisfied:

1) Lorentz invariance: there must exist unitary operators $U(a, \Lambda)$ such that

$$\phi(\Lambda x + a) = U(a, \Lambda) \, \phi(x) \, U^{-1}(a, \Lambda)$$

for every proper orthochronous inhomogeneous Lorentz transformation.

2) Absence of negative energy states: the spectrum of the Hamiltonian operator must be nonnegative, the Hamiltonian being defined as the hermitian generator of time translations.

3) Local commutativity: the commutator of two field operators at space like points must vanish:

$$[\phi(x), \phi(y)] = 0 \qquad for \ (x-y)^2 < 0$$

4) The existence of the "vacuum" state: there exists a   state invariant under all $U(a, \Lambda)$.

If we now form the vacuum expectation values of products of n field operators labelled by the points $x_1, \cdots, x_n$:

$$W^{(n)}(x_1, \cdots, x_n) \equiv W^{(n)}(\{x\}) = \langle 0| \, \phi(x_1) \cdots \phi(x_n) |0\rangle \qquad (1)$$

It can then be shown that as a consequence of the conditions imposed on the Hilbert space $\mathcal{H}$ and the linear operators $\phi(x)$ that this set of functions labelled by the four-vector variables (hereafter called Wightman functions) has the following properties:

(i) $\qquad W^{(n)}(\{\Lambda x + a\}) = W^{(n)}(\{x\})$ (Lorentz invariance)

(ii) $W^{(n)}(\{x\})$ is the boundary value of a complex function $W^{(n)}(\{z\})$ analytic for $Im\{z\}$ in the backward light cone (absence of negative energies).

(iii) $W^{(n)}(\{x\}) = W^{(n)}(\{x'\})$ where $\{x'\}$ is any permutation of the n variables $\{x\}$, provided the permuted variables have space-like separations (local commutativity).

(iv)
$$\sum_{r,s} \int \cdots \int f_r^x(x_1, \cdots, x_r)\, W^{(r+s)}(x_1, \cdots, x_r, y_1, \cdots, y_s)$$

$$f_s(y_1, \cdots, y_s)\, d^4x_1 \cdots d^4x_r\, d^4y_1 \cdots d^4y_s \gtreqless 0 \tag{2}$$

where f_r are suitable arbitrary functions. (positive definite metric) Wightman has also shown[3] that these conditions are sufficient: that is, given a set of functions $W^{(n)}(\{x\})$ satisfying these conditions one can construct a theory of a (neutral scalar) field satisfying the four conditions stated at the beginning of this section which has these functions for its vacuum expectation values.

Before the field theory so defined can be used to describe a model of relativistic quantum theory of particles one must introduce some particle concepts. The structure satisfying only the conditions introduced in this section is a more general system; we shall refer to this structure as a "Wightman field."

We now state two obvious properties of a Wightman field in terms of its Wightman functions in the form of two theorems.

THEOREM I (Scale Change). If $W^{(n)}(\{x\})$ are a set of Wightman functions, the set of functions $k^n W^{(n)}(\{x\})$ define a Wightman field for every real number k.

This statement is immediately verified by noting that if $\phi(x)$ is the Wightman field which corresponds to $W^{(n)}(\{x\})$, then $k\phi(x)$ corresponds to $k^n W^{(n)}(\{x\})$.

THEOREM II (Convexity). If $W_1^{(n)}(\{x\})$ and $W_2^{(n)}(\{x\})$ are two sets of Wightman functions, the convex set

$$W^{(n)}(\{x\}) = \lambda W_1^{(n)}(\{x\}) + (1-\lambda) W_2^{(n)}(\{x\}) \tag{3}$$

defines a Wightman field provided the real number λ lies between 0 and 1.

The theorem is proved by noting that the functions $W^{(n)}(\xi \times \xi)$ satisfy all the conditions imposed on Wightman functions: Lorentz invariance, analyticity in the future tube, permutation symmetry for spacelike separated arguments and finally the condition specified by Eq. (2). Hence they define a Wightman field. Note that in this case it is not easy to construct the field operator in a simple manner but these functions satisfy all the conditions imposed on Wightman functions; hence they define a Wightman field. If   is real but not necessarily in the interval $0 \leqq \lambda \leqq 1$ then all conditions are satisfied except positive definiteness; even this last condition may be satisfied in special cases (as is seen by considering $W^{(n)}(\xi \times \xi)$ and $W_1^{(n)}(\xi \times \xi)$).

Thus given one Wightman field we can construct an infinite number of distinct Wightman fields using Theorem I; however out of this infinite set a specific choice can be made by stating a normalization condition. We shall state such a condition in the next section. Theorem II allows us to construct an infinite set of Wightman fields (normalized, if so required) from two (or more) distinct Wightman fields. Let us call the set of all Wightman fields $W^{(n)}(\xi \times \xi)$ generated by $W_1^{(n)}(\xi \times \xi)$ and $W_2^{(n)}(\xi \times \xi)$ the "family"; every point in this family is labelled by a parameter λ . We have remarked above that while $0 \leqq \lambda \leqq 1$ is allowed in all cases, values of λ outside this interval are not necessarily forbidden. It is then interesting to state the following theorem regarding the boundedness of the allowed values of λ :

THEOREM III (Semi-bounded families). There exists either a lower limit λ_1 or an upper limit λ_2 (or both) such that for either $\lambda < \lambda_1$ or $\lambda_2 < \lambda$ (or both) the combinations

$$V^n(\xi \times \xi) = \lambda \, W_1^{(n)}(\xi \times \xi) + (1-\lambda) \, W_2^{(n)}(\xi \times \xi)$$

cannot be a set of Wightman functions.

To prove the existence of such limits, we use the positive definiteness condition showing that for sufficiently large negative or positive values of they are violated. Consider in particular $W_1^{(2)}(\xi \times \xi)$

which is nonnegative according to (2). It cannot be everywhere zero without making the field operator $\phi_i(x)$ trivial. Choose any suitable testing function $f(y)$ such that

$$\int d^4y \; f^x(y) \; W_1^{(2)}(x+y/2, x-y/2) \, f(y) = 1$$

and let

$$\int f^x(y) \; W_2^{(3)}(x+y/2, x-y/2) \, f(y) \, d^4y = a \not\gtreqless 0$$

Then

$$\int f^x(y) \; V^{(2)}(x+y/2, x-y/2) \, f(y) \, d^4y = \lambda(1-a) + a$$

which becomes negative for $\lambda < -\dfrac{a}{|1-a|}$ or for $\dfrac{a}{|a-1|} > \lambda$ according as a is less than or greater than unity. Hence the statement made in the theorem is proved.

This demonstration however does not guarantee that provided $\lambda_1 < \lambda < \lambda_2$ the set $V^{(n)}(\{x\})$ are Wightman functions since the positive definiteness condition in its complete form may still be violated; it may even be violated for other testing functions using $W^{(2)}(\{x\})$ only. However from Theorem II we know that there exists the nontrivial family $0 \leqq \lambda \leqq 1$ at least. In general the family is, of course larger.

It is also remarkable that of the original fields obeyed canonical commutation relations the family of Wightman fields so generated also satisfy canonical commutation relations. This statement is consequent upon the identification of all matrix elements of the commutator of the field and its time derivative (at the same time) in terms of the Wightman functions.

3. Asymptotic Particle Interpretation and the Scattering Matrix

If this _field_ theory is to become a theory of interacting _particles,_ one must introduce particle variables into the theory and identify at least some subspace of the Hilbert space $\mathcal{H}$ as being associated with the particle states. Such a program[4] has so far not been carried out except for free fields. There is however another type of particle interpretation which is less ambitious in the sense that certain linear combinations of vacuum expectation values of the

fields are identified with a scattering amplitude for "asymptotically free" particles. Since there are certain properties to be satisfied by the scattering amplitude this identification in turn imposes some restrictions on the Wightman fields. However the scattering amplitudes themselves provide only an incomplete characterization of the field; and it appears that without the use of sufficiently strong additional postulates, the scattering amplitudes do _not_ determine the Wightman field[5]. In support of this, it is known that one can construct several distinct Wightman fields with a trivial associated scattering amplitude.[6]

It is conventional[2] to state the requirement of an asymptotic particle interpretation in terms of an appropriately stated "asymptotic condition" and then to "derive" the scattering amplitude in terms of certain linear combinations of vacuum expectation values. We shall follow the alternative method of stating the connection between the scattering amplitude and the vacuum expectation values as the additional axiom. This apparently arbitrary procedure has certain advantages: first of all, unlike the other axioms of quantum field theory, the asymptotic condition has so far been stated only in unsatisfactory forms and their plausibility is not immediately obvious. The best defence seems to be that it leads to a covariant expression for the scattering amplitude; but the expression itself could be obtained by other means, say for example by a formal summation of the perturbation series.[7] Secondly the question of completeness of the particle scattering states which is generally a prerequisite to the axiomatization of the asymptotic condition seems too strong; it is conceivable that the field Hilbert space is considerably larger than the particle Hilbert space.

We shall hence take as an axiom the following condition:[8]

The scattering matrix element related to the transition to a state containing Γ particles with four-momenta $p_1, \ldots, p_r$ from a state containing S particles with four-momenta $q_1, \ldots, q_s$ (with $p_1^2 = \cdots = q_s^2 = N^2$) is given by the expression:

$$S(p_1, \ldots, p_r; q_1, \ldots, q_s) = \int d^4 x_1 \cdots d^4 x_r \, d^4 y_1 \cdots d^4 y_s \; \Lambda(p_1, x_1)$$

$$\Lambda(p_r, x_r) \, \Lambda(-q_1, y_1) \cdots \Lambda(-q_s, y_s) \langle 0| T\{\phi(x_1) \cdots \phi(x_r) \phi(y_1) \cdots \phi(y_s)\} |0\rangle \quad (4a)$$

where

$$\Lambda(p,x) = \frac{-i}{(2\pi)^4} e^{ipx} \left(\Box_x^2 - \mu^2 \right) \tag{4b}$$

and μ is a "mass" parameter. Here the T product vacuum expectation value is defined in terms of the Wightman functions by the equations

$$\langle 0| T\{\phi(x_1) \cdots \phi(x_n)\} |0\rangle = W^{(n)}(x_1, \cdots, x_n) \tag{5a}$$
$$\text{for} \quad x_1^0 > x_2^0 > \cdots > x_n^0$$

$$\langle 0| T\{\phi(x_1) \cdots \phi(x_n)\} |0\rangle = \langle 0| T\{\phi(x_1') \cdots \phi(x_n')\} |0\rangle \tag{5b}$$

where $x_1', \cdots, x_n'$ are any permutations of $x_1, \cdots, x_n$. (Asymptotic Particle Interpretation)

Given any Wightman field we can now calculate the particle scattering matrix in terms of this identification; but there is no guarantee that the scattering matrix so defined satisfies the conditions imposed on a scattering matrix, in particular unitarity. It is considered further necessary that the one-particle states are "steady" so that the S-matrix elements connecting one-particle states to any other state vanish identically (and that the two-particle scattering is elastic below the three-particle threshold). This condition can be used to normalize the field operator:

$$\int d^4y \, \Lambda(-q,y) \int d^4x \, \Lambda(p,x) \langle 0| T\{\phi(x)\phi(y)\} |0\rangle = (2\pi)^4 \, (p^2 - \mu^2) \, \delta(p-q) \tag{6}$$

with $p^2 = q^2 = \mu^2$. It then follows that if $W^{(n)}(\{x\})$ denotes the Wightman functions for this normalized field of mass μ then $k^n W^{(n)}(\{x\})$ defines a field which is not normalized except for the special case $k = \pm 1$. The fields defined in terms of two (or more) sets of normalized Wightman functions in the form $W^{(n)} = \lambda W_1^{(n)} + (1-\lambda) W_2^{(n)}$ is also normalized in the above manner if and only if the masses are identical.

The axiom of asymptotic particle interpretation introduced here is weaker than the usual asymptotic condition in the sense that we do not assume either the completeness of the many particle states nor the existence of asymptotic fields. But if the asymptotic condition is postulated as an axiom of the theory in addition

to the axioms for a Wightman field, we can <u>derive</u> the expression for the particle

scattering matrix yielding the so-called reduction formulas.[9] Thus the axiom

of asymptotic particle interpretation for a Wightman field yields a more general

system than the Wightman field with the stronger axiom of asymptotic condition.

Needless to say everything we have proved in the following sections apply

<u>a forteori</u> to fields satisfying the usual system of axioms including the

asymptotic condition. We now proceed to show that Wightman fields in general do

<u>not</u> have an asymptotic particle interpretation.

4. Wightman Fields without Asymptotic Particle — Interpretation.

In terms of the scattering matrix S one may now define the scattering

amplitude f in the standard manner; and then note that the scattering amplitude

so defined is <u>linearly</u> related to the Wightman functions. The unitarity relation

imposed on $f(p_1 \cdots p_r; q_1 \cdots q_s)$ is

$$f(p_1 \cdots p_r; q_1 \cdots q_s) - f^x(q_1 \cdots q_s; p_1 \cdots p_r) = i \sum_{n=0}^{\infty} \int d^4 k_1 \, \delta(k_1^2 - N^2)$$

(7)

$$\theta(k_1^0) \cdots \int d^4 k_n \, \delta(k_n^2 - N^2) \, \theta(k_n^0) \, f(p_1 \cdots p_r; k_1 \cdots k_n) \, f^x(q_1 \cdots q_s; k_1 \cdots k_n)$$

or symbolically,

$$(f - f^+) = i \, f \, f^+$$

(7')

In the summation most of the terms contribute nothing since energy and momentum

must be conserved if the scattering amplitude is not to vanish. Let f_1 and f_2

be the scattering amplitudes for two Wightman fields with asymptotic particle

interpretation defined by their Wightman functions $W_1^{(n)}$ and $W_2^{(n)}$. We

shall further specialize than to correspond to the same "mass". If we now define

a field in terms of the Wightman functions

$$W^{(n)} = \lambda \, W_1^{(n)} + (1 - \lambda) \, W_2^{(n)}$$

in view of the linear relation between the Wightman function and the scattering

amplitude, it now follows that the scattering amplitude f for this Wightman field

is simply given by

$$f = \lambda f_1 + (1-\lambda) f_2$$

Using the unitarity condition, Eq. (7), twice it is now possible to derive the relation

$$\{\lambda f_1 + (1-\lambda) f_2\} \{\lambda f_1^+ + (1-\lambda) f_2^+\} = \lambda f_1 f_1^+ + (1-\lambda) f_2 f_2^+ \quad (8')$$

which may be written,

$$\lambda (1-\lambda) \sum_{n=0}^{\infty} \int d^4 k_1 \, \delta(k_1^2 - N^2) \, \theta(k_1^0) \cdots \int d^4 k_n \, \delta(k_n^2 - N^2) \, \theta(k_n^0)$$

$$g(p_1, \ldots, p_r; k_1, \ldots, k_n) \, g^x(q_1, \ldots, q_s; k_1, \ldots, k_n) \qquad (8a)$$

with

$$g = f_1 - f_2 \qquad (8b)$$

If we now specialize to the case of elastic scattering, the integrand is nonnegative and the vanishing of the integral implies that either g = 0 identically or $\lambda (1-\lambda) = 0$. In the first case the two Wightman fields must have the same scattering matrix and all the Wightman fields in the allowed family yield the same scattering matrix; the second case is trivial. We may now prove the following theorem:

THEOREM IV (Equivalent Scattering Matrices): A Wightman field defined in terms of the Wightman functions

$$W^{(n)} = \sum_\alpha \lambda_\alpha W_\alpha^{(n)} \qquad \sum \lambda_\alpha = 1 \qquad \lambda_\alpha \geqq 0$$

the functions $W_\alpha^{(n)}$ admitting asymptotic particle interpretations with the same "mass", has an asymptotic particle interpretation if and only if all the Wightman fields have the same scattering matrix.

This more general statement is proved essentially the same way as used above; one derives in place of (8') the equation

$$\sum_{\alpha > \beta} \lambda_\alpha \lambda_\beta \, (f_\alpha^+ - f_\beta^+) (f_\alpha - f_\beta) = 0$$

from which it follows that $f_\alpha = f_\beta$ unless λ_α or λ_β vanishes provided all the λ_α are nonnegative. Note that unlike the case of two fields only, here the condition $\lambda_\alpha > 0$ cannot be simply relaxed; in general, on grounds of continuity, one expects the domain of values of λ_α (with sum unity) for which

the theorem holds is somewhat larger in view of the demonstration above regarding
only two fields.

5. Discussion

We have thus shown that the axiom of asymptotic particle interpretation is
independent of the other axioms of field theory and is not derivable from it.
We have actually used only a weaker axiom in this connection in the sense that
we have neither required detailed properties of the field mass spectrum nor the
completeness of the many-particle states. Our systems are correspondingly more
general and the "unitarity conditions" are imposed only on the Fourier transforms
of the time-ordered combinations $\langle 0 | T \{ \phi(x_1) \cdots \phi(x_n) \} | 0 \rangle$ of the Wightman
functions for momenta on the mass shell; without additional restrictions this is
not sufficient to determine the field in any sense. Yet here we see that the
unitarity requirement on the particle scattering matrix excludes most Wightman
fields from having an asymptotic particle interpretation.

Perhaps the weakest point of the present investigation is that it has not
provided any example of a field theory with asymptotic particle interpretation
with a non-trivial scattering matrix; rather it asserts that if there exists at
least one such theory there exists an infinity of Wightman fields not having an
asymptotic particle interpretation belonging to the family generated by this one
field together with the free field of the same mass.

We have worked here within the framework of the conventional axiomatization
of quantum field theory. If the purpose of the field theory is only to provide a
quantum theory of interacting particles invariant under the complex Lorentz group,
the conventional axiomatization is too rigid in that it imposes "physical
requirements" on the field. This is most easily seen in the case of the axiom of
positive definiteness: in a theory where the physical particle states do not form
a complete set of states in the generalized Hilbert space in which the field
operators are defined, it is sufficient of the particle states constitute a subspace

with positive definite metric. That these considerations are not devoid of physical interest is seen from the example of the quantized Maxwell field. One of the present authors has discussed[10] examples of quantum field theories formulated in terms of a generalized Hilbert space with an indefinite metric where again the physical particle states are not complete in the generalized space but constitute only a subspace with positive definite metric. In such theories the physical interpretation requires an interpretive postulate and the precise form of this postulate depends on the dynamics of the field.

Our investigations also provide several examples of Wightman fields with a trivial scattering matrix. In addition to a trivial scale change $W^{(n)} \to k^n W^{(n)}$ we also have more generally:

$$W^{(n)} = \sum_\alpha \lambda_\alpha \, k_\alpha^n \, W_\alpha^{(n)} \qquad \sum_\alpha \lambda_\alpha = 1 \qquad \lambda_\alpha \gtrless 0 \qquad (9)$$

which provide Wightman fields, the functions $W_\alpha^{(n)}$ corresponding to known theories; say either free fields with arbitrary masses, or the Wick polynomials of free fields or terminating Haag expansions.[6] By a limiting procedure in forming such linear combinations one can produce any two-point function

$$\langle 0 | \phi(x) \, \phi(y) | 0 \rangle = \int d\rho(m^2) \, \Delta^{(1)}(m, x-y) \qquad (10)$$

(where $\Delta^{(1)}(m, x-y)$ is the two-point Wightman function for a free field of mass m) by taking for the Wightman functions:

$$W^{(n)}(\{x\}) = \int d\rho(m^2) \, W^{(n)}(m, \{x\}) \qquad (11)$$

where $W^{(n)}(m, \{x\})$ are the Wightman functions for a free field of mass m, and $\rho(m^2)$ is a nonnegative measure. But all these fields have a trivial scattering matrix.

Finally the present study illustrates the validity of Wightman's statement[3] that the consequences of positive definiteness are distinct from the consequences of unitarity. The Wightman fields constructed above satisfy positive definiteness but do not yield unitary scattering matrices, while certain indefinite metric theories (including quantum electrodynamics) provide[10] examples of theories in which the field operators are defined in a generalized Hilbert space but the

scattering matrices are unitary.

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