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CYLINDRICAL LATTICE COLLISION PROBABILITY CODES, B692/RP

by

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ABSTRACT

First-flight collision probabilities for a regular lattice of cylindrical fuel rods and moderator are calculated by means of the assumption that neutrons reaching the outer boundary of the unit cell are returned isotropically. The Fortran program written to calculate these collision probabilities is described, and comparisons are made with results of Fukai's exact method and other approximate methods. The treatment is then extended to lattices with cladding on the fuel, and a description is given of the corresponding three-region Fortran program. Collision probabilities calculated by the method presented here are in rather good agreement with exact values.

I. INTRODUCTION

First-flight collision probabilities for a lattice are useful in calculations of fast fission, resonance escape probability, and thermal disadvantage factors for closely packed lattices.

Fukai(1) has presented a calculation of first collision probabilities in a lattice of cylindrical rods which is exact, but which requires large amounts of computer time. Various authors have given approximations which are simple in form, but involve the Dancoff factor C, which is difficult to calculate accurately. A review by Nordheim(2) summarizes several of these methods. Takahashi(3) has derived collision probabilities by an equivalent-cell method. This method is inaccurate, however, because neutrons incident on the boundary of the unit cell are assumed to be returned by mirror-image reflection. As has been pointed out by Newmarch,(4) this reflective boundary condition is unrealistic since neutrons originating in the moderator and travelling within a certain finite range of directions cannot have a first collision in the fuel.

In this paper an equivalent-cell method is used, but the assumption is made that neutrons reaching the boundary of the cell are returned isotropically. Thus, a neutron originating in the moderator with any initial direction can have a first collision in the fuel. This equivalent-cell method with isotropic return has previously been used(5) in determining collision probabilities for fast-fission calculations. However, in Reference 5, approximations are made so that the calculations can be accomplished through use of tables of collision probabilities(6) for the case in which the total cross sections of fuel and moderator regions are equal.

In the following sections the theory for a two-region cell of unclad fuel and moderator is developed, and the Fortran code B692/RP, written for the IBM-704 computer, is described. Collision probabilities from B692/RP are compared with those from other methods, including Fukai's exact method. Then the formulae for a three-region unit cell consisting of fuel, clad, and moderator are presented, and a description is given of the corresponding Fortran program.

II. THEORY FOR A TWO-REGION UNIT CELL

A. Derivation of Two-region Equations

We consider a unit cell consisting of an infinitely long cylindrical fuel rod, of radius a and total cross section Σ_1, surrounded by an annular moderator region with outer radius b and total cross section Σ_2. The collision probability $P_{ij} (i,j = 1,2)$ is the probability that a neutron born uniformly and isotropically in region i will have its first collision in region j before reaching the boundary of the unit cell. Correspondingly, P^*_{ij} is the probability that a neutron born uniformly and isotropically in region i will have its first collision in region j either before reaching the unit cell boundary or after reaching the boundary and being returned isotropically one or more times. The quantity G_{3i} $(i = 1,2)$ is the probability that a neutron incident isotropically on the unit cell from the outside (region 3) will have a first collision in region i of the cell before leaving the cell again. Thus P_{ij}, P^*_{ij}, and G_{3i} are related by the equations

$$P^*_{ij} = P_{ij} + P_{i3} \frac{G_{3j}}{G_{31} + G_{32}}; \qquad (i,j = 1,2), \tag{1}$$

where P_{i3} is the probability that a neutron born in region i will leave the unit cell before having a first collision. The various probabilities are also connected by the relations

$$\sum_{j=1}^{3} P_{ij} = 1; \qquad (i = 1,2), \tag{2}$$

$$\sum_{j=1}^{2} P^*_{ij} = 1; \qquad (i = 1,2), \tag{3}$$

the reciprocity relations:

$$\Sigma_i V_i P_{ij} = \Sigma_j V_j P_{ji}; \quad (i,j = 1,2), \tag{4}$$

and

$$\Sigma_i V_i P_{ij}^{*} = \Sigma_j V_j P_{ji}^{*}; \quad (i,j = 1,2), \tag{5}$$

where V_i is the volume of region i, and the relation

$$G_{3i} = \frac{4V_i}{S_2} \Sigma_i P_{i3}, \tag{6}$$

where S_2 is the area of the outer surface of region 2. Equations (4) and (6) are derived, for example, in Reference 5, whereas Eq. (5) follows from Eqs. (1), (4), and (6). It is necessary to determine only P_{11}, P_{12}, and P_{22} from first principles, as the other quantities can be derived from these.

Derivation of expressions for P_{11} and P_{12} may be carried out with reference to Fig. 1. The line PQR is the projection in a plane perpendicular to the fuel rod axis of a typical path for a neutron originating in the fuel. The collision probability P_{11} is given by

$$P_{11} = 1 - 4\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{2a \csc\theta \cos\phi} \left(\frac{1}{\pi a^2}\right)\left(r^2 \sin\theta \, dr d\theta d\phi\right)\left(e^{-\Sigma_1 r}\right)$$

$$\times \int_0^{2\pi} \left(\frac{\sin\theta \cos\phi}{4\pi r^2}\right) a \, d\alpha$$

$$= 1 - \frac{1}{2a\Sigma_1}\left[1 - \frac{4}{\pi}\int_0^{\pi/2} K_{i3}\left(2a\,\Sigma_1 \cos\phi\right) \cos\phi \, d\phi\right], \tag{7}$$

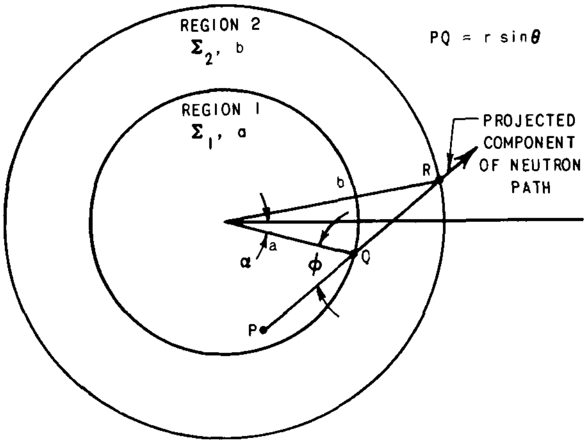

Fig. 1

Projection of Neutron Path in Plane Perpendicular to Fuel Rod Axis for Calculation of P_{11} and P_{12}.

where the definition

$$K_{i3}(x) = \int_0^{\pi/2} e^{-x \csc \theta} \sin^2\theta \, d\theta \tag{8}$$

has been used. Similarly,

$$P_{12} = 4 \int_0^{\pi/2} \int_0^{\pi/2} \int_0^{2a \csc \theta \cos \phi} \left(\frac{1}{\pi a^2}\right) (r^2 \sin \theta \, dr \, d\theta \, d\phi) \left(e^{-\Sigma_1 r}\right)$$

$$\times \left\{1 - e^{-\Sigma_2 \csc \theta [(b^2 - a^2 \sin^2\phi)^{1/2} - a \cos \phi]}\right\} \int_0^{2\pi} \left(\frac{\sin \theta \cos \phi}{4\pi r^2}\right) a \, d\alpha$$

$$= 1 - P_{11} - \frac{2}{\pi a \Sigma_1} \int_0^{\pi/2} \left[K_{i3} \{\Sigma_2 (b^2 - a^2 \sin^2\phi)^{1/2} - \Sigma_2 a \cos \phi\}\right.$$

$$\left. - K_{i3} \{\Sigma_2 (b^2 - a^2 \sin^2\phi)^{1/2} + (2 \Sigma_1 - \Sigma_2) a \cos \phi\}\right] \cos \phi \, d\phi. \tag{9}$$

For the derivation of P_{22}, see Fig. 2, where PQ is the projected component of a typical neutron path. It is seen that P_{22} may be expressed as the sum of four terms which are distinguished by the region of origin and region of collision for neutrons proceeding toward the point Q. These terms represent neutrons starting in A and colliding in A, starting in B and colliding in B, starting in C and colliding in B, and starting in C and colliding in C. Thus, P_{22} is given by

$$P_{22} = \frac{2b}{\pi(b^2 - a^2)} \left[\int_0^{\pi/2} \int_{\sin^{-1}(a/b)}^{\pi/2} \int_0^{2b \csc \theta \cos \phi} \left(1 - e^{-\Sigma_2 r}\right) \sin^2\theta \cos \phi \, dr \, d\theta \, d\phi \right.$$

$$+ \int_0^{\pi/2} \int_0^{\sin^{-1}(a/b)} \int_0^{b \csc \theta \cos \phi - \csc \theta (a^2 - b^2 \sin^2\phi)^{1/2}}$$

$$(1 - e^{-\Sigma_2 r}) \sin^2\theta \cos \phi \, dr \, d\theta \, d\phi + \int_0^{\pi/2} \int_0^{\sin^{-1}(a/b)} \int_{b \csc\theta \cos\phi + \csc\theta (a^2 - b^2 \sin^2\phi)^{1/2}}^{2b \csc \theta \cos \phi}$$

$$\left\{e^{-\Sigma_2 [r - \csc \theta (a^2 - b^2 \sin^2\phi)^{1/2} - b \csc \theta \cos \phi] - 2\Sigma_1 \csc \theta (a^2 - b^2 \sin^2\phi)^{1/2}}\right\}$$

$$\times \left\{1 - e^{-\Sigma_2 \csc \theta [b \cos \phi - (a^2 - b^2 \sin^2\phi)^{1/2}]}\right\} \sin^2\theta \cos \phi \, dr \, d\theta \, d\phi$$

$$+ \int_0^{\pi/2} \int_0^{\sin^{-1}(a/b)} \int_{b \csc \theta \cos \phi + \csc \theta (a^2 - b^2 \sin^2\phi)^{1/2}}^{2b \csc \theta \cos \phi}$$

$$\left. \left\{1 - e^{-\Sigma_2 [r - \csc \theta (a^2 - b^2 \sin^2\phi)^{1/2} - b \csc \theta \cos \phi]}\right\} \sin^2\theta \cos \phi \, dr \, d\theta \, d\phi \right] \tag{10}$$

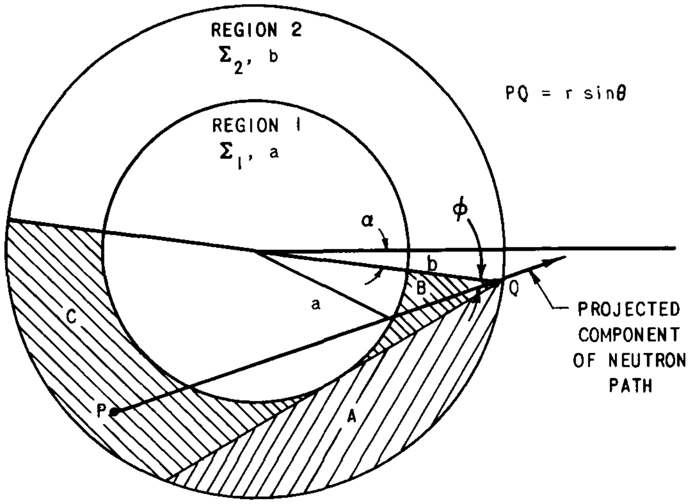

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Fig. 2. Projection of Neutron Path in Plane Perpendicular to Fuel Rod Axis for Calculation of P_{22}.

The first integral over ϕ is split into integrals from 0 to $\pi/2$ and from 0 to $\sin^{-1}(a/b)$. Then the upper limits of $\sin^{-1}(a/b)$ in the ϕ integrals are changed to $\pi/2$ by writing $\sin \phi' = (b/a) \sin \phi$. After straightforward calculation the result

$$\begin{aligned} P_{22} = 1 - \frac{1}{2(b-a)\Sigma_2} + \frac{2a}{\pi(b^2-a^2)\Sigma_2} \int_0^{\pi/2} \Bigg[\frac{b}{a} K_{i3}\,(2\,\Sigma_2 b \cos\phi) \\ - K_{i3}\left\{ 2\Sigma_2(b^2 - a^2\sin^2\phi)^{1/2} \right\} + 2K_{i3}\left\{ \Sigma_2(b^2 - a^2\sin^2\phi)^{1/2} - \Sigma_2 a\cos\phi \right\} \\ + K_{i3}\,(2\Sigma_1 a\cos\phi) - 2K_{i3}\left\{ \Sigma_2(b^2 - a^2\sin^2\theta)^{1/2} + (2\Sigma_1 - \Sigma_2)\,a\cos\phi \right\} \\ + K_{i3}\left\{ 2\Sigma_2(b^2 - a^2\sin^2\phi)^{1/2} + 2(\Sigma_1 - \Sigma_2)\,a\cos\phi \right\} \Bigg] \cos\phi\, d\phi \quad (11) \end{aligned}$$

is obtained, where the prime on ϕ' has been dropped.

Now set

$$P_0(t) = \frac{1}{2t}\left[1 - \frac{4}{\pi}\int_0^{\pi/2} K_{i3}\,(2t\cos\phi)\cos\phi\, d\phi \right], \qquad (12)$$

$$\alpha = \frac{2a}{\pi(b^2 - a^2)\Sigma_2} \int_0^{\pi/2} \Big[K_{i3} \left\{ \Sigma_2 (b^2 - a^2 \sin \phi)^{1/2} - \Sigma_2 a \cos \phi \right\} - K_{i3} \Big\{ \Sigma_2 (b^2 - a^2 \sin^2 \phi)^{1/2}$$

$$+ (2\Sigma_1 - \Sigma_2)\, a \cos \phi \Big\} \Big] \cos \phi \, d\phi, \tag{13}$$

and

$$\beta = \frac{2a}{\pi(b^2 - a^2)\Sigma_2} \int_0^{\pi/2} \Big[K_{i3} \left\{ 2\Sigma_2 (b^2 - a^2 \sin^2 \phi)^{1/2} \right\} - K_{i3} \Big\{ 2\Sigma_2 (b^2 - a^2 \sin^2 \phi)^{1/2}$$

$$+ 2(\Sigma_1 - \Sigma_2)\, a \cos \phi \Big\} \Big] \cos \phi \, d\phi. \tag{14}$$

Then the equations below may be written, use being made of Eqs. (2), (4), and (6) in obtaining P_{13}, P_{21}, P_{23}, G_{31} and G_{32}:

$$P_{11} = 1 - P_0(\Sigma_1 a); \tag{15}$$

$$P_{12} = P_0(\Sigma_1 a) - \frac{\Sigma_2 (b^2 - a^2)}{\Sigma_1 a^2}\, \alpha; \tag{16}$$

$$P_{13} = 1 - P_{11} - P_{12}; \tag{17}$$

$$P_{21} = \frac{\Sigma_1 a^2}{\Sigma_2 (b^2 - a^2)}\, P_{12}; \tag{18}$$

$$P_{22} = 1 - \frac{\Sigma_1 a^2}{\Sigma_2 (b^2 - a^2)}\, P_0(\Sigma_1 a) - \frac{b^2}{b^2 - a^2}\, P_0(\Sigma_2 b) + 2\alpha - \beta; \tag{19}$$

$$P_{23} = 1 - P_{21} - P_{22}; \tag{20}$$

$$G_{31} = \frac{2\Sigma_1 a^2}{b}\, P_{13}; \tag{21}$$

$$G_{32} = \frac{2\Sigma_2 (b^2 - a^2)}{b}\, P_{23}. \tag{22}$$

The collision probabilities P^*_{ij} may now be determined from Eq. (1).

B. The Two-region Fortran Program

The K_{i3} functions involved in the integrals are calculated from

$$K_{i3}(x) = \int_0^{\infty} \frac{e^{-x \cosh v}}{\cosh^3 v}\, dv, \tag{23}$$

which is obtained by writing csc θ = cosh v in Eq. (8). The code first computes values of cosh v and $1/\cosh^3 v$ for values of v from 0 to 8.0 at intervals of Δv, which is an input number, such that $0.01 \leq \Delta v \leq 0.1$. The arguments of the K_{i3} functions involved in Integral 1 through Integral 6 for $\phi = 0$ are then evaluated. Integrals 1 through 4 are those appearing in α and β in order of their appearance in Eqs. (13) and (14); Integrals 5 and 6 are those appearing in the expressions for $P_0(\Sigma_1 a)$ and $P_0(\Sigma_2 b)$, respectively. Normalization is such that all integrals are of the form

$$\int_0^{\pi/2} K_{i3} \text{ (function of } \phi) \cos \phi \, d\phi.$$

After evaluation of the arguments, the six corresponding values of the K_{i3} functions are computed by numerical integration by means of Weddle's rule:

$$\int_{v_0}^{v_6} f(v) \, dv = \frac{3\Delta v}{10} \left[f_0 + 5f_1 + f_2 + 6f_3 + f_4 + 5f_5 + f_6 \right].$$

The convergence criterion for the K_{i3} functions is that Weddle's rule is applied J times with J being determined by the fact that the integrand at $v = 6J\Delta v$ must be less than ϵ times its value at v = 0, where the input number ϵ is $\geq 10^{-8}$. For the function $K_{i3}(0)$, which can be integrated analytically, even with the infinite upper limit replaced by a finite number, a value of $\epsilon = 10^{-8}$ gives an error in the integral of roughly $(1/3) \times 10^{-8}$ or a relative error of about $(4/3\pi) \times 10^{-8}$. This is the error made by stopping the integration at a finite number rather than at infinity. Of course, Δv must be small enough that Weddle's rule yields a good approximation to the actual value of the integral.

After calculation of the six K_{i3} functions and the corresponding integrands for $\phi = 0$, the code repeats the process at intervals of $\Delta\phi$ until $\phi = \pi/2$ is reached. The ϕ integration is also carried out by Weddle's rule, which is repeated ν times, where ν is an input number ≤ 80. Thus $\Delta\phi = \pi/12\nu$. After the computation of the six integrals involving K_{i3} functions, only minor algebra is left in determining the various collision probabilities.

It might be noted that the functions $P_0(\Sigma_1 a)$ or $P_0(\Sigma_2 b)$ can be expressed in terms of Bessel functions without any integrals being involved (e.g., p. 8 of Ref. 5). In fact, this is how the tables of $P_c = 1 - P_0$ cylinders in Reference 7 were calculated. However, values of P_0 were calculated here by integration of K_{i3} functions in order to help determine how small the mesh interval $\Delta\phi$ should be.

The Fortran listing and input and output sheets for a sample problem are given in Appendix A.

The input to the code has FORMAT (I3, I2/6E12.6) and consists of the problem number, the number of times (ν) that Weddle's rule is repeated in the ϕ integration, the total cross section of the fuel, the total cross section of the moderator, the radius of the fuel, the outer radius of the moderator, the mesh interval (Δv) for the integrals yielding the K_{i3} functions, and the convergence criterion (ϵ) for the integrals yielding the K_{i3} functions, in the order.

The labelling of the quantities on the output sheet in Appendix A is self-explanatory except that ϕ is called U, and P_{11}^*, P_{12}^*, P_{22}^*, and P_{21}^* are labelled PFF, PFD, PDD, and PDF respectively

The code has been written to read input from tape 7 and write output on tape 6 in order to conform to the 704 monitor system. Also, a version to be run without the monitor is available, this can print all the K_{i3} functions and corresponding integrands under sense switch control.

It is suggested that the value 10^{-8} be used for ϵ in all problems. For the sample problem which has input chosen so that some of the K_{i3} functions have arguments corresponding to values in the table of Reference 8, it was found that the calculated K_{i3} functions agreed with the tabulated values to at least six significant figures and sometimes to seven or eight figures. In the sample problem a value of 0 1 was used for Δv.

To check the effect of changing the mesh interval in the ϕ integration, the sample problem was run with $\nu = 2, 4, 8$, and 16. The results are given in Table I The results are fairly good, even for $\nu = 2$, and there is no change in the six figures printed out for any of the quantities in going from $\nu = 8$ to $\nu = 16$. The values of $P_0(\Sigma_1 a)$ and $P_0(\Sigma_2 b)$ agree with the five-digit values given in Reference 7.

The probability $P_0(\Sigma_2 b)$ depends on Integral 6, which might be expected to be the most difficult to calculate accurately because, for this problem, the argument of its K_{i3} function varies more rapidly than the arguments of the other K_{i3} functions near $\phi = \pi/2$.

Originally, the program was written with the variable $\sin \phi = u$ as the integration variable for the six integrals involving K_{i3} functions. However, the rapid variations of the integrands near $u = 1$, especially for Integral 6, made this an unsatisfactory method For $\nu = 2$ there was almost 9% error in Integral 6, and even for $\nu = 16$ there was still about 0.4% error in this integral. However, the change in the integration variable from u to ϕ has eliminated the undesirable effects of rapid variation in the integrands and, as can be seen from Table I. there is only about 0.01% error in Integral 6 even for $\nu = 2$.

Table I

OUTPUT FOR A SAMPLE TWO-REGION PROBLEM

$\Sigma_1 = 1.0,\ \Sigma_2 = 2.0,\ a = 0.5,\ b = 0.75,\ \Delta v = 0.1,\ \epsilon = 10^{-8}$

ν	Integral 1	Integral 2	Integral 3	Integral 4	Integral 5	Integral 6
2	0.384628	0.155501	0.0344028	0.0792263	0.317337	0.0748268
4	0.384628	0.155501	0.0344029	0.0792263	0.317338	0.0748341
8	0.384628	0.155501	0.0344029	0.0792264	0.317338	0.0748343
16	0.384628	0.155501	0.0344029	0.0792264	0.317338	0.0748343

α	β	$P_0(\Sigma_1 a)$	$P_0(\Sigma_2 b)$	P_{11}	P_{12}	P_{13}
0.116694	-0.0228284	0.595953	0.301576	0.404047	0.304219	0.291734
0.116694	-0.0228284	0.595953	0.301573	0.404047	0.304219	0.291734
0.116694	-0.0228284	0.595953	0.301573	0.404047	0.304219	0.291734
0.116694	-0.0228284	0.595953	0.301573	0.404047	0.304219	0.291734

P_{21}	P_{22}	P_{23}	G_{31}	G_{32}
0.121688	0.474998	0.403314	0.194490	0.672191
0.121688	0.475003	0.403309	0.194489	0.672182
0.121688	0.475004	0.403309	0.194489	0.672181
0.121688	0.475004	0.403309	0.194489	0.672181

P^*_{11}	P^*_{12}	P^*_{22}	P^*_{21}
0.469514	0.530486	0.787806	0.212194
0.469515	0.530485	0.787806	0.212194
0.469515	0.530485	0.787806	0.212194
0.469515	0.530485	0.787806	0.212194

The running time for a problem with $\Delta v = 0.1$ and $\nu = 16$ is around 2 min on the IBM-704. The time should be roughly proportional to $\nu/\Delta v$.

C. Comparisons with Other Methods

Values of P^*_{12} calculated by four methods are listed in Table II. Both the Fukai and Nordheim values are taken from Reference (1) The expression (2) for the Nordheim method is

$$P^*_{12} = \frac{P_0(\Sigma_1 a)}{1 + \frac{2\Sigma_1 a C}{1 - C} P_0(\Sigma_1 a)}, \tag{24}$$

where C is the Dancoff factor. In the fourth method used in Table II, P^*_{12} is calculated from

$$P^*_{12} = P_0\left(\frac{\Sigma_1 a}{1 - C}\right). \tag{25}$$

The values of P_0 for Eq. (25) were found by nonlinear interpolation in the tables of $P_c = 1 - P_0$ in Reference (7). For all lattices in Table II, $a = 0.46482$ cm, $b = \sqrt{2}a$, and $\Sigma_2 = 1.4916$ cm^{-1}. The exact Dancoff factor for these rectangular lattices was found to be $C = 0.34587$ by Fukai$^{(1)}$ and was used in Eqs. (24) and (25).

Table II

ESCAPE PROBABILITIES BY VARIOUS METHODS

$\Sigma_1 a$	B692/RP	Fukai	Nordheim	Eq. (25)
0.1	0.81364	0.80745	0.80925	0.83414
0.3	0.58855	0.57988	0.58378	0.61799
0.5	0.45695	0.44745	0.45316	0.48137
0.8	0.33839	0.32973	0.33564	0.35320
1.0	0.28694	0.27918	0.28461	0.29697
2.0	0.15912	0.15490	0.15762	0.15995
4.0	0.082044	0.080363	0.081104	0.081356
10.0	0.033088	0.032600	0.032667	0.032680

Inspection of Table II shows that for these lattices the Nordheim method gives a somewhat better approximation to Fukai's exact values than does the B692/RP method. However, the Nordheim formula requires the calculation of an exact Dancoff factor to give such accurate results. The B692/RP code yields better results than Eq. (25) except for fuel rods with large Σ_1. All of these methods are better than the approximations of Wigner, Roe, or Takahashi as is shown by comparing Table II values with those in Table 2 of Reference 1.

III. THEORY FOR A THREE-REGION UNIT CELL

A. The Three-region Equations

1. The Collision Probabilities

The derivation of the collision probability formulae for a three-region unit cell proceeds in the same manner as for a two-region cell. Since no new complications are involved, only the results are given here. The fuel, clad, and moderator regions have radii a, b, and c, and total cross sections Σ_1, Σ_2, and Σ_3 respectively. We use the definitions of $P_0(t)$, α, and β given in Eqs. (12), (13), and (14), and of γ, δ, and ϵ as defined below.

$$\gamma = \frac{2a}{\pi(b^2 - a^2)\Sigma_2}\int_0^{\pi/2}\Big[K_{i3}\Big\{\Sigma_3(c^2 - a^2\sin^2\phi)^{1/2} + (\Sigma_2 - \Sigma_3)(b^2 - a^2\sin^2\phi)^{1/2} - \Sigma_2 a\cos\phi\Big\}$$

$$- K_{i3}\Big\{\Sigma_3(c^2 - a^2\sin^2\phi)^{1/2} + (\Sigma_2 - \Sigma_3)(b^2 - a^2\sin^2\phi)^{1/2} + (2\Sigma_1 - \Sigma_2)a\cos\phi\Big\}\Big]\cos\phi\, d\phi, \tag{26}$$

$$\delta = \frac{2a}{\pi(b^2 - a^2)\Sigma_2}\int_0^{\pi/2}\Big[K_{i3}\Big\{\Sigma_3(c^2 - a^2\sin^2\phi)^{1/2} + (2\Sigma_2 - \Sigma_3)(b^2 - a^2\sin^2\phi)^{1/2}\Big\}$$

$$- K_{i3}\Big\{\Sigma_3(c^2 - a^2\sin^2\phi)^{1/2} + (2\Sigma_2 - \Sigma_3)(b^2 - a^2\sin^2\phi)^{1/2} + 2(\Sigma_1 - \Sigma_2)\, a\cos\phi\Big\}$$

$$+ \frac{b}{a}K_{i3}\Big\{\Sigma_3(c^2 - b^2\sin^2\phi)^{1/2} - \Sigma_3 b\cos\phi\Big\} - \frac{b}{a}K_{i3}\Big\{\Sigma_3(c^2 - b^2\sin^2\phi)^{1/2}$$

$$+ (2\Sigma_2 - \Sigma_3)\, b\cos\phi\Big\}\Big]\cos\phi\, d\phi, \tag{27}$$

$$\epsilon = \frac{2a}{\pi(b^2 - a^2)\Sigma_2}\int_0^{\pi/2}\Big[K_{i3}\Big\{2\Sigma_3(c^2 - a^2\sin^2\phi)^{1/2} + 2(\Sigma_2 - \Sigma_3)(b^2 - a^2\sin^2\phi)^{1/2}\Big\}$$

$$- K_{i3}\Big\{2\Sigma_3(c^2 - a^2\sin^2\phi)^{1/2} + 2(\Sigma_2 - \Sigma_3)(b^2 - a^2\sin^2\phi)^{1/2} + 2(\Sigma_1 - \Sigma_2)\, a\cos\phi\Big\}$$

$$+ \frac{b}{a}K_{i3}\Big\{2\Sigma_3(c^2 - b^2\sin^2\phi)^{1/2}\Big\} - \frac{b}{a}K_{i3}\Big\{2\Sigma_3(c^2 - b^2\sin^2\phi)^{1/2}$$

$$+ 2(\Sigma_2 - \Sigma_3)\, b\cos\phi\Big\}\Big]\cos\phi\, d\phi. \tag{28}$$

Only P_{11}, P_{12}, P_{13}, P_{22}, P_{23}, and P_{33} need be calculated from first principles since all other quantities may be derived from these by using equations for a three-region unit cell analogous to Eqs. (1) through (6) for a two-region cell. The quantities P_{11}, P_{12}, and P_{22} have been given above in Eqs. (15), (16), and (19); the quantities P_{13}, P_{23}, and P_{33} are given below.

$$P_{13} = \frac{\Sigma_2(b^2 - a^2)}{\Sigma_1 a^2}(\alpha - \gamma), \tag{29}$$

$$P_{23} = \frac{b^2}{b^2 - a^2}P_0(\Sigma_2 b) - \alpha + \beta + \gamma - \delta, \tag{30}$$

$$P_{33} = 1 - \frac{\Sigma_2 b^2}{\Sigma_3(c^2 - b^2)}P_0(\Sigma_2 b) - \frac{c^2}{c^2 - b^2}P_0(\Sigma_3 c) - \frac{\Sigma_2(b^2 - a^2)}{\Sigma_3(c^2 - b^2)}(\beta - 2\delta + \epsilon). \tag{31}$$

2. Calculation of Average Thermal Fluxes

The three-region Fortran program described in the next section has an option providing for calculation of average thermal fluxes by the method described below.

The thermal flux, $\phi(r)$, in a lattice where all scattering is isotropic satisfies the equation

$$\phi(\underline{r})\,\Sigma(\underline{r}) = \int_{\text{lattice}} P(\underline{r}'\rightarrow\underline{r})\Big[q(\underline{r}') + \Sigma_s(\underline{r}')\phi(\underline{r}')\Big]dV(\underline{r}'), \tag{32}$$

where $P(\underline{r}'\rightarrow\underline{r})$ is the probability that a neutron originating at $\underline{r}'$ has a first collision at $\underline{r}$, and $q(\underline{r}')$ is the slowing-down source at $\underline{r}'$. In Eq. (32) replace the flux in each region i of the lattice by its average value $\bar{\phi}_i$ (flat-flux approximation) and integrate over each of the N regions in turn. This procedure gives

$$\sum_{j=1}^{N} \bar{\phi}_j A_{ij} = S_i \qquad (i = 1,2, \ldots N) \tag{33}$$

as the equations determining the average fluxes, where

$$A_{ij} = V_j(\delta_{ij}\,\Sigma_i - \Sigma_{sj}\,P^*_{ji}) \tag{34}$$

and

$$S_i = \sum_{j=1}^{N} q_j V_j P^*_{ji}\,. \tag{35}$$

It has been assumed that the source q_i in each region is spatially constant.

The accuracy of this method depends on the validity of the flat-flux approximation. One may say roughly that the flat-flux approximation is reasonably good if the thickness of each region in the cell is less than one mean free path. Calculations for two-region slab lattices by Fukai(9) show that the errors in $(\bar{\phi}_2/\bar{\phi}_1) - 1$ are less than 7% for cases in which both fuel and moderator slabs are one mean free path thick.

B. The Three-region Fortran Program

The computations of the collision probabilities in the three-region version of B692/RP are carried out in essentially the same manner as in the two-region code. Since there are now 17 integrals over ϕ to be evaluated rather than six, a three-region problem requires about three times as much computer time as a two-region problem.

Appendix B contains the Fortran listing along with input and output sheets for a three-region problem. Input is read from tape 7 and output is written on tape 6 as required by the monitor system.

The input is as follows:

Card 1: MU, problem number	FORMAT, I3
NU, number of times Weddle's rule is repeated in the ϕ integration	FORMAT, I3
NF, flux calculation is done if NF > 0	FORMAT, I3
NP, K_{i3} functions are printed if NP > 0	FORMAT, I3
Δv, mesh interval for integrals yielding K_{i3} functions	FORMAT, E 12.6
E, convergence criterion for integrals yielding K_{i3} functions	FORMAT, E 12.6
Card 2: a, b, c, Σ_1, Σ_2, Σ_3	FORMAT, 6 E 12.6
Card 3 (if NF > 0): Σ_{s1}, Σ_{s2}, Σ_{s3}, q_1, q_2, q_3 [see Eq. (34) and (35)]	FORMAT, 6 E 12.6.

In the headings on the output sheet, Integrals 1 through 14 are those appearing in α, β, γ, δ, and ϵ in their order of appearance in Eqs. (13), (14), (26), (27), and (28). Their normalization is such that they are all of the form

$$\int_0^{\pi/2} K_{i3}(\text{function of } \phi) \cos \phi \, d\phi.$$

Likewise, Integrals 15, 16, and 17 are those appearing in the expressions for $P_0(\Sigma_1 a)$, $P_0(\Sigma_2 b)$, and $P_0(\Sigma_3 c)$, respectively, and have the same normalization. The collision probabilities P^*_{ij} are labelled PFF, etc., where F denotes fuel (region 1), C denotes clad (region 2), and M denotes moderator (region 3). In the optional flux calculation, the sources q_i are labelled SOURCE, and the fluxes ϕ_i calculated from Eq. (33) are renormalized to give a value of unity for ϕ_1. If the input had called for a printout of K_{i3} functions (NP > 0), these would have appeared before the 17 related integrals in $6\nu + 1$ groups of 17 corresponding to the values of ϕ from 0 to $\pi/2$.

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APPENDIX A

Fortran Listing, Input and Output Sheets for the Two-region Case

704 INPUT DATA

FORM I

COST CODE

PROGRAM B692/RP PROBLEM TWO-REGION ORIGINATOR DATE PAGE 1 OF 1

```
         1         2         3         4         5         6         7         8
123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
00102                                                                        1-1
+.100000E+01+.200000E+01+.500000E+00+.750000E+00+.100000E+00+.100000E-07     1-2
```

```
C     CYLINDRICAL CELL COLLISION PROBABILITY PROGRAM
      DIMENSIONV(810),A(810),B(810),U(500),F(500),Y(500),G(500),X(500),
     XZ(6,500),XK3(6,500),H(6,500),S(6),T(6),C(6,810),XINT(6),
     XXSUM(6,80)
    2 READ INPUT TAPE 7,5,MU,NU,SIGMA1,SIGMA2,R1,R2,DELTAV,E
    5 FORMAT(I3,I2/6E12.6)
      XNU=NU
      DELTAU=1.5707963/(6.*XNU)
      WRITE OUTPUT TAPE 6,10,MU,SIGMA1,SIGMA2,R1,R2,DELTAV,E,DELTAU
   10 FORMAT(64H1CYLINDRICAL CELL COLLISION PROBABILITY PROGRAM  PROBLEM
     X NUMBER I3/104H0      SIGMA 1       SIGMA 2           A
     X    B          KI3 MESH       EPSILON        DELTA U/1H 7E15.6)
   11 S1A=SIGMA1*R1
      S2B=SIGMA2*R2
      S2A=SIGMA2*R1
      D1=1.-(2.*SIGMA1/SIGMA2)
      D2=1.-(SIGMA1/SIGMA2)
      D3=2.*SIGMA1/SIGMA2
      D4=2.*R2/R1
      D5=R2**2./R1**2.
      D6=0.3*DELTAV
      D7=(0.63661977*R1)/(SIGMA2*(R2**2.-R1**2.))
      D8=(SIGMA2)*(R2**2.-R1**2.)/(SIGMA1*(R1**2.))
      D9=R2**2./(R2**2.-R1**2.)
      D10=2.*SIGMA1*(R1**2.)/R2
      D11=2.*SIGMA2*(R2**2.-R1**2.)/R2
      D12=0.3*DELTAU
      D13=(R2**2.-R1**2.)/(R1**2.)
      V(1)=0.0
      A(1)=1.0
      B(1)=1.0
      W=0.0
      XLU=8.0/DELTAV
      LU=XLU
      DO12I=2,LU
      V(I)=W+DELTAV
      A(I)=COSHF(V(I))
      B(I)=1./((A(I))**3.)
   12 W=V(I)
      L=6*NU+1
      Q=0.0
      DO50K=1,L
      U(K)=Q
      IF(K-L)14,13,14
   13 F(K)=0.0
      Y(K)=0.0
      G(K)=0.0
      GO TO 15
   14 F(K)=COSF(U(K))
      Y(K)=S2A*F(K)
      G(K)=(F(K))**2.
   15 X(K)=S2A*SQRTF(D13+G(K))
   17 Z(1,K)=X(K)-Y(K)
      Z(2,K)=X(K)-D1*Y(K)
      Z(3,K)=2.*X(K)
      Z(4,K)=2.*(X(K)-(D2*Y(K)))
      Z(5,K)=D3*Y(K)
      Z(6,K)=D4*Y(K)
      DO36N=1,6
      S(N)=1./EXPF(Z(N,K))
      T(N)=S(N)
      J=1
```

```
     M1=6*J-4
     M2=6*J+1
  20 DO25I=M1,M2
  25 C(N,I)=B(I)/EXPF(Z(N,K)*A(I))
     S(N)=S(N)+5.*C(N,6*J-4)+C(N,6*J-3)+6.*C(N,6*J-2)+C(N,6*J-1)
    X+5.*C(N,6*J)+2.*C(N,6*J+1)
     IF(C(N,6*J+1)-E*T(N))35,30,30
  30 J=J+1
     M1=6*J-4
     M2=6*J+1
     GO TO 20
  35 XK3(N,K)=D6*(S(N)-C(N,6*J+1))
  36 H(N,K)=(XK3(N,K))*(F(K))
  50 Q=U(K)+DELTAU
     DO60N=1,6
     XINT(N)=0.0
     DO60M=1,NU
     XSUM(N,M)=D12*(H(N,6*M-5)+5.*H(N,6*M-4)+H(N,6*M-3)
    X+6.*H(N,6*M-2)+H(N,6*M-1)+5.*H(N,6*M)+H(N,6*M+1))
  60 XINT(N)=XINT(N)+XSUM(N,M)
     ALPHA=D7*(XINT(1)-XINT(2))
     BETA=D7*(XINT(3)-XINT(4))
     POS1A=(1./(2.*S1A))-(0.63661977*XINT(5)/S1A)
     POS2B=(1./(2.*S2B))-(0.63661977*XINT(6)/S2B)
     P11=1.-POS1A
     P12=POS1A-(D8*ALPHA)
     P13=D8*ALPHA
     G31=D10*P13
     P21=(POS1A/D8)-ALPHA
     P22=1.-(D9*POS2B)-(POS1A/D8)+(2.*ALPHA)-BETA
     P23=(D9*POS2B)-ALPHA+BETA
     G32=D11*P23
     PFF=P11+(P13*G31)/(G31+G32)
     PFD=P12+(P13*G32)/(G31+G32)
     PDD=P22+(P23*G32)/(G31+G32)
     PDF=P21+(P23*G31)/(G31+G32)
     WRITE OUTPUT TAPE 6,70,(XINT(N),N=1,6),ALPHA,BETA,POS1A,
    XPOS2B,P11,P12,P13,G31,P21,P22,P23,G32,PFF,PFD,PDD,PDF
  70 FORMAT(90H0   INTEGRAL 1     INTEGRAL 2    INTEGRAL 3     INTEGR
    XAL 4     INTEGRAL 5    INTEGRAL 6/1H 6E15.6/62H0      ALPHA
    X     BETA      PO(SIGMA 1 A)  PO(SIGMA 2 B)/1H 4E15.6/
    X57H0        P11           P12           P13           G31/
    X1H 4E15.6/57H0       P21           P22           P23
    X G32/1H 4E15.6/57H0       PFF           PFD           PDD
    X      PDF/1H 4E15.6)
  75 GO TO 2
     END  (   1 ,   1 ,   0 ,   1 ,   0   )                        ANL-30
```

```
CYLINDRICAL CELL COLLISION PROBABILITY PROGRAM  PROBLEM NUMBER    1

      SIGMA 1         SIGMA 2            A               B           KI3 MESH        EPSILON         DELTA U
    0.100000E 01    0.200000E 01    0.500000E 00    0.750000E 00    1.000000E-01    1.000000E-08    0.130900E-00

     INTEGRAL 1      INTEGRAL 2      INTEGRAL 3      INTEGRAL 4      INTEGRAL 5      INTEGRAL 6
    0.384628E-00    0.155501E-00    0.344028E-01    0.792263E-01    0.317337E-00    0.748268E-01

       ALPHA            BETA       PO(SIGMA 1 A)   PO(SIGMA 2 B)
    0.116694E-00   -0.228284E-01    0.595953E 00    0.301576E-00

        P11             P12             P13             G31
    0.404047E-00    0.304219E-00    0.291734E-00    0.194490E-00

        P21             P22             P23             G32
    0.121688E-00    0.474998E-00    0.403314E-00    0.672191E 00

        PFF             PFD             PDD             PDF
    0.469514E-00    0.530486E 00    0.787806E 00    0.212194E-00
* * ALL DATA PROCESSED.
```

APPENDIX B

Fortran Listing, Input and Output Sheets for the Three-region Case

704 INPUT DATA

FORM I

COST CODE

PROGRAM B692/RP PROBLEM THREE-REGION ORIGINATOR DATE PAGE 1 OF 1

001004001000	+.100000E+00	+.100000E-07				1-1
+.435610E+00	+.477520E+00	+.716500E+00	+.866540E+00	+.940500E+00	+.269085E+01	1-2
+.365110E+00	+.794080E+00	+.267574E+01	+.000000E+00	+.000000E+00	+.100000E+01	1-3

```
C     COLLISION PROBABILITIES THREE REGION CYLINDRICAL CELL
      DIMENSION SIGMA(3),SIGS(3),SOURCE(3),G4(3),VOL(3),RT(3),DEL(3),
     XFLUX(3),P(3,4),PSTAR(3,3),AM(3,3),S(17),T(17),XINT(17),
     XXSUM(17,40),Z(17,250),XK3(17,250),H(17,250),C(17,410),U(250),
     XF(250),G(250),X1(250),X2(250),X3(250),V(410),A(410),B(410)
    2 READ INPUT TAPE7,5,MU,NU,NF,NP,DELTAV,E,R1,R2,R3,(SIGMA(I),I=1,3)
    5 FORMAT(4I3,2E12.6/6E12.6)
      IF(NF)9,9,6
    6 READ INPUT TAPE7,8,(SIGS(I),I=1,3),(SOURCE(I),I=1,3)
    8 FORMAT(6E12.6)
    9 XNU=NU
      DELTAU=1.5707963/(6.*XNU)
      WRITE OUTPUT TAPE6,10,MU,DELTAV,E,DELTAU,R1,R2,R3,(SIGMA(I),I=1,3)
   10 FORMAT(75H1COLLISION PROBABILITIES FOR THREE REGION CYLINDRICAL CE
     XLL  PROBLEM NUMBER I3/44HC     KI3 MESH     CONVERGENCE     DELT
     XA U/1H 3E15.6/89H0         A              B              C
     X   SIGMA 1        SIGMA 2        SIGMA 3/1H 6E15.6)
C     CALCULATE VARIOUS CONSTANTS
      D1=R1**2.
      D2=R2**2.
      D3=R3**2.
      D4=D2-D1
      D5=D3-D1
      D6=D3-D2
      D7=2.*SIGMA(1)*R1
      D8=2.*SIGMA(2)
      D9=SIGMA(2)*R1
      D10=2.*D9
      D11=2.*SIGMA(2)*R2
      D12=2.*SIGMA(3)
      D13=SIGMA(3)*R2
      D15=2.*SIGMA(3)*R3
      D17=D7-D10
      D21=D8-D12
      D22=D21*R2
      D23=0.3*DELTAV
      D24=0.3*DELTAU
      D25=(0.63661977*R1)/(SIGMA(2)*D4)
      D26=(SIGMA(2)*D4)/(SIGMA(1)*D1)
      D27=(SIGMA(2)*D4)/(SIGMA(3)*D6)
C     CALCULATE QUANTITIES COMMON TO KI3 INTEGRANDS
      V(1)=0.0
      A(1)=1.0
      B(1)=1.0
      W=0.0
      XLU=8.0/DELTAV
      LU=XLU
      DO12I=2,LU
      V(I)=W+DELTAV
      A(I)=COSHF(V(I))
      B(I)=1./((A(I))**3.)
   12 W=V(I)
C     CALCULATE ARGUMENTS OF KI3 FUNCTIONS
      L=6*NU+1
      Q=0.0
      DO50K=1,L
      U(K)=Q
      IF(K-L)14,13,14
   13 F(K)=0.0
      G(K)=0.0
      GO TO15
   14 F(K)=COSF(U(K))
```

```
      G(K)=(F(K))**2.
   15 X1(K)=SQRTF(D4+D1*G(K))
      X2(K)=SQRTF(D5+D1*G(K))
      X3(K)=SQRTF(D6+D2*G(K))
   17 Z(1,K)=SIGMA(2)*X1(K)-D9*F(K)
      Z(2,K)=Z(1,K)+D7*F(K)
      Z(3,K)=D8*X1(K)
      Z(4,K)=Z(1,K)+Z(2,K)
      Z(5,K)=SIGMA(3)*(X2(K)-X1(K))+Z(1,K)
      Z(6,K)=Z(5,K)+D7*F(K)
      Z(7,K)=Z(1,K)+Z(5,K)+D10*F(K)
      Z(8,K)=Z(7,K)+D17*F(K)
      Z(9,K)=SIGMA(3)*X3(K)-D13*F(K)
      Z(10,K)=Z(9,K)+D11*F(K)
      Z(11,K)=D12*X2(K)+D21*X1(K)
      Z(12,K)=Z(11,K)+D17*F(K)
      Z(13,K)=D12*X3(K)
      Z(14,K)=Z(13,K)+D22*F(K)
      Z(15,K)=D7*F(K)
      Z(16,K)=D11*F(K)
      Z(17,K)=D15*F(K)
C     EVALUATE INTEGRALS WHICH GIVE KI3 FUNCTIONS
      DO36N=1,17
      S(N)=1./EXPF(Z(N,K))
      T(N)=S(N)
      J=1
   18 M1=6*J-4
      M2=6*J+1
   20 DO25I=M1,M2
   25 C(N,I)=B(I)/EXPF(Z(N,K)*A(I))
      S(N)=S(N)+5.*C(N,6*J-4)+C(N,6*J-3)+6.*C(N,6*J-2)+C(N,6*J-1)
     X+5.*C(N,6*J)+2.*C(N,6*J+1)
      IF(C(N,6*J+1)-E*T(N))35,30,30
   30 J=J+1
      GO TO18
   35 XK3(N,K)=D23*(S(N)-C(N,6*J+1))
   36 H(N,K)=XK3(N,K)*F(K)
   50 Q=U(K)+DELTAU
C     OPTIONAL OUTPUT OF KI3 FUNCTIONS
      IF(NP)58,58,52
   52 WRITE OUTPUT TAPE6,55,((XK3(N,K),N=1,17),K=1,L)
   55 FORMAT(1H06E15.8/6E15.8/5E15.8)
C     EVALUATE INTEGRALS WHICH INVOLVE KI3 FUNCTIONS
   58 DO60N=1,17
      XINT(N)=0.0
      DO60M=1,NU
      XSUM(N,M)=D24*(H(N,6*M-5)+5.*H(N,6*M-4)+H(N,6*M-3)
     X+6.*H(N,6*M-2)+H(N,6*M-1)+5.*H(N,6*M)+H(N,6*M+1))
   60 XINT(N)=XINT(N)+XSUM(N,M)
C     CALCULATE COMBINATIONS OF KI3 INTEGRALS
   65 ALPHA=D25*(XINT(1)-XINT(2))
      BETA=D25*(XINT(3)-XINT(4))
      GAMMA=D25*(XINT(5)-XINT(6))
      DELTA=D25*(XINT(7)-XINT(8)+(R2/R1)*(XINT(9)-XINT(10)))
      EPS=D25*(XINT(11)-XINT(12)+(R2/R1)*(XINT(13)-XINT(14)))
      POS1A=(1.-1.27323954*XINT(15))/D7
      POS2B=(1.-1.27323954*XINT(16))/D11
      POS3C=(1.-1.27323954*XINT(17))/D15
C     CALCULATE P(I,J) AND G4(I)
   70 P(1,1)=1.-POS1A
      P(1,2)=POS1A-D26*ALPHA
      P(1,3)=D26*(ALPHA-GAMMA)
```

```
      P(1,4)=1.-P(1,1)-P(1,2)-P(1,3)
      P(2,1)=P(1,2)/D26
      P(2,2)=1.-(POS1A/D26)-(D2/D4)*(POS2B)+2.*ALPHA-BETA
      P(2,3)=(D2/D4)*POS2B-ALPHA+BETA+GAMMA-DELTA
      P(2,4)=1.-P(2,1)-P(2,2)-P(2,3)
   72 P(3,1)=SIGMA(1)*D1*P(1,3)/(SIGMA(3)*D6)
      P(3,2)=D27*P(2,3)
      P(3,3)=1.-SIGMA(2)*D2*POS2B/(SIGMA(3)*D6)-D3*POS3C/D6
     X-D27*(BETA-2.*DELTA+EPS)
      P(3,4)=1.-P(3,1)-P(3,2)-P(3,3)
      G4(1)=2.*D1*SIGMA(1)*P(1,4)/R3
      G4(2)=2.*D4*SIGMA(2)*P(2,4)/R3
      G4(3)=2.*D6*SIGMA(3)*P(3,4)/R3
      GSUM=G4(1)+G4(2)+G4(3)
C     CALCULATE PSTAR(I,J)
   80 DO85I=1,3
      DO85J=1,3
   85 PSTAR(I,J)=P(I,J)+P(I,4)*G4(J)/GSUM
C     OPTIONAL CALCULATION OF AVERAGE FLUXES
      IF(NF)135,135,90
   90 VOL(1)=D1
      VOL(2)=D4
      VOL(3)=D6
      DO100I=1,3
      RT(I)=0.0
      DO100J=1,3
  100 RT(I)=RT(I)+SOURCE(J)*VOL(J)*PSTAR(J,I)
      DO110I=1,3
      DO110J=1,3
      IF(I-J)104,102,104
  102 AM(I,J)=VOL(J)*(SIGMA(I)-SIGS(J)*PSTAR(J,I))
      GO TO110
  104 AM(I,J)=-VOL(J)*SIGS(J)*PSTAR(J,I)
  110 CONTINUE
      DEL(1)=RT(1)*(AM(2,2)*AM(3,3)-AM(2,3)*AM(3,2))
     X+RT(2)*(AM(1,3)*AM(3,2)-AM(1,2)*AM(3,3))
     X+RT(3)*(AM(1,2)*AM(2,3)-AM(2,2)*AM(1,3))
      DEL(2)=RT(1)*(AM(2,3)*AM(3,1)-AM(2,1)*AM(3,3))
     X+RT(2)*(AM(1,1)*AM(3,3)-AM(1,3)*AM(3,1))
     X+RT(3)*(AM(1,3)*AM(2,1)-AM(1,1)*AM(2,3))
      DEL(3)=RT(1)*(AM(2,1)*AM(3,2)-AM(2,2)*AM(3,1))
     X+RT(2)*(AM(1,2)*AM(3,1)-AM(1,1)*AM(3,2))
     X+RT(3)*(AM(1,1)*AM(2,2)-AM(1,2)*AM(2,1))
      DO120I=1,3
  120 FLUX(I)=DEL(I)/DEL(1)
C     WRITE OUTPUT DATA
  135 WRITE OUTPUT TAPE6,145,(XINT(N),N=1,17),ALPHA,BETA,GAMMA,
     XDELTA,EPS,POS1A,POS2B,POS3C
      WRITE OUTPUT TAPE6,150,((P(I,J),J=1,4),I=1,3)
      WRITE OUTPUT TAPE6,155,(G4(I),I=1,3)
      WRITE OUTPUT TAPE6,160,((PSTAR(I,J),J=1,3),I=1,3)
  145 FORMAT(90H0    INTEGRAL 1     INTEGRAL 2     INTEGRAL 3     INTEGR
     XAL 4     INTEGRAL 5     INTEGRAL 6/1H 6E15.6/90H0    INTEGRAL 7
     X  INTEGRAL 8     INTEGRAL 9     INTEGRAL10     INTEGRAL11     INTE
     XGRAL12/1H 6E15.6/75H0    INTEGRAL13     INTEGRAL14     INTEGRAL15
     X    INTEGRAL16     INTEGRAL17/1H 5E15.6/74H0       ALPHA
     XBETA           GAMMA          DELTA          EPSILON/1H 5E15.6/47H0
     X   P0(SIGMA 1 A)  P0(SIGMA 2 B)  P0(SIGMA 3 C)/1H 3E15.6)
  150 FORMAT(57H0        P11            P12            P13            P1
     X4/1H 4E15.6/57H0        P21            P22            P23
     X   P24/1H 4E15.6/57H0        P31            P32            P33
     X        P34/1H 4E15.6)
  155 FORMAT(42H0        G41            G42            G43/1H 3E15.6)
  160 FORMAT(42H0        PFF            PFC            PFM/1H 3E15.6/42H
     X0        PCF            PCC            PCM/1H 3E15.6/42H0        P
     XMF            PMC            PMM/1H 3E15.6)
      IF(NF)180,180,165
  165 WRITE OUTPUT TAPE6,170,(I,SIGMA(I),SIGS(I),SOURCE(I),FLUX(I),
     XI=1,3)
  170 FORMAT(61H0REGION     SIGMA         SIGMA S        SOURCE
     X FLUX/(1H I4,4E15.6))
  180 GO TO2
      END  (   1 ,   1 ,   0 ,   1 ,   0   )                            ANL-30
```

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COLLISION PROBABILITIES FOR THREE REGION CYLINDRICAL CELL  PROBLEM NUMBER  1

      KI3 MESH       CONVERGENCE       DELTA U
   1.000000E-01   1.000000E-08   0.654498E-01

         A               B               C            SIGMA 1         SIGMA 2         SIGMA 3
   0.435610E-00   0.477520E-00   0.716500E 00   0.866540E 00   0.940500E 00   0.269085E 01

     INTEGRAL 1      INTEGRAL 2      INTEGRAL 3      INTEGRAL 4      INTEGRAL 5      INTEGRAL 6
   0.734757E 00   0.365256E-00   0.323070E-00   0.341727E-00   0.308525E-00   0.155355E-00

     INTEGRAL 7      INTEGRAL 8      INTEGRAL 9      INTEGRAL10      INTEGRAL11      INTEGRAL12
   0.138153E-00   0.146150E-00   0.316725E-00   0.139581E-00   0.611729E-01   0.647470E-01

     INTEGRAL13      INTEGRAL14      INTEGRAL15      INTEGRAL16      INTEGRAL17
   0.149130E-01   0.596776E-01   0.390888E-00   0.345721E-00   0.458966E-01

       ALPHA            BETA            GAMMA           DELTA          EPSILON
   0.284698E 01  -0.143749E-00   0.118017E 01   0.143458E 01  -0.405630E-00

   PO(SIGMA 1 A)  PO(SIGMA 2 B)  PO(SIGMA 3 C)
   0.665352E 00   0.623252E 00   0.244182E-00

        P11             P12             P13             P14
   0.334648E-00   0.421779E-01   0.364848E-00   0.258327E-00

        P21             P22             P23             P24
   0.192690E-00   0.844302E-01   0.468465E-00   0.254414E-00

        P31             P32             P33             P34
   0.781328E-01   0.219596E-01   0.546857E 00   0.353051E-00

        G41             G42             G43
   0.118568E-00   0.255602E-01   0.756682E 00

        PFF             PFC             PFM
   0.368650E-00   0.495078E-01   0.581842E 00

        PCF             PCC             PCM
   0.226177E-00   0.916491E-01   0.682174E 00

        PMF             PMC             PMM
   0.124603E-00   0.319773E-01   0.843420E 00

REGION     SIGMA           SIGMA S         SOURCE          FLUX
   1    0.866540E 00    0.365110E-00    0.              0.100000E 01
   2    0.940500E 00    0.794080E 00    0.              0.111732E 01
   3    0.269085E 01    0.267574E 01    0.100000E 01    0.123938E 01
* * ALL DATA PROCESSED.
```