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RESEARCH MEMORANDUM

THE EFFECT OF PLASTICITY ON DECOUPLING OF UNDERGROUND EXPLOSIONS

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SUMMARY

The effect of plasticity, including work hardening, on decoupling underground explosions has been studied both for cavities designed to give full decoupling according to the Geneva specification (70 cubic meters per ton of explosive energy) as well as small (overdriven) cavities designed to give partial decoupling. An important result is that plasticity plays no role whatsoever for full-decoupling cavities, even those at great depth in which some plastic flow occurs during construction of the cavity. For overdriven cavities at great depth plasticity affects the decoupling factor by an amount which depends upon the degree of overdriving and the depth as well as the detailed stress-strain curve of the medium. A further result of the study is that for cavities at a depth of about one kilometer and in a medium like salt, which exhibits a reasonable amount of work hardening, the decoupling factor will be at least as great as that obtained in the overdriven Cowboy experiments and could be appreciably greater. To obtain more quantitative conclusions better stress-strain data are needed for loading conditions appropriate to the decoupling problem. Plastic flow associated with pressure transients, ignored here, should also be examined.

CONTENTS

I. INTRODUCTION

This report grew out of a number of questions concerning the influence of plasticity on the seismic signal from decoupled explosions. First, it is well known that around cavities at great depth where the overburden pressure is comparable to the yield stress of the medium, there is a zone in which plastic flow occurs during the making of the cavity. The question arises: how does the presence of this plastic zone affect the decoupling factor?

Secondly, we recall that the volume of a decoupling cavity (in salt at a depth of about 1 kilometer) was set at 7×10^4 cubic meters per kiloton by requiring that the medium respond elastically to the explosive forces.(1) There are two kinds of inelasticity which are important: cracking and plastic flow. To avoid making cracks, the hoop stresses must remain compressive because rock-like materials have little or no strength in tension, and this requires that the average pressure in the cavity be less than three times the overburden pressure. If the medium is already cracked, the average pressure in the cavity must be less than the overburden pressure itself in order to keep the crack from opening up and propagating. Finally, to avoid plastic flow, stress differences must not become large compared to the yield stress of the medium, even during the passage of the large pressure pulse associated with the reflection of the shock wave from the cavity wall. This latter effect--plasticity due to the pressure spike--imposes the most stringent limit and fixes the volume quoted above.*

* At a depth of about a kilometer the volume requirement of 7×10^4 cubic meters per kiloton is equivalent to requiring that the average pressure in the cavity not exceed one-half of the overburden pressure.

However this volume could clearly be reduced if the pressure spike were eliminated, as in fact it can be by methods already suggested.[2] In this case, there would be a new limit on the volume due to plastic flow associated with the _average_ pressure in the cavity. The question then arises: at what depth does this limit become more stringent than the cracking limit?

Finally we note that the Cowboy experiments were conducted at a depth of ~250 meters, where the overburden is only 50 bars. Therefore, even at five times the overburden--the greatest overdriving in Cowboy--the pressure barely exceeds the elastic limit of the salt and plasticity plays essentially no role in reducing the decoupling.* However, for the nominal cavity at a depth of 1 kilometer, five times the overburden is ~1000 bars--clearly exceeding the elastic limit of salt--and therefore plastic effects cannot be ignored. The question is: how can the results of Cowboy for the overdriven shots be extrapolated to deeper cavities?

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* The reduction in decoupling observed in the overdriven Cowboy experiments implies inelastic behavior of the medium, but this inelasticity was most probably due to cracking.

II. STRESS AND STRAIN DISTRIBUTION BEFORE THE EXPLOSION

It is convenient to state the equations in a form which applies to both plasticity and elasticity. Throughout we make the infinitesimal strain approximation.

The stress equilibrium equation, assuming spherical symmetry, is

$$\frac{\partial \sigma_r}{\partial r} + \frac{2}{r} (\sigma_r - \sigma_t) = 0, \qquad (1)$$

where σ_r is the radial and σ_t the tangential stress. The stresses σ_r and σ_t are related to the displacement ξ (which is purely radial) in the following way:

$$\frac{1}{3} (\sigma_r + 2\sigma_t) = k \left(\frac{\partial \xi}{\partial r} + \frac{2\xi}{r} \right) , \qquad (2)$$

where k, the incompressibility, is assumed to be constant and the same for both elastic and plastic deformations. Equation (2) merely states that the incompressibility times the dilatation is equal to the average stress.

In addition it is assumed that the shear stress $\sigma_r - \sigma_t$ is a function of the shear strain ε only, i.e.,

$$\sigma_r - \sigma_t = \sigma(\varepsilon). \qquad (3)$$

The shear strain is of course just the difference between the radial and tangential strains:

$$\varepsilon \equiv \frac{\partial \xi}{\partial r} - \frac{\xi}{r} . \qquad (4)$$

In the case of elasticity $\sigma(\varepsilon)$ is given by

$$\sigma(\varepsilon) = 2\mu\varepsilon, \qquad (5)$$

where μ is the modulus of rigidity. Since for small enough strains the medium is elastic, any actual function $\sigma(\varepsilon)$ must approach $2\mu\varepsilon$ near the origin. It should also be noted that Eq. (3) implies that the shear strain is independent of the average pressure, which is only roughly true for real materials.

It is mathematically convenient to approximate the stress-strain relation $\sigma(\varepsilon)$ by a pair of straightline segments as shown in the solid lines of Fig. 1. In this approximation

$$\sigma_r - \sigma_t = \begin{cases} 2\mu\varepsilon & (\sigma_r - \sigma_t \leqslant \sigma_o) \\ 2\alpha\varepsilon + \sigma_o(1 - \frac{\alpha}{\mu}) & (\sigma_r - \sigma_t \geqslant \sigma_o), \end{cases} \qquad (6)$$

where σ_o is the elastic limit, and α is a constant less than μ, chosen to give a reasonable fit to the measured stress-strain relation $\sigma(\varepsilon)$. The approximation of Eq. (6) exhibits the essential elastoplastic features of our problem and includes ''ideal'' plasticity (α=0) as a special case.

Equation (3) applies for processes in which the shear stress does not decrease. The forming of a cavity by excavation or washing is such a process. However, for an explosion, which involves increasing the pressure in the cavity and therefore decreasing the existing shear stress, Eq. (3) is not applicable. In this case, experience shows that the shear stress decreases along a line paralleling the elastic portion of the

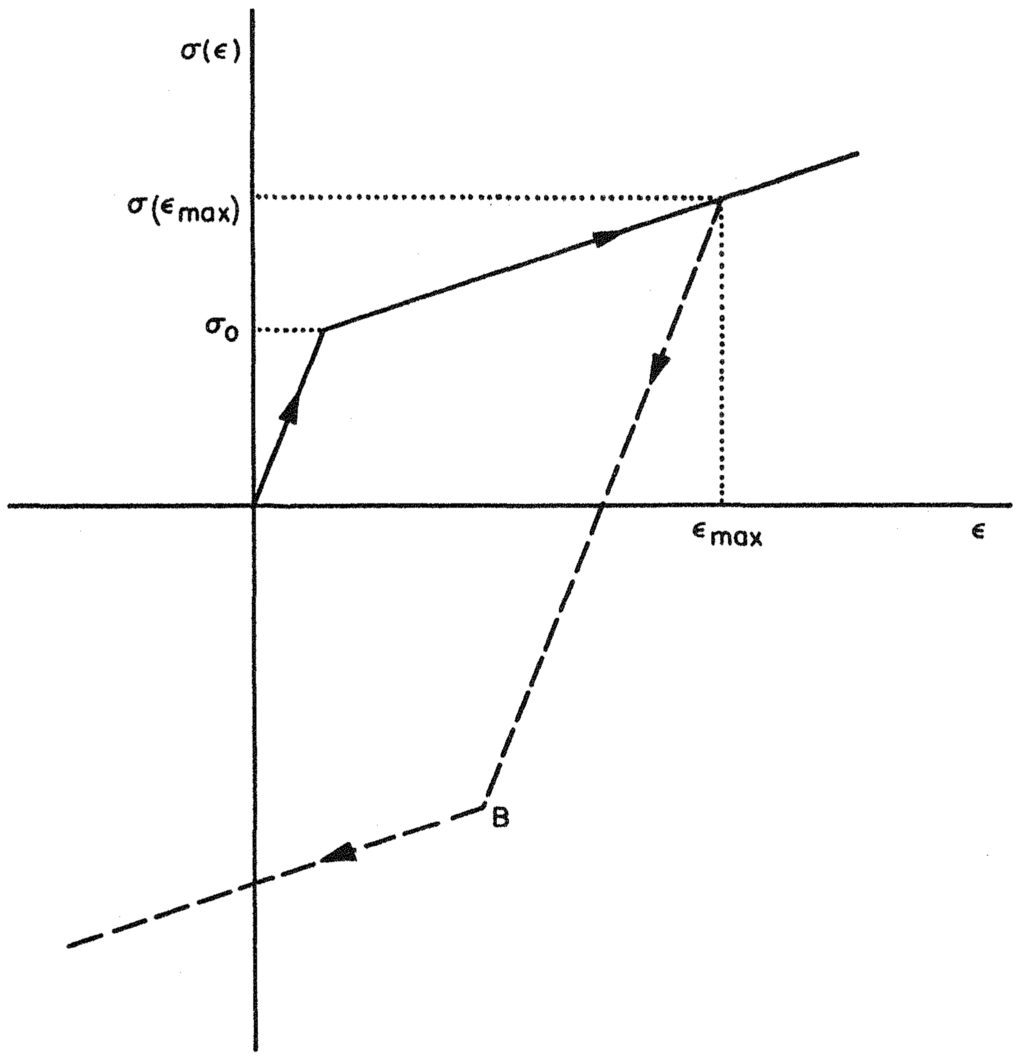

Fig. 1—The approximate stress-strain relations given by Eqs. (6) and (7). The solid portion applies to unworked material. The dashed line to the point B shows a typical unloading path. Beyond B plasticity sets in again.

stress-strain curve, so that

$$\sigma_r - \sigma_t = 2\mu(\varepsilon - \varepsilon_{max}) + \sigma(\varepsilon_{max}), \qquad (7)$$

where μ is the same modulus of rigidity as before, and ε_{max} is the maximum strain reached by the medium at the location in question. Equation (7) is a simple description of the work-hardening process and means that the medium behaves elastically on stress reduction but relative to a permanent strain state. The dashed line in Fig. 1 represents Eq. (7), with arrows appropriately indicating the loading path. Note that plasticity occurs again at the break point B where $\sigma(\varepsilon) = -\sigma(\varepsilon_{max})$.

To calculate the stresses resulting from the creation of a hole, the above equations must be solved subject to appropriate boundary conditions. We assume that the radius a of the hole is small compared to its depth and that the stress in the medium before the existence of the hole is hydrostatic and equal to $-p_o$, that is, the negative of the lithostatic pressure at that depth. It follows that far from the hole the stress becomes hydrostatic and equal to $-p_o$, i.e.,

$$\sigma_r = \sigma_t = -p_o \quad (r \gg a). \qquad (8)$$

A second condition is that at the boundary of the (empty) hole the radial stress is equal to zero, i.e.,

$$\sigma_r = 0 \qquad (r = a). \qquad (9)$$

We assume that initially the medium is everywhere at the origin of the stress-strain curve, i.e., that there are no shear stresses or shear strains prior to the formation of the hole. In forming the hole each point of the medium moves along the stress-strain curve $\sigma(\epsilon)$ to some final state ϵ_{max} which depends upon the radius r of the point and the hydrostatic pressure p_o.

The maximum shear stress $(\sigma_r - \sigma_t)$ occurs of course at the cavity surface $r = a$. If the lithostatic pressure p_o is less than some critical value, the shear stress at $r = a$ is less than σ_o and the entire medium is elastic. For p_o greater than the critical value there is a radius $\rho(>a)$ at which the shear stress is exactly σ_o, beyond which the medium is elastic, and inside which the shear stress is determined by the plastic branch of the stress-strain curve. Requiring stresses and displacements to be continuous across the radius $r = \rho$ completes the specification of the boundary conditions.

The algebra is complex and uninstructive, but leads to the following equation for the elastic-plastic radius ρ:

$$\ln \frac{\rho}{a} + \frac{1}{3} - \frac{p_o}{2\sigma_o} + \frac{k+4\mu/3}{k} \frac{(\alpha/\mu)}{1-(\alpha/\mu)} \left[\frac{1}{3}\left(\frac{\rho}{a}\right)^3 - \frac{p_o}{2\sigma_o} \right] = 0. \quad (10)$$

Figure 2 shows the solutions for ρ/a as a function of $p_o/2\sigma_o$ for several values of α/μ and for a Poisson ratio equal to 1/4. In the limiting cases $\alpha/\mu = 0$ and 1 (ideal plasticity and elasticity, respectively) ρ/a is independent of the Poisson ratio, as is evident from Eq. (10). Note that for a Poisson ratio of 1/4 the factor $(k+4\mu/3)/k$ is equal to 9/5.

All curves start out with the same slope at the critical pressure, the pressure at which $\rho = a$, which is equal to $2\sigma_o/3$. Below this pressure there is no plasticity.

The quantities of interest can be obtained from the value of ρ/a and the following formulae:

$$\sigma = \left(\frac{\rho}{r}\right)^3 \sigma_o \qquad (r \geqslant \rho), \quad (11a)$$

$$= \frac{k}{k+4\alpha/3} \frac{\alpha}{\mu} \sigma_o \left[\frac{k+4\mu/3}{k} \left(\frac{\rho}{r}\right)^3 + \frac{\mu}{\alpha} - 1\right] \qquad (r \leqslant \rho); \quad (11b)$$

$$\sigma_r = - p_o + 2\sigma/3 \qquad (r \geqslant \rho), \quad (12a)$$

$$= - p_o + 2\sigma/3 + \frac{2k}{k+4\alpha/3} \left(1 - \frac{\alpha}{\mu}\right)\sigma_o \ln \frac{\rho}{r} \qquad (r \leqslant \rho); \quad (12b)$$

$$\xi = - \frac{\sigma_o \rho}{6\mu} \left(\frac{\rho}{r}\right)^2 \qquad (r \geqslant \rho), \quad (13a)$$

$$= - \frac{k+4\mu/3}{k+4\alpha/3} \frac{\sigma_o \rho}{6\mu} \left(\frac{\rho}{r}\right)^2 + \frac{2}{3}\left(1 - \frac{\alpha}{\mu}\right) \frac{\sigma_o}{k+4\alpha/3} r\left(\ln \frac{\rho}{r} + \frac{1}{3}\right) \qquad (r \leqslant \rho). \quad (13b)$$

Of particular interest are the displacement $\xi(a)$ and the shear stress $\sigma(a)$ at the cavity wall $r = a$. These quantities are shown in Figs. 3 and 4 as a function of $p_o/2\sigma_o$ for various values of α/μ, again for a Poisson ratio of 1/4. In these figures the unit of displacement is the elastic value $p_o a/4\mu$, and the unit of shear stress is the elastic value $3p_o/2$.

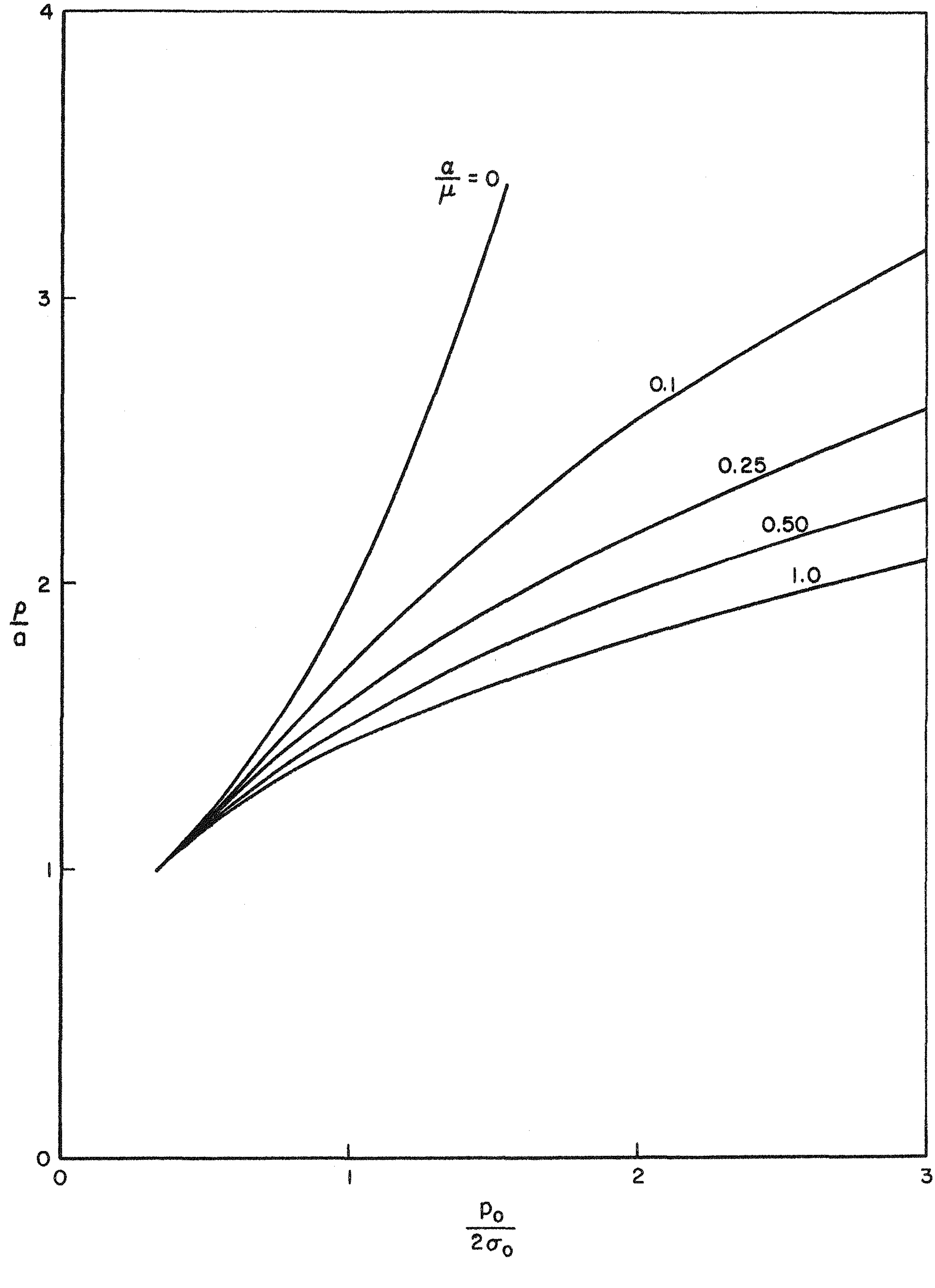

Fig. 2—The elastic-plastic radius as a function of $p_o/2\sigma_o$ for various values of α/μ and a Poisson ratio of 1/4

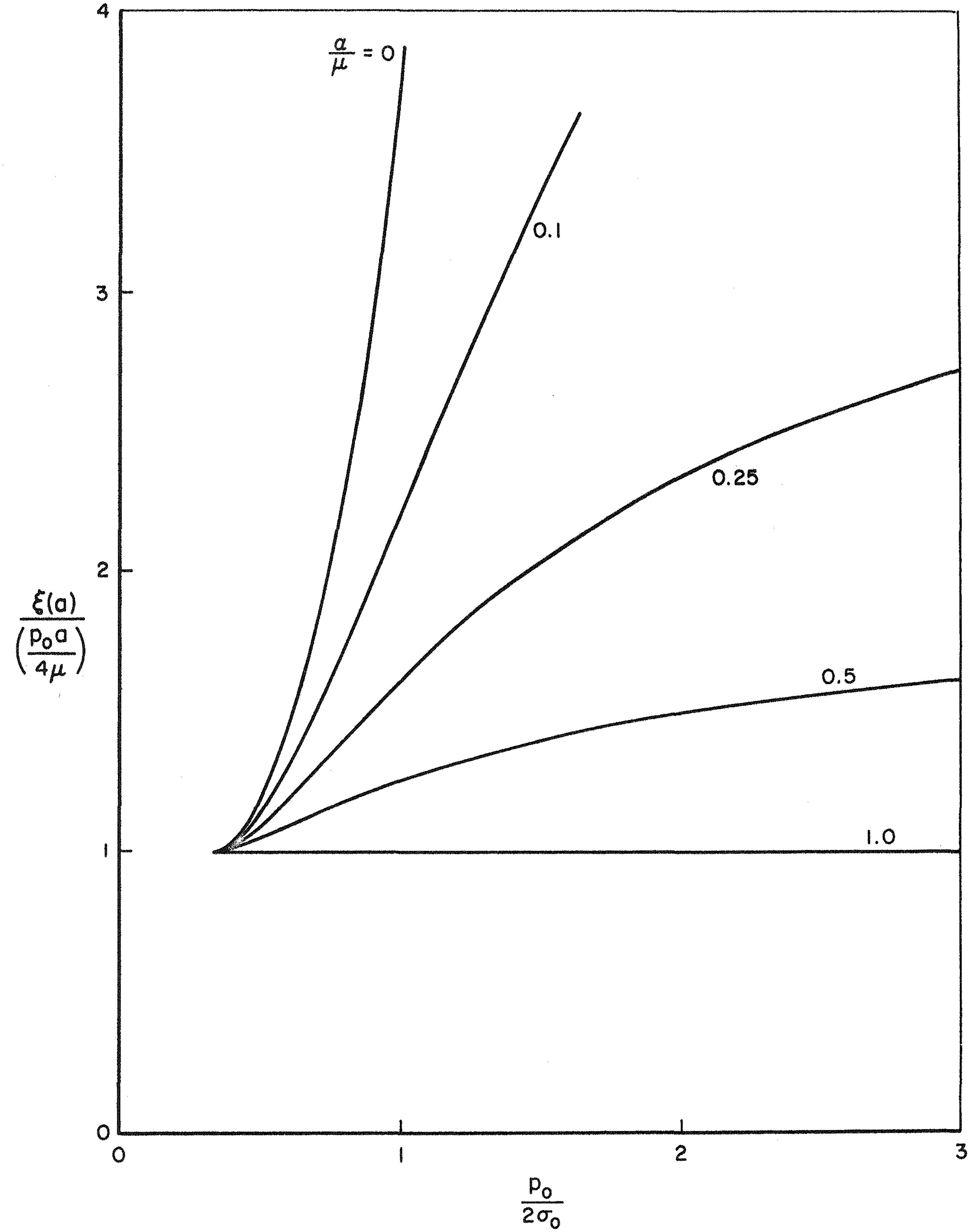

Fig. 3—The (inward) displacement at the cavity wall ($r = a$) as a function of $p_o/2\sigma_o$ for various values of a/μ and a Poisson ratio of 1/4. Note that the displacement is given as a ratio to the elastic value $p_o a/4\mu$.

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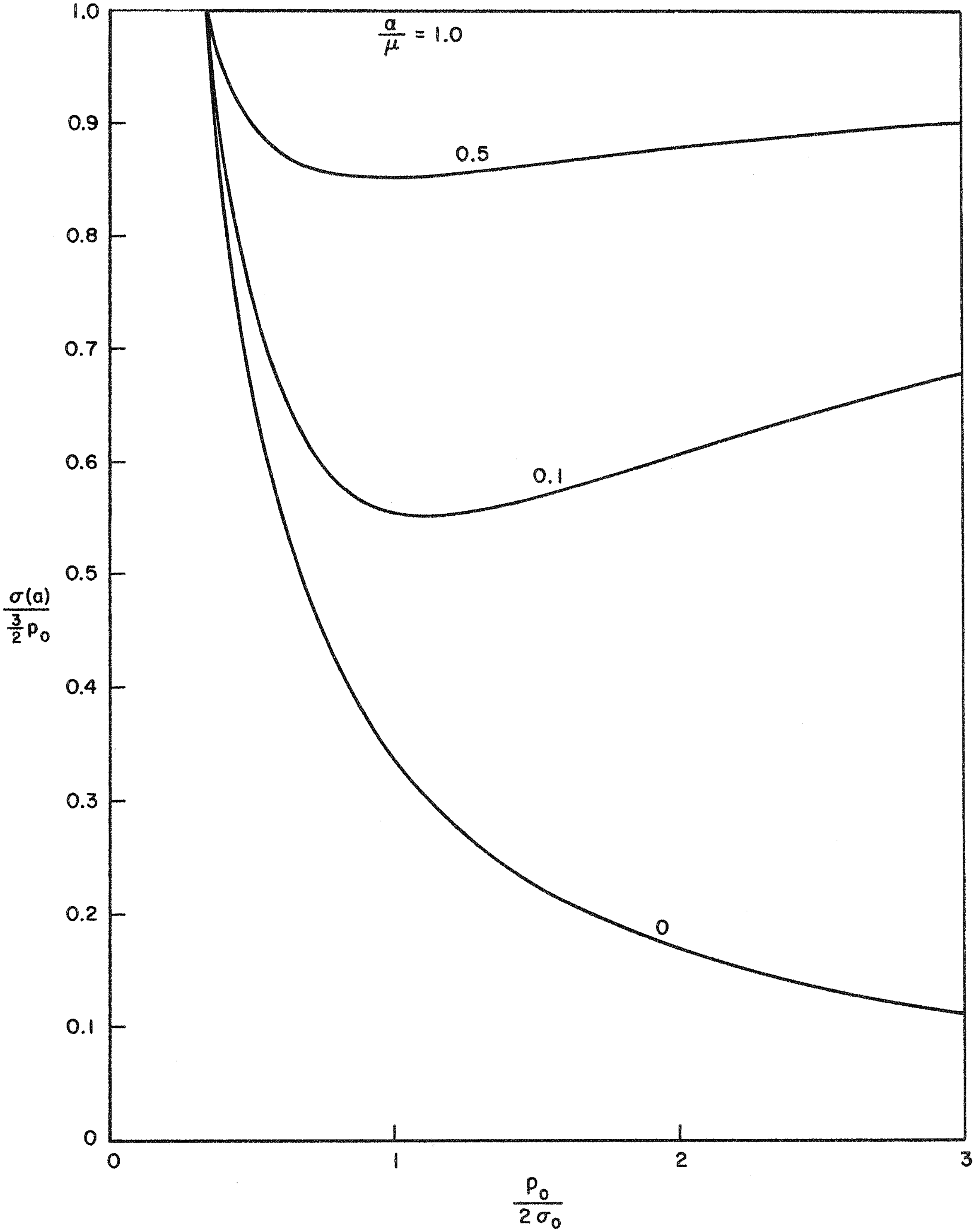

Fig. 4 — The shear stress at $r = a$ as a function of $p_0/2\sigma_0$ for various values of α/μ and a Poisson ratio of 1/4

III. AFTER THE EXPLOSION

The explosion subjects the cavity wall to a pressure-time history, which can be calculated quite accurately from the equations of hydrodynamics assuming the cavity is a rigid spherical chamber. Typically the pressure as a function of time looks like that shown in Fig. 5 with a very short duration spike caused by the reflected shock wave, settling rapidly to a steady pressure p_{∞}.

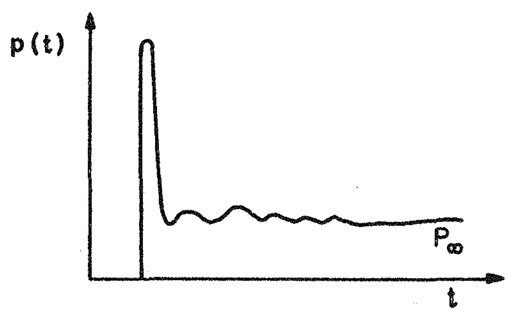

Fig. 5—Pressure vs time on cavity wall

If it were necessary to solve the time-dependent elasto-plastic equations in order to find the seismic signal corresponding to this pressure-time history, we would be faced with a very difficult mathematical problem. Moreover, we do not have the physical data needed to include the effects of strain rate, which are probably important for the transients. Fortunately, the seismic signal at a great distance from the explosion undergoes the filtering action of the earth, which allows only low frequency components of the signal to be transmitted. According to the theory of Latter, Martinelli and Teller[(3)] the amplitude of this low frequency signal is simply proportional to the permanent volume expansion of any spherical surface which completely encloses all the inelastic behavior. It follows that to obtain a good approximation for the distant seismic signal all we need to do is solve the static elasto-plastic equations corresponding to a nonzero pressure in the cavity, specifically the pressure p_{∞} (Fig. 5).

The approximation here amounts to ignoring work hardening caused by the transients. Because of the nonlinearity of plasticity, this work hardening can affect the static solution and hence the distant signal. The magnitude of this effect is being investigated, but at the present time we believe it to be small and it will be ignored in what follows. For our purposes then we will start with the stresses and strains existing around the empty cavity as calculated in the previous section and then turn on the p_{∞} adiabatically and monotonely, each point of the medium moving in the stress-strain diagram along paths similar to those shown dashed in Fig. 1.

Evidently if p_{∞} is small enough, the point B of Fig. 1 will not be exceeded anywhere in the medium, in which case the entire medium responds elastically, and the plasticity which occurred during the making of the cavity is irrelevant to the distant seismic signal. The stress and strain distribution around the cavity is affected by the plastic zone, and can be obtained by adding to the plastic solutions of Section II the appropriate elastic solutions, namely

$$\sigma_r = - p\left(\frac{a}{r}\right)^3 \qquad (14a)$$

$$\sigma = - 3/2 \cdot p\left(\frac{a}{r}\right)^3 . \qquad (14b)$$

We have now answered the first question raised in the introduction, concerning the influence of the plastic zone around the cavity, by showing that it has no effect whatsoever on the decoupling factor provided the pressure in the hole is small enough. This leads naturally to the second question, namely what is the maximum pressure in the hole so that the point B (Fig. 1) is not exceeded anywhere in the medium? As the pressure in the hole (p_{∞}) is increased, each point of the medium moves toward (its) point B on the stress-strain diagram, but clearly the radius $r = a$ gets there first; we denote the value of p_{∞} at which this occurs by p_{pl}. From Fig. 1 and Eq. (14b) we see that

$$- \frac{3}{2} p_{pl} + \sigma(a) = - \sigma(a), \qquad (15)$$

and hence, referring to Eq. (11b) for $\sigma(a)$,

$$\frac{p_{pl}}{2\sigma_o} = \frac{2}{3} \left[\left(\frac{k+4\mu/3}{k+4\alpha/3} \right) \frac{\alpha}{\mu} \left(\frac{\rho}{a}\right)^3 + \frac{k}{k+4\alpha/3} \left(1 - \frac{\alpha}{\mu}\right) \right] . \qquad (16)$$

The quantity ρ/a is a function $p_o/2\sigma_o$ through Eq. (10) (cf. Fig. 2). Values of $p_{pl}/2\sigma_o$ are plotted in Fig. 6 as a function of $p_o/2\sigma_o$ for various values of α/μ and a Poisson ratio of 1/4. Referring to the figure, we see that if $p_o < 4\sigma_o/3$, p_{pl} is greater than p_o for all values of α/μ. If α/μ is greater than about 0.1, p_{pl} is greater than p_o for any p_o.

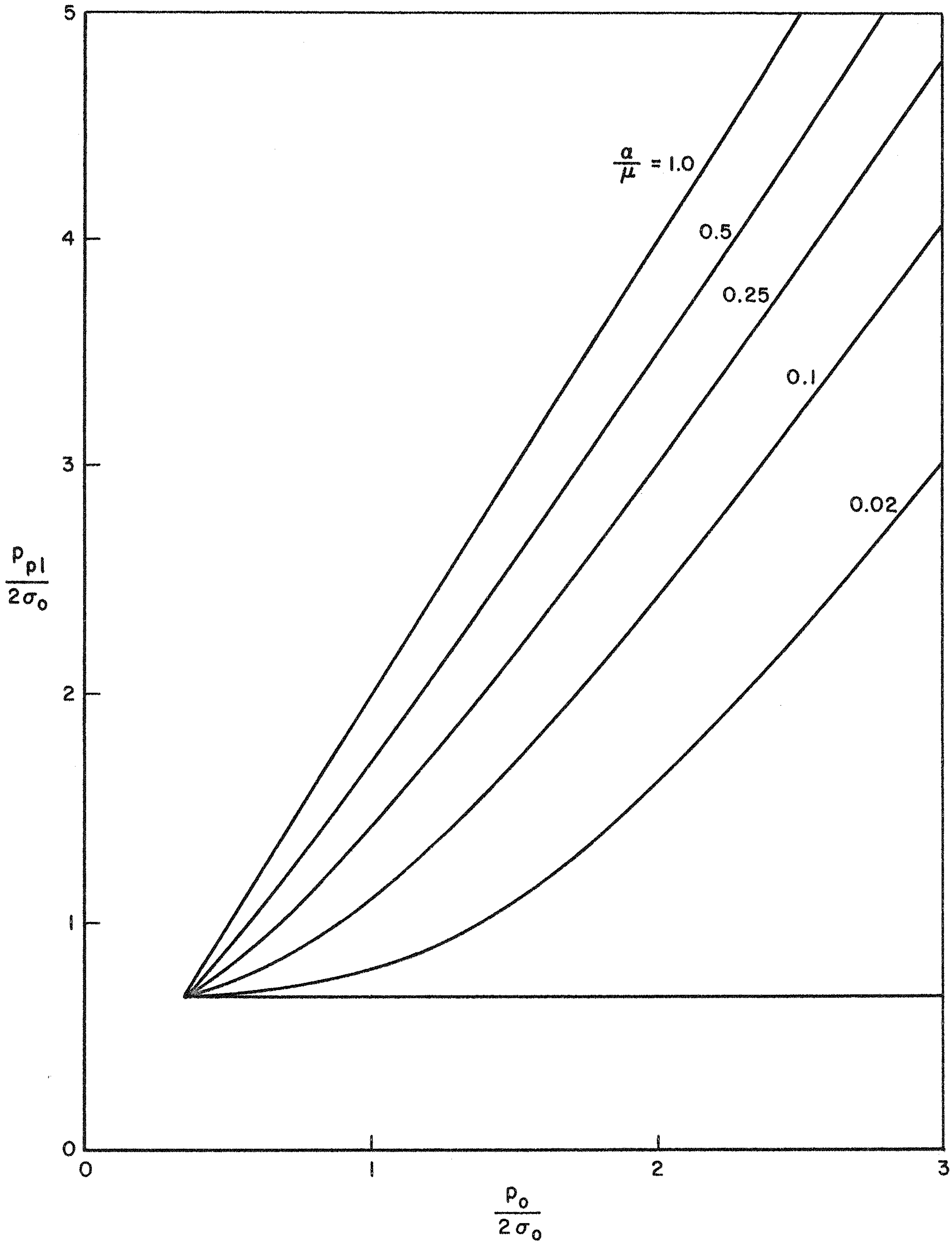

Fig. 6 — Pressure (p_{pl}) in the cavity at which the medium stops responding elastically to the explosion pressure, as a function of $p_0/2\sigma_0$ for various values of a/μ and a Poisson ratio of 1/4

To understand the significance of these results we must know the stress-strain diagram quantitatively. Since salt is a medium of particular interest for decoupling cavities, we have plotted its stress-strain diagram as given by Handin[4] in Fig. 7. Handin's data are actually for many different cases of triaxial loading and we have chosen the curve that seems to us most appropriate to our problem (but see the section on recommendations below). Handin's data are not adequate to determine the elastic portion of the stress-strain curve and for this purpose we have used some measurements from the Bureau of Mines,[5] specifically, a value $\mu \sim 100$ kilobars, and a Poisson ratio of 1/4. A reasonable two-segment fit to the curve in Fig. 7, assuming σ's of interest do not exceed a few hundred bars, is σ_o about 150 bars and α/μ roughly equal to 0.1.

With these values of σ_o and α/μ it follows from the results in the last paragraph that plasticity sets a less stringent limit on the cavity volume than the requirement that the average pressure in the cavity be less than p_o (a *fortiori* if the average pressure is less than $1/2 \cdot p_o$). Therefore the answer to the second question in the introduction is that for fully decoupled explosions, plasticity is unimportant at any depth.

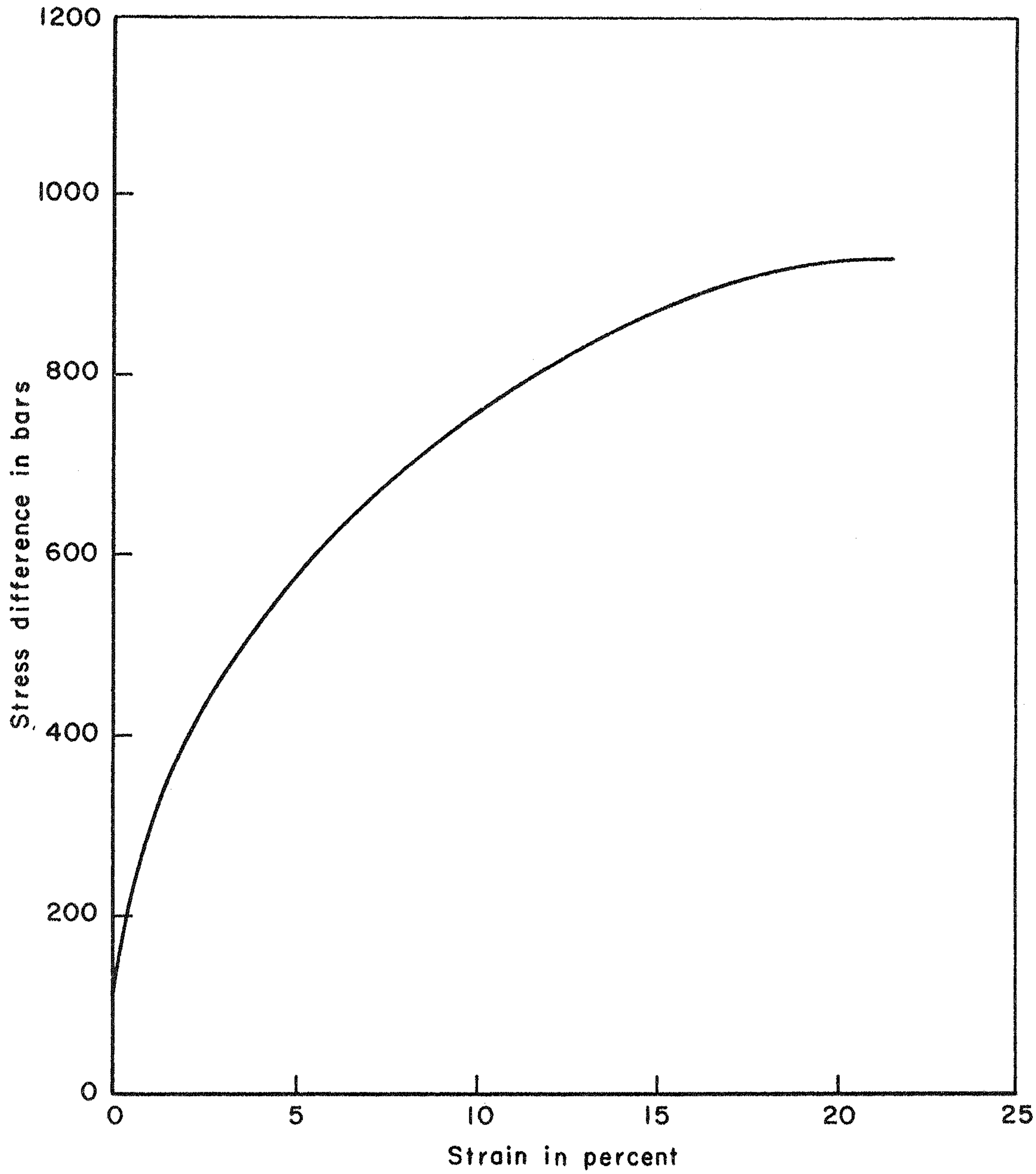

Fig. 7—Stress-strain curve for rock salt (after Handin)

IV. OVERDRIVEN HOLES

We come now to the last question, concerning the effect of plasticity on decoupling with overdriven holes. As was mentioned in the introduction plasticity did not play an important role in the Cowboy experiments because of their relatively shallow depth. However for the same degree of overdriving, i.e., the same value of p_∞/p_o, for cavities at a depth of $\sim$ 1 km, plasticity cannot be ignored. In this case the reduction in the decoupling factor may be due to plasticity rather than to cracking.

That plasticity can lead to a big reduction in decoupling can be seen from the following simple argument. Because of plasticity the value of the stress differences $|\sigma_r - \sigma_t|$ is limited to something in the neighborhood of the yield stress σ_o. If σ_r is large compared to σ_o, as it will be for overdriven shots at great depth, it follows that σ_r will be approximately equal to σ_t—which is the characteristic property of a liquid. For a liquid,(static) pressures are transmitted virtually without attenuation, which implies large displacements.

Of course a plastic medium has more strength than a liquid, but how much more depends on the detailed stress-strain diagram or, in our two-segment fit, on the values of α/μ and σ_o. To obtain a value for the reduction in the decoupling factor, i.e., the ''undecoupling factor,'' due to plasticity, we have to go through an analysis similar to that of Section III but with p_∞ greater than p_{pl}. The algebra is complicated, involving bookkeeping on the zones in which plasticity has occurred once, twice or not at all, and we merely quote the results here. The details are to be given in a separate report.[6]

Table 1

UNDECOUPLING FACTOR FOR OVERDRIVEN CAVITIES
AT A DEPTH OF 1 KM (p_o = 200 bars)
WITH VARIOUS STRESS-STRAIN RELATIONS APPROXIMATING THAT OF SALT

Case \ p_∞/p_o	1	2.5	5
1. $\sigma_o = 150$, $\alpha/\mu = 0.1$	~1	1.4	2.8
2. $\sigma_o = 150$, $\alpha/\mu = 0.02$	~1	1.8	8.2
3. $\sigma_o = 150$, $\alpha/\mu = 0$ (ideal)	~1	2.0	110.
4. Cowboy (p = 50 bars)	~1	~4	~10

Table 1 gives the undecoupling factor for cavities at a depth of 1 km in salt, assuming several different fits to Handin's[4] stress-strain diagram given in Fig. 7. As is evident in Fig. 7 the best value of α/μ depends on the value of p_∞ and hence on the degree of overdriving. For a small amount of overdriving a reasonable value of α/μ is ~0.1. For the greatest overdriving a better value of α/μ is ~0.02. The extreme plastic behavior, $\alpha/\mu = 0$, has also been included for comparison. Finally the table contains the undecoupling factor experimentally determined in the overdriven Cowboy shots.

In interpreting the data of Table 1 it is important to inquire whether in Cases 1 to 3 the medium has at any time gone into tension, i.e., $\sigma_t > 0$, since in that event cracking as well as plasticity would have to be considered. A detailed examination of the plastic solutions shows that the medium does not go into tension for values of p_∞/p_o between 1 and 5, and therefore cracking due to tension does not occur. We note in passing that in the case of ideal plasticity, $\alpha/\mu = 0$, the medium never does go into tension no matter how large p_∞/p_o; in Case 2, $\alpha/\mu = 0.02$, tension occurs when $p_\infty/p_o = 12.5$; and in Case 1, $\alpha/\mu = 0.1$, tension occurs when $p_\infty/p_o = 7.5$.

Besides cracking due to tension, there is the possibility previously mentioned of crack propagation due to the explosion gases penetrating into pre-existing cracks, which sets a limit for elastic behavior at $p_\infty/p_o \sim 1$. If this mechanism was the controlling form of inelasticity in Cowboy and if the mechanism continues to operate for overdriven shots at a depth of ~ 1 kilometer, then the appropriate undecoupling factor is probably close to that observed in the Cowboy experiments. If, however, the Cowboy inelasticity was due to something else, for instance the large transient pressure, or if the occurrence of plasticity has a sealing effect on the pre-existing cracks, which seems possible, then the appropriate undecoupling factor is determined by plasticity. In this case Table 1 shows that the undecoupling factor for deep overdriven cavities may be even smaller than that observed in the Cowboy experiments.

V. COWBOY INELASTICITY

We have said a number of times that the undecoupling observed in the Cowboy experiments was probably due not to plasticity but rather to cracking either because the medium was subjected to tension or because gases leaked into pre-existing cracks. In this section we will support this statement by showing that the actual observed undecoupling factors are consistent with this picture.

Making use of the theory of Latter, Martinelli and Teller,(3) we can write the undecoupling factor as

$$U = \frac{\Delta p r_{el}^3}{p_{\infty} a^3}, \qquad (17)$$

where r_{el} is the smallest radius outside which the medium is elastic, and Δp is the increase in pressure at the radius r_{el}; p_{∞} is the average pressure in the cavity and a is the radius of the cavity, as previously defined. The quantity Δp--the maximum radial stress which the medium can sustain without becoming inelastic--for rock-like materials which have no strength in tension, must be something in the neighborhood of the overburden pressure p_o, i.e.,

$$\Delta p = k\, p_o, \qquad (18)$$

where $$k \sim 1.$$

To relate r_{el} to Δp and p_{∞} we must know how the radial stress depends on the radius. If we assume that the pressure varies inversely with the n-th power of the radius, the connection for given p_{∞} is given by

$$\Delta p = p_{\infty} \left(\frac{a}{r_{el}}\right)^{n}, \qquad (19)$$

where the exponent n may itself depend on p_{∞}. From Eqs. (17), (18) and (19) the undecoupling factor becomes

$$U = \left(\frac{p_{\infty}}{kp_{o}}\right)^{\frac{3}{n} - 1}. \qquad (20)$$

By inserting the values of the Cowboy undecoupling factors (Table 1) we obtain an equation for each of the two cases in which $p_{\infty}/p_{o} = 2.5$ and 5. These two equations are not sufficient to determine uniquely the value of k and the two values of n. However, for a reasonable value of k, in the range $p_{o}/2$ to $3p_{o}/2$, it turns out that the two values of n are roughly equal to each other and lie in the range 1 to 1.5.

What does such a value of n, between 1 and 1.5, imply? From Eq. (1), assuming the power-law dependence for the radial stress, we see that the ratio of the hoop stress to the radial stress is simply

$$\frac{\sigma_{t}}{\sigma_{r}} = 1 - \frac{n}{2}. \qquad (21)$$

Now there are two well-known cases in which the power-law dependence of the radial stress on distance is exact. One of these is elasticity, for which n = 3 and therefore $\sigma_{t}/\sigma_{r} = -1/2$. At the other extreme is (liquid) hydrostatics for which n = 0 and $\sigma_{t}/\sigma_{r} = 1$. Between these two cases of an elastic solid and a liquid ($n = 3$ and $n = 0$) one can imagine all kinds of media with varying degrees of strength leading to values of n between 0 and 3. In particular if $n = 2$, $\sigma_t/\sigma_r = 0$, i.e., the hoop stress is zero, which in some ideal sense describes a medium with open cracks such as might be constructed with building blocks.

We are now in a position to interpret a value of n which lies in the range of 1 to 1.5. Such a value of n implies that σ_t/σ_r is between 1/3 and 1/2 and corresponds to a medium which is intermediate between the building-blocks model and a liquid. We take this result to be evidence for cracking of the medium either with a little plasticity or more probably with some leakage of the gas into the cracks to provide the small amount of compressive hoop stress corresponding to σ_t/σ_r lying between 1/3 and 1/2.

VI. CONCLUSIONS

We have shown that there is a possibility that plasticity plays an important role in decoupling with overdriven cavities. To estimate how important this effect may be it is necessary to know the stress-strain relation of the medium under appropriate loading conditions. Handin's work[4] provides some stress-strain data for salt, which is a medium of particular interest, but this data is not directly applicable to decoupling because the loading conditions were not appropriate.

In order to get directly applicable stress-strain data, an experiment should be made, preferably in the laboratory, to determine directly the dilatation of the volume of a cavity subjected to internal pressure. It is important that the cavity be under the influence of a confining pressure, and that this pressure be sufficiently large in some cases so that plastic flow occurs during the construction of the cavity. The results of such an experiment can be used in conjunction with the theory developed in the previous sections to infer the proper stress-strain relationship.

In addition, in order to learn more about the transient effects, which have not been studied in this report, small HE explosions should be conducted. In our opinion such a facility would be extremely useful in studying many phases of the decoupling problem, in particular for obtaining criteria for the onset of inelastic behavior. For instance, the influence of cracks could be investigated by using a medium which has been precracked in a deliberate and controlled way.

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