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A THEORY OF HYPERFRAGMENTS. II MESIC DECAY OF HYPERFRAGMENTS

by

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SUMMARY

Mesic decay of hyperfragments is discussed systematically on the basis of our previous model for hyperfragments. The general formalism for the two-body and three-body mesic decay is developed. The polarization-direction correlation and the angular correlation for the two-body and the three-body decays are discussed together with the decay probability. The formalism is developed so as to include the isotopic spin selection rule (ΔI = 1/2 and 3/2) for the mesic decays. The theory developed here is applied especially for the low mass number hyperfragments where we found that the branching ratios of the two-body and the three-body mesic decays of ${}^3H_\Lambda$ and ${}^4H_\Lambda$, $({}^3H_\Lambda \longrightarrow {}^3He + \pi^-)/({}^3H_\Lambda \longrightarrow D + p + \pi^-)$ and $({}^4H_\Lambda \longrightarrow {}^4He + \pi^-)/({}^4H_\Lambda \longrightarrow {}^3H + p + \pi^-)$, can be used for the determination of the spins of both hyperfragments. The fraction of the p-wave decay rate for the free Λ decay obtained from the ${}^5He_\Lambda \longrightarrow {}^4He + p + \pi^-$ where the decay proceeds through two-resonant states ($p_{3/2}$ and $p_{1/2}$) is given by $p^2/(s^2 + p^2) \approx .4$ which gives the spin zero of ${}^4H_\Lambda$ in connection with the Dalitz and Liu plot and hence odd parity for the kaon. The decay rate of the charged and the neutral modes is always 2/1 if and only if the condition obtained by Okubo, Marshak and Sudarshan is satisfied. Finally we show that the final state interaction for the two-body mesic decay can be described by the pion and residual nucleus scattering phase shifts by making use of invariance of the total S-matrix of the decay processes under the Wigner (weak) time reversal.

1. INTRODUCTION

We had, in earlier papers[1,2] investigated the properties of Λ hyperfragments in relation to the binding energies of the Λ hyperon and it was pointed out that the experimental binding energies of the Λ hyperon for p-shell hyper-nuclei can be explained by both antiparallel and parallel spins. The main results of our previous papers are the evidence for the preference for the antiparallel spin couplings for s-shell hypernuclei and hence the odd kaon parity[1,2]. The internal consistency of our model and method were discussed in succeeding reports[3,4]. The method applied for the ordinary nuclei shows that the qualitative features (e.g. the ground state spins and the total binding energies) are in agreement with the experiments but a precise fit, (e.g. for the detailed level structure) cannot be obtained. It can be improved by making use of the intermediate coupling wave function for nucleons obtained from the current shell model calculations and couple the Λ wave function to it. This is rather satisfactory because the Λ hyperon is loosely bound (with a strength of about 1/2.5 of that of N-N forces) to the individual nucleons (compare the parameters given in papers cited above). But this is a rather complicated procedure and misses the essential point of the problem. The precise calculation should be undertaken after arriving at the essential results. Therefore in this paper we will continue to confine ourselves to the extreme j-j scheme and study the main features of the mesic decays of hyperfragments. Among several investigations of this problem Dalitz and Liu[5] have calculated the ratio of two-body mesic decays to all decay modes making use of their variational wave functions for A = 3 and 4 hyperfragments although they assumed the isotopic spin selection rule $\Delta I = 1/2$ for Λ decay. The mesic decay of p-shell hyperfragment has been investigated by Lawson and Rotenberg[6] but it is confined to two-body decays alone. The purpose of this paper is to derive a theory for the mesic decay of hyperfragments and to determine the properties of hyperfragments as well as the nature of the decay interactions.

It is rather hard to determine the absolute lifetime of the bound Λ particle experimentally because it is hard to discriminate between hyperfragment decays in flight and decays at rest (for low energy production) and hence the ambiguity arises from the measurement of the track length.* The decays occur through pi-mesic and

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* It was pointed out by Professors Harth and Leitner that the measurement of the absolute lifetime by a bubble chamber (private communication) has an ambiguity of about a factor of three.

---

non-mesic channels with several decay modes. The former is identified more easily than the latter because the maximum charge state and the suitable energy release make easier the identification. We want to investigate the spin states of parent hyperfragments, s-wave and p-wave admixture ratio of the free Λ decay, the isotopic spin selection rule, $\Delta I = 1/2$ and $3/2$ including the phase difference of both amplitudes (we do not confine ourselves to the $\Delta I = 1/2$ rule) and so on.

The experimental information for the decay processes are rather poor at present except for a few hyperfragments; we summarize the possible topics for the theoretical investigations in the following:

1) Two-body mesic decay: decay branching ratio to the various residual states of the nucleus, branching ratio of π^--mesic and

and π^0-mesic decay, polarization direction correlation of the produced hyperfragment and the decay pion.

2) Three-body mesic decay: angular correlation of the successive emission of the pion and the nucleon if the decay occurs through the intermediate state with a definite angular momentum, parity and isotopic spin (decay through a compound state of the nucleus).

3) Branching ratio of the two-body and the three-body mesic decays.

In order to correlate the experimental information with our method of analysis we will develop a formalism in section 2. In section 3 we will derive a numerical table suitable for the treatment of the two-body mesic decay branching probabilities for various residual states for configurations with both parallel and antiparallel spins of the parent hyperfragments. General comments for each mesic decay are given. It is pointed out that in some cases the observation of decay modes will uniquely determine the spin of the hyperfragment. In section 4 we will discuss the isotopic spin selection rule from the two-body mesic decay. The poor experimental statistics of the neutral decay mode gives us no definite information. In section 5 we study three-body mesic decays but restrict ourselves to the $A = 3,4$ and 5 hyperfragments and gave a convenient table for the applications. In section 6 we compared the calculated two-body and the three-body mesic decay branching ratios of $A = 3$ and 4 with experiment. The proton decay mode of the $A = 5$ hyperfragment through two intermediate states $p_{3/2}$ and $p_{1/2}$ is compared with experiment. Conclusions about the spin of s-shell hyperfragments and the fraction of the p-wave probability of the free Λ decay are derived from the comparison given in this section. In all these discussions we always consider the branching ratio: hence only the kinematical parameters are significant quantities in our treatment. In the last section we derive a relation which takes into account the final state interaction of the pion in the two-body mesic decay making use of the weak time reversal invariance for the S-matrix (section 7). In Appendix 1 we will summarize the calculation of the phase volume of the three-body mesic decay including the matrix elements. In Appendix 2 we pointed out that the parameters derived from s-shell hyperfragment and p-shell hyperfragment binding energies favor the antiparallel coupling of spins for p-shell hyperfragments provided that there is no anomalously strong spin-orbit coupling of N-Λ forces. A brief communication on the principal results of this part of our investigation has already been published.[7]

2. FORMALISM

Spin 1/2 of the Λ hyperon, parity non-conservation in Λ decay[8] and the possible isotopic spin changes $\Delta I = 1/2$ and $3/2$ for the decays into strongly interacting particles[9] (we will consider both possibilities without refering to the usual $\Delta I = 1/2$ rule[10]) lead to the following form of operator for the decay of the Λ particle

$$H = \left\{ a_{1/2s} D^{(1/2)} + a_{3/2s} D^{(3/2)} \right\}$$

$$-i \left\{ a_{1/2p} D^{(1/2)} + a_{3/2p} D^{(3/2)} \right\} \underline{\sigma} \cdot \underline{\nabla} / p_\Lambda$$

$$= \sum_{\Delta I, \Delta J} a_{\Delta I, \Delta J} D^{(\Delta I)} \quad \underline{\sigma}^{(\Delta J)} \cdot \left(\frac{-i\, \underline{\nabla}}{p_\Lambda} \right)^{(\Delta J)} \qquad (1)$$

where $a_{\Delta I, \Delta J}$, $D^{(\Delta I)}$, $\underline{\sigma}$, $\underline{\nabla}$ and p_Λ are the relative amplitudes of ΔI, ΔJ-contribution, the operator in isotopic spin space, Pauli spin matrix, gradient operator for the pion wave function and the magnitude of the relative momentum of the pion in free Λ decay at rest, respectively. From the conservation of angular momentum $\Delta J = 0$ and 1. (We put p_Λ in the expression for the p-wave contribution so as to give the same dimensionality for both channels; later on we will introduce more suitable names for both contributions for hyperfragment decay, see below.)

A) Two-body Mesic Decay

We will develop the theory in such a form that the pion wave function (including its isotopic spin wave function) appears as an operator. The space part of the outgoing pion wave function is expressed by a spherical wave. The final state interaction can be taken into account in this expression either as a mean potential of residual nucleus for the pion[6] or as a pion nucleus phase shift (see section 7). We do not want to include this problem in this first investigation because the kinetic energy of the pion is at most 40 Mev which gives only a small phase shift for pion-nucleon scattering. The spherical wave can be expressed as

$$e^{i\underline{k}\underline{r}} = 4\pi \sum_{\ell,m} (i)^{\ell}\, j_{\ell}(kr)\, Y_{\ell m}(\underline{r})^{*}\, Y_{\ell m}(\underline{k}), \qquad (2)$$

where $j_\ell(kr)$, $Y_{\ell m}(\underline{r})$ and $Y_{\ell m}(\underline{k})$ are spherical Bessel function, and spherical harmonics of angles for $\underline{r}$ and $\underline{k}$ respectively. The part of the operator $j_\ell(kr)Y_{\ell m}(\underline{r})$ makes the rearrangement of decay nucleon so as to overlap the residual nuclear states. For the rearrangement leading to the s-shell and p-shell nucleons in the residual nucleus the term with $\ell = 0$ and $\ell = 1$ remain; and $\ell = 2$ for the d-shell nucleon and so on. Thus the outgoing pion wave relative to the residual nucleus does not have $\ell = 0$ for s-wave pion of the <u>free</u> lambda decay; similarly for the p-wave pion. For this reason it will be convenient to introduce new names "direct term" and "derivative term" corresponding terms of s-wave and p-wave pion in the free Λ decay. The derivation of the "derivative term" requires some algebraic manipulation; we will carry it out shortly in the following.

The scalar product of $\underline{\sigma}\cdot\underline{k}$ is written in terms of irreducible tensors by making use of the identity:

$$\underline{T}^{(k)}\cdot\underline{U}^{(k)} = \sum_q (-)^q T_q^{(k)} U_{-q}^{(k)} \tag{3}$$

where $\underline{T}^{(k)}$ and $\underline{U}^{(k)}$ are tensor operators of rank k.

$$\underline{\sigma}\cdot\underline{k}/p_\Lambda = k/p_\Lambda \ (4\pi/3)^{1/3} \sum_q \sigma_q^{(1)} Y_q^{(1)*}(\underline{k}) \tag{4}$$

where we used the relation $Y_m^{(\ell)*}(\underline{r}) = (-)^m Y_{-m}^{(\ell)}$. Introduce the convenient tensor operator forms for spherical harmonics:

$$Y_m^{(\ell)}(\underline{r}) = ([\ell]/4\pi)^{1/2} C_m^{(\ell)} \tag{5}$$

and

$$Y_m^{(\ell)*}(\underline{k}) = ([\ell]/4\pi)^{1/2} D(\ell,m0;\underline{k}) \tag{6}$$

The former relation is convenient for calculating the matrix element in coordinate space and the latter relation is convenient for the angular correlation problem[11]. One other form of product of tensor operator will be introduced by

$$T_{q_1}^{(k_1)} U_{q_2}^{(k_2)} = \sum_k \left[T^{(k_1)} \times U^{(k_2)}\right]_q^{(k)} C_{q_1\ q_2\ q}^{k_1\ k_2\ k} \tag{7}$$

Using these relations we obtain

$$\text{direct term:}\quad e^{i\,\underline{k}\cdot\underline{r}} = \sum_{\ell,m} [\ell] i^\ell \ j_\ell(kr)\ C_m^{(\ell)}(\underline{r})\ D(\ell,m0;\underline{k}) \tag{8}$$

Derivative term:

$$\underline{\sigma}\cdot\underline{\nabla}/ip_\Lambda\ e^{i\underline{k}\cdot\underline{r}} = k/p_\Lambda \sum_{\lambda,m+q}\ \sum_{\ell} [\ell] \left[\frac{\ell}{\lambda}\right]^{1/2} C\begin{matrix}\ell & 1 & \lambda\\ 0 & 0 & 0\end{matrix}\ i^{\ell} j_{\ell}(kr)$$

$$\left[C^{(\ell)}(\underline{r}) \times \sigma^{(1)}\right]^{(\lambda)}_{m+q}\ D(\lambda,m+q0;\underline{k}) \qquad (9)$$

Thus far we have separated out the operator into the coordinate space part and the momentum space part. The formalism we are going to establish for the two-body mesic decay is generalized in a straight-forward manner for the three-body mesic decay. The reader will find out that the notation introduced here is a very convenient one at that stage.

Introducing the isotopic spin wave function of pion $t^{(1)}_{m}$ and treating it as a tensor operator we can find out the total isotopic spin operator by the help of the tensor algebra introduced in (7). The total expression of the decay interaction is given by

$$H = \Big\{a_{1/2s}\sqrt{2/3}\left[t^{(1)} \times D^{(1/2)}\right]^{(1/2)}_{1/2} + a_{3/2s}\sqrt{1/6}\left[t^{(1)} \times D^{(3/2)}\right]^{(1/2)}_{1/2}\Big\}.$$

$$\sum_{\ell,m} [\ell]\, i^{\ell}\ j_{\ell}(kr)\, C^{(\ell)}_{m}(\underline{r})\, D(\ell,m0;\underline{k})$$

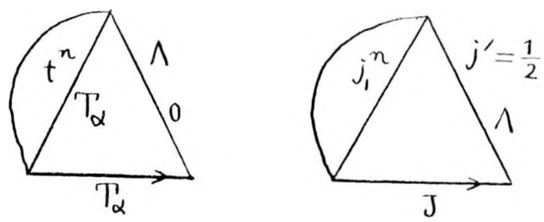
$$+\Big\{a_{1/2p}\sqrt{2/3}\left[t^{(1)} \times D^{(1/2)}\right]^{(1/2)}_{1/2} + a_{3/2p}\sqrt{1/6}\left[t^{(1)} \times D^{(3/2)}\right]^{(1/2)}_{1/2}$$

$$k/p_\Lambda \sum_{\ell}[\ell] \sum_{\lambda,m+q}[\ell\,\lambda]^{1/2} C^{\ell 1\lambda}_{000}\ i^{\ell} j_{\ell}(kr)\left[C^{(\ell)} \times \sigma^{(1)}\right]^{(\lambda)}_{m+q}$$

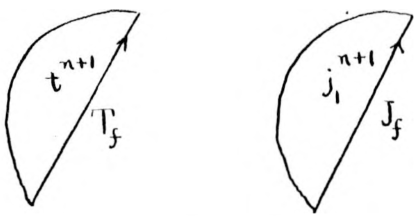
$$D(\lambda,m+q0;\underline{k}) \qquad (10)$$

This is the tensor operator form for the lambda decay into a proton and a negative pion. It must be remarked that the operator $t^{(1)}_{-1}$ for π^- state changed into $t^{(1)}_{1}$ in the above because it turned into the destruction operator (i.e. the Hermitian conjugate of the π field). We can obtain the corresponding expression for the neutral pion and neutron decay by replacing $\sqrt{2/3}$ and $\sqrt{1/6}$ in front of the isotopic spin operator by $\sqrt{1/3}$ and $-\sqrt{1/3}$ respectively and the magnetic quantum number 1/2 for the isotopic spin operator by $-1/2$.

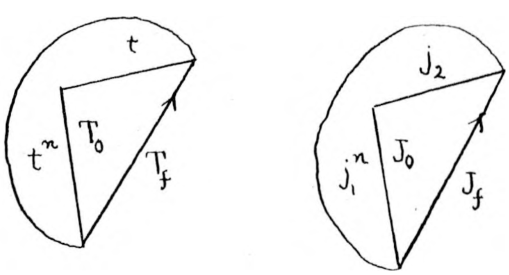
Wave function for the initial and final states Ψ_i and Ψ_f are given in terms of French diagrams by

$\Psi_i =$  (11)

and

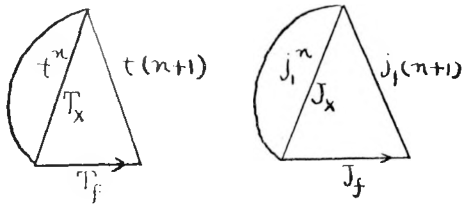
$\Psi_f =$  (12)

for the rearrangement of decay nucleon in j_1-shell which is not occupied by nucleons of the original hyperfragment; and by

$\Psi_f =$  (13)

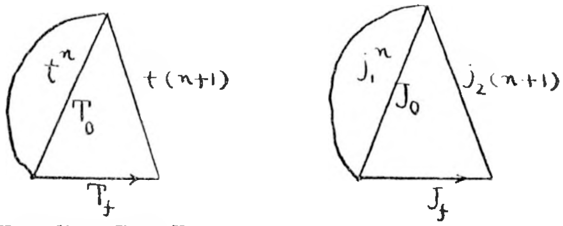
for the case where there is rearrangement of decay nucleon in j_2-shell (which is a different shell from the shell occupied by the nucleons in the hyperfragment). The curly bracket on the French diagram show that nucleons in it are totally antisymmetrized. The decay operator is not isoscalar so we have to introduce the wave function in the isotopic spin space (compare the notation in paper I). Equations (12) and (13) are rewritten in convenient forms by making use of the coefficient of fractional parentage (c.f.p.)

$$\Psi_f = \sum_{T_x, J_x} \langle T_f J_f \mid T_x J_x 1/2\, j_1 \rangle \tag{14}$$



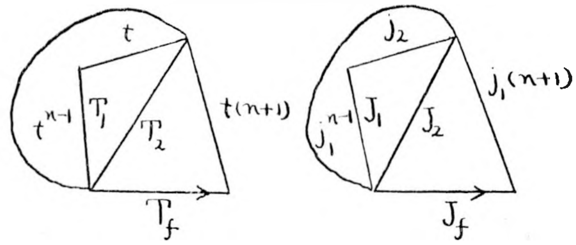
and

$$\Psi_f = 1/\sqrt{n+1}$$



$$-\sqrt{n/(n+1)} \sum_{\substack{T_1, T_2 \\ J_1, J_2}} (-)^{J_1+J_f-J_0-J_2} \; U(j_1 J_1 J_f j_2 ; J_0 J_2) \cdot$$

$$\cdot \langle T_0 J_0 \mid T_1 J_1 1/2\, j_1 \rangle \tag{15}$$



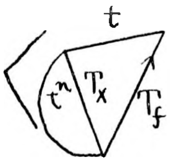
where the c.f.p. is given in charge-space-spin states. The coefficient U is related to the Racah coefficient by

$$U(abcd;ef) = [ef] \; W(abcd;ef), \tag{16}$$

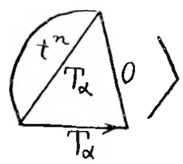
where $[ef] = (2e+1)\,(2f+1)$. It is clear that the second term of (15) does not contribute to the matrix element for two-body mesic decay because the n-th nucleon has a different configuration from that of the hyperfragment. As mentioned before s-shell hyperfragment takes the same expression in both L-S and j-j scheme. We will first calculate the matrix element in isotopic spin space then in space-spin space. A general form of the operator in isotopic spin space takes the form

$$\left[t^{(1)} \times D^{(a)} \right]^{(b)}_{1/2} \qquad (17)$$

for the decay into the proton and the negative pion. We have

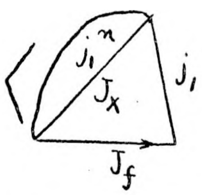


$$\left| \left[t^{(1)} \times D^{(a)} \right]^{(b)}_{1/2} \right|$$

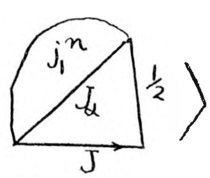


$$= 1/\sqrt{2}\; C^{T\ \ 1/2\ \ T_f}_{m_T\ 1/2\ m_{T_f}} \left\langle 1/2 \left\| \left[t^{(1)} \times D^{(a)} \right]^{(1/2)} \right\| 0 \right\rangle \delta_{b1/2}\, \delta_{T_x T_a} \qquad (18)$$

The space-spin matrix element of the direct term is given by



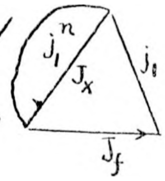
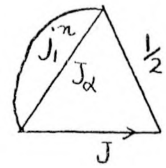
$$\left| i^{\ell} j_\ell (kr)\, C_m(\underline{r}) \right|$$



$$= -i^{\ell} \langle R_p \left| j_\ell(kr) \right| R_\Lambda \rangle \sqrt{[J]}\; C^{J\ \ \ell\ \ J_f}_{m_J\, m\ m_{J_f}} (-)^{J_a+1/2-J}\, W(1/2\, j_1 J J_f ; \ell\, J_a)$$

$$[j_1/\ell]^{1/2} \delta_{\ell_1 \ell}\, \delta_{J_x J_a} \qquad (19)$$

where R_p and R_Λ are the radial wave function of the bound proton and the bound lambda respectively; ℓ_1 is the orbital quantum number of the bound nucleon. The same for the derivative term is given by

$\Bigg\langle\Bigg($  $\Bigg| \; i^{\ell} j_{\ell}(kr) \left[C^{(\ell)}(\underline{r}) \times \sigma^{(1)} \right]^{(\ell)}_{m+q} \Bigg|$  $\Bigg\rangle$

$$= \sqrt{6}\; i^{\ell} \langle R_p \,|\, j_{\ell}\,(kr) \,|\, R_{\Lambda} \rangle \sqrt{[J]} \quad C^{J\ \lambda\ J_f}_{m_J\, m+q\ m_{J_f}} \quad (-)^{J_\alpha + 1/2 - J}$$

$$W(1/2\,J\,j_1\,J_f; J_\alpha\,\lambda\,)\; W(\ell\,\lambda\; 1/2\,1/2; 1\,j_1) \quad \left[\lambda\, j_1/\ell\,\right]^{1/2} \delta_{\ell_1 \ell}\; \delta_{J_x J_\alpha} \qquad (20)$$

where $\left[ab\cdots \quad /ef\cdots \quad\right] = (2a+1)(2b+1)\cdots /(2e+1)(2f+1)\cdots$.
Combining all the calculations done above we get the complete matrix element in the form

$$\langle f \,|\, H \,|\, i \rangle = \sqrt{n+1} \quad C^{T_\alpha\ \ 1/2\ \ T_f}_{m_T\ 1/2\ m_{T_f}} \Big/ \sqrt{2} \quad \sum_{\ell} i^{\ell} \langle R_p \,|\, j_{\ell}\,(kr) \,|\, R_{\Lambda} \rangle$$

$$(-)^{1/2-J} \quad [\,J\,]^{1/2}\, \delta_{\ell_1 \ell} \langle T_f J_f \,|\, T_\alpha J_\alpha \quad 1/2\, j_1 \rangle \; (-)^{J_\alpha}$$

$$\Bigg[-\left\{ \sqrt{2/3}\; a_{1/2s} + \sqrt{1/6}\; a_{3/2s} \right\} W(1/2\,j_1\,J\,J_f; \ell\; J_\alpha) \quad \sum_{m} C^{J\quad \ell\quad J_f}_{m_J\ \ m\ \ m_{J_f}} D(\ell, m0; \underline{k})$$

$$+ \sqrt{6\,[\ell_1]} \quad k/p_{\Lambda} \left\{ \sqrt{2/3} a_{1/2p} + \sqrt{1/6} a_{3/2p} \right\} \sum_{\lambda} W(1/2\,J\,j_1\,J_f; J_\alpha\,\lambda\,)$$

$$W(\ell\,\lambda\, 1/2\, 1/2; 1\, j_1)\; C_{0\ \ 0\ \ 0}^{\ell\ \ 1\ \ \lambda} \quad \sum_{m+q} C^{J\quad \lambda\quad J_f}_{m_J\ m+q\ m_{J_f}} \quad D(\lambda\,, m+q\ 0; \underline{k}) \qquad (21)$$

The factor $\sqrt{(n+1)}$ in the front of the right hand side comes from the indistinguishability of the nucleons in the final state in the j_1-shell. The corresponding matrix element for the neutral mesic decay is obtained by replacing the coefficient of the decay amplitude and the magnetic quantum number for the Clebsch-Gordan coefficient as we discussed for the derivation of the operator (see the discussion under eq. (10)).

The decay into final state (15) is obtained by replacing (n+1) by 1 for both mesic decays. Other kinematical factors should also be changed into the corresponding quantities, for instance, j_1 into j_2 etc.

(a) Polarization-Direction Correlation

Before calculating the decay probability we wish to talk about the polarization-direction correlation which is possible only if the hyperfragment produced has a non-zero spin and a definite polarization. The kaon absorption in flight by the nucleus and the consequent production of hyperfragment and pion can produce such a state. The polarized hyperfragment decay gives an asymmetry for the momentum distribution of the decay pion. This will become one of the independent test for the determination of the spin of $^4He_\Lambda$ from its decay in $K^- + {}^4He \longrightarrow {}^4He_\Lambda + \pi^-$ experiment[12] and hence the relative parity of the kaon. Although the above experiment was done for the kaon absorpiton at rest up to the present. Most of the hyperfragments are observed in nuclear plates and hence an associated emission of many nucleons makes ambiguous the polarization of the hyperfragment. The state of the polarization is specified by the magnetic quantum number m_J of the angular momentum J of the hyperfragment. Thus in the completely polarized state only a definite value of m_J is allowed. The absolute square of (21) with no summation with respect to m_J gives the probability of the correlation. The example discussed above will be shown explicitly at the end of Section 3.

(b) Decay Probability

In computing the total mesic decay probability we will first "rotate" the matrix element (21) in the momentum space so that the argument of the third component of the D-function changes from zero to M and then we will take the average over the initial spin for the absolute square of the matrix element and sum over the magnetic quantum number of the hyperfragment. (No summation should be taken for m_J in the correlation problem discussed in (i) above.) In carrying out the above procedure we will employ a few mathematical tricks; firstly for the absolute square of the D-function, where we will use the following relations

$$D^*(\ell,m,M;R) = (-)^{M-m}D(\ell,-m,-M;R) \tag{22}$$

where R is the rotation with respect to Euler angle $\alpha\beta\gamma$, such that the coordinate system describing the pion propagation vector is carried over into the quantization coordinate system,

$$D^*(\ell,m,M;R)D(\ell,m,M;R) = (-)^{M-m}C^{\ell\ \ \ell'\ \ \nu_1}_{-m\ m'}\ C^{\ell\ \ \ \ell'\ \ \nu_1}_{-M\ M'}\ D(\nu_1,m',M'-M;R) \tag{23}$$

etc. where ν_1 is the possible value of the triangle (ℓ,ℓ',ν_1). Secondly one has to take the summation over the triple product of the Clebsch-Gordan coefficients of the type

$$\sum_{m_J}(-)^{m_J}C^{J\ \ \ \ \ell\ \ \ J_f}_{m_J m_{J_f}-m_J}\ C^{J\ \ \ \ \ \ell\ \ \ \ \ \ J_f}_{m_J\ \ J_f-m_J}\ C^{\ell\ \ \ \ \ \ \ \ \ \ell'\ \ \ \ \ \ \ \ \ \ \nu_1}_{m_J-m_{J_f}\ m'_{J_f}-m_J}$$

$$= (-)^J[J_f]C^{J_f\ \ \ \ J_f\ \ \ \ \nu_1}_{m_{J_f}\ -m'_{J_f}}\ W(J_fJ_f\ \ell\ \ell';\ \nu_1\ J) \tag{24}$$

etc. Performing the type of the calculations shown in (22)-(24) we obtain the correlation function, $E(m_{J_f}, m'_{J_f})$ in the form

$$E(m_{J_f}, m'_{J_f}) = 1/[J] \sum_{m_J} |\langle f| \; H \; | i \rangle|^2$$

$$= (n+1) \; [J_f] \left(C^{T_a \; 1/2 \; T_f}_{m_T \; 1/2 \; m_{T_f}} \right)^2 /2 \langle R_p | j_{\ell_1}(kr) | R_\Lambda \rangle^* \langle R_p | j_{\ell'_1}(kr) | R_\Lambda \rangle$$

$$(-)^{J-m_{J_f}+M} [j_1][\ell_1 \ell'_1]^{1/2} \langle T_f J_f | T_a J_a \; 1/2 j_1 \rangle^2$$

$$\Big[\left| \sqrt{2/3}\, a_{1/2s} + \sqrt{1/6}\, a_{3/2s} \right|^2 W(1/2 j_1 J J_f ; \ell_1 J_a)\, W(1/2 j_1 J J_f ; \ell'_1 J_a)$$

$$C^{\ell_1 \ell'_1 \nu_1}_{-M M'} \; C^{J_f J_f \nu_1}_{m_{J_f} - m'_{J_f}} \; W(J_f J_f \ell_1 \ell'_1 ; \nu_1 J)\; D(\nu_1, m'_{J_f} - m_{J_f}, M'-M; \mathcal{R})$$

$$- \sqrt{6[\ell_1]}\; k/p_\Lambda \; (\sqrt{2/3}\, a_{1/2s} + \sqrt{1/6}\; a_{3/2s})^* \; (\sqrt{2/3}\; a_{1/2p} + \sqrt{1/6}\; a_{3/2p})$$

$$W(1/2 j_1 J J_f ; \ell_1 J_a) W(1/2 J j_1 J_f ; J_a \lambda') W(\ell'_1 \lambda' \; 1/2 1/2 ; 1 j_1) [\lambda'] \; C^{\ell'_1 1 \lambda'}_{0\;0\;0} C^{\ell_1 \lambda' \nu_2}_{-M M'} C^{J_f J_f \nu_2}_{m_{J_f} - m'_{J_f}}$$

$$W(J_f J_f \ell_1 \lambda' ; \nu_2 \; J)\; D(\nu_2, m'_{J_f} - m_{J_f}, M'-M; \mathcal{R})$$

$$- \sqrt{6[\ell_1]}\; k/p_\Lambda \; (\sqrt{2/3} a_{1/2p} + \sqrt{1/6} a_{3/2p})^* (\sqrt{2/3} a_{1/2s} + \sqrt{1/6} a_{3/2s})$$

$$W(1/2 J j_1 J_f ; J_a \lambda) W(\ell_1 \lambda 1/2 1/2 ; 1 j_1) W(1/2 j_1 J J_f ; \ell'_1 J_a) [\lambda] C^{\ell_1 1 \lambda}_{000} C^{\ell_1 \lambda \nu_3}_{-M M'} C^{J_f J_f \nu_3}_{m_{J_f} - m'_{J_f}}$$

$$W(J_f J_f \; \lambda \ell'_1 ; \nu_3 \; J)\; D(\nu_3, m'_{J_F} - m_{J_f}, M'-M; \mathcal{R})$$

$$+6 \; [\ell_1 \ell'_1]^{1/2} \; k^2/p_\Lambda^2 \; \left| \sqrt{2/3}\; a_{1/2p} + \sqrt{1/6} a_{\;3/2p} \right|^2$$

$$W(1/2 J j_1 J_f ; J_a \lambda) W(1/2 J j_1 J_f ; J_a \lambda') \; W(\ell_1 \lambda 1/2 1/2 ; 1 j_1) W(\ell'_1 \; \lambda' 1/2 1/2 ; 1 j_1)$$

$$C^{\ell_1 \; 1 \; \lambda}_{0 \;\; 0 \;\; 0} \; C^{\ell'_1 1 \lambda'}_{0\;0\;0} [\lambda \lambda'] C^{\lambda \lambda' \nu_4}_{-M M'} C^{J_f J_f \nu_4}_{m_{J_f} - m'_{J_f}} \;\; E(J_f J \lambda \lambda' ; \nu_4 \;\; J).$$

$$\cdot D(\nu_4, m'_{J_f} - m_{J_f}\; M'-M; \mathcal{R}) \Big]. \qquad (25)$$

Summations here on the right hand side are over $\ell_1, \ell_1', \lambda, \lambda', \nu_1, \nu_2, \nu_3, \nu_4$, M and M'. As seen from this expression the correlation function is independent of the radial integrals if a pure configuration is important for the residual nuclear state. There is no preferential direction in the two-body mesic decay; so we can put M'-M = $m'_{J_f} - m_{J_f} = 0$ and thus the D-function is reduced to a Legendre function:

$$D(\nu, 00; \alpha\beta\, 0) = P_\nu(\cos\beta) \tag{26}$$

The decay probability is obtained by integrating over the pion momentum $\underline{k}$ multiplying delta function of the conservation of energy to the correlation function deduced above, (25), summed over the magnetic quantum number of the residual nuclear states in non-relativistic limit:

$$w_{fi} = \int d\underline{k}\ \delta(E_i - E_f) \sum_{m_{J_f}, m'_{J_f}} E(m_{J_f}, m'_{J_f}) \tag{27}$$

Integrating over cosβ we will find out that only $\nu = 0$ term contributes to (27). The cross terms between direct and derivative term in (25) vanish by the conservation of angular momentum. Using the relations

$$C^{\ell_1 \ell'_1 0}_{-MM'} = (-)^{\ell_1 - M}\ \delta_{\ell_1 \ell'_1} / [\ell_1]^{1/2}$$
$$W(J_f J_f \ell_1 \ell_1'; 0J) = (-)^{J_f + \ell_1 - J}\ \delta_{\ell_1 \ell_1'} / [\ell_1 J_f]^{1/2} \tag{28}$$

etc., the sign factor after the radial overlap integral in (25) exactly cancels out and we obtain w_{fi} in a simple form.

$$w_{fi} = (n+1)\ [J_f] \left(C^{T\ 1/2\ T_f}_{m_T 1/2\ m_{T_f}} \right)^2 \langle R_p \,|\, j_{\ell_1}(kr) \,|\, R_\Lambda \rangle^2 \ 2\pi\mu k$$
$$\left| \sqrt{2/3}\ a_{1/2s} + \sqrt{1/6}\ a_{3/2s} \right|^2 \left| [j_1]^{1/2} \langle T_f J_f | T_a J_a 1/2 j_1 \rangle\ W(1/2 j_1 J J_f; \ell_1 J_a) \right|^2$$
$$+ 6\ k^2/p_\Lambda^2 \ \left| \sqrt{2/3}\ a_{1/2p} + \sqrt{1/6}\ a_{3/2p} \right|^2 \sum_\lambda [\lambda] \left(C^{\ell_1\ 1\ \lambda}_{0\ \ 0\ 0} \right)^2$$
$$\left| [j_1 \ell_1]^{1/2} \langle T_f J_f | T_a J_a 1/2 j_1 \rangle \ W(1/2 J j_1 J_f; J_a \lambda) W(\ell_1 \lambda\, 1/2 1/2; 1 j_1) \right|^2 \quad (29)$$

where μ and k are pion mass and relative momentum of the pion and the residual nucleus. We made an approximation that the pion mass is small compared to that of the residual nucleus. In the exact non-relativistic approach we have to use the reduced mass $M\mu/M+\mu)$ instead of μ (where M is the mass of the residual nucleus).

B) Three-body Mesic Decay through "Compound Nucleus"

(1) Angular Correlation

In most of the three-body mesic decay the pion will be emitted without appreciable final state interaction because of the small phase shifts of the low energy pion-nucleon interaction. On the other hand the nucleon will have a strong final state interaction with the residual nucleus because of the existence of the various compound states of the nucleus. This confines the spin and the parity of the outgoing nucleon and the residual nucleus. It will provide means for the investigation of the unknown nature of hyperfragments and the structure of the decay interaction of the Λ hyperon. Therefore we will first formulate the case in which the outgoing nucleon decays through the compound state. The lifetime of the resonance state is of the order of the nuclear lifetime so we will expect the disturbance in the intermediate state of the problem is very little[13]. The (differential) probability for this channel will be given by the product of two correlation functions by taking the complex conjugate for one of them and summing over the magnetic quantum numbers in the intermediate state, one of the correlation functions being for the pion emission and the other for the nucleon emission. The first one is given by (25) replacing the isotopic spin T_f, the spin J_f by their intermediate values T_I and J_I. Of course all other quantities appearing in the final states should be taken the same as the intermediate ones. In order to obtain the second one we expand the nucleon wave function in plane waves:

$$\psi(\underline{r}) = u(p)\, e^{i\,\underline{p}\cdot\underline{r}}$$

$$= u(p)\sum_{L,M} (4\pi[L])^{1/2}\, i^L\, j_L(pr)\, Y_{LM}(\underline{r})\, D(L,M0;\underline{p})$$

$$= u(p)\sum_{L,M} (4\pi[L])^{1/2}\, i^L\, j_L(pr) \sum_{\substack{j \\ M+m_s}} C^{L\ 1/2\ j}_{M\ \ m_s}\ D(L,M0;\underline{p}), \tag{30}$$

where u(p) is the Pauli spinor. The correlation function, $E^{(2)}(m_{J_I}, m'_{J_I})$, is given by

$$E^{(2)}(m_{J_I}, m'_{J_I}) = (-)^{N-m_{J_I}-J_f} \left(C^{T_I\ \ 1/2\ \ T_f}_{m_{T_I}\ 1/2\ m_{T_f}}\right)^2 4\pi[L]$$

$$\langle R_p | j_L(pr)\rangle^2 \sum_{\substack{J_f \\ M_J+M+m_s}} C^{J_\alpha\, j_1\, J_f}_{M_{J_\alpha}\, M+m_s}\, C^{J_\alpha\, j_1'\, J_f}_{M_{J_\alpha}\, M+m_s} \sum_{\substack{j \\ M+m_s}} C^{L\,1/2\,j_1}_{M\,m_s}\, C^{L'\,1/2\,j_1'}_{M\ \ m_s}$$

$$C^{L\ \ L'\ \nu}_{-N\ N'}\, C^{J_I\, J_I\ \nu}_{m_{J_I}\, m'_{J_I}}\, W(J_I J_I L L'; \nu J_f)\, D(\nu, m'_{J_I}-m_{J_I}, N'-N; \mathcal{R}_2), \tag{31}$$

where we have chosen a general coordinate system in momentum space and $\mathcal{R}_2$ corresponds to the general rotation. Making use of the orthogonality of the Clebsch-Gordan coefficients

$$\sum_{m_{J_I}} C^{J_I\, J_I\ \nu}_{m_{J_I}\, -m'_{J_I}}\, C^{J_I\, J_I\ \nu'}_{m_{J_I}\, -m'_{J_I}} = +\,\delta_{\nu\nu'} \tag{32}$$

and the relation for the product of D-functions

$$D(\nu,\mu t_1;\mathcal{R}_1)\, D^*(\nu,t_2\mu;\mathcal{R}_2^{-1})$$

$$= \sum_{\mu} D(\nu,\mu t_1;\mathcal{R}_1) D(\nu,t_2\mu;\mathcal{R}_2^{-1})$$

$$= D(\nu,t_2 t_1;\mathcal{R}_2^{-1}\mathcal{R}_1), \tag{33}$$

where $\mathcal{R}_2^{-1}$ is the inverse rotation to $\mathcal{R}_2$ and the rotation $\mathcal{R}_2^{-1}\mathcal{R}_1$ is that rotation which carries the coordinate system of the pion into the coordinate system of the nucleon. We have the double correlation function, W, in the form

$$W = \sum_{m_{J_I},m'_{J_I}} E^{(1)}(m_{J_I},m'_{J_I})\, E^{(2)*}(m_{J_I},m'_{J_I})$$

$$= (n+1)\left\{C^{T\ \frac{1}{2}\ T_I}_{m_T\ \frac{1}{2}}\right\}^2 \left\{C^{T_I\ \frac{1}{2}\ T_f}_{m_{T_I}\ \frac{1}{2}}\right\}^2 \Big/ 2 \quad \langle R_p| j_{l_1}(kr)|R_\Lambda\rangle^* \langle R_p| j_{l_1'}(kr)|R_\Lambda\rangle$$

$$\langle R_p|j_L(pr)\rangle^* \langle R_p|j_{L'}(pr)\rangle\ (-)^{J-m_{J_I}+J_f-m_{J_I}}\ [j_1 j_1' l_1 l_1']^{\frac{1}{2}}$$

$$4\pi\,[LL']^{\frac{1}{2}} \sum_{\substack{J_f \\ M_{J_\alpha}+M+m_s}} C^{J_\alpha\ j_1\ J_f}_{m_{J_\alpha}\ M+m_s}\ C^{J_\alpha\ j_1'\ J_f}_{m_{J_\alpha}\ M+m_s} \sum_{M+m_s} C^{L\ \frac{1}{2}\ j_1}_{M\ m_s}\ C^{L'\ \frac{1}{2}\ j_1'}_{M\ m_s}$$

$$\langle T_I\, J_I|\, T_\alpha\, J_\alpha\, \tfrac{1}{2}\, j_1\rangle \langle T_I\, J_I|\, T_\alpha\, J_\alpha\, \tfrac{1}{2}\, j_1'\rangle$$

$$\left[\,\left|\sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}s} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}s}\right|^2 W(\tfrac{1}{2} j_1 J J_I;\, l_1 J_\alpha)\ W(\tfrac{1}{2} j_1' J J_I;\, l_1' J_\alpha)\right.$$

$$(-)^n\, C^{l_1\ l_1'\ \nu}_{-n\ \tau_1+n}\ W(J_I J_I l_1 l_1';\, \nu J)$$

$$-\sqrt{6[l_1']}\ k/p_\Lambda \left(\sqrt{\tfrac{2}{3}}\,a_{\frac{1}{2}s}+\sqrt{\tfrac{1}{6}}\,a_{\frac{3}{2}s}\right)^*\left(\sqrt{\tfrac{2}{3}}\,a_{\frac{1}{2}p}+\sqrt{\tfrac{1}{6}}\,a_{\frac{3}{2}p}\right)$$

$$W(\tfrac{1}{2}j_1JJ_I;l_1J_\alpha)\,W(\tfrac{1}{2}Jj_1J_I;J_\alpha\lambda')\,W(l_1'\lambda'\tfrac{1}{2}\tfrac{1}{2};1j_1)\,C^{l_1'1\lambda'}_{0\ 0}(-)^n C^{l_1\lambda'\nu}_{-n\tau_1+n}$$

$$W(J_IJ_Il_1\lambda';\nu J)$$

$$-\sqrt{6[l_1]}\ k/p_\Lambda \left(\sqrt{\tfrac{2}{3}}\,a_{\frac{1}{2}p}+\sqrt{\tfrac{1}{6}}\,a_{\frac{3}{2}p}\right)^*\left(\sqrt{\tfrac{2}{3}}\,a_{\frac{1}{2}s}+\sqrt{\tfrac{1}{6}}\,a_{\frac{3}{2}s}\right)$$

$$W(\tfrac{1}{2}Jj_1J_I;J_\alpha\lambda)\,W(l_1\lambda\tfrac{1}{2}\tfrac{1}{2};1j_1)\,W(\tfrac{1}{2}j_1JJ_I;l_1'J_\alpha)\,C^{l_1 1\lambda}_{0\ 0}(-)^n C^{l_1\lambda\nu}_{-n\tau_1+n}$$

$$W(J_IJ_I\lambda l_1';\nu J) \tag{34}$$

$$+6\,[l_1l_1']^{\frac{1}{2}}\ k^2/p_\Lambda^2\left|\sqrt{\tfrac{2}{3}}\,a_{\frac{1}{2}p}+\sqrt{\tfrac{1}{6}}\,a_{\frac{3}{2}p}\right|^2$$

$$W(\tfrac{1}{2}Jj_1J_I;J_\alpha\lambda)\,W(\tfrac{1}{2}Jj_1'J_I;J_\alpha\lambda')\,W(l_1\lambda\tfrac{1}{2}\tfrac{1}{2};1j_1)\,W(l_1'\lambda'\tfrac{1}{2}\tfrac{1}{2};1j_1')$$

$$C^{l_1 1\lambda}_{0\ 0}\,C^{l_1'1\lambda'}_{0\ 0}\,[\lambda\lambda']\,(-)^n\,C^{\lambda\lambda'\nu}_{-n\tau_1+n}\,W(J_IJ_I\lambda\lambda';\nu J)\Big].$$

$$\cdot(-)^N C^{LL'\nu}_{-N\tau_2+N}\,W(J_IJ_ILL';\nu J_f)\,D(\nu,\tau_2\tau_1;R_2^{-1}R_1),$$

where D-function is the function of the relative angle between pion and nucleon momentums in the general coordinate system. The knowledge of the direction-direction correlation of momentum distribution of pion and nucleon is obtained by putting $t_1 = t_2 = 0$. As seen from (34) the correlation is completely determined by the angular momentum if the decay occurs through the pure intermediate state. The multiplication factor arising from the radial dependence is unimportant for the correlation.

(ii) Decay Probability

The three-body mesic decay probability, w_{fi}, is obtained by integrating correlation function (34) (keeping $t_1=t_2=0$) over the momentums of the pion $\underline{k}$ and the nucleon $\underline{p}$ multiplying delta-function for the conservation of the total energy.

$$w_{fi} = \int d\underline{k} \int d\underline{p}\, \delta(E_i - E_f)\, W, \tag{35}$$

where the multiplicative constant such as the weight factor with respect to spin is neglected. Fixing the direction of the prōton momentum as a reference system the argument of the Legendre polynomial p(arising from D-function in(34)) is exactly the angle between $\underline{k}$ and $\underline{p}$ in the new system. Then the integration for the angle of $\underline{p}$ will give 4π and the total integration reduces to the integral for the relative angle and the magnitude of k and p. Rewriting the delta-function in a suitable form we obtain w_{fi} in the form

$$W_{fi} = 2(2\pi)^2 M \int p\,dp \int k\,dk\; d(\cos\beta)$$
$$\left(\cos\beta + (M+m)/_{2m} \cdot p/_{k} + (M+\mu)/_{2\mu} \cdot k/_{p} - ME/_{pk}\right) W, \tag{36}$$

where μ, m and M are masses of the pion, nucleon and residual nucleus respectively. This integration can be done analytically if we specify the rank ν of the Legendre polynomial (see Appendix 1).

(C) Three-body Mesic Decay without Compound Nucleus Formation.

The p-shell nuclei have so many level that the outgoing nucleon will have final state interactions corresponding to each level of the compound state. Even if there is no appreciable level structure the ground state can contribute to the final state interaction. In some of the decay process the outgoing nucleon has a proper energy and no appreciable final state interaction appears, for instance, for s-shell hyperfragment decay. In order to treat this type of problem we will develop here formulae for the three-body mesic decay without final state interactions. The decay Hamiltonian is given by (10) and the outgoing nucleon is described by (30). The wave function for the final state with the angular momentum J_f is given by the decoupling of the nucleon wave function with the angular momentum j and the residual wave function with the angular momentum J_a. Thus the Clebsch-Gordan coefficients give the possible weights of the admixtures. The space-spin part of the contribution of the direct and derivative term is given by

$$\text{Direct term} = -\sum_{\ell} i^{2\ell}\sqrt{4\pi}\,\langle j_\ell(pr)|j_\ell(kr)|R_\Lambda\rangle\,[J]^{\frac{1}{2}} \sum_{\substack{J_f \\ M_{J_\alpha}+M+m_s}} C^{J_\alpha\, j\, J_f}_{M_{J_\alpha}\, M+m_s} \sum_{j,\, M+m_s} C^{\ell\, \frac{1}{2}\, j}_{M\, m_s}\, C^{J\, \ell\, J_f}_{m_J\, m\, m_{J_f}}\, (-)^{J_\alpha+\frac{1}{2}-J} \tag{37}$$

$$W(\tfrac{1}{2} j\, J\, J_f;\ell\, J_\alpha)\,[j]^{\frac{1}{2}}\,\delta_{\ell L}\,\delta_{J_x J_\alpha}\;;$$

and

$$\text{Derivative term} = \sqrt{6}\sqrt{4\pi}\sum_{\ell} i^{2\ell}\langle j_\ell(pr)|j_\ell(kr)|R_\Lambda\rangle\,[J]^{\frac{1}{2}} \sum_{\substack{J_f \\ M_{J_\alpha}+M+m_s}} C^{J_\alpha\, j\, J_f}_{M_{J_\alpha}\, M+m_s} \sum_{j,\, M+m_s} C^{\lambda\, \frac{1}{2}\, j}_{M\, m_s}\, C^{J\, \lambda\, J_f}_{m_J\, m+f}\, (-)^{J_\alpha+\frac{1}{2}-J}$$

$$W(\tfrac{1}{2} J\, j\, J_f;\, J_\alpha\, \lambda)\, W(\ell\, \lambda\, \tfrac{1}{2}\tfrac{1}{2};1 j)\,[\lambda j]^{\frac{1}{2}}\,\delta_{\ell L}\,\delta_{J_x J_\alpha} \tag{38}$$

The matrix element for the process is given by

$$\langle f|H|i\rangle = \sum \left(C^{T_\alpha\ \frac{1}{2}\ T_f}_{m_{T_\alpha}\ \frac{1}{2}\ m_{T_f}} \right)^2 \Big/ \sqrt{2}\ \sum_{\ell} i^{2\ell} \sqrt{4\pi}$$

$$\langle j_\ell(pr)|j_\ell(kr)|R_\Lambda\rangle (-)^{\frac{1}{2}-J} \sum_{j} [Jj]^{\frac{1}{2}} [\ell] (-)^{J_\alpha} \sum C^{J_\alpha\ j\ J_f}_{M_{J_\alpha}\ M+m_s}\ C^{\ell\ \frac{1}{2}\ j}_{M\ m_s}$$

$$\Big[-\left\{ \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}s} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}s} \right\} W(\tfrac{1}{2}\, j\, J\, J_f;\ \ell\, J_\alpha) \sum_{m} C^{J\ \ell\ J_f}_{m_J\ m}\ D(\ell, m0; \underline{k}) \cdot D(\ell, M0; \underline{p})$$

$$-\sqrt{6[\ell]}\ \frac{k}{k_\Lambda} \left\{ \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}p} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}p} \right\} \sum_{\lambda, m+q} W(\tfrac{1}{2}\, J\, j\, J_f;\ J_\alpha\, \lambda)\ W(\ell\, \lambda\, \tfrac{1}{2}\, \tfrac{1}{2};\ 1\, j)$$

$$C^{\ell\ 1\ \lambda}_{0\ 0}\ [\lambda]\ C^{J\ \lambda\ J_f}_{m_J\ m+q}\ D(\lambda, m+q\, 0; \underline{k})\ D(\ell, M0; \underline{p}) \Big] \qquad (39)$$

where the first sum on the right is over the possible value of the isotopic spin for the residual nucleus and the outgoing nucleon. The summation for the angular momentum state is not given explicitly. In order to calculate the decay probability we have to take the absolute square of eq. (39) and summation over the final magnetic quantum numbers and average over the initial spin. The procedure of the calculation of the D-function will be done in the same way as we did in paragraph B). The result is given by

$$W = \sum \left(C^{T\ \frac{1}{2}\ T_f}_{m_T\ \frac{1}{2}\ m_{T_f}} \right)^4 (2\pi) \sum_{\substack{m_{J_f} \\ \ell, \ell'}} \langle j_\ell(pr)|j_\ell(kr)|R_\Lambda\rangle^*$$

$$\langle j_{\ell'}(pr)|j_{\ell'}(kr)|R_\Lambda\rangle (-)^{J-m_{J_f}+\mu} [jj']^{\frac{1}{2}} \sum C^{J_\alpha\ j\ J_f}_{M_{J_\alpha}\ M+m_s}\ C^{J_\alpha\ j'\ J_f}_{M_{J_\alpha}\ M'+m_s'}.$$

$$\cdot \sum C^{\ell \frac12 j}_{M m_s} \; C^{\ell' \frac12 j'}_{M' m_s'} \Bigg[\left| \sqrt{\tfrac23}\, a_{\frac12 s} + \sqrt{\tfrac16}\, a_{\frac32 s} \right|^2 W(\tfrac12 j J J_f ; \ell J_\alpha)$$

$$W(\tfrac12 j' J J_f ; \ell' J_\alpha)\, C^{\ell \ell' \nu_1}_{-\mu \mu'} \; C^{J_f J_f \nu_1}_{m_{J_f} -m'_{J_f}} \; W(J_f J_f \ell \ell' ; \nu_1 J) \sum_{\beta_1} C^{\ell \ell'' \beta_1}_{-\kappa \kappa'} \; C^{\ell \ell'' \beta_1}_{-M M'}$$

$$D(\nu_1, m'_{J_f} - m_{J_f}, \mu' - \mu ; \mathcal{R}_1) \; D^*(\beta_1, M' - M, \kappa' - \kappa ; \mathcal{R}_2)$$

$$-\sqrt{6}\,[\ell'] \; {}^{k}/_{p_\Lambda} \left(\sqrt{\tfrac23}\, a_{\frac12 s} + \sqrt{\tfrac16}\, a_{\frac32 s} \right)^* \left(\sqrt{\tfrac23}\, a_{\frac12 p} + \sqrt{\tfrac16}\, a_{\frac32 p} \right)$$

$$W(\tfrac12 j J J_f ; \ell J_\alpha)\, W(\tfrac12 J j J_f ; J_\alpha \lambda')\, W(\ell' \lambda' \tfrac12 \tfrac12 ; 1 j)\, [\lambda']\, C^{\ell' 1 \lambda'}_{0\;0} \, C^{\ell \lambda' \nu_2}_{-\mu \mu'} \, C^{J_f J_f \nu_2}_{m_{J_f} -m'_{J_f}}$$

$$W(J_f J_f \ell \lambda' ; \nu_2 J) \sum_{\beta_2} C^{\ell \ell'' \beta_2}_{-\kappa \kappa'} \; C^{\ell \ell'' \beta_2}_{-M M'} \; D(\nu_2, m'_{J_f} - m_{J_f}, \mu' - \mu ; \mathcal{R}_1)\, D^*(\beta_2, M' - M, \kappa' - \kappa ; \mathcal{R}_2)$$

$$-\sqrt{6}\,[\ell] \; {}^{k}/_{p_\Lambda} \left(\sqrt{\tfrac23}\, a_{\frac12 p} + \sqrt{\tfrac16}\, a_{\frac32 p} \right)^* \left(\sqrt{\tfrac23}\, a_{\frac12 s} + \sqrt{\tfrac16}\, a_{\frac32 s} \right)$$

$$W(\tfrac12 J j J_f ; J_\alpha \lambda)\, W(\ell \lambda \tfrac12 \tfrac12 ; 1 j)\, W(\tfrac12 j J J_f ; \ell' J_\alpha)\, [\lambda]\, C^{\ell 1 \lambda}_{0\;0} \, C^{\ell \lambda \nu_3}_{-\mu \mu'} \, C^{J_f J_f \nu_3}_{m_{J_f} -m'_{J_f}}$$

$$W(J_f J_f \lambda \ell' ; \nu_3 J) \sum_{\beta_3} C^{\ell \ell'' \beta_3}_{-\kappa \kappa'} \; C^{\ell \ell'' \beta_3}_{-M M'} \; D(\nu_3, m'_{J_f} - m_{J_f}, \mu' - \mu ; \mathcal{R}_1)\, D^*(\beta_3, M' - M, \kappa' - \kappa ; \mathcal{R}_2)$$

$$+6\,[\ell \ell'] \; {}^{k^2}/_{p_\Lambda^2} \left| \sqrt{\tfrac23}\, a_{\frac12 p} + \sqrt{\tfrac16}\, a_{\frac32 p} \right|^2$$

$$W(\tfrac12 J j J_f ; J_\alpha \lambda)\, W(\tfrac12 J j' J_f ; J_\alpha \lambda')\, W(\ell \lambda \tfrac12 \tfrac12 ; 1 j)\, W(\ell' \lambda' \tfrac12 \tfrac12 ; 1 j')$$

$$C^{\ell 1 \lambda}_{0\;0} \; C^{\ell' 1 \lambda'}_{0\;0} \, [\lambda \lambda'] \; C^{\lambda \lambda' \nu_4}_{-\mu \mu'} \; C^{J_f J_f \nu_4}_{m_{J_f} -m'_{J_f}} \; W(J_f J_f \lambda \lambda' ; \nu_4 J)$$

$$\sum_{\beta_4} C^{\ell \ell'' \beta_4}_{-\kappa \kappa'} \; C^{\ell \ell'' \beta_4}_{-M M'} \; D(\nu_4, m'_{J_f} - m_{J_f}, \mu' - \mu ; \mathcal{R}_1) \; D^*(\beta_4, M' - M, \kappa' - \kappa ; \mathcal{R}_2) \Bigg], \qquad (40)$$

where ν and β are now quantum numbers appearing after the absolute square of the D-functions for the pion and nucleons respectively. The probability w_{fi} is obtained in the same way discussed in paragraph B.

3. Two-body Mesic Decay: Applications.

We want to apply the theory developed in the preceeding section to the two-body mesic decay processes in this section.

(1) Decay probability

As seen in section 3 all the mesic decays are described by the absolute square of the radial matrix element with numerical coefficients arising from the number of nucleons, the spin of the residual nucleus, the weight factor from the charge space $2\pi\mu k$ and the amplitudes multiplied by the kinematical factors (see eq. (29)). The amplitudes($\sqrt{2/3}\, a_{1/2s} + \sqrt{1/6}\, a_{3/2s}$) and ($\sqrt{2/3}\, a_{1/2p} + \sqrt{1/6}\, a_{3/2p}$) contain the reduced matrix elements in charge space. These are for the charged mesic decay amplitudes. For the neutral mesic decay we have to change numerical factors of the amplitudes as we discussed in section 2 (see the discussion under eq. (10)). We will first tabulate the kinematical coefficients and then discuss for individual hyperfragments.

Table 1. Two-body Mesic Decay Probabilities

Decay Scheme	J	J_f	Coefficients of Radial Term	Coefficients of Direct Term	Coefficients of Derivative Term
${}^3H_\Lambda \rightarrow \pi^- + {}^3He$	1/2	1/2	6	1/4	1/12
	3/2	1/2	6	0	1/3
${}^3H_\Lambda \rightarrow \pi^0 + {}^3H$	1/2	1/2	6	1/4	1/12
	3/2	1/2	6	0	1/3
${}^4H_\Lambda \rightarrow \pi^- + {}^4He$	0	0	2	1	0
	1	0	2	0	1
${}^4He_\Lambda \rightarrow \pi^0 + {}^4He$	0	0	2	1	0
	1	0	2	0	1
${}^6He_\Lambda \rightarrow \pi^- + {}^6Li$	1	0	1	1/3	0
	1	1	3	5/18	1/6
	1	2	5	1/6	1/2
	1	3	7	0	1
	2	0	1	0	1
	2	1	3	1/30	9/10
	2	2	5	1/10	7/10
	2	3	7	1/5	2/5
${}^7He_\Lambda \rightarrow \pi^- + {}^7Li$	1/2	3/2	4	5/24	25/24
${}^7Li_\Lambda \rightarrow \pi^- + {}^7Be$	1/2	1/2	6	2/9	0
	1/2	3/2	12	1/24	3/40
	3/2	1/2	6	1/72	3/16
	3/2	3/2	12	1/18	3/20
${}^8Li_\Lambda \rightarrow \pi^0 + {}^8Li$	1	2	20	1/30	1/10
	2	2	20	1/50	7/50

Table 1 (continued)

Decay Scheme	J	J_f	Coefficients of Radial Term	Direct Term	Derivative Term
${}^{9}Li_{\Lambda} \rightarrow \pi^{-} + {}^{9}Be$	3/2	3/2	40/3	1/5	1/5
	5/2	3/2	40/3	1/30	1/10
${}^{9}Be_{\Lambda} \rightarrow \pi^{0} + {}^{9}Be$	1/2	3/2	20	5/24	25/24
${}^{10}Be_{\Lambda} \rightarrow \pi^{-} + {}^{10}B$	1	0	6	1/3	0
	1	1	9	5/18	1/6
	1	2	30	1/6	1/2
	1	3	21	0	1
	2	0	6	0	1
	2	1	9	1/30	9/10
	2	2	30	1/10	7/10
	2	3	21	1/5	2/5
${}^{10}Be_{\Lambda} \rightarrow \pi^{0} + {}^{10}Be$	1	0	6	1/3	0
	1	2	30	1/6	1/2
	2	0	6	0	1
	2	2	30	1/10	7/10
${}^{10}B_{\Lambda} \rightarrow \pi^{-} + {}^{10}C$	1	0	6	1/3	0
	2	0	6	0	1
${}^{11}Be_{\Lambda} \rightarrow \pi^{-} + {}^{11}B$	1/2	3/2	56/3	1/2	5/2
	1/2	3/2	4/3	1/14	5/14
${}^{11}B_{\Lambda} \rightarrow \pi^{-} + {}^{11}C$	5/2	3/2	28	1/6	1/14
	5/2	1/2		0	0
	7/2	3/2		0	0
	7/2	1/2		0	0

Table 1 (continued)

Decay Scheme	J	J_f	Coefficients of Radial Term	Coefficients of Direct Term	Coefficients of Derivative Term
${}^{11}B_\Lambda \to \pi^0 + {}^{11}B$	5/2	3/2	28	1/6	1/14
	5/2	1/2		0	0
	7/2	3/2		0	0
	7/2	1/2		0	0
${}^{11}C_\Lambda \to \pi^0 + {}^{11}C$	1/2	3/2	56/3	1/2	5/2
	1/2	1/2	4/3	1/14	5/14
${}^{12}B_\Lambda \to \pi^- + {}^{12}C$	1	0	4	1/3	0
	1	2	5/2	1/3	0
	2	0	4	0	1
	2	2	5/2	1/10	1/5
${}^{12}B_\Lambda \to \pi^0 + {}^{12}B$	1	1	3/2	1/18	1/3
	1	2	5/2	1/6	0
	2	1	3/2	1/6	0
	2	2	5/2	1/10	0
${}^{13}C_\Lambda \to \pi^- + {}^{13}N$	1/2	1/2	2	1/2	1/2
${}^{13}C_\Lambda \to \pi^0 + {}^{13}C$	1/2	1/2(p)	2	1/2	1/2
	1/2	1/2(s)	2	1/2	3/2
${}^{14}C_\Lambda \to \pi^- + {}^{14}N$	0	1	6	1/3	0
	0	0	4	0	0
	1	1	6	2/9	1/2
	1	0	4	1/3	0

Table 1 (continued)

Decay Scheme	J	J_f	Coefficients of		
			Radial Term	Direct Term	Derivative Term
${}^{16}O_\Lambda \to \pi^0 + {}^{16}O$	0	0	2	0	1
	1	0	2	1/3	0
${}^{17}O_\Lambda \to \pi^- + {}^{17}F$	1/2	5/2(d)	6	1/2	7/2
	1/2	1/2(s)	2	1/2	3/2
${}^{17}O_\Lambda \to \pi^0 + {}^{17}O$	1/2	5/2(d)	6	1/2	7/2
	1/2	1/2(s)	2	1/2	3/2

Notations s and d in the bracket after J_f show the configuration of the decay nucleons which are different from the original configuration of nucleons.

Assuming the same s-wave and p-wave rate in the decay interaction we can discuss about the decay branching ratio using Table 1. We will give an example how to use the Table and analyze the more interesting examples. Let us take ${}^{10}Be_\Lambda$ negative mesic decay which leads to ${}^{10}B+\pi^-$. The hole-hole configuration of $p_{3/2}$-shell and all four states belonging to this configuration are stable; hence the branching ratios to the spins 0,1,2 and 3 are very interesting. They are obtained to be 2:4:20:21 and 90:140:360:189 for the anti-parallel spin and parallel spin of ${}^{10}Be_\Lambda$ respectively. These ratios are not sufficiently sensitive so it is better to find out more crucial examples. One sees from Table 1 that ${}^{11}B_\Lambda$ decay into the negative and the neutral two-body decay modes are allowed only for spin 5/2. Thus the observation of the two-body decay mode uniquely determines the antiparallel-favoured spin for $p_{3/2}$-shell hyperfragments. Another less definite example is the two-body decay mode of $^{14}N_\Lambda$ where the decay into $J_f = 0$ state is forbidden for the antiparallel favoured spin configuration. This will be used for the determination of the relative preferences for parallel-spin and antiparallel-spin configurations for $p_{1/2}$-shell hyperfragments.

(ii) Polarization and Direction Correlation

The case of the cascade decay

$$K^- + {}^4He \longrightarrow {}^4He_\Lambda + \pi^- \tag{41}$$

$$\hspace{4em}\searrow {}^4He + \pi^0 \tag{42}$$

may be examined. The correlation will be obtained only for J = 1 for $^4He_\Lambda$. Assuming a complete polarized state for $^4He_\Lambda$, $J = 1, m_J = 1$ leads to the correlation, $W(\theta)$:

$$W(\theta) = \tfrac{1}{6}\, k^2/p_\Lambda^2 \left| \sqrt{\tfrac{2}{3}}\, a_{1/2\,p} + \sqrt{\tfrac{1}{6}}\, a_{3/2\,p} \right|^2 \sin^2\theta \tag{43}$$

The distribution of the pion momentum with respect to the direction of the polarization of $^4He_\Lambda$ is symmetric with respect to the decay plane. There is no up-down asymmetry hence the correlation probability, X, is defined by

$$X \equiv \alpha = \int_{\pi/4}^{3/4\pi} W(\theta)\sin\theta\, d\theta \Big/ \int_0^{\pi} W(\theta)\sin\theta\, d\theta = \sqrt{2}/2. \tag{44}$$

The value αP_1 is compared with the experimental value where P_1 is the polarization with magnetic quantum number 1.

4. Isotopic Spin Selection Rule

We will consider the isotopic spin selection rule in this section. This can be done from the decay branching ratio of the negative and the neutral pion modes. Unfortunately we do not have enough data for this purpose so we will confine ourselves to the theoretical investigation and show by illustration how to apply our ideas to the problem. In order to express the formula in a compact form let us introduce the following short notation

$$
\begin{aligned}
s &= \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}s} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}s} , \qquad p = \tfrac{k}{p_\Lambda}\left(\sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}p} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}p}\right) \\
s' &= \sqrt{\tfrac{1}{3}}\, a_{\frac{1}{2}s} - \sqrt{\tfrac{1}{3}}\, a_{\frac{3}{2}s} , \qquad p' = \tfrac{k}{p_\Lambda}\left(\sqrt{\tfrac{1}{3}}\, a_{\frac{1}{2}p} - \sqrt{\tfrac{1}{3}}\, a_{\frac{3}{2}p}\right)
\end{aligned}
\tag{45}
$$

The branching ratios of interests are given by

$$
\frac{w\,({}^3H_\Lambda \to {}^3He + \pi^-)}{w\,({}^3H_\Lambda \to {}^3H + \pi^0)} = \frac{|s|^2 + \frac{1}{3}|p|^2}{|s'|^2 + \frac{1}{3}|p'|^2} \qquad \text{for } J = J_f = \tfrac{1}{2} \tag{46.a}
$$

$$
= \frac{|p|^2}{|p'|^2} \qquad \text{for } J = \tfrac{3}{2},\ J_f = \tfrac{1}{2} \tag{46.b}
$$

$$
\frac{w\,({}^7Li_\Lambda \to {}^7Be + \pi^-)}{w\,({}^7Li_\Lambda \to {}^7Li + \pi^0)} = \frac{|s|^2}{|s'|^2} \qquad \text{for } J = J_f = \tfrac{1}{2} \tag{46.c}
$$

$$
= \frac{\frac{1}{3}|s|^2 + \frac{3}{5}|p|^2}{\frac{1}{3}|s'|^2 + \frac{3}{5}|p'|^2} \qquad \text{for } J = \tfrac{1}{2},\ J_f = \tfrac{3}{2} \tag{46.d}
$$

$$= \frac{\frac{1}{9}|s|^2 + \frac{3}{2}|p|^2}{\frac{1}{9}|s'|^2 + \frac{3}{2}|p'|^2} \quad \text{for } J = \tfrac{3}{2},\ J_f = \tfrac{1}{2} \qquad (46.e)$$

$$= \frac{\frac{1}{9}|s|^2 + \frac{3}{10}|p|^2}{\frac{1}{9}|s'|^2 + \frac{3}{10}|p'|^2} \quad \text{for } J = \tfrac{3}{2},\ J_f = \tfrac{3}{2} \qquad (46.f)$$

$$\frac{w({}^{10}Be_\Lambda \to {}^{10}B + \pi^-)}{w({}^{10}Be_\Lambda \to {}^{10}Be + \pi^0)} = \frac{|s|^2}{|s'|^2} \quad \text{for } J = 1,\ J_f = 0 \qquad (46.g)$$

$$= \frac{|s|^2 + 3|p|^2}{|s'|^2 + 3|p'|^2} \quad \text{for } J = 1,\ J_f = 2 \qquad (46.h)$$

$$= \frac{|p|^2}{|p'|^2} \quad \text{for } J = 2,\ J_f = 0 \qquad (46.i)$$

$$= \frac{|s|^2 + 7|p|^2}{|s'|^2 + 7|p'|^2} \quad \text{for } J = 2,\ J_f = 2 \qquad (46.j)$$

$$\frac{w({}^{10}B_\Lambda \to {}^{10}C + \pi^-)}{w({}^{10}B_\Lambda \to {}^{10}B + \pi^0)} = \frac{|s|^2}{|s'|^2} \quad \text{for } J = 1,\ J_f = 0 \qquad (46.k)$$

$$= \frac{|p|^2}{|p'|^2} \quad \text{for } J = 2,\ J_f = 0 \qquad (46.l)$$

$$\frac{w({}^{11}B_\Lambda \to {}^{11}C + \pi^-)}{w({}^{11}B_\Lambda \to {}^{11}B + \pi^0)} = \frac{7|s|^2 + 3|p|^2}{7|s'|^2 + 3|p'|^2} \quad \text{for } J = \tfrac{3}{2},\ J_f = \tfrac{3}{2} \qquad (46.m)$$

$$\frac{w({}^{13}C_\Lambda \to {}^{13}N + \pi^-)}{w({}^{13}C_\Lambda \to {}^{13}C + \pi^0)} = \frac{|s|^2 + |p|^2}{|s'|^2 + |p'|^2} \qquad \text{for } J = \tfrac{1}{2},\ J_f = \tfrac{1}{2} \qquad (46.n)$$

As stated earlier the parameters which we are using are related to those for free Λ decay of Okubo et al[9] by

$$\begin{aligned} a_{\frac{1}{2}s} &= A_1 \langle \tfrac{1}{2} \| [t^{(1)} \times D^{(\frac{1}{2})}]^{(\frac{1}{2})} \| 0 \rangle \\ a_{\frac{3}{2}s} &= A_3 \langle \tfrac{1}{2} \| [t^{(1)} \times D^{(\frac{3}{2})}]^{(\frac{1}{2})} \| 0 \rangle \\ a_{\frac{1}{2}p} &= B_1 \langle \tfrac{1}{2} \| [t^{(1)} \times D^{(\frac{1}{2})}]^{(\frac{1}{2})} \| 0 \rangle \\ a_{\frac{3}{2}p} &= B_3 \langle \tfrac{1}{2} \| [t^{(1)} \times D^{(\frac{3}{2})}]^{(\frac{1}{2})} \| 0 \rangle \end{aligned} \qquad (47.a)$$

The reality of A, B and π^-, π^0 branching ratio of the free Λ decay, 2/1, give

$$A_3 = 2\sqrt{2}\, A_1 , \qquad B_3 = 2\sqrt{2}\, B_1 \qquad (47.b)$$

If we write reduced matrix element in the form

$$\sqrt{2} \langle \tfrac{1}{2} \| [t^{(1)} \times D^{(\frac{1}{2})}]^{(\frac{1}{2})} \| 0 \rangle = \langle \tfrac{1}{2} \| [t^{(1)} \times D^{(\frac{3}{2})}]^{(\frac{1}{2})} \| 0 \rangle \qquad (47.c)$$

we see that conditions (47) give the ratio 2/1 for all the cases of (46). One may consider also the decay branching ratios of the same hyperfragment to the different residual nuclear state for π^- and π^0 decay which is not always 2/1 even if we confine $\Delta I = 1/2$ or 3/2 rule. But the ratio does not change in the case with the simultaneous contribution of $\Delta I = 1/2$ and 3/2 rule.

5. Three-Body Mesic Decay (Numerical)

(a) Nucleon through Definite States in Decay

--Directional Correlation--

It is well known that the directional correlation is a good technique to determine the spin of the nuclear states. We will consider the pion and proton directional correlation for assignments of the various possible spins of hyperfragments. Now we are considering the decay through the definite spin (parity) of the nucleus so that the single j_1 and $\ell_1(\ell_1 = L = \ell)$ contribute as the intermediate state of the proton and the residual nucleus interaction. Assuming the angle θ between the directions of the pion and the proton we have the correlation function in the form

$$W(\theta) = a \langle j_\ell(pr)|R_p\rangle^2 \langle R_p| j_\ell(kr)|R_\Lambda\rangle^2 \cdot \Big[b\,|s|^2 \{ c\, D(0,00;\mathcal{R}) + d\, D(2,00;\mathcal{R})\} + e\,\frac{k^2}{p_\Lambda^2}|p|^2 \{ f\, D(0,00;\mathcal{R}) + g\, D(2,00;\mathcal{R})\}\Big], \tag{48}$$

where

$$D(k,00;\mathcal{R}) = P_k(\cos\theta)$$

and $\mathcal{R}$ is the short notation of $\mathcal{R}_2^{-1}\mathcal{R}_1$ (see (33)). The cross term between the direct and derivative terms vanishes for the cases which we are interested. The doefficients a,b,..., g are numbers given by

$$a = (n+1)\left(C^{T_\alpha \frac{1}{2} T_I}_{m_{T_\alpha} \frac{1}{2} m_{T_I}} \, C^{T_I \frac{1}{2} T_f}_{m_{T_I} \frac{1}{2} m_{T_f}} \right)^2 \Big/ 2 \; [j\ell]\, 4\pi\, [L] \tag{49.a}$$

$$b = W(\tfrac{1}{2} j J J_I ; \ell J_\alpha)\, W(\tfrac{1}{2} j J J_I ; \ell J_\alpha) \tag{49.b}$$

$$c = \sum_{J_f} [J_f]/2 \; (-)^n C^{\ell\ell 0}_{-n\, n} \, W(J_I J_I \ell\ell ; 0 J)(-)^N C^{\ell\ell 0}_{-N\, N} \cdot W(J_I J_I \ell\ell ; 0 J_f), \tag{49.c}$$

where we used the relation

$$\sum_{\substack{J_f \\ M_{J_\alpha}+M+m_s}} \left(C^{J_\alpha\, j\, J_f}_{m_{J_\alpha} M+m_s} \right)^2 \sum_{M+m_s} \left(C^{L\, \frac{1}{2}\, j}_{M\, m_s} \right)^2 = \sum_{J_f} [J_f]/2 \,.$$

$$d = \sum_{J_f} [J_f]/2 \, (-)^n C^{\ell \ell 0}_{-n\, n} \, W(J_I J_I \ell\ell ; 2 J) (-)^N C^{\ell \ell 0}_{-N N} \, W(J_I J_I \ell\ell ; 2\, J_f) \tag{49.d}$$

$$e = W(\tfrac{1}{2} J j J_I ; J_\alpha \lambda) \, W(\tfrac{1}{2} J j J_I ; J_\alpha \lambda') \, W(\ell \lambda \tfrac{1}{2} \tfrac{1}{2} ; 1 j) \, W(\ell \lambda' \tfrac{1}{2} \tfrac{1}{2} ; 1 j) \cdot C^{\ell 1 \lambda}_{0\, 0} \, C^{\ell 1 \lambda'}_{0\, 0} \, [\lambda \lambda'] \tag{49.e}$$

$$f = \sum_{J_f} [J_f]/2 \, (-)^n C^{\lambda \lambda' 0}_{-n\, n} \, W(J_I J_I \lambda \lambda' ; 0 J) (-)^N C^{\ell \ell 2}_{-N N} \, W(J_I J_I \ell \ell ; 0\, J_f) \tag{49.f}$$

$$\gamma = \sum_{J_f} [J_f]/2 \, (-)^n C^{\lambda \lambda' 2}_{-n\, n} \, W(J_I J_I \lambda \lambda' ; 2 J) (-)^N C^{\ell \ell 2}_{-N N} \, W(J_I J_I \ell \ell ; 2\, J_f). \tag{49.g}$$

The classification of the coefficients given here is convenient for the calculation. We will calculate these values for the various intermediate spins J_I for the decay of the three hyperfragments with A = 3,4 and 5. The three-body mesic decay of ${}^3H_\Lambda$ is given by

$${}^3H_\Lambda \longrightarrow D + p + \pi^- \tag{50}$$

The bound state of D + p, ^3He, is $^2S_{1/2}$ which gives rather strong effect because it is strongly bound state. Other intermediate states can only be possible for p-wave nucleon. We neglected d-wave nucleons in computing the table (see Table 2). The decay of $^4H_\Lambda$ and $^4He_\Lambda$ is given respectively by

$$^4H_\Lambda \rightarrow {}^3H + p + \pi^- \tag{51}$$

and

$$^4He_\Lambda \rightarrow {}^3He + p + \pi^- \tag{52}$$

In these decays we have also considered up to p-wave protons. The decay of $^5He_\Lambda$ is given by

$$^5He_\Lambda \rightarrow {}^4He + p + \pi^- \tag{53}$$

This is a typical hyperfragment decay with strong final state interaction of the nucleon and the residual nucleus through $p_{3/2}$ and $p_{1/2}$ channel. The spin of ^4He is zero so the proton state is equivalent to the total J_I. We tabulate all the cases discussed above in Table 2.

Table 2. Three-body Mesic Decay-Nucleon through Definite State in Decay--

$^3H_\Lambda \rightarrow D + p + \pi^-$

J	J_I	J_f	State of p	a	b	c	d	e	f	g
1/2	1/2	1/2, 3/2	$s_{1/2}$	6	$1/4\sqrt{2}$	1	0	$3/4\sqrt{6}$	1	0
3/2	1/2	1/2, 3/2	$s_{1/2}$	6	0	0	0	1/2	1	0
1/2	1/2	1/2, 3/2	$p_{3/2}$	24	1/9	1/6	0	0	0	0
1/2	3/2	1/2, 3/2	$p_{3/2}$	24	5/72	1/12	4/45	5/8	$\sqrt{3}/120$	13/300
1/2	1/2	1/2, 3/2	$p_{1/2}$	12	1/36	1/6	0	1/8	$\sqrt{3}/6$	0
1/2	3/2	1/2, 3/2	$p_{1/2}$	12	1/9	1/12	13/180	0	0	0
3/2	1/2	1/2, 3/2	$p_{3/2}$	24	1/144	1/12	0	25/144	1/20	0
3/2	3/2	1/2, 3/2	$p_{3/2}$	24	1/36	1/6	16/225	5/4	$1/80\sqrt{15}$	0
3/2	5/2	3/2, 5/2	$p_{3/2}$	24	1/256	5/54	4/135	35/48	1/18	38/2205
3/2	1/2	1/2, 3/2	$p_{1/2}$	12	1/9	1/6	0	0	0	0
3/2	3/2	1/2, 3/2	$p_{1/2}$	12	5/72	1/6 2	0	5/24	1/4 3	0

$^4H_\Lambda \rightarrow {}^3H + p + \pi^-$

J	J_I	J_f	State of p	a	b	c	d	e	f	g
0	0	0	$s_{1/2}$	4	1/2	1	0	0	0	0
0	1	1, 2	$p_{3/2}$	9	1/6	4/27	2/27	0	0	0
0	2	1, 2	$p_{3/2}$	9	0	0	0	5/2	4/75	4/75
0	0	0, 1	$p_{1/2}$	9/2	0	0	0	1/2	2/3	0
0	1	0, 1	$p_{1/2}$	9/2	1	2/27	2/27	0	0	0
1	1	1, 2	$p_{3/2}$	9	1/36	4/27	1/27	25/12	4/45	$1/9\sqrt{105}$
1	2	1, 2	$p_{3/2}$	9	1/12	4/45	28/45	5/4	$4/25\sqrt{15}$	1/150
1	0	0, 1	$p_{1/2}$	9/2	1/6	2/9	0	0	0	0
1	1	0, 1	$p_{1/2}$	9/2	1/9	2/27	5/108	25/12	2/45	1/12

Table 2 (continued)

				$^5He_\Lambda \rightarrow {}^4He + p + \pi^-$						
J	J_I	J_f	State of p	a	b	c	d	e	f	g
1/2	3/2	1/2,3/2	$p_{3/2}$	72π	1/8	1/12	1/12	25/8	1/20	1/20
1/2	1/2	1/2,3/2	$p_{1/2}$	36π	1/4	1/6	0	3/4	1/2	0

(b) Nucleon with no Definite State

In this case we consider only s-wave proton because the decay proton from s-state Λ stays in s-state without having interaction. The correlation function for $^3H_\Lambda$, $^4H_\Lambda$ and $^4He_\Lambda$ decays are given by

$$W(\theta) = a' \langle j_0(pr) | j_0(kr) | R_\Lambda \rangle^2 \Big[|s|^2 \{ b' D(0,00;\mathcal{R}) + c' D(2,00;\mathcal{R}) \} + 2\, k/p_\Lambda \, \mathcal{R}e\,(s^* p)\; d' D(1,00;\mathcal{R}) + k^2/p_\Lambda^2 \, |p|^2 \{ e' D(0,00;\mathcal{R}) + f' D(2,00;\mathcal{R}) \} \Big]. \qquad (54)$$

The results of the calculation are given in Table 3.

Table 3. Three-body Mesic Decay--Nucleon with No Definite State--

$^3H_\Lambda \to D + p + \pi^-$							
J	J_f	a'	b'	c'	d'	e'	f'
1/2	1/2,3/2	2	$1/4\sqrt{2}$	0	$-1/8\sqrt{3}$	1/24	1/48
3/2	1/2,3/2	2	0	0	0	7/32	1/24
$^4H_\Lambda \to {}^3H + p + \pi^-$							
0	0,1	1/2	1/2	0	0	0	0
1	0,1	1/2	0	0	0	11/54	1/27

It is seen from the table that there is an asymmetry for the pion and proton angular correlation for the spin J = 1/2 state decay of $^3H_\Lambda$ but no such term for the J = 3/2 state. For $^4H_\Lambda$ decay J = 1 state contains P_2 term about one-fifth of P_0 term but no P_2 term for J = 0 state.

6. Mesic Decay of A = 3,4 and 5 Hyperfragments and their Spins

For the mesic decay of hyperfragments with low mass number there are comparatively adequate data in contrast to the case of high mass numbers; so we will confine ourselves to mass number 3,4 and 5 hypernuclei in this section and discuss them rather in detail. The important data is the ratio between the two-body and three-body mesic decays of A = 3 and 4 and the three-body mesic decay rate of $^5He_\Lambda$ through the unstable ground state and first excited state of ^5He. Assuming no final state interaction for the former we can determine whether the parallel spin or antiparallel spin coupling is favoured for $^3H_\Lambda$ and $^4H_\Lambda$. From the latter we can determine the mixing rate of s- and p-wave for Λ decay. We will start from $^4H_\Lambda$ decay. For J = 0 only the s-wave pion and nucleon contribute. For J = 1 p-wave pion and s-wave nucleon contribute. Thus the numerical constants arising from the arbitrary normalization in our formalism for the relative decay rate of two-body and three-body are determined from the decay of $^4H_\Lambda$ both for the parallel and antiparallel spins of $^4H_\Lambda$ decay. This simple minded result arises only if the outgoing nucleon is in s-state both for $^3H_\Lambda$ and $^4H_\Lambda$ decays. We will arbitrarily introduce the constants C_1 and C_2 for $^4H_\Lambda$ decays by

$$w(^4H_\Lambda \rightarrow {}^4He + \pi^-) = 4\pi k\, C_1 \left| \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}s} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}s} \right|^2 \quad \text{for } J = 0 \tag{55.a}$$

$$= 4\pi k\, C_1\, 1.29 \left| \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}p} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}p} \right|^2 \quad \text{for } J = 1 \tag{55.b}$$

and

$$w(^4H_\Lambda \rightarrow {}^3H + p + \pi^-) = 4\pi p_\Lambda\, C_2\, 0.17 \left| \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}s} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}s} \right|^2 \quad \text{for } J = 0 \tag{56.a}$$

$$= 4\pi p_\Lambda\, C_2\, 0.12 \left| \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}p} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}p} \right|^2 \quad \text{for } J = 1, \tag{56.b}$$

where $k/p_\Lambda \cong 1.32$. We can determine the ratio C_1/C_2 comparing with the experimental rate[14] of

$$\frac{w(^4H_\Lambda \rightarrow {}^4He + \pi^-)}{w(^4H_\Lambda \rightarrow {}^3H + p + \pi^-)} = 28/6 .$$

This value fixes $C_1/C_2 \doteq 3/5$ and $1/3$ for $J = 0$ and 1 respectively. Using the same definition of C_1 and C_2 we have,

$$w({}^3H_\Lambda \to {}^3He + \pi^-) = 12\pi k C_1 \left[0.25 \left| \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}s} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}s} \right|^2 + 0.11 \left| \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}p} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}p} \right|^2 \right] \quad \text{for } J = \tfrac{1}{2} \tag{57.a}$$

$$= 12\pi k C_1 \left[0.33 \left| \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}p} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}p} \right|^2 \right] \quad \text{for } J = \tfrac{3}{2} \tag{57.b}$$

$$w({}^3H_\Lambda \to D + p + \pi^-) = 16\pi p_\Lambda C_2 \left[0.13 \left| \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}s} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}s} \right|^2 + 0.014\, \mathrm{Re} \left(\sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}s} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}s} \right)^* \left(\sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}p} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}p} \right) + 0.05 \left| \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}p} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}p} \right|^2 \right] \quad \text{for } J = \tfrac{1}{2} \tag{58.a}$$

$$= 16\pi p_\Lambda C_2\ 0.27 \left| \sqrt{\tfrac{2}{3}}\, a_{\frac{1}{2}p} + \sqrt{\tfrac{1}{6}}\, a_{\frac{3}{2}p} \right|^2 \quad \text{for } J = \tfrac{3}{2}. \tag{58.b}$$

It must be remarked that the asymmetry term with respect to pion-nucleon correlation does not vanish even for the total decay probability; we obtain from (57) and (58) the relations

$$\frac{w({}^3H_\Lambda \to {}^3He + \pi^-)}{w({}^3H_\Lambda \to D + p + \pi^-)} \approx \frac{(11 + 4.6\, x^2)\, C_1}{(6.5 + 0.7\, \mathrm{Re}\, x + 2.5\, x^2)\, C_2} \quad \text{for } J = \tfrac{1}{2} \tag{59.a}$$

$$\approx \frac{C_1}{C_2} \quad \text{for } J = \tfrac{3}{2}. \tag{59.b}$$

where

$$x = \frac{\sqrt{2/3}\, a_{\frac{1}{2}p} + \sqrt{1/6}\, a_{\frac{3}{2}p}}{\sqrt{2/3}\, a_{\frac{1}{2}s} + \sqrt{1/6}\, a_{\frac{3}{2}s}} \tag{60}$$

If we use C_1/C_2 obtained from the $^4H_\Lambda$ decay we have $\simeq$ 1 and 1/3 for (59.a) and (59.b) respectively. Experimental data for (59) is 4/8 = .5 thus we cannot give a definite conclusion from this investigation but the ratio 1 and 1/3 is big enough for a final decision (compare also our previous report[7]).

The decay rate of $^5He_\Lambda \rightarrow {}^4He + p + \pi^-$ through the ground state $p_{3/2}$ and the first excited state $p_{1/2}$ of ^5He is given by

$$w/w^* = (17 + 55x^2)/(20 - 6x^2) \tag{61}$$

where w and w^* are the relative probability of the three-body mesic decay through the ground and the first excited states respectively. The experimental rate of (67) is $\simeq$ 54/18=3/1[15] which gives $x^2 \simeq 0.6$ and $p^2/(p^2 + s^2) \simeq 0.4$. This is consistent with the value obtained from the free Λ decay, 0.2 - 0.8[5]. If we accept the value 0.4 Dalitz and Liu's calculation for the ratio of two-body decays to all the mesic decays for $^4H_\Lambda$ definitely gives the spin 0 of the $^4H_\Lambda$ and hence spin zero for $^4He_\Lambda$ by the charge independence of N-Λ forces. Thus we are again led to the conclusion that the kaon parity is odd from the observed

$$K^- + {}^4He \longrightarrow {}^4He_\Lambda + \pi^- \tag{62}$$

reaction.[12]

7. Final State Interaction in Two-body Mesic Decay

Assuming the weak time reversal invariance[16] for the decay of Λ hyperon[17] we can express the matrix element of reaction matrix of hyperfragment decay in terms of the absolute value of the matrix element times an appropriate phase factor from the pion-nucleus scattering[18]. Expanding the amplitude corresponding to the two-body mesic decay in partial waves (as we did in our formulation) we can write the partial amplitude in the form

$$\pm |R_T(J,L)| \exp i(\delta_T(J,L)) \tag{63}$$

where $R_T(J,L)$ is the absolute value of the particle amplitude, $\delta_T(J,L)$ is the pion-nucleus scattering phase shift, T, J and L are the total isotopic spin in the final state, total angular momentum and the relative orbit of the pion and the residual nucleus.

It is especially interesting to see the effect to the isotopic spin selection rule. As an example let us take ${}^3H_\Lambda$ decay; we have the following relation from (46.a) and (47):

$$\frac{w({}^3H_\Lambda \to {}^3He + \pi^-)}{w({}^3H_\Lambda \to {}^3H + \pi^0)} = \frac{2|e^{i\delta_1'} + 2e^{i\delta_3'}|^2 + \frac{2}{3}y^2|e^{i\delta_{11}'} + 2e^{i\delta_{31}'}|^2}{|e^{i\delta_1'} - 4e^{i\delta_3'}|^2 + \frac{1}{3}y^2|e^{i\delta_{11}'} - 4e^{i\delta_{31}'}|^2} \tag{64}$$

where $\delta_1', \delta_3', \delta_{11}', \delta_{31}'$ are the pion-${}^3He$ scattering s-wave, p-wave phase shifts in isotopic spins 1/2, 3/2 respectively and $y = |B_1/A_1|$. We have no abailable data for pion-nucleus phase-shifts at about 40 Mev. If we assume that they are about equal to the mass number of the residual nucleus times the pion-single nucleon phase shift[19] we obtain an appreciable effect to the ratio (64). Using the experimental values of pion-nucleon phase shifts[19] $\delta_1 = 8^\circ$, $\delta_3 = -4^\circ$, $\delta_{11} \simeq 0$ and $\delta_{31} \simeq 0$ the ratio of the s-wave contribution changes from 2/1 to 5/4 which is an appreciable change. Thus we conclude that the pion-nucleus scattering phase shifts at about 40 Mev are very important for the determination of the isotopic spin selection rule from the two-body mesic decay of hyperfragments.

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Appendix 1. Integrals for the Three-body Mesic Decay Probabilities

The integrals appearing for the three-body mesic decay probabilities are given by

$$A = \int d\underline{Q}\, d\underline{p}\, d\underline{k}\ \delta(\underline{Q}+\underline{p}+\underline{k})\ \delta(E - Q^2/2m_1 - p^2/2m_2 - k^2/2m_3)\cdot \tag{A.1}$$
$$\cdot F(p, k, \cos\theta),$$

where $\underline{Q}$, $\underline{p}$, $\underline{k}$ and m_1, m_2, m_3 are momentums and masses of the residual nucleus, the proton and the pion respectively. Θ is an angle between the momentums of proton and the pion. The scalar function $F(p,k,\cos\Theta)$ takes 1, k^2p^2, $k^2p^2\cos\Theta$, $k\cos\Theta$, $k^3p^2\cos\Theta$, k^2, k^4p^2, $k^2\cos\Theta$ and $k^4p^2\cos^2\Theta$ for the cases in which we are interested. They come out from the spherical wave expansion of the pion wave function. The argument of the Bessel functions appearing in the expansion is small so that we may take the lowest terms in an expansion with respect to their arguement. After performing an integration with respect to $\underline{Q}$ we obtain,

$$A = 2(2\pi)^2 m_1 \int p\,dp\ k\,dk\ d(\cos\theta)$$
$$\delta\left(\cos\theta - m_1E/pk + (m_1+m_2)p/2m_2k + (m_1+m_3)k/2m_3p\right)\cdot \tag{A.2}$$
$$\cdot F(p, k, \cos\theta).$$

The upper and the lower limit of the integral will be determined by the delta function given by (1). To find out the limit of p and k in Eq. (2) we proceed as follows

$$p^2/m_2 + k^2/m_3 + (\underline{p}+\underline{k})^2/m_1 = 2E \tag{A.3}$$

$$k^2(1/m_1 + 1/m_3) + 2kp\cos\theta/m_1 + p^2(1/m_1 + 1/m_2) - 2E = 0 \tag{A.4}$$

Hence

$$k = \frac{-p\cos\theta/m_1 \pm \sqrt{p^2\cos^2\theta/m_1^2 - (1/m_3 + 1/m_1)\left[(1/m_1 + 1/m_2)p^2 - 2E\right]}}{1/m_1 + 1/m_3}$$

$$= \frac{-p\cos\theta \pm \sqrt{2(1+m_1/m_3)Em_1 + p^2\left(\cos^2\theta - (1+m_1/m_2)(1+m_1/m_3)\right)}}{1+m_1/m_3} \tag{A.5}$$

Since k is always real,

$$p^2\left\{(1+m_1/m_2)(1+m_1/m_3) - \cos^2\theta\right\} \leq 2m_1E\,(1+m_1/m_3)$$

or

$$p \leq \sqrt{2m_1E\left(1+m_1/m_3\right)/\left[(1+m_1/m_2)(1+m_1/m_3) - \cos^2\theta\right]} \tag{A.6}$$

To find the maximum value of k for a fixed value of p we vary cos θ in(4) or (5) and maximize k if the value cosθ so obtained is inside the range $-1 \leq \cos\theta \leq 1$ otherwise it is on the boundary $\cos\theta = \pm 1$. Differentiating (5) with respect to cosθ for fixed p we get $\partial k/\partial(\cos\theta)$ to zero for extreme we find either p = 0; or,

$$\sqrt{2m_1E\,(1+m_1/m_3) + p^2\left[\cos^2\theta - (1+m_1/m_2)(1+m_1/m_3)\right]} = p\cos\theta$$

or

$$2m_1E\,(1+m_1/m_3) + p^2\cos^2\theta - p^2(1+m_1/m_2)(1+m_1/m_3) = p^2\cos^2\theta$$

or

$$p^2 = 2m_1E/(1+m_1/m_2) = 2m_1m_2E/(m_1+m_2)$$

and arbitrary cos θ. But for this value of p^2

$$k = (-p\cos\theta \pm p\cos\theta)/(1+m_1/m_3) = 0 \qquad \text{since} \quad k \geq 0.$$

Hence an extremum is for k = 0 or p = 0 and arbitrary cos θ. In all other cases we have to choose the boundary values cos θ = ± 1.

Thus

$$k_{min} \leq k \leq k_{max}$$

with

$$k_{\substack{min\\max}} = \frac{\mp p + \sqrt{2m_1E(1+m_1/m_3) - p^2\left[(1+m_1/m_2)(1+m_1/m_3)-1\right]}}{1+m_1/m_3} \tag{A.7}$$

and

$$0 \leq p \leq \sqrt{2m_1E\,(1+m_1/m_3)\Big/\left[(1+m_1/m_2)(1+m_1/m_3)-1\right]} \tag{A.8}$$

Using these values we obtain the integral A in the form

$F = 1$:

$$A_1 \equiv 2\,(2\pi)^2 m_1 \left(m_3/(m_1+m_3)\right)^2 I_1 \tag{A.9}$$

$F = k^2p^2$:

$$A_2 \equiv (4\pi)^2 m_1 \left(m_3/(m_1+m_3)\right)^4 \left[2m_1(m_1+m_3)E\, I_2/m_3 - \frac{m_1(m_1+m_2+m_3)-m_2m_3}{m_2m_3} I_3\right] \tag{A.10}$$

$F = k^2p^2\cos^2\theta$:

$$A_3 \equiv 2(2\pi)^2 m_1 \left(m_3/(m_1+m_3)\right)^2 \Big[\tfrac{4}{3}\, m_1m_3\, E\, I_2/(m_1+m_3) + \left\{(m_1+m_2)\,m_3\,(1-\beta)/m_2(m_1+m_3) + \tfrac{1}{6}\left(m_3/(m_1+m_3)\right)^2(3-10\beta+3\beta^2)\right\} I$$

(A.11)

$F = k\cos\theta$:

$$A_4 \equiv -(4\pi)^2\, m_1 \left(m_3/(m_1+m_3) \right)^3 J_2 \qquad (A.12)$$

$F = k^3 p^2 \cos\theta$:

$$A_5 \equiv 2(2\pi)^2\, m_1 \left(m_3/(m_1+m_3) \right)^4 \Big[\alpha/m_2(m_1+m_3) \left\{ m_2 m_3 (3\beta-1) - (m_1+m_2)(m_1+m_3) \right\} J_3 + \left\{ (\beta-1)(m_1+m_2)/m_2 - \tfrac{1}{3}\left(m_3/(m_1+m_3) \right)(3-10\beta+3\beta^2) \right\} J_4 \Big]$$

(A.13)

$F = k^2$:

$$A_6 \equiv (4\pi)^2\, m_1 \left(m_3/(m_1+m_3) \right)^4 \Big[2(m_1+m_3)\, m_1 E\, I_1/m_3 - \frac{m_1(m_1+m_2+m_3) - m_2 m_3}{m_2 m_3}$$

(A.14)

$F = k^4 p^2$:

$$A_7 \equiv 2(2\pi)^2/3\;\, m_1 \left(m_3/(m_1+m_3) \right)^6 \Big[(6-20\beta+6\beta^2)\, I_4 + \alpha(20-12\beta)\, I_3 + 6\beta^2 I_2 \Big]$$

(A.15)

$F = k^2\cos^2\theta$:

$$A_8 \equiv \tfrac{4}{3}(2\pi)^2\, m_1 \left(m_3/(m_1+m_3) \right)^3 \Big[2 m_1 E\, I_1 + m_3/(m_1+m_3) \cdot \left\{ 3 - m_1(m_2+m_3)/m_2 m_3 - m_1^2/m_2 m_3 \right\} I_2 \Big]$$

(A.16)

$F = k^4 p^2 \cos^2\theta$:

$$A_9 \equiv 2(2\pi)^2\, m_1 \left(m_3/(m_1+m_3) \right)^4 \Big[\tfrac{17}{3} m_1^2 E^2 I_2 - \tfrac{2}{3}\, m_1 m_3/(m_1+m_3)\, E\,(16-7 + \tfrac{2}{3}\left(m_3/(m_1+m_3) \right)^2 (3-8\beta+\beta^2)\, I_4 \Big],$$

(A.17)

where I_n and J_n are defined by

$$I_n = \int_0^{p_{max}} x^{2n}\,dx\,\sqrt{\alpha - \beta x^2} \qquad n = 0, 1, 2, \ldots \tag{A.18}$$

and

$$J_n = \int_0^{p_{max}} x^{2n-1}\,dx\,\sqrt{\alpha - \beta x^2} \qquad n = 1, 2, \ldots\ , \tag{A.19}$$

where α and β are given by

$$\alpha = 2m_1(m_1+m_3)E/m_3 \quad \text{and} \quad \beta = m_1(m_1+m_2+m_3)/m_2 m_3 .$$

These integrals become simple if we use the fact that the pion mass is small compared to the mass of the hyperfragment. They are given by

$$I_0 \approx 2\left((A-1)/A\right)^{\frac{1}{2}}\left((m_N+m_\pi)/m_\pi\right)^{\frac{1}{2}} m_N E \left\{\left(\frac{1}{A(A-1)}\right)^{\frac{1}{2}}\left(\frac{m_\pi}{m_N}\right)^{\frac{1}{2}}\frac{1}{2} + \frac{\pi}{2}\right\}$$

$$I_1 \approx 2^2\left((A-1)/A\right)^{\frac{3}{2}}\left((m_N+m_\pi)/m_\pi\right)^{\frac{1}{2}} m_N^2 E^2 \left\{\left(1/A(A-1)\right)^{\frac{1}{2}}\left(\frac{m_\pi}{m_N}\right)^{\frac{1}{2}}\frac{1}{12} + \frac{\pi}{12}\right\}$$

$$I_2 \approx 2^2\left((A-1)/A\right)^{\frac{5}{2}}\left((m_N+m_\pi)/m_\pi\right)^{\frac{1}{2}} m_N^3 E^3 \left\{\left(1/A(A-1)\right)^{\frac{1}{2}}\left(\frac{m_\pi}{m_N}\right)^{\frac{1}{2}}\frac{1}{12} + \frac{\pi}{24}\right\}$$

$$I_3 \approx 2^3\left((A-1)/A\right)^{\frac{7}{2}}\left((m_N+m_\pi)/m_\pi\right)^{\frac{1}{2}} m_N^4 E^4 \left\{\left(1/A(A-1)\right)^{\frac{1}{2}}\left(\frac{m_\pi}{m_N}\right)^{\frac{1}{2}}\frac{1}{13} + \frac{\pi}{30}\right\}$$

$$I_4 \approx 2^4\left((A-1)/A\right)^{\frac{9}{2}}\left((m_N+m_\pi)/m_\pi\right)^{\frac{1}{2}} m_N^5 E^5 \left\{\left(1/A(A-1)\right)^{\frac{1}{2}}\left(\frac{m_\pi}{m_N}\right)^{\frac{1}{2}}\frac{1}{26} + \frac{\pi}{33}\right\},$$

where A is the mass number of the hyperfragment.

$$J_1 \approx \tfrac{1}{3} \left(\alpha/\beta \right) \alpha^{\frac{1}{2}}$$

$$J_2 \approx \tfrac{2}{15} \left(\alpha/\beta \right)^2 \alpha^{\frac{1}{2}}$$

$$J_3 \approx \tfrac{8}{105} \left(\alpha/\beta \right)^3 \alpha^{\frac{1}{2}}$$

$$J_4 \approx \tfrac{16}{315} \left(\alpha/\beta \right)^4 \alpha^{\frac{1}{2}}$$

Appendix 2. Spin of $p_{3/2}$-shell Hyperfragments

In the previous papers[1-3] we pointed out that the binding energies of the $p_{3/2}$-shell hyperfragments are expalined equally well by the parameters derived for both antiparallel and parallel favoured spins. We want to point out here that from a comparison of the parameters of s-shell and p-shell hyperfragments with no extremely strong spin-orbit interaction can give the antiparallel favoured spins for $p_{3/2}$-shell hyperfragments. As is well known the effective nucleon-nucleon forces in the nucleus give very small expectation value of tensor interaction which is effectively almost zero[20]. If we accept this concept for the effective N-Λ forces we can determine the parameter for the spin-orbit interaction of N-Λ forces from the realtions derived before. The actual potential is defined in the form

$$V_{N\Lambda} = \left(1 + a\, \sigma_N \cdot \sigma_\Lambda + b\, S_{N\Lambda} + c\, \underline{L} \cdot \underline{S} \right) V(r) \qquad \text{(A.20)}$$

where a $\simeq$ -.24. Using the relations (38) and (39) in paper I and b $\simeq$ 0 we have c $\simeq$ -.22 and 3.8 for the antiparallel and parallel spin respectively. Thus for a not too strong spin-orbit force we have the antiparallel spins of $p_{3/2}$-shell hyperfragments.

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