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by

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It is well known that there is a great difficulty in explaining the lack of the radiative muon decay in a theory with an intermediate charged vector meson. Such a theory was discussed by many authors(1). Experiments provided the upper limit for the ratio of this decay to the usual muon decay(2). $\rho = \frac{R(\mu \to e+\gamma)}{R(\mu \to e+\nu+\bar{\nu})} < 1 2 \; 10^{-6}$

This problem was examined by several authors(3). The usual theory with an intermediate meson is nonrenormalizable and therefore they had to introduce a cut-off Λ (4) in order to compute ρ even in the lowest order of perturbation theory. The result is strongly dependent on Λ. If the vector meson has no anomalous magnetic moment one must take Λ equal approximately 0,2 of the vector meson mass to obtain ρ in agreement with experiment. One can obtain a reasonable value of Λ if the anomalous magnetic moment 0,75 is given to the vector meson.

The purpose of this note is to test another possibility of introducing the charged vector meson to the theory of weak interactions. The following form of free Lagrangian for the meson ϕ_μ is assumed

$$\mathcal{L}_0 = -\partial^\mu \phi^+_\nu \partial_\mu \phi^\nu + M^2 \phi^+_\mu \phi^\mu \tag{1}$$

which leads to the meson propagator in the form

$$\langle 0| T \phi_\mu(x) \phi^+_\nu(y) |0\rangle = i g_{\mu\nu} \Delta_F(x-y) \tag{2}$$

instead of the usually adopted form $i(g_{\mu\nu} - M^{-2}\partial_\mu \partial_\nu)\Delta_F(x-y)$ The choice of such a Lagragian makes the theory of weak interactions full renormalizable, introducing however an indefinite metric into the Hilbert space. As long as there are no vector mesons in the initial and final states our scheme gives unique and finite transition amplitudes for every weak process. To deal with the vector mesons appearing in the final or initial states this scheme must be supplemented by some methods of handling states with negative norm as for example the one proposed by E. C. G. Sudarshan(5).

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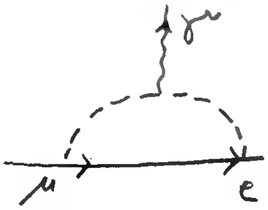
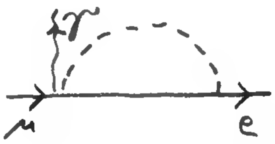
From the Lagragian(1) one may derive the electromagnetic interaction of the meson ϕ_μ by assuming the "minimal" electromagnetic interaction. This procedure does not give however the interaction of the meson magnetic moment and the corresponding interaction term must be added separately. The electromagnetic Lagrangian for the vector meson has finally the form

$$\mathcal{L}_{em}^{vm} = ie\phi_\nu^+ \overset{\leftrightarrow}{\partial}_\mu \phi^\nu A^\mu - e^2 A^\mu A_\mu \phi_\nu^+ \phi^\nu + ie(1+\lambda)\phi_\nu^+ \phi_\mu F^{\nu\mu} \tag{3}$$

where λ is an anomalous magnetic moment in the units of the vector meson magnetor. The inclusion of this interaction does not spoil the renormalizability of our theory.

The probability for the process $\mu \to e + \gamma$ is computed from the following diagrams.





The Lagrangian giving contribution to this process has the form

$$\mathcal{L} = M\sqrt{G}\,\bar{\Psi}_{(e)}\gamma_\lambda(1-i\gamma_5)\Psi_{(\nu)}\phi^\lambda + M\sqrt{G}\,\bar{\Psi}_{(\nu)}\gamma_\lambda(1-i\gamma_5)\Psi_{(\mu)}\phi^{\lambda +} + \mathcal{L}_{em} \tag{4}$$

The divergent contributions cancel in the sum as a result of gauge invariance of the theory. The transition amplitude was computed in the approximation in which (m_e/m_μ) and $(m_\mu/M)^2$ were neglected in comparison with unity. The result is

$$S_{i\to f} = \delta_4(p-q-k)(1+3\lambda)C\,\bar{u}_{(e)}(\bar{q})(1+i\gamma_5)\gamma_{\mu\nu}k^\mu e^\nu(\bar{k})(1+i\gamma_5)u_{(\mu)}(\bar{p}) \tag{5}$$

where $u_{(e)}(\bar{q})$ and $u_{(\mu)}(\bar{p})$ are the wave functions of electron and mmon with momenta $\bar{q}$ and $\bar{p}$ respectively, $e^\nu(\bar{k})$ describes the polarization of photon, C is a constant depending on e, 0, m_μ and m_e only. This amplitude does not vanish unless we put $\lambda = -1/3$. The

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value of ρ obtained for $\lambda = 0$ is 10^3 times larger than the experimental upper limit.

No interactions are known which could produce a large anomalous magnetic moment of the vector meson and therefore our result should be regarded as a negative one. There are two ways of saving the above proposed theory of the weak interactions: to assume the existance of two different kinds of neutrinos or to consider the magnetic moment $Z + \lambda = 2/3$ to be the normal one for this kind of meson.

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FOOTNOTES

* On leave of absence from the Institute of Physics, Polish Academy of Sciences, Warsaw, Poland.

(1) J. Schwinger, Ann. Phys. 2, 407 (1957)

R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958)

E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. 109, 1860 (1958)

(2) The lowest limit was obtained by S. Frankel, V. Hagopian, J. Halpern and A. L. Whetstone, Phys. Rev. 118, 589 (1960)

(3) C. Feinberg, Phys. Rev. 110, 1482 (1958)

Ph. Meyer and G. Salzman, Nuovo Cimento 14, 1310 (1959)

M. E. Ebel and F. J. Ernst, Nuovo Cimento 15, 173 (1960)

(4) The result is of course dependent on the form of the cut-off factor. In all papers mentioned in (3) it was chosen to be Λ^2/Λ^2+p^2

(5) E. C. G. Sudarshan preprint

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