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SU(3) SYMMETRY IN LEPTONIC DECAYS OF PSEUDOSCALAR MESONS

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Talk presented by J. Werle

at the International Symposium on Mathematical Physics,

January 5 - 8, 1976 Mexico City

* Work supported in part by the National Science Foundation Contract GF - 42060 and the ERDA Contract No. AT(40-1)3992.

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This is a very short report on the present status of the work performed under a collaboration program between CPT and IFP. Since we have obtained not only some new results which were not contained in the former paper by A. Bohm and J. Werle [1)], but also some changes in the assumptions of the model were introduced recently, we shall start from a short general introduction and presentation of the main assumptions of our model. However, we shall skip all the more involved calculations.

The usual treatment of the SU(3) symmetry of the matrix elements of the hadronic currents is full of mathematical and physical inconsistencies. Eg., one assumes that the initial and final momentum eigenvectors describing hadrons, which belong to the same SU(3) multiplet, transform irreducibly under SU(3). Assuming, furthermore, that the hadronic currents belong to the same SU(3) octet, one makes use of the Wigner-Eckart theorem in order to obtain the most convenient parametrization of the decay amplitudes and definite relations between different decays. Such a procedure is certainly not correct because of large mass splitting within the SU(3) multiplets. Another possible source of error is the breaking

of SU(3) covariance by the current operators themselves. The latter effect is usually taken into account by certain additional phenomenological suppression factors which are expressed by the Cabibbo angles θ_V and θ_A.

The aim of our work was to show that at least in the case of leptonic decays of pseudoscalar mesons, one can remove the mentioned inconsistencies and obtain several interesting theoretical predictions which at the moment are in good agreement with experiment even without introducing any Cabibbo angles.

The much more difficult problem of baryonic decays is still being investigated by A. Bohm, A. Garcia and B. Teese. The report on the present status of this problem has been just presented by B. Teese.

The basic assumptions of our model are:

1. Velocity eigenvectors corresponding to single π and K mesons transform irreducibly both with respect to the Poincaré group $\mathcal{P}$ and to SU(3). In other words we assume that although the mass operator M and the momentum operators P_μ don't commute with the generators E^b of SU(3), the velocity operators $\hat{P}_\mu = {}^{P_\mu}/_{M}$ and the generators $L_{\mu\nu}$ of the proper Lorentz group commute with all E^b $(b = 1,\ldots,8)$ when acting on one-hadron states, i.e.

$$\left[\hat{P}_\mu, E^b\right] |\hat{p}\lambda;\alpha a> = \left[L_{\mu\nu}, E^b\right] |\hat{p}\lambda;\alpha a> = 0 \, . \qquad (1)$$

Here α is the SU(3) representation (or better: multiplet classifying) index and $a = (II_3Y)$ is the corresponding SU(3) vector index. It follows from (1) that the velocity eigenvectors can be written in the form of a direct product

$$|\hat{p}\lambda;\alpha a> = |\hat{p}\lambda>|\alpha a> \, . \qquad (2)$$

They are very simply related to the momentum eigenvectors

$$|\hat{p}\lambda;\alpha a> = m(\alpha,a)\ |p\lambda;\alpha a> \, . \qquad (3)$$

The multiplicative mass factor is, however, relevant for symmetry considerations as it is not SU(3) invariant.

II. The SU(3) breaking of the physical mesonic vector and axial vector currents V_ρ^b and A_ρ^b is expressed by the assumptions that the anticommutators

$$\{V_\rho^b, M^{-1}\} = \hat{V}_\rho^b \quad , \quad \{A_\rho^b, M^{-1}\} = \hat{A}_\rho^b \qquad (4)$$

are strict octet operators. Using (3) one can express the physical SU(3) symmetry breaking matrix elements of the respective currents in terms of well behaving matrix elements of $\hat{V}$ and $\hat{A}$ between velocity eigenstates.

$$<p';\alpha c|V_\rho^b|p;\alpha a> = (m_a+m_c)^{-1} <\hat{p}';\alpha c|\hat{V}_\rho^b|\hat{p};\alpha a>$$
$$<p';\alpha c|A_\rho^b|p;\alpha a> = (m_a+m_c)^{-1} <\hat{p}';\alpha c|\hat{A}_\rho^b|\hat{p};\alpha a> \qquad (5)$$

III. CVC hypothesis. We assume that the vector octet of currents contains the electromagnetic current which is conserved. This assumption applied now to the well behaving matrix elements appearing on the right hand side of (5) implies vanishing of some form factors [1]. After simple calculations we obtain for the matrix elements of the vector currents which describe all $K_{\ell 3}$ and $\pi_{\ell 3}$ decays the following simple expression

$$<p';c|V_\rho^b|pa> = C(abc)\ (2m_a m_c)^{-1}$$
$$\left[(p+p')_\rho - \frac{m_a-m_c}{m_a+m_c}\,(p-p')_\rho\right] F(\hat{q}^2) \qquad (6)$$

Here C(abc) denotes suitable Clebsch-Gordan coefficient. The SU(3) invariant form factor $F(\hat{q}^2)$ which is thus the same for all the mentioned decays is an unknown function of the squared four-velocity transfer $\hat{q}^2 = \hat{q}_\mu \hat{q}^\mu$, $\hat{q}_\mu = \hat{p}'_\mu - \hat{p}_\mu$. Comparing our expression (6) with the conventional form expressed in terms of the momentum dependent form factors f_+ and f_- we find

$$f_+(q^2) = (2m_a m_c)^{-1}\ F(\hat{q}^2)$$
$$f_-(q^2) = (2m_a m_c)^{-1}\ \frac{m_c-m_a}{m_c+m_a}\ F(\hat{q}^2) \qquad (7)$$

where $q_\mu = p'_\mu - p_\mu$.

For the matrix elements of the axial currents between one meson states $|p;a\rangle$ and no-hadron state $|0\rangle$, which describe all the $K_{\ell 2}$ and $\pi_{\ell 2}$ decays, one gets from (5)

$$\langle 0|A^b_\rho|pa\rangle = \varkappa m_a^{-1}\, C(ab0)\, p_\rho \qquad (8)$$

Here $\varkappa$ is an SU(3) invariant constant, the same for all the mentioned leptonic decays.

The common (in some sense universal) form factor $F(\tilde{q}^2)$ can be approximated by a function linear in $\tilde{q}^2$ which will be also linear in q^2. After a simple calculation one finds

$$f_+(q^2) = (2m_a m_c)^{-1}\, F(\tilde{q}^2) = (2m_a m_c)^{-1}\,(1+b\tilde{q}^2) = \\ = f_+(0)\,(1+\lambda_+\, q^2/m_\pi^2\,) \qquad (9)$$

where

$$f_+(0) = (2m_a m_c)^{-1}\left[1-b\,\frac{(m_a-m_c)^2}{m_a m_c}\right] , \\ \lambda_+ = \frac{m_\pi^2}{m_a m_c}\quad \frac{b}{1-b\dfrac{(m_a-m_c)^2}{m_a m_c}} \qquad (10)$$

The coefficient b is not fixed by our model. We can use expression (10) for determining its value from the value of λ_+ taken for some particular $K_{\ell 3}$ decay. Taking the recent

experimental value $\lambda_+ = 0.0285 \pm 0.0093$ obtained for $K^+_{\ell 3}$ decays we find $b = 0.088 \pm 0.013$. Taking this value of b as the only experimental input parameter we obtain the following results for the most important experimentally verifiable parameters expressed in terms of masses of mesons:

Our Model General expression;	Value	Experimental Value
$s_{\ell 2} = m_\pi / m_k$	0.283	0.275
$s_{\ell 3} = \frac{m_\pi}{m_k}\left(1-b\frac{(m_k-m_\pi)^2}{m_k m_\pi}\right)$	0.235	0.224
$\xi = f_- / f_+ = \frac{m_\pi - m_k}{m_\pi + m_k}$	-0.57	-0.57 (?)
$\zeta = \frac{\Gamma(k_{\mu 3})}{\Gamma(k_{\ell 3})}$	0.615	0.664
$\lambda_- = \lambda_+$	0.0285	(?)

If the input experimental value of λ_+ increases, the agreement between experiment and predictions of our model becomes even better both in the case of the suppression factor $s_{\ell 3}$ and the branching ratio ζ. It is worthwhile stressing that if the form factor F = constant, then $s_{\ell 2} = s_{\ell 3}$, which

corresponds in the Cabibbo theory to taking $\theta_V = \theta_A$.
In our model the two suppression factors are definite functions of masses and the q^2 dependence of the form factor $F(q^2)$ is the only source of the difference between $s_{\ell 3}$ and $s_{\ell 2}$. It is to be stressed that our model predicts that f_- is proportional to f_+, and thus the parameter ξ must be a q^2 independent constant expressed in a definite manner by the initial and final meson masses. Although one of the recent published experimental results obtained by Merlan et. al. [2] gives exactly the value of ξ predicted by our model, the experimental situation is still rather confused because other groups obtained other values [3]. Practically nothing is known about the experimental value of λ_-. Our prediction is $\lambda_- = \lambda_+$.

It seems that our simple model works quite well in the case of leptonic decays of pseudoscalar mesons. Without introducing any Cabibbo angles we have obtained good results for the suppression factors and the branching ratio ζ. The model gives also some other definite predictions which should be soon verified by experiment.

References

[1.] A. Bohm, J. Werle, SU(3) Symmetry in Leptonic Decays of Pseudoscalar Mesons (sent for publication).

[2.] S. Merlan et. al. PRL 30 (1973) 102.

[3.] V. Chaloupka et. al., Review of Particle Properties, P.L. 50B (1974).