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COUPLED EQUATIONS METHOD FOR PION SCATTERING FROM THE TWO-NUCLEON SYSTEM

by

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This Technical Report contains the text of the doctoral dissertation described on the following page, with the exception of Appendices A-E, pp. 180-230, which have been omitted. Appendices F-H have been retained.

COUPLED EQUATIONS METHOD FOR PION SCATTERING FROM THE TWO-NUCLEON SYSTEM

by

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VITAE

The author was [redacted] On completion of his high school education he attended the University of Tokyo where he received B.Sc. in March 1968 and M.Sc. in March 1970, both in Astronomy.

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ABSTRACT

An attempt is made to clarify the structure of pion-nucleus scattering taking into account the fact that the pions in scattering and in nuclear forces are identical. To understand the essential points, a simple pion plus two-nucleon system is chosen. The processes considered are $\pi NN \to \pi NN$ and $\pi NN \leftrightarrow NN$. Both relativistic and non-relativistic approaches are adopted. In the relativistic approach Taylor's method is used, whereas in the non-relativistic approach a Hamiltonian (Schrödinger equation) method is utilized together with a projection technique. Both approaches give a finite set of amplitude equations respectively, and the formal correspondence between these sets of equations is observed. Due to the proper consideration of pions, the problem of pion overcounting does not occur and the amplitudes are shown to satisfy at least two- and three-particle unitarity.

Through the study of the scattering in the πNN system, several important aspects common in general pion-nucleus scattering have been observed, which would not be possible through commonly used approaches in the study of pi-nucleus interactions. For instance, this observation gives some new insight into the proper structure of pion-nucleus optical potentials.

For practical applications the non-relativistic πNN amplitudes are reduced in angular momentum-isospin eigen-states together with the inclusion of bound states.

The reduced amplitudes are then used to study the pion absorption effect on the pion-deuteron scattering length. The result is consistent with experiment. Some possible future applications of the formulations developed in the thesis are discussed.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Once my study was nuclear physica

Dalton's atomisms and other such mystica

But from such nonsense or every particular

I've now struck a course perfectly perpendicular

Pioneering now in the sublively ridicula

The uncharted lands of un-clear physica

A distorted version of

"Lines on the Islands of Utopia"

by Bob Miola

CHAPTER I

INTRODUCTION

A. Pion-nucleus Interactions

There has recently been a growing interest in the study of pion-nucleus interactions. This is due to the fact that (i) pions are quite useful candidates in probing nuclear structure including basic nucleon-nucleon interactions and, (ii) on the practical side, high intensity pion beams are now available at several "meson factories", which is of primary importance for a better understanding of pi-nucleus phenomena.

There are several other particles useful for probing nuclear structure; the electron, photon, nucleon, muon, etc. Each particle can claim its own merit as a nuclear probe, and pion's usefulness in the study of nuclear structure can be understood through its physical properties[(1)] including the interaction with the nucleon. For example, the pion interacts strongly with nucleons, but it also interacts with them electromagnetically. So negative pions can form pi-mesic atoms whose Bohr orbits are about 1/250 of those in the usual "electronic" atoms due to the mass ratio of pions and electrons ($m_\pi/m_e \sim 250$).

Those orbital pions then can see the hadronic surface structure of the nucleus through strong interactions, which is impossible for the leptons. Since pions have charge states; +, 0, -, it is possible (1) in principle to separate the electromagnetic interaction from the strong interaction in pi-nucleus scattering to see the pure strong interaction effect and (2) to have pi-nucleus double-charge exchange scattering to observe some exotic nuclear states. Another important feature is that through the basic coupling, $N \leftrightarrow N+\pi$, pions can be produced or absorbed by the nucleus. Since the pion absorption (or production) is accompanied by at least ~140 MeV of energy transfer to or from the nucleus, it may be useful to obtain some information about short range correlations of the nucleons inside the nucleus (as well as basic N-N interactions). There are several other nice features with pi-nucleus interactions which make pions attractive nuclear probes. However, we shall not discuss them here. For details, see several articles.[2-4]

B. Our Studies in Pi-nucleus Interactions

It is well known that the long range part of the N-N interaction is mediated by the one-pion-exchange (OPE) mechanism which comes from the basic $N \leftrightarrow N+\pi$ process mentioned in the last section. (Multi-pion exchanges are expected to contribute to the shorter range part of the N-N interaction.) Therefore, when we consider the scattering of a pion by the nucleus, there should be no difference, in principle, between scattered and exchanged pions in the nucleus.

There are several commonly used methods for describing pi-nucleus scattering. Both Watson's[5] and Glauber's[6] methods belong to the multiple scattering theory. Essentially the description in those methods is in terms of two-body π-N and N-N t-matrices (or potentials) which are assumed to be given from outside. Therefore, although they could include nuclear dynamics in a proper way, an equivalent treatment of the scattered and exchanged pions is not possible.

On the other hand, the use of dispersion relations[7] and of current algebra techniques[8] seem to take a correct account of the pions because they do not begin with two-body inputs. However, the inclusion of proper nuclear dynamics is awfully difficult; even practically impossible, in these methods.

In this thesis, we make an attempt to take a proper account of the pions in pi-nucleus scattering, i.e., the pions in scattering and in N-N interactions are considered on the same basis.

To understand the point easily and transparently, we have chosen the πNN scattering problem, including πNN→πNN and πNN↔NN processes. The methods adopted are (1) one due to Taylor[10,11] for our relativistic approach and (2) a non-relativistic Hamiltonian approach equipped with the projection technique of the type utilized, for example, by Feshbach.[23] These methods have allowed us to treat the scattered and exchanged pions in the nucleus equally, and we have obtained a finite set of amplitude equations. These equations may be coupled or uncoupled depending upon what are given to them as known input functions (or sub-amplitudes).

We then reduce the non-relativistic equations for the πNN amplitudes to the form useful for practical applications. With a suitable set of input functions, they are of the same degree of difficulty in numerical procedure as the non-relativistic Faddeev equations.

The reduced equations are applied with some approximations to obtain the contribution to the π-d scattering length from the intermediate pion absorption process in π-d elastic scattering. The calculation is new except for the recent one by Afnan and Thomas[28] using the Faddeev equation.

C. Outline of the Chapters

The organization of the chapters is as follows: In Chapter II, we describe our motivation more in detail by reviewing the methods frequently used in pi-nucleus scattering problems. Then several questions are raised as to the appropriateness of the potential description of the πN interaction in the multiple scattering method, how to treat pion production problems, etc. from the point of view of the proper account of pions. Some discussion is given for the problem of pion overcounting.

Chapter III is rather long but is the central part of our formal studies. We have chosen the πNN system as the simplest representative of pi-nucleus system to study the structure of the scattering amplitudes. First we adopt Taylor's method in our relativistic approach to the problem, and second a non-relativistic Hamiltonian approach is used where we make use of the pion number projection operators to obtain a set of coupled equations. Finally a set of t-matrix equations, describing the processes $\pi NN \to \pi NN$ and $\pi NN \leftrightarrow NN$, are obtained for both relativistic and non-relativistic methods, which for given suitable input functions are effectively decoupled. In connection with this chapter we give a proof of unitarity for our relativistic amplitudes in Appendix C.

In Chapter IV, we first make a comparison between the relativistic and non-relativistic πNN amplitudes that we have obtained in Chap. III. Formal one-to-one correspondence is observed. Then we answer the questions raised in Chap. II. Finally, from an observation of the structure of πNN amplitudes so far obtained, we make a plausible guess on the possible structure of general pi-nucleus amplitudes.

For some practical applications, the non-relativistic πNN amplitudes are put into a reduced form in Chapter V. First we make an isobar approximation to the two-body input t-matrices. Then the amplitudes are antisymmetrized with respect to two nucleons and are decomposed in angular momentum-isospin eigenstates.

Chapters VI, VII and VIII constitute the second part of the thesis, and are aimed at the application of our non-relativistic formulation of πNN scattering. For this purpose we have chosen to study the effect of intermediate pion absorption, in the elastic $\pi d \to \pi d$ process, on the pion-deuteron scattering length; $a_{\pi d}$. This effect is expected to be rather sensitive to the details of N-N interactions, but so far has only been studied a little. As the effect of pion multiple scattering seems less important there in comparison with that of pion absorption, the problem is well suited for the application of our formulation.

In Chapter VI we review the theory and experiment on the pi-deuteron scattering length; $a\pi d$, with a special emphasis on the effect of pion absorption contribution to it; $\Delta a\pi d$.

Chapter VII is aimed at describing the method of calculating $\Delta a\pi d$, and in Chapter VIII we summarize the result of the calculation.

Finally we draw some conclusions on our studies contained in this thesis, in Chapter IX. A selection of auxilliary topics, and some extensions and details of the subject matter of these chapters, are given in the appendices.

CHAPTER II

THE MOTIVATION OF OUR STUDY

A. Introduction

One day a friend of mine asked me "Why are you studying nuclear physics? It's useless because we don't know the basic structure of nucleon-nucleon interactions in field theory". Of course he was in theoretical particle physics. My answer was then, "If you actually show me how to determine the orbit of the moon around the earth, as accurately as possible by solving the Schrödinger equation, then I may stop doing nuclear physics."

We take it for granted that at least the physical world possesses a structure with several strata, and each of them looks more or less closed in itself. As may be known, classical mechanics can predict the position of the moon around the earth within a few meters in accuracy without any help of "Quantum Mechanics". Low energy nuclear physics is also self consistent, apart from possible electromagnetic and weak interactions involved, in that it can describe many low energy nuclear phenomena very beautifully without asking any help from particle physics, especially in the aspect of strong interaction.

Sometimes, however, the structure belonging to one stratum happens to appear explicitly in another. Superconductivity (or superfluidity) is a macroscopic phenomenom which, however, is totally governed by quantum microscopic principles without understanding them, nothing can be said to explain it.

Now we get into the problem of pion-nucleus scattering. Then what do we know about its features? A nucleus is a substance in which we assume some forces acting among its constituent nucleons to bind them together. By itself it does not seem that a nucleus has to have any other explicit degrees of freedom to be taken into account apart from electromagnetic property. But once we try to consider the interaction of pions with nuclei, we have to look at things more microscopically. We know that the pion can be absorbed and emitted by a nucleon in many-body systems. We also know that the long range part of the nucleon-nucleon force is carried by one pion exchange (OPE) and its shorter range part, possibly by multi-pion exchanges. Then if we hit a nucleus with a pion, how can we distinguish the scattered pion from those pions exchanged among the nucleons in the nucleus? This necessitates us to regard a nucleus not only as an ensemble of pion scatterers but also as an ensemble of sources and sinks of pions.

Of course it is practically impossible to solve pion-nucleus problems by taking "fundamentalist's attitude", to start with "basic" Lagrangian or Hamiltonian to generate everything; N-N force, π-N scattering etc. at the same time. But looking at problems in this manner may provide some new important aspects which would otherwise never appear. So we shall take an attitude which is a little closer to "fundamentalist's point of view" but not equal in order not to become sterile.

B. A Look at Some Conventional Approaches to Pion-nucleus Scattering

(i) Let us look at the most common method in elastic pi-nucleus scattering which also is used to construct pi-nucleus optical potentials for the applications to pi-production or absorption problems. It usually begins with a Hamiltonian of the form

$$H = K + \sum_{i>j} V_{NN}^{ij} + \sum V_{\pi N}^{i} , \quad (4) \qquad \text{(2B-1)}$$

where K; kinetic energy of the pion and nucleons

V_{NN}^{ij}; potential between i-th and j-th nucleons

$V_{\pi N}^{i}$; potential between the pion and i-th nucleon.

Then writing $H_o \equiv K + \sum_{i>j} V_{NN}^{ij}$, and with the t-matrix equation for the pion and i-th nucleon scattering under nuclear effect;

$$t_{\pi N}^{i} = V_{\pi N}^{i} + V_{\pi N}^{i} [E^{+} - H_0]^{-1} t_{\pi N}^{i} , \qquad \text{(2B-2)}$$

the Watson multiple scattering series[5] is derived which takes the form

$$T_{\pi\text{-nucleus}}^{(E)} = \sum_{i} t_{\pi N}^{i} + \sum_{i \neq j} t_{\pi N}^{i} [E^{+} - H_0]^{-1} t_{\pi N}^{j} + \cdots\cdots . \qquad \text{(2B-3)}$$

This series is approximated in practical calculations; for example, by keeping at most up to the second term, which allows one to include some nuclear dynamics.

Looking at (2B-1) we normally wonder whether we can claim the possible existence of pion-nucleon potentials or not. Or we can ask:

(1) Whether the Hamiltonian makes sence, or not, from field theoretical viewpoint?

(ii) We now turn to the problem of pion production and absorption by nucleus which has become very fashionable in recent years. The common recipe is to adopt the distorted wave method[9] used in direct nuclear reaction theory. According to that method, the initial- and final-state Hamiltonians are taken, in a pion production problem for example, as

$$H_I = K_0 + \sum_{i>j} V_{NN}^{ij} , \qquad \text{(2B-4)}$$

and

$$H_F = K + \sum_{i>j} V_{NN}^{ij} + \sum_i V_{\pi N}^{i} + \sum_i (h_i + h_i^\dagger), \qquad \text{(2B-4')}$$

where H_I ; initial state total Hamiltonian

H_F ; final state total Hamiltonian

K_o ; kinetic energy of nucleons

$h_i(h_i^+)$; pion absorption (emission) operator by i-th nucleon.

Other quantities are already used in (2B-1).

One then evaluates the t-matrix for the process using distorted wave Born approximation[9] (DWBA),

$$T_{fi} = \langle \psi_f^{(-)} | \sum_j h_j^\dagger | \phi_i^{(+)} \rangle, \qquad \text{(2B-5)}$$

where $\phi_i^{(+)}$; outgoing scattering eigenstate of H_I.

$\psi_f^{(-)}$; incoming scattering eigenstate of H in (2B-1) or of H_F without $\sum_i (h_i+h_i^+)$. This state corresponds to the pion distortion by the nucleus.

H_i seems reasonable because in the initial state only nucleons appear.(Strictly speaking, in pion production problem, the initial state should implicitly contain pion degrees of freedom. Otherwise there is no pion production.) On the other hand, the form of H_F is questionable. First of all, in DW approach H_I should be equal to H_F, which, however, cannot be shown without knowing an explicit pionic structure of H_I and especially or H_F. A careful look at H_F shows that it is inconsistent in its own structure; both V_{NN}^{ij} and $V_{\pi N}^{i}$ in H_F contain certain parts which can be generated from h_i and h_i^+. This inconsistency becomes more transparent when we try to obtain T_{fi} for pion production by going beyond DWBA. Then we find that H_F produces serious pion overcounting as well as nucleon self-energy which should be thought to have been renormalized. So we should ask the second question:

(2) What is the correct approach and expression to describe the problem of pion production (and absorption) by nucleus?

(iii) Let us notice the following fact; as pions can be absorbed and emitted by nucleons, there should exist, even in elastic pi-nucleus processes, some intermediate states with no explicit pion present. A process like this cannot be taken care of by a Hamiltonian of the type we have in (2B-1) even though we try to admit the existence of $V^i_{\pi N}$. Then we arrive at the third question:

(3) How to include the effect of intermediate pion absorption in elastic pi-nucleus scattering?

Connected with what is mentioned in (iii) is the fact that in addition to zero pion intermediate state there are various multi-pion intermediate states, for example, in elastic pi-nucleus amplitudes. Therefore every state with a definite number of pions should couple to others corresponding to definite but different numbers of pions in a unitary way. This means that we have a coupled set of equations with an infinite number of unknown amplitudes. So the question goes:

(4) Is there any appropriate method which eliminates all those amplitudes that are not of our direct concern, to get an effective set of equations?

C. Pion Overcounting and the Proper Structure of the Pion-nucleus Amplitudes

Before going to the next chapter for a detailed study of πNN scattering (as a simple model of general pi-nucleus scattering), let us have a brief look at the structure of pi-nucleus scattering in connection with the problem of pion overcounting. As in later chapters, we have chosen the πNN system for our study here. This provides us a basic insight into the proper structure of other general cases of pi-nucleus interactions.

Let us consider an elastic πNN scattering and pick up several processes in it where possible pion overcounting might creep in. Here we do not assume any specific Hamiltonian to begin with but simply look at the diagrammatic structure of the whole process.

(i) Consider a process shown in Fig. 2-1.

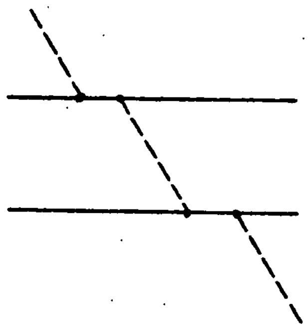


Fig. 2-1

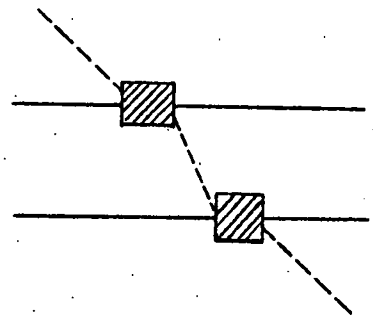


Fig. 2-2

As one may easily notice, this can be viewed in two different ways; (1) two successive direct Born scatterings of a pion by two nucleons, or (2) pion absorption by a nucleon → one pion exchange N-N interaction → pion emission by the other nucleon. Similar dual interpretation happens in other large classes of constituent process (or graphs).

(a) (b)

Fig. 2-3

When we take the viewpoint (1), then a graph like in Fig. 2-1 belongs to the one shown in Fig. 2-2 where hatched squares mean total πN t-matrices and hence we do not have any explicit state with two nucleons alone. However, we also have graphs like in Fig. 2-3 (the processes shown are supposed to go off-shell in general) and these processes force us to take the viewpoint (2). Of course, if we adopt both viewpoints at the same time, there appears an ambiguity in classifying the diagrams like the ones in Fig. 2-1, and 2-3(b), which may be considered to belong to the diagrams in Fig. 2-4. There in the figure, the circles represent the total N-N scattering t-matrices including pion exchanges.

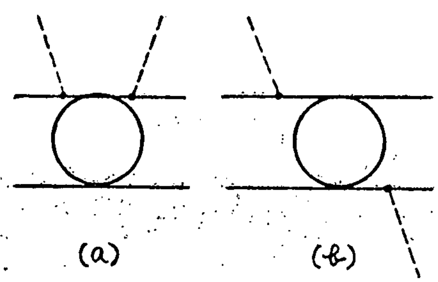


Fig. 2-4

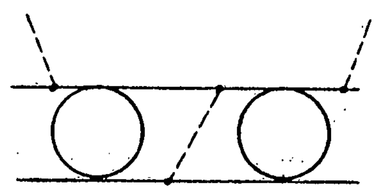


Fig. 2-5

(ii) The next thing to be examined is whether the two-nucleon states with possible N-N interactions appear as many times as possible in πNN→πNN process. In other words we should ask whether a process represented in Fig. 2-5, which is a combination of diagrams (a) and (b) in Fig. 2-4, can happen in the total πNN→πNN amplitude. The answer is "no", because every possible process connecting initial and final two-nucleon states in (a) and (b) in Fig. 2-4 is already in the circles so the diagram in Fig. 2-5 overcounts the pions as well as other mesons exchanged. Therefore we may conclude that in the πNN elastic amplitude, the pure two-nucleon intermediate state (with possible N-N interaction) should appear once and only once. Of course we also have the contribution to the elastic πNN scattering from the process with no intermediate pi-absorption. Thus the total πNN amplitude takes the form; $T^{TOT}_{\pi NN} = T^{SCAT}_{\pi NN} + T^{ABS}_{\pi NN}$, where the second term is the contribution from the pion absorption. This structure seems also to be present in general pi-nucleus elastic amplitudes.

Now the question is how to obtain complete expressions with the proper structure mentioned above without pion overcounting.

First, if we begin with some adequate field theoretical Hamiltonian containing the nucleon field, several independent meson fields and the interaction terms among them (its concrete form does not necessarily have to be known), then the overcounting problem would never appear, in principle. All we have to do in this case is to classify all possible processes properly to identify sub-amplitudes: the π-N t-matrices, N-N t-matrices etc. in the total amplitude. For this purpose Taylor's method has been found useful and we apply it for the study of πNN amplitudes in the next chapter. Second, we may ask if there is any approach which has a similar form of Hamiltonian as appears in (2B-4') but is free from pion overcounting. As is obvious, Hamiltonian (2B-4') counts processes like diagram "b" in Fig. 2-3 more than once. This is because the N-N potential is assumed to contain the effect of pion exchanges. So if the N-N potential there is replaced by the one without pion exchanges, it will become free from the overcounting of pions. In our non-relativistic approach to the πNN problem, we have adopted this picture, which will be found also in the next chapter.

CHAPTER III

THE STRUCTURE OF πNN SCATTERING AMPLITUDES

The aim of this chapter is to study the structure of πNN amplitudes and through this study we shall find a set of amplitude equations. First we consider the problem in the framework of relativistic quantum field theory and later we adopt a non-relativistic Hamiltonian model.

A. Relativistic Approach

The method adopted here is due to Taylor(10,11) who tried to have an alternative approach to relativistic quantum field theory without any explicit use of Lagrangian or Hamiltonian. Later this method was utilized by himself in constructing relativistic three-body equations in their most general form.(12) Since the development there was formal, the equations do not seem to have been studied further nor applied. We have found that his method is useful in studying the structure of pi-nucleus scattering; πNN problem here.

I. The Taylor Method (preliminaries)

The prototype of Taylor's method is found in Symanzik's work[13] on the many-particle structure of Green's functions. Symanzik's approach is to use functional differentiation with respect to the source field in S-matrix and obtain retarded Green's functions for which the structure is analyzed. Taylor extended this in order that causal propagators can be used as well.

The reason why we have adopted Taylor's method in our study is as follows. As mentioned above it is developed in the framework of relativistic field theory but as will be mentioned, it does not require any specific Lagrangian nor Hamiltonian to start with. This is because the analysis of scattering amplitudes in his approach is a combinatorial problem associated with diagrams. This analysis is done through the "cutting lemma" which will also be mentioned later and this lemma enables us to decompose a given amplitude in terms of a combination of more reduced amplitudes in a unique way and hence leads to the amplitude with no overcounting. With the cutting lemma and the notion of "complete unitarity" introduced later, it is possible to reduce every possible amplitude on and on (in principle infinite number of times) and thus we can construct a field theory alternative to the Lagrangian approach. For our purpose, however, it is not necessary to do that. Since

Taylor's method gives us a relation among various scattering amplitudes, we are able to get the structure of a certain amplitude in terms of a finite number of amplitudes which are more reduced in their forms. Actually, we shall see in the next subsection, i.e. equations (3A-III-1) - (3A-IV-15'), that we can obtain a set of equations for the scattering involving πNN with a finite number of given input amplitudes; πNN vertex functions, generalized NN potential, propagators, etc. all of them may be renormalized quantities, to be given.

In Taylor's approach to relativistic field theory there are several basic assumptions which are common to every relativistic field theory. They are (i) existence of fields, (ii) Lorentz covariance (iii) existence of physical vacuum (iv) existence of asymptotic fields (v) renormalizability etc. Just to know the structure of a given scattering amplitude or to obtain a set of equations among scattering amplitudes in question through Taylor's method, we may not need those assumptions above because as has been said, it only requires to solve a combinatorial problem.

In order to simplify our analysis we consider that particles are all spinless and distinguishable. This does not lose any important points in actual πNN scattering that we try to understand as the essence in Taylor's method is in combinatorials but not in the detailed kinematics nor internal symmetries.

Let us remember some basics in quantum field theory. The S-matrix describes a scattering process, which is unitary, i.e. $SS^+ = S^+S = 1$ and it is usually written as

$$S \equiv 1 + R \equiv 1 + iT. \qquad (3A\text{-}I\text{-}1)$$

Its many-particle matrix element is

$$S_{fi} = \langle \vec{q}_1, \vec{q}_2 \cdots \vec{q}_m; \text{out} | \vec{p}_1, \vec{p}_2 \cdots \vec{p}_n; \text{in} \rangle. \qquad (3A\text{-}I\text{-}2)$$

When we use the reduction formula of Lehmann-Symanzik-Zimmermann (LSZ), we can relate S_{fi} with a many-particle causal Green's function (of course S_{fi} can be related to retarded or advanced Green's function as well),

$$S_{fi} = (1)_{fi} + (i)^{n+m} \prod_{j,k} \int d^4z_j d^4x_k \, e^{iq_j z_j} e^{-ip_k x_k} D_{z_j} D_{x_k} \qquad (3A\text{-}I\text{-}3)$$
$$\times \langle 0 | T(\phi_1'(z_1) \cdots \phi_m'(z_m) \phi_1^\dagger(x_1) \cdots \phi_m^\dagger(x_n)) | 0 \rangle,$$

where $P \cdot x \equiv p_\mu x^\mu$ (four scalar product)

$Dx_i \equiv \Box_{x_i} + m_i^2$; Klein-Gordon operator

The reason why we put "primes" to some operators is to show that the field ϕ_1' can be different from ϕ_1. Also to make things definite we should mention that the normalization convention in which

$$\langle \vec{p}' | \vec{p} \rangle = (2\pi)^3 E_p \delta^3(p' - p), \quad (E_p = \sqrt{m^2 + p^2}),$$
$$\langle 0 | \phi(x) | \vec{p} \rangle = e^{-ipx}, \quad \text{etc.}$$

is adopted.

Now because of (3A-I-3) the center of our study is in the analysis of time-ordered many-particle Green's functions (or sometimes called propagators). We are most concerned with the structure of their Fourier transforms after multiplied by several Klein-Gordon operators and these quantities are expressed in terms of "propagator - amputated Green's functions"; τ, which is defined through

$$\langle 0|T(\phi_1'(z_1)\cdots\phi_n'(z_n)\,\phi_1^{+}(x_1)\cdots\phi_m^{+}(x_m))|0\rangle \equiv \prod_{j,k}\int d^4v_j\, d^4u_k \times [i\Delta'_{F_j'}(v_j-z_j)][i\Delta'_{F_k}(u_k-x_k)]\,\tau(v_1\cdots v_n;u_1\cdots u_m), \qquad (3A\text{-}I\text{-}4)$$

where $i\Delta_F'(x-y)\equiv\langle 0|T(\phi(x)\phi^{+}(y))|0\rangle$ is a single-particle causal propagator. (Note a "free" causal propagator is defined as $i\Delta_F(x-y)\equiv\langle\tilde{0}|T(\tilde{\phi}(x)\tilde{\phi}^{+}(y))|\tilde{0}\rangle$, where states and operators are in Dirac picture). We then plug (3A-I-4) into (3A-I-3) and obtain the expression for R. In doing so we regard those momenta in the exponent in (3A-I-3) to be general four-vectors; they are not necessarily on-mass-shell. Thus utilizing the relation

$$\int e^{\pm ipx} D_x \Delta_F'(x-y)\, d^4x = -e^{\pm ipy}\,\Delta_F'(p)/\Delta_F(p), \qquad (3A\text{-}I\text{-}4')$$

where $\Delta_F'(p)$ etc. are Fourier transform of $\Delta_F'(x)$ etc., we arrive at

$$\mathcal{R}(q_1 \cdots q_n; p_1 \cdots p_m) \equiv (2\pi)^4 \delta^4(\Sigma q - \Sigma p)\, \mathcal{W}(q_1 \cdots q_n; p_1 \cdots p_m) \quad \text{(3A-I-5)}$$

and

$$\mathcal{W}(q_1 \cdots q_n; p_1 \cdots p_m) = \prod_j^n \prod_k^m \left\{ \Delta'_{F_j'}(q_j) / \Delta_{F_j'}(q_j) \right\} \left\{ \Delta'_{F_k}(p_k) / \Delta_{F_k}(p_k) \right\}$$
$$\times \hat{\tau}(q_1 \cdots q_n; p_1 \cdots p_m). \quad \text{(3A-I-5')}$$

Here $\hat{\tau}$ is defined as

$$\tau(z_1 \cdots z_n, x_1 \cdots x_m) \equiv 1/(2\pi)^{4n+4m} \int \prod_j^n \prod_k^m e^{iq_j z_j} e^{-ip_k x_k} \hat{\tau}(q_1 \cdots q_n; p_1 \cdots p_m)(2\pi)^4 \delta^4(\Sigma q - \Sigma p).$$
(3A-I-5")

Since $\Delta_F'(p)/\Delta_F(p) = 1$ for on-mass-shell p, the Fourier transform of the "propagator-amputated Green's function"; $\hat{\tau}$, becomes equal to R when all particles are on-mass-shell and therefore our study finally converges to the point where the analysis is in the structure of $\hat{\tau}$ functions.

Next step is to assume the possible cluster decomposition property of $\hat{\tau}$, which is reasonable as long as the interactions involved are of short range (we neglect Coulomb interaction). Then we may assume

$$\hat{\tau}(q_1\cdots q_n;p_1\cdots p_m) \equiv M_{nm}(q_1\cdots q_n;p_1\cdots p_m) + \sum \hat{\tau}(q_{j_1}\cdots q_{j_\alpha};p_{k_1}\cdots p_{k_\beta}) \times M_{n-\alpha,m-\beta}(q_{j_{\alpha+1}}\cdots q_{j_n};p_{k_{\beta+1}}\cdots p_{k_m}). \tag{3A-I-6}$$

In the above expression $M_{x,y}$ is a completely connected part of $\hat{\tau}(r_1\ldots r_\alpha;s_1\ldots s_y)$ and the summation is over all possible combinations $(j_1,\ldots j_\alpha;k_1\ldots,k_\beta)$ where $(j_1\cdots j_\alpha)$ is a set of numbers arbitrarily chosen from $(1,\ldots,n)$ for $\alpha=1$ up to $\alpha=n-1$ whereas $(k_1\ldots k_\beta)$ is a similar set from $(1,\ldots,m)$ for $\beta=1$ up to $\beta=m-1$.

By "completely connected" amplitude we mean the one which does not contain any contribution associated with spectator particles, nor can it be written in terms of a product of several amplitudes which contain fewer particles in them. It is this connected amplitude M_{nm} (or connected par of $\hat{\tau}_{nm}$) which is of physical importance and our task is thus to study its structure and relate this to the S-matrix through (3A-I-1)-(3A-I-6). The study of the structure of M_{nm} is done by exposing intermediate states with a certain number of particles by the method associated with "cutting lemma". We shall study it in the next subsection.

II. The Taylor Method (exposing intermediate states in the amplitudes)

We shall adopt an approach in which a given scattering amplitude M_{nm} is regarded as a formal summation of graphs. This picture seems to be necessarily connected with the perturbation theory. So if the perturbation series does not converge in a given field theory, it does not seem to work. But as t'Hooft and Veltman stated,[15] it may be appropriate to use the summation of graphs formally to obtain amplitudes or amplitude equations and then start with them. (Of course all possible consistency checks should be done with regard to those amplitudes or equations once obtained.)

As has been stated at the beginning of subsection I, no detailed specification of Hamiltonian nor Lagrangian is required. To go one step further into the method several definitions may be due here which are due to Taylor.

(i) A graph is defined to be a two-dimentional Feynman type figure consisting of internal propagators, point vertices and external particle legs. A graph should be connected. For our purpose the initial and final states are to be specified, but in the analysis we are going to do, it is not necessary to specify particle momenta etc. So the specification may be just the number of particle lines in the initial and final states. We assign a graph as in Fig. 3A-1, where the external legs may or may not be explicitly shown, i.e.:

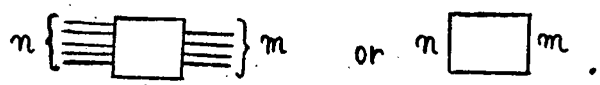


Fig. 3A-1

(ii) A diagram is defined to be a formal summation of all possible connected, topologically distinct (perturbation) graphs (as defined in (i)), which are associated with a process under consideration. We identify a diagram as a connected amplitude M_{nm}. A diagram is depicted as:

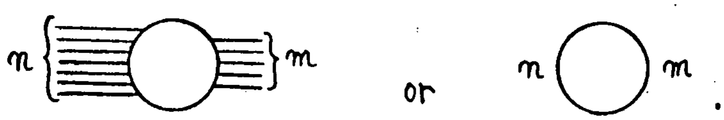


Fig. 3A-2

The reason why we distinguish a diagram from a graph is that we may need that distinction when we discuss the last cut lemma and things connected with it.

It may be adequate to stress here that the outer legs in graphs and diagrams are generally off-mass-shell.

(iii) A cut is defined to be an arc with no multiple points and intersects particle lines in a given graph (or a diagram) to separate the initial and final states. It may intersect several external lines but should intersect at least one internal line. It should not intersect one particle line more than once.

(iv) An r-cut is a cut intersecting r lines in a graph (or a diagram).

(v) A graph (or a diagram) is called r-irreducible (or r-particle irreducible) if no k-cut ($k \leq r$) can be drawn in it. This is expressed as in Fig. 3A-3 (graph) and Fig. 3A-3' (diagram).

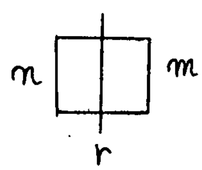


Fig. 3A-3

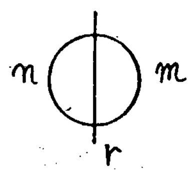
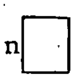


Fig. 3A-3'

We shall use cuts to expose intermediate particle lines in a given graph,which leads to the structure analysis of the graph. For this purpose we need the "last cut lemma" of Taylor. It is explained briefly in Appendix A and here we just state the result. [Last cut lemma] For a given graph n  m, in whch we can draw r-cuts, it is always possible to find a unique r-cut which is nearest either n or m, provided the graph is already r-1 irreducible.

As is stated in Appendix A, a given graph  which is r-1 irreducible belongs to one of the several sets; one without r-cut, one admitting r-cuts intersecting r internal lines, one in which any r-cut intersects external lines from m, etc. A careful study shows that to any case this "last cut lemma" applies.

Now using the lemma just stated, we can expose intermediate particle lines uniquely; particle lines are exposed where they are cut by the last cut. Let us consider a graph shown in Fig.3A-4 which is r-1 irreducible and admits the existence of r-cuts intersecting r internal lines. Then the "last cut lemma" leads to the unique

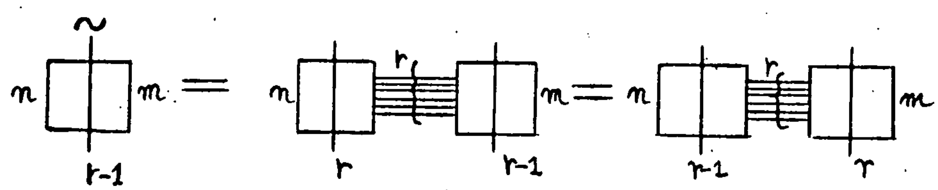


Fig. 3A-4

exposure of intermediate r particles (either closest to n or m) as shown in Fig. 3A-4. When we sum all similar graphs, admitting r-cuts cutting r internal lines, we obtain

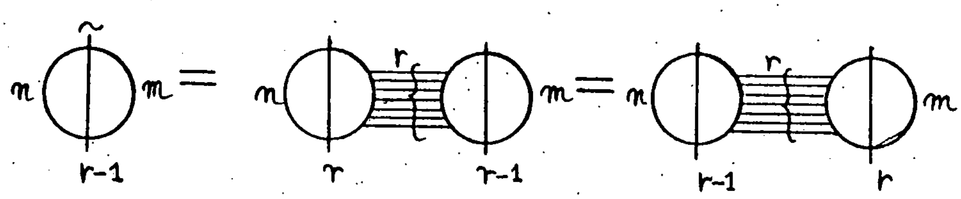
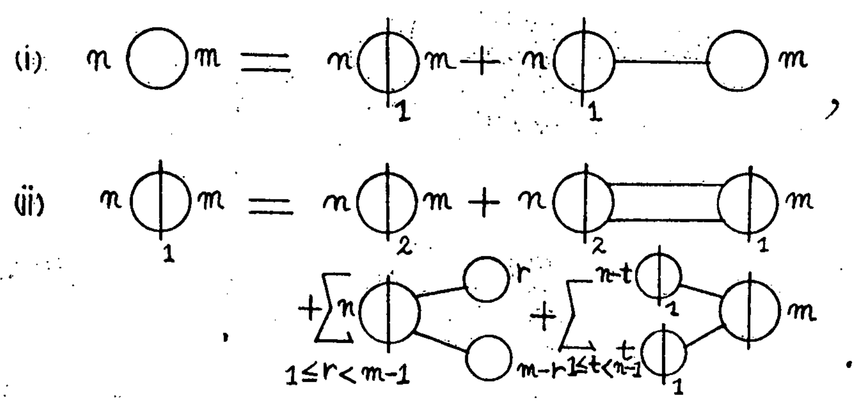


Fig. 3A-5

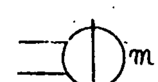
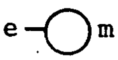
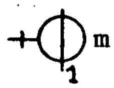
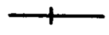
Here in Fig. 3A-5 we only have diagrams but not graphs. Similar exposure can be done in those r-1 irreducible graphs in which r-cuts intersect at least one external line. We classify all these (r-1) irreducible graphs according to the "last cut" structure. When we sum all these classes of graphs (by this time each class is expressed in terms of diagrams only) including $n\,\tilde{\phi}_{r-1}m$, we obtain an exposure of r-1 irreducible diagram $n\,\phi_{r-1}m$ ($=\Sigma\, n\,\tilde{\phi}_{r-1}m$) in terms of the combination of other irreducible diagrams.

As it is difficult to write down an exposed form (or we shall call it a cut structure) of a diagram in general, we show some examples in Fig. 3A-6 .

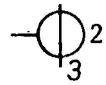
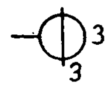


(ii)

Fig. 3A-6

We should note here that, for example  in Fig. 3A-6 (ii) can be different from $M^{(1)}_{2,m}$, which is a one-particle irreducible part of the connected $m\to 2$ amplitude $M_{2,m}$ defined in a manner shown in (3A-I-6). This happens if "free" propagators are exposed by the last cut. Therefore our cutting is understood to expose "dressed" propagators from now on. Thus we obtain  $= M^{(1)}_{2,m}$. One should be careful, however, in adopting this viewpoint; we should replace  by  where  is a dressed propagator since this dressed propagator "pulls out" ~~with it one-~~ particle reducible part from .

Equipped with the last cut lemma, a study of the structure of given amplitudes starts with exposing single-particle intermediate state, two-particle state, and on and on to expose higher-particle sectors in principle.

There is one more step for us to take to complete Taylor's method. We are changing our former definition of diagrams. It now goes as follows. A physical amplitude is defined to be a diagram and a diagram is then defined as an all possible summation of topologically distinct perturbation diagrams. A perturbation diagram is defined in terms of a perturbation graph, by replacing (1) every point vertex by a diagram with corresponding number of legs and (2) free propagators by corresponding dressed propagators. The replacement just mentioned above is, according to Taylor, a requirement from "complete unitarity". This complete unitarity means that physically, all possible intermediate states occur in any given process. The cut structure of diagrams does not change by this requirement. But this complete unitarity makes it possible to expose any higher-particle intermediate states in any amplitude and connect that amplitude with other amplitudes. Taylor has emphasized that it is possible to accept the "last cut lemma" and "complete unitarity" as two basic guiding principles and forget about the perturbation concept utilized to establish the lemma to start with the field theory. Actually, he has shown, in the case of self coupling field, that when approximating  and  by constants λ_1 and λ_2, a canonical form of field equation (or equation in terms of t-matrix) which would have been obtained from the current operator $j=\lambda_1 A^2 + \lambda_2 A^3$ can be derived.

One thing which we want to see is if the last cut lemma plus complete unitarity leads to the unitary structure of the amplitudes. Taylor did it for some simpler cases like $2\to2$ amplitudes. We give an explicit and probably an original proof of the unitarity of the amplitudes derived from Taylor's approach for the case of πNN amplitudes including those for $\pi NN \rightleftarrows NN$ at least in the elastic region of NN and πNN states. (This is shown in Appendix C.)

To end this section we should make a couple of remarks.

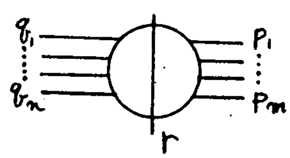
(i) The cutting can be done in every possible channel, s,t,u etc.. So we could in principle get equations of scattering amplitudes which exhibit explicit crossing symmetry. But as may be expected, the equations will become non-linear and very complicated.

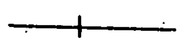
(ii) The concept of particle irreducibility is not very new. For example, it is (often implicitly) adopted to derive Bethe-Salpeter equation[19]. Also it was used in a less rigorous manner by Zachariasen[16] and later by Thomas[17].

III. The Structure of πNN Amplitudes

By means of Taylor's approach we can expose intermediate states of various πNN amplitudes; amplitudes for $\pi NN \to \pi NN$ and $\pi NN \rightleftarrows NN$, and the result is a set of relations or equations which expresses the amplitudes in question in terms of the combination of several higher irreducible amplitudes. As has been mentioned in subsection I, if we regard these higher irreducible amplitudes as given imputs, we then can solve the equations to get the amplitudes for those processes stated above. For our purpose the intermediate states to be exposed should be two-nucleon with no pion, and two-nucleon plus one-pion states.

First we set up a convention connecting amplitudes and diagrams. The rules are as follows:

(1) $M^{(r)}_{nm}(q_1 \dots q_n ; p_1 \dots p_m) \equiv$ 

(2) $d(p) \equiv i\Delta'_F(p) \equiv$ 

(3) each internal line carries $\int \frac{d^4p}{(2\pi)^4}$

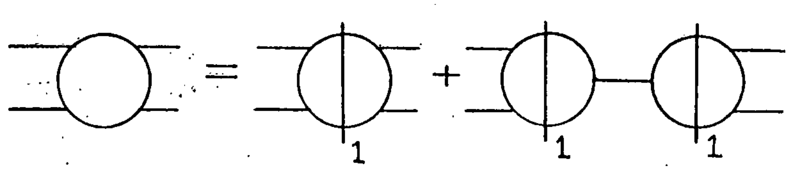
(4) four-momentum conservation factor $(2\pi)^4 \delta(\Sigma q-\Sigma p)$ carried by each amplitude

Properly speaking we should use different lines for pions and nucleons respectively. For diagrammatic simplicity we shall not use two different lines unless required for clarity.

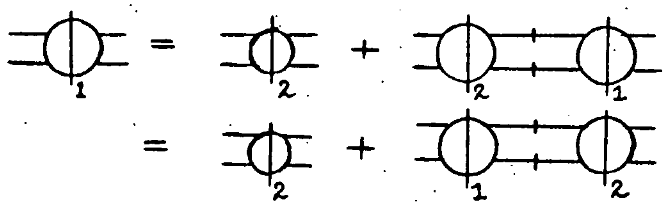
We shall start our cut structure analysis after one appropriate definition.

[Definition] A generalized n-body potential is defined as n Φ_n n; n outer legs both initial and final state of an n-particle irreducible amplitude. As will be understood, this is consistent with our picture of nucleon-nucleon potentials, the lowest order contribution which comes from single pion exchange. This is readily seen to be a two-particle irreducible 2→2 graph.

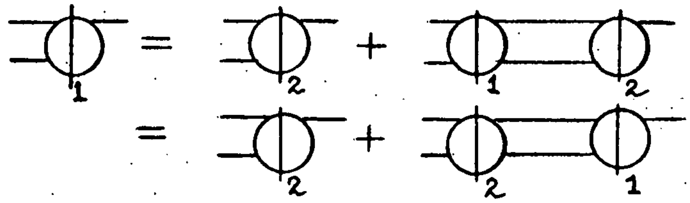
First we should know the structure of two-particle amplitudes which are expected to appear in three-body amplitudes. The cut structure analysis gives



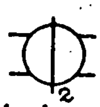
(3A-III-1)

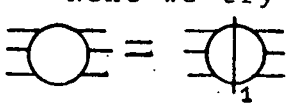
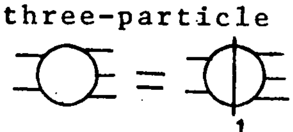


(3A-III-1')

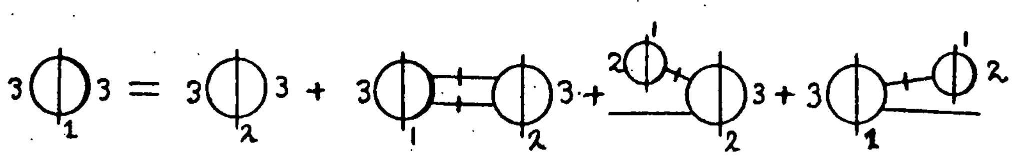


(3A-III-1")

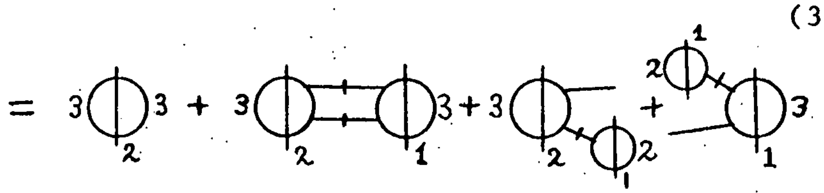
In (3A-III-1) the second term on the righthand side disappears for the case of N-N amplitudes as we expose only nucleon and pion intermediate lines. In (3A-III-1') we immediately notice that the relation is of Lippmann-Schwinger[18] form or more precisely of Bethe-Salpeter[19] type equation and  actually corresponds to the potential in non-relativistic scattering there.

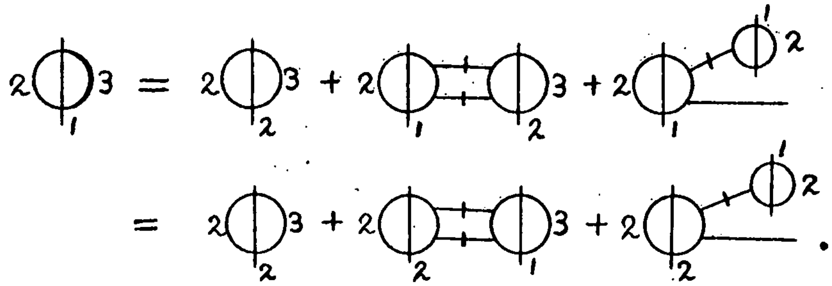
Next we try to expose three-particle (πNN) amplitudes. As  and  (same for ) in our πNN amplitudes we find the following structure:

(i) decomposition of one-particle irreducible amplitudes

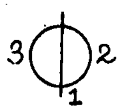


(3A-III-2)



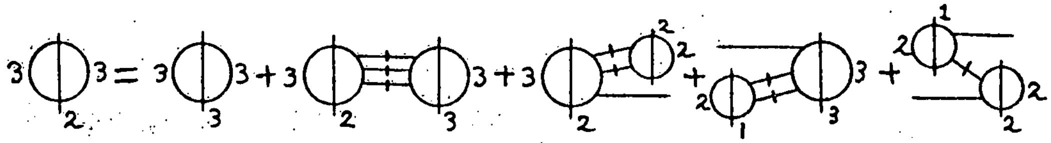


(3A-III-3)

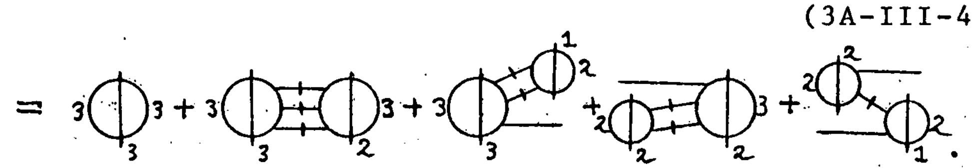
 has a similar expression to (3A-III-3).

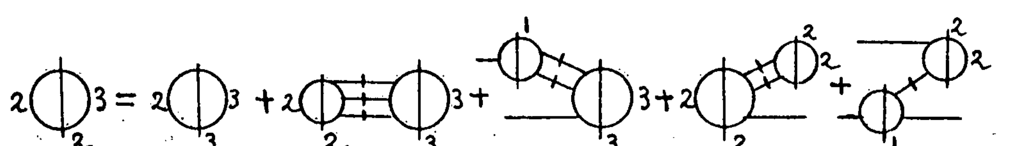
On the righthand side of both (3A-III-2) and (3A-III-3) the first line is due to the last cut nearest the initial state and second line due to the last cut nearest the final state. Note that there is no appearance of potentials here.

(ii) decomposition of two-particle irreducible amplitudes

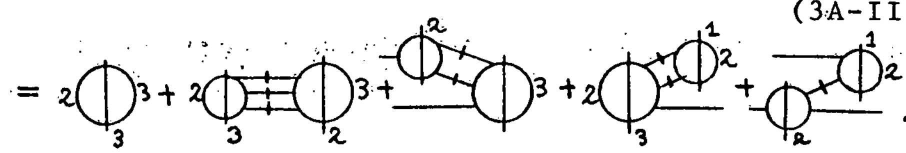


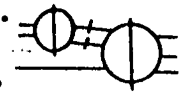
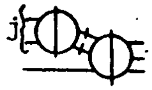
(3A-III-4)





(3A-III-5)

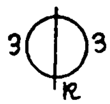
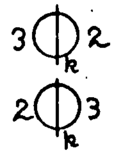


 has a similar expression to (3A-IV-5). 
Note that throughout (3A-III-2) to (3A-III-5), etc. mean $\sum_j$  where j is a label for a pair or a spectator.

At this point let us look at those amplitudes which are cut analyzed. Especially (3A-III-2) shows that the amplitude contains a pure two nucleon (+N-N interaction) intermediate state explicitly just once. This is in agreement with the result of our qualitative study on the structure of π-nucleus elastic amplitudes at the end of the last chapter.

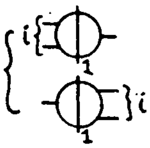
IV. Equations for πNN amplitudes

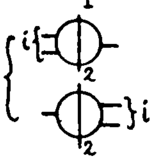
We set up a "dictionary" for the translation from diagrams to standard expressions. The translation scheme is as follows:

$m^{(k)} \equiv$  , $\Gamma_{\pm}^{(k)} \equiv \{$ 

$\tilde{\mu}_i \equiv$  } i-th pair (πN) , $\mu_i \equiv \tilde{\mu}_i d_i^{-1}$

$\tilde{M}_i^{(k)} \equiv$  , $M_i^{(k)} \equiv \tilde{M}_i^{(k)} d_i^{-1}$

$\tilde{\gamma}_{\pm}^{i} \equiv \{$  , $\gamma_{\pm}^{i} \equiv \tilde{\gamma}_{\pm}^{i} d_i^{-1}$

$\tilde{g}_{\pm}^{i} \equiv \{$  , $g_{\pm}^{i} \equiv \tilde{g}_{\pm}^{i} d_i^{-1}$

$G_i \equiv d_j d_k \ (i \neq j, i \neq k)$, $G_3 \equiv d_1 d_2 d_3$

$d_{k\ell}$; nucleon propagator resulting from a pion absorbed by a nucleon; either "k" or "ℓ" indicates the pion and "ℓ" or "k" the absorbing nucleon

$\hat{G}_i \equiv d_i d_{k\ell}$

Using these notations, we write down the relations among amplitudes depicted in (3A-III-1)-(3A-III-5).

(i) (3A-III-1)-(3A-III-1")

$$\tilde{\mathcal{M}}_i = \tilde{M}_i^{(1)} + \tilde{\gamma}_+^i d_{k\ell} \tilde{\gamma}_-^i \qquad \text{(3A-IV-1)}$$

$$\begin{aligned} \tilde{M}_i^{(1)} &= \tilde{M}_i^{(2)} + \tilde{M}_i^{(2)} G_i \tilde{M}_i^{(1)} \\ &= \tilde{M}_i^{(2)} + \tilde{M}_i^{(1)} G_i \tilde{M}_i^{(2)} \end{aligned} \qquad \text{(3A-IV-1')}$$

$$\begin{aligned} \tilde{\gamma}_+^i &= \tilde{g}_+^i + \tilde{M}_i^{(1)} G_i \tilde{g}_+^i \\ &= \tilde{g}_+^i + \tilde{M}_i^{(2)} G_i \tilde{\gamma}_+^i . \end{aligned} \qquad \text{(3A-IV-1")}$$

$\tilde{\gamma}_-^i$ has a similar expression to (3A-IV-1").

For our later purpose the following expressions are more useful for the three-body amplitudes as we shall see soon.

$$\left.\begin{aligned} \mathcal{M}_i &= M_i^{(1)} + \gamma_+^i \hat{G}_{Ti} \gamma_-^i \\ M_i^{(1)} &= M_i^{(2)} + M_i^{(2)} G_3 M_i^{(1)} \\ &= M_i^{(2)} + M_i^{(1)} G_{T_3} M_i^{(2)} \\ \gamma_+^i &= g_+^i + M_i^{(1)} G_{T_3} g_+^i \\ &= g_+^i + M_i^{(2)} G_{T_3} \gamma_+^i \end{aligned}\right\} .$$

(3A-IV-2)

(ii) (3A-III-4)

$$\mathcal{M}^{(2)} = \mathcal{M}^{(3)} + \mathcal{M}^{(3)} G_3 \mathcal{M}^{(2)} + \mathcal{M}^{(3)} G_3 \sum_i M_i^{(1)} + \sum_i M_i^{(2)} G_3 \mathcal{M}^{(2)} + \sum_{i \neq j} M_i^{(2)} G_3 M_j^{(1)}$$

$$= \mathcal{M}^{(3)} + \mathcal{M}^{(2)} G_3 \mathcal{M}^{(3)} + \mathcal{M}^{(2)} G_3 \sum_i M_i^{(2)} + \sum_i M_i^{(1)} G_3 \mathcal{M}^{(3)} + \sum_{i \neq j} M_i^{(1)} G_3 M_j^{(2)} .$$

(3A-IV-3)

When we define $\hat{\mathcal{M}}^{(2)} \equiv \mathcal{M}^{(2)} + \sum_i M_i^{(1)}$ and $\hat{\mathcal{M}}^{(3)} \equiv \mathcal{M}^{(3)} + \sum_i M_i^{(2)}$, (3A-IV-3) becomes

$$\hat{\mathcal{M}}^{(2)} = \hat{\mathcal{M}}^{(3)} + \hat{\mathcal{M}}^{(3)} G_3 \hat{\mathcal{M}}^{(2)}$$
$$= \hat{\mathcal{M}}^{(3)} + \hat{\mathcal{M}}^{(2)} G_3 \hat{\mathcal{M}}^{(3)} . \qquad \text{(3A-IV-3')}$$

This is of Lippmann-Schwinger or Bethe-Salpeter type equations with $\hat{m}^{(3)}$ being the generalized potential. In its cut structure $\hat{m}^{(3)}$ indeed is consistent with our conventional notion of potentials. We now try to obtain an expression for $\hat{m}^{(2)}$ (or $m^{(2)}$) analogous to the non-relativistic Faddeev equations[21] (see Appendix B). We first decompose $\hat{m}^{(2)}$ as

$$\hat{\mathcal{M}}^{(2)} = \sum_{j=0}^{3} \hat{\mathcal{M}}_{j\cdot}^{(2)} = \sum_{j=0}^{3} \hat{\mathcal{M}}_{\cdot j}^{(2)} , \qquad \text{(3A-IV-4)}$$

where $\hat{m}^{(2)}_{j\bullet}$; a part of $\hat{m}^{(2)}$ in which the final state interaction occurs among j-th pair (j=1,2,3).

$\hat{m}^{(2)}_{o\bullet}$; similar to $\hat{m}^{(2)}_{j\bullet}$ (j=1,..3) but the final state interaction is due to three-body potentials.

Similar definitions apply for $\hat{m}^{(2)}_{\bullet\mu}$ (μ=0,..3) except that the specifications are concerned with the initial state interaction. Then with $U_o \equiv m^{(3)}$ and U_j $(j=1,..3) \equiv M_j^{(2)}$ we obtain from (3A-IV-3') and (3A-IV-4) that

$$\left.\begin{aligned} \hat{m}^{(2)}_{j\bullet} &= U_j + U_j G_3 \hat{m}^{(2)} = U_j + \hat{m}^{(2)}_{j\bullet} G_3 \hat{m}^{(3)} \\ \hat{m}^{(2)}_{\bullet j} &= U_j + \hat{m}^{(2)} G_3 U_j = U_j + \hat{m}^{(3)} G_3 \hat{m}^{(2)}_{\bullet j} \\ &\qquad (j=0,1,2,3) \end{aligned}\right\}. \qquad \text{(3A-IV-5)}$$

Using (3A-IV-2) and the conventional technique to eliminate potentials in non-relativistic Faddeev equation formulation (see Appendix B), (3A-IV-5) is modified to become

$$\left.\begin{aligned} \hat{m}^{(2)}_{j\bullet} &= M_j^{(1)} + M_j^{(1)} G_3 \sum_{k\neq 0,j} \hat{m}^{(2)}_{k\bullet} + M_j^{(1)} G_3 \hat{m}^{(2)}_{0\bullet} \quad (j=1,2,3) \\ \hat{m}^{(2)}_{0\bullet} &= M_0 + M_0 G_3 \sum_{k=1}^{3} \hat{m}^{(2)}_{k\bullet} \end{aligned}\right\} \qquad \text{(3A-IV-5')}$$

and similarly

$$\left.\begin{array}{l}\hat{m}^{(2)}_{\cdot j} = M^{(1)}_j + \sum_{k\neq j,0} \hat{m}^{(2)}_{\cdot k} G_3 M^{(1)}_j + \hat{m}^{(2)}_{\cdot 0} G_3 M^{(1)}_j, (j=1,2,3) \\ \hat{m}^{(2)}_{\cdot 0} = M_0 + \sum_{k=1}^{3} \hat{m}^{(2)}_{\cdot k} G_3 M_0 \end{array}\right\}, \qquad (3A\text{-}IV\text{-}5'')$$

where $M_o=U_o+U_oG_3M_o=U_o+M_oG_3U_o$. (3A-IV-5') and (3A-IV-5") are of "Faddeev type".

We next try to eliminate $\hat{m}^{(2)}_{\bullet o}$ (or $\hat{m}^{(2)}_{o\bullet}$) from the integral equations for $\hat{m}^{(2)}_{\bullet j}$ and $\hat{m}^{(2)}_{j\bullet}$ (j-1,2,3). The result is

$$\left.\begin{array}{l}\hat{m}^{(2)}_{j\cdot} = M^{(1)}_j + M^{(1)}_j G_3 \sum_{k=1}^{3} (\bar{\delta}_{kj} + M_0 G_3) \hat{m}^{(2)}_{k\cdot} + M^{(1)}_j G_3 M_0 \\ \qquad\qquad (j=1,2,3) \\ \hat{m}^{(2)}_{0\cdot} = M_0 + \sum_{k=1}^{3} M_0 G_3 \hat{m}^{(2)}_{k\cdot} \end{array}\right\}, \qquad (3A\text{-}IV\text{-}6)$$

$$(\bar{\delta}_{\alpha\beta} \equiv 1 - \delta_{\alpha\beta})$$

and

$$\left.\begin{array}{l}\hat{m}^{(2)}_{\cdot j} = M^{(1)}_j + \sum_{k=1}^{3} \hat{m}^{(2)}_{\cdot k} (\bar{\delta}_{kj} + G_3 M_0) G_3 M^{(1)}_j + M_0 G_3 M^{(1)}_j \\ \qquad\qquad (j=1,2,3) \\ \hat{m}^{(2)}_{\cdot 0} = M_0 + \sum_{k=1}^{3} \hat{m}^{(2)}_{\cdot k} G_3 M_0 \end{array}\right\}, \qquad (3A\text{-}IV\text{-}6')$$

where $\bar{\delta}_{kj}=1-\delta_{kj}$. (3A-IV-6) and (3A-IV-6') show that $\hat{m}_{o\bullet}^{(2)}$ and $\hat{m}_{\bullet o}^{(2)}$ can be obtained just by integrations once we solve for $\hat{m}_{j\bullet}^{(2)}$ and $\hat{m}_{\bullet j}^{(2)}$ respectively.

We may decompose $\hat{m}_{j\cdot}$, $\hat{m}_{\cdot j}$ etc. further:

$$\hat{m}_{j\cdot}^{(2)} \equiv \sum_{\nu=0}^{3} \hat{m}_{j\nu}^{(2)} , \quad (j=0,1,2,3), \qquad \text{(3A-IV-7)}$$

and

$$\hat{m}_{\cdot j}^{(2)} \equiv \sum_{\nu=0}^{3} \hat{m}_{\nu j}^{(2)} , \quad (j=0,1,2,3), \qquad \text{(3A-IV-7')}$$

with the similar meaning to be given to the second suffices as explained in connection with (3A-IV-4). When we put (3A-IV-7) and (3A-IV-7') into (3A-IV-6) and (3A-IV-6') we obtain:

$$\begin{cases} \hat{m}_{jk}^{(2)} = M_j^{(1)}\delta_{jk} + M_j^{(1)} G_3 \sum_{\ell=1}^{3} (\bar{\delta}_{j\ell} + M_0 G_3)\hat{m}_{\ell k}^{(2)}, (j,k=1,2,3) \\ \hat{m}_{j0}^{(2)} = \sum_{k=1}^{3} \hat{m}_{jk}^{(2)} G_3 M_0 , \quad (j=1,2,3) \\ \hat{m}_{0k}^{(2)} = M_0 G_3 \sum_{j=1}^{3} \hat{m}_{jk}^{(2)} , \quad (k=1,2,3) \\ \hat{m}_{00}^{(2)} = M_0 + M_0 G_3 \sum_{j=1}^{3} \hat{m}_{j0}^{(2)} . \end{cases} \qquad \text{(3A-IV-8)}$$

Expression (3A-IV-8) shows that we have only to solve the first equation for $\hat{m}^{(2)}_{jk}$ there and other amplitudes are obtainable from $\hat{m}^{(2)}_{jk}$ $(j,k=1,2,3)$ by integrations.

When physically significant connected part only is considered, the equation to be solved for it is

$$\mathcal{M}^{(2)}_{jk} = M^{(1)}_j G_3 (\bar{\delta}_{jk} + M_0 G_3) M^{(1)}_k + M^{(1)}_j G_3 \sum_{\ell=1}^{3} (\bar{\delta}_{j\ell} + M_0 G_3)\, \mathcal{M}^{(2)}_{\ell k} \qquad (j,k=1,2,3)\,. \tag{3A-IV-9}$$

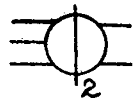
The rest are

$$\left.\begin{aligned} \mathcal{M}^{(2)}_{0k} &= M_0 G_3 M^{(1)}_k + M_0 G_3 \sum_{\ell=1}^{3} \mathcal{M}^{(2)}_{\ell k} \\ \mathcal{M}^{(2)}_{k0} &= M^{(1)}_k G_3 M_0 + \sum_{\ell=1}^{3} \mathcal{M}^{(2)}_{k\ell} G_3 M_0 \\ \mathcal{M}^{(2)}_{00} &= M_0 + M_0 G_3 \sum_{\ell=1}^{3} \mathcal{M}^{(2)}_{\ell 0} = M_0 + \sum_{\ell=1}^{3} \mathcal{M}^{(2)}_{0\ell} G_3 M_0 \end{aligned}\right\}. \qquad (k=1,2,3) \tag{3A-IV-10}$$

Equation(3A-IV-9) is a Faddeev-like equation which is obtained by replacing $\bar{\delta}_{j\ell}$ with $\bar{\delta}_{j\ell}+M_0G_3$;putting the effect of the three-body potential.[22] Of course the connected total amplitude is

$$\mathcal{M}^{(2)} = \sum_{\mu,\nu=0}^{3} \mathcal{M}^{(2)}_{\mu\nu}\ .$$

(iii) (3A-III-5)

We define $\hat{\Gamma}_{\pm}^{(2)} \equiv \Gamma_{\pm}^{(2)} + \sum_{j=1,2} \gamma_{\pm}^{J}$ and $L_{\pm} = \Gamma_{\pm}^{(3)} + \sum_{k} g_{\pm}^{j}$. Then (3A-III-5) and similar relation for  are written as

$$\hat{\Gamma}_{-}^{(2)} = L_{-} + L_{-} G_3 \hat{m}^{(2)} , \tag{3A-IV-11}$$

and

$$\hat{\Gamma}_{+}^{(2)} = L_{+} + \hat{m}^{(2)} G_3 L_{+} . \tag{3A-IV-11'}$$

(3A-IV-11) and (3A-IV-11') show that both $\hat{\Gamma}_{-}^{(2)}$ and $\hat{\Gamma}_{+}^{(2)}$ can be calculated rather easily (by integrations) once $\hat{m}^{(2)}$ and hence $\hat{m}_{jk}^{(2)}$ (j,k=1,2,3) has been obtained. It is possible that we can set up some integral equations for $\Gamma_{\pm}^{(2)}$ as Taylor did, but it is less practical to do that.

(iv) Expression for (3A-III-3) and (3A-III-2)

Introducing $\hat{\Gamma}_{\pm}^{(1)} \equiv \Gamma_{\pm}^{(1)} + \sum_{j} \gamma_{\pm}^{i}$, it is easy to show that they are written as

$$\hat{\Gamma}_{-}^{(1)} = \hat{\Gamma}_{-}^{(2)} + M^{(1)} G_2 \hat{\Gamma}_{-}^{(2)} , \tag{3A-IV-12}$$

and

$$\hat{\Gamma}_{+}^{(1)} = \hat{\Gamma}_{+}^{(2)} + \hat{\Gamma}_{+}^{(2)} G_2 M^{(1)} . \tag{3A-IV-13}$$

As for (3A-III-2), it can be expressed as

$$\mathcal{M}^{(1)} = \mathcal{M}^{(2)} + \Gamma_+^{(1)} G_2 \Gamma_-^{(2)} + \sum_j \gamma_+^j G_2 \Gamma_-^{(2)} + \Gamma_+^{(1)} G_2 \sum_j \gamma_-^j . \qquad \text{(3A-IV-14)}$$

Introduction of $\hat{m}^{(1)} \equiv m^{(1)} + \sum_{j=1}^{3} M_j^{(1)} + \sum_j \gamma_+^j G_2 \gamma_-^j$ makes (3A-IV-14) written as

$$\hat{\mathcal{M}}^{(1)} = \hat{\mathcal{M}}^{(2)} + \hat{\Gamma}_+^{(1)} G_2 \hat{\Gamma}_-^{(2)} = \hat{\mathcal{M}}^{(2)} + \hat{\Gamma}_+^{(2)} G_2 \hat{\Gamma}_-^{(1)} . \qquad \text{(3A-IV-15)}$$

This expression is at least more compact.

V. Summary

Physical $3\to3$ and $3\rightleftarrows2$ processes in πNN system are one-particle irreducible and the structure of the amplitudes corresponding to them is shown in (3A-III-2)-(3A-III-5) or more compactly in (3A-IV-3')-(3A-IV-15). As for the corresponding two-particle irreducible amplitudes, cut structure analysis, i.e. exposure process of intermediate states, has lead to the equations that those amplitudes should satisfy. Notice that the unique cut structure due to the last cut lemma leads to the correct counting of pions and thus makes the amplitude overcounting-free. For $m^{(2)}$ the equation is of extended relativistic Faddeev equation with a three-body potential in it. This could be solved in principle for given $M_j^{(1)}$ and M_o (or $M_j^{(2)}$ and $m^{(3)}$). But of course these inputs have complicated structure that can be exposed if we try to continue the cutting analysis further.

As we look at those relations; (3A-IV-1)-(3A-IV-15), we understand that the amplitudes describing $\pi NN \rightarrow \pi NN$ and $NN \rightleftarrows \pi$ can be obtained by first solving relativistic Faddeev equation and later by integrating that Faddeev solution; $m^{(2)}$, with given input $g_{\pm}^{J}$ (or $\gamma_{\pm}^{j}$) and $\Gamma_{\pm}^{(3)}$. So the central problem is to solve the Faddeev equation.

In the relations or equations for these amplitudes we notice that we do not have to have any renormalization to be done because it is hidden in higher irreducible structure of the amplitudes as well as of propagators. For our purpose we do not need to have the exposure of the amplitudes where more than three particles are exposed.

In the process of our analysis it seems to be true that Taylor's method really may be an alternative approach to a specific field theory. So as has been mentioned, we try to see if it actually can give unitarity structure in a given amplitude. We examine it by deriving the (generalized) unitarity of the amplitudes associated with our πNN system. That is shown in Appendix C. Our final remark is that although we leave out spin structure, it seems rather easy to put it in because what we have done is more of combinatorials than kinematics.

What we shall study next is the non-relativistic version of πNN problem. It seems that with some suitable approximations and assumptions we could reduce our equations for relativistic amplitudes to the ones in non-relativistic domain. Or we can use cut analysis in non-relativistic field theory.* But we shall take a different approach to obtain non-relativistic scattering equations for πNN system.

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*When cut analysis or irreducible decomposition of the amplitudes is considered in the framework of using global propagators, the concept of irreducibility changes slightly in some diagrams. The reason is that the time order is essential there and hence a cut should be drawn "perpendicular" to time coordinate axis.

B. Non-relativistic Construction of πNN Scattering Equations

Our approach in this section is through Hamiltonian-Schrödinger or Lippmann-Schwinger equation method. We first start with a non-relativistic model Hamiltonian which includes pion emission-absorption vertices. We then use a projection technique and obtain an infinite set of coupled equations where the quantities to be solved are the wave functions for two physical nucleons plus n pions ($n=0,1,2,\ldots$). The next procedure is to eliminate two-nucleon plus n-pion states for $n\geq 2$ to obtain an effective operator describing the effect coming from those multi-pion states and the coupled-equation reduces to the one where only two-nucleon state and two-nucleon plus one pion state couple. To avoid the problem of pion overcountion we should start with a Hamiltonian where N-N interactions are assumed to be purely non-pionic. For simplicity we regard this non-pionic N-N interaction to be given in terms of a static potential. (For simplicity we shall not consider "non-pionic" π-N interactions. This, however, can be included rather easily.) In connection with our non-relativistic approach there is a problem of Galilean invariance associated with the particle emission and absorption.(27) This will not be discussed in this thesis but will be reported elsewhere.

I. Preliminaries

Our assumptions on the form of the Hamiltonian are as follows:

1. For simplicity nucleons are considered distinguishable; we can put the effect of particle identity rather easily afterwards.
2. Spin-isospin degrees of freedom are not explicitly considered as this also can be taken into account easily.
3. π-π direct interaction is not considered. This may be taken into account in the form of heavy-meson exchange.
4. πN interaction is only through Yukawa type πNN coupling.
5. N-N interaction by heavy meson exchanges are taken care of by a static potential.

There is an additional thing that we should keep in mind which is connected with the non-relativistic treatment; anti-nucleons are ruled out because they correspond to the small components of the spinor in positive energy which disappear in the non-relativistic limit.

II Non-relativistic Hamiltonian Approach

Our Hamiltonian is assumed to take the form,

$$H = H_0 + \tilde{W} + U + U^{\dagger} \qquad \text{(3B-II-1)}$$

where H_o; kinetic energy of pions and nucleons

$\tilde{W}$; heavy meson exchange static N-N potential

$U(U^+)$; $U= \sum_i h_i$, h_i is a pi-absorption vertex for i-th nucleon

$U^+ = \sum_i h_i^+$, h_i^+ is a pi-emission vertex for i-th nucleon

This Hamiltonian is essentially what we have mentioned in Chap. 2. In our development from now on, no detailed forms are required for h_i (h_i^+) and $\tilde{W}$. The only restriction upon the form of h_i (h_i^+) is that it should correspond at least to two-particle irreducible πNN vertex, i.e. its irreducibility is not lower than one. The easiest realization of this requirement is that h_i (h_i^+) is independent of energy and depends only upon momentum transfer (relative momentum), etc. Whenever necessary, nucleon self-energy counter term may be explicitly added and subtracted in (3B-II-1).

We now begin with the Schrödinger equation:

$$(E - H)|\Psi\rangle = 0 \qquad \text{(3B-II-2)}$$

where $|\Psi\rangle$ contains, as an initial incoming state, two nucleons with or without a pion, and all of them are in plane wave states. $|\Psi\rangle$ also includes an out-going state with two nucleons and in principle an arbitrary number of pions. We then introduce a set of pion number projection operators $\{P_N\}$. The projection operator technique in general is fairly common in nuclear reaction theory as well as in the theory of effective interactions in nuclei (see, for example, Feshbach).[23-iii]

We define our set of projection operators as follows: A projection operator P_N picks up from $|\Psi\rangle$, an asymptotic state that contains N pions at large distance from two physical nucleons. This definition is not unique since it does not specify the separation of states or in other words channels at short distance. This ambiguity is fixed only in the final model where several input subamplitudes or potentials are given for the amplitude equations. The set of projection operators satisfies when acting on $|\Psi\rangle$:

$$\left.\begin{aligned} &\sum_{j=0}^{\infty} P_j = 1 \\ &P_n P_m = \delta_{nm} P_n \end{aligned}\right\} \qquad \text{(3B-II-3)}$$

Setting $P_n|\Psi\rangle \equiv |\phi_n\rangle$, we obtain from (3B-II-2):

$$(E - P_n H P_n)|\phi_n\rangle = -\sum_{m \neq n} P_n H P_m |\phi_m\rangle, \qquad \text{(3B-II-4)}$$
$$(n = 0, 1, \cdots\cdots).$$

Since $\tilde{W}$ does not contain pion variables, $P_n W P_m = \delta_{nm} W P_n$ holds. Also $P_n H_o P_m = \delta_{nm} H_o P_n$ and we set $H_o P_n \equiv h_n$. As for $P_n U P_m$ and $P_n U^+ P_m$ we know that

$$P_n U P_m = 0 \quad (n \neq m-1),$$

and

$$P_n U^+ P_m = 0 \quad (n \neq m+1).$$

Thus we set

$$\left.\begin{aligned} P_n U P_m &\equiv R_{n,m}\,\delta_{n,m-1} \\ P_n U^\dagger P_m &\equiv R^\dagger_{m,n}\,\delta_{n,m+1} \end{aligned}\right\}. \tag{3B-II-5}$$

When setting $\tilde{H}_n \equiv h_n + \tilde{W} P_n$ which is defined in $2N+n\pi$ Hilbert space, a set of projected equations are obtained by just rewriting (3B-II-4),

$$\left.\begin{aligned} (E-\tilde{H}_0)|\phi_0\rangle &= R_{0,1}|\phi_1\rangle \\ (E-\tilde{H}_1)|\phi_1\rangle &= R_{1,2}|\phi_2\rangle + R^\dagger_{0,1}|\phi_0\rangle \\ &\text{-------------} \\ (E-\tilde{H}_n)|\phi_n\rangle &= R_{n,n+1}|\phi_{n+1}\rangle + R^\dagger_{n-1,n}|\phi_{n-1}\rangle \\ &\text{-------------} \end{aligned}\right\}. \tag{3B-II-6}$$

This obviously is a set of infinite number of coupled equations for $|\phi_n\rangle$ $(n=0,1,\ldots\infty)$.

What we try to do next is to eliminate $|\phi_n\rangle$ $(n\geq 2)$ and obtain an "effective" set of coupled equations between $|\phi_0\rangle$ and $|\phi_1\rangle$. The procedure is similar to what is practiced in the theory of effective nuclear interactions or in optical potential theory.[23] First, let us set $|\phi_k\rangle=0$ for $k>N$ and see what we can get. Then (3B-II-6) can be solved algebraically and we find (taking into account the fact that there is no incoming wave with pion numbers more than one) that we can eliminate $|\phi_j\rangle$ for $j>2$ and obtain

$$|\phi_2\rangle = (E^+ - \tilde{H}_2 - D_2)^{-1} R^\dagger_{1,2} |\phi_1\rangle, \tag{3B-II-7}$$

where $D_j = R_{j,j+1} \frac{1}{E^+ - H_{j+1} - D_{j+1}} R^+_{j,j+1}$ $(j=2,\ldots,N)$ and $D_k=0$ for $k\geq N+1$. $|\phi_0\rangle$ and $|\phi_1\rangle$ satisfy the equations in (3B-II-6). We suppose then that (3B-II-7) is valid for $N\to\infty$ although there seems no way to establish it with mathematical rigor especially in operator relations. Physically, as long as the energy of the process in a given situation is finite, we can expect that $|\phi_k\rangle$ for large k has a very small norm and this may justify our assumption. Then D_2 can be expressed as a continued fraction in terms of operators. (Note that this continued fraction form can be obtained without assuming $|\phi_k\rangle=0$ for $k>N$). So if we go one step further and define $D_1 \equiv R_{1,2} \frac{1}{E^+ - H_2 - D_2} R^+_{1,2}$, the effective coupled equations become

$$\left.\begin{aligned} (E-\tilde{H}_0)|\phi_0\rangle &= R_{0,1}|\phi_1\rangle \\ (E-\tilde{H}_1-D_1)|\phi_1\rangle &= R^\dagger_{0,1}|\phi_0\rangle \end{aligned}\right\}, \tag{3B-II-8}$$

and

$$D_j = R_{j,j+1}(E^+ - \tilde{H}_{j+1} - D_{j+1})^{-1} R^\dagger_{j,j+1}, \quad (j=1,2,\cdots\cdots).$$

Formally what we have done is to set

$$R_{1,2}|\phi_2\rangle \equiv D_1(E)|\phi_1\rangle, \qquad (3B\text{-}II\text{-}8')$$

and thus put every information coming from $|\phi_k\rangle$ $(k>1)$ in $D_1(E)$. (3B-II-8') may be compared with the definition of scattering t-matrix; $T|\phi\rangle \equiv V|\psi^+\rangle$, or that of the effective interaction in nuclear shell model; $v|\tilde{\phi}\rangle = V|\Psi\rangle$, where ϕ and ψ^+ are plane wave and the total scattered plus incoming wave respectively, whereas $\tilde{\phi}$ and Ψ are the finite model space wave function and the exact eigenstate of the problem.

The coupled-equation (3B-II-8) is closed if our "effective interaction" D_1 is known and given from outside. In practice D_1 cannot be calculated exactly because D_∞ has to be known in the first place, which is impossible. In analogy with what we have in the last section, to know D_∞ is to expose in a given diagram every intermediate state up until an infinite number of intermediate particles appear. So in our spirit, what is necessary is to know the structure of D_1; its connectedness structure and irreducibility. This can be observed without a good deal of painful expansion of D_1 by its perturbation (or iteration) expansions. We think this to be plausible physically and shall put the study in D_i in Appendix D, thus only quote the result here. It says that we can write:

$$\mathcal{D}_1(E) = \sum_{j=1,2} \tilde{v}^{j}_{\pi N}(E) + \tilde{V}^{\pi}_{NN}(E) + C(E), \qquad (3B\text{-}II\text{-}9)$$

where (i) $\tilde{v}^{j}_{\pi N}(E)$ is an operator in πNN Hilbert space which is two-particle irreducible if restricted to πN Hilbert space. This we may regard as a πN potential (between a pion and j-th nucleon).

(ii) $\tilde{V}^{\pi}_{NN}(E)$ is identified as a N-N potential coming from pure pi exchanges and also from the mixed pi and heavy meson exchanges. $\tilde{V}^{\pi}_{NN}(E)$ is also in πNN Hilbert space.

(iii) C(E) is regarded as a three-body πNN potential as it is three-particle irreducible.

Note that because these operators are identified in terms of particle irreducibility, $\tilde{v}^{j}_{\pi N}$ does not contain contributions like direct Born term. Note also that they are energy dependent and off-energy-shell in general. One point to be mentioned before going forward is that we may use the same notations for operators both in two-particle and three-particle spaces. There will not occur any inconvenience due to that convention, but in the case where it introduces some confusion, we shall differentiate two operators acting in different spaces.

When D_1 is added to $\tilde{W}$, we obtain

$$V_3(E) \equiv V_{NN}(E) + \sum_j \tilde{v}^{j}_{\pi N}(E) + C(E), \qquad \text{(3B-II-10)}$$

where

$$V_{NN}(E) \equiv \tilde{W} + \tilde{V}^{\pi}_{NN}(E) \qquad \text{(3B-II-10')}$$

is the total N-N potential. Now (3B-II-8) becomes

$$\left.\begin{aligned} (E-\tilde{H}_0)|\phi_0\rangle &= R_{0,1}|\phi_1\rangle \\ (E-h_1-V_3(E))|\phi_1\rangle &= R^{+}_{0,1}|\phi_0\rangle \end{aligned}\right\}. \qquad \text{(3B-II-11)}$$

This is the basis towards our next step; obtaining various πNN amplitudes.

III. Wave Function Description of Various πNN Processes

(i) Wave functions for 3→3 and 3→2 processes

We take the initial state to be of πNN plane waves denoted as $|i_3\rangle$. Let the solution to $[E-h_1-V_3(E)]|\phi_1\rangle=0$ with incoming $|i_3\rangle$ be $|\chi_i^+\rangle$. $|\chi_i^+\rangle$ then satisfies

$$|\chi_i^+\rangle = |i_3\rangle + [E^+-h_1-V_3(E)]^{-1}V_3(E)|i_3\rangle. \qquad \text{(3B-III-1)}$$

We call $|\chi_i^+\rangle$ a "Faddeev state". In terms of this Faddeev state, $|\phi_1\rangle$ is expressed as

$$|\phi_1\rangle = |\chi_i^+\rangle + [E^+-h_1-V_3(E)]^{-1}R^{+}_{0,1}|\phi_0\rangle, \qquad \text{(3B-III-2)}$$

and the equation for $|\phi_0\rangle$ becomes

$$(E-\tilde{H}_0)|\phi_0\rangle = R_{0,1}|\chi_i^+\rangle + R_{0,1}[E^+-h_1-V_3(E)]^{-1}R_{0,1}^+|\phi_0\rangle. \qquad (3B\text{-}III\text{-}3)$$

In (3B-III-3) we pick up $X(E)\equiv R_{o,1}[E^+-h_1-V_3(E)]^{-1}R_{0,1}^+$ and study its structure. First it should effectively be an operator in two-nucleon Hilbert space. Its structure can be examined by utilizing the relation:

$$[E^+-h_1-V_3(E)]^{-1} = [E^+-h_1]^{-1} + [E^+-h_1]^{-1}T^F(E)[E^+-h_1]^{-1}, \qquad (3B\text{-}III\text{-}4)$$

where "Faddeev" amplitude $T^F(E)$ satisfies

$$T^F(E) = V_3(E) + V_3(E)[E^+-h_1]^{-1}T^F(E). \qquad (3B\text{-}III\text{-}4')$$

As $T^F(E)$ is written as (see Appendix B)

$$T^F(E) = \sum_i t_{\pi N}^i(E) + t_{NN}(E) + T^C(E), \qquad (3B\text{-}III\text{-}5)$$

where $T^C(E)$ is the connected part of T^F, we can calculate $X(E)$ and separate it into two parts (for detailed calculations, see Appendix E).

$$X(E) = X_u(E) + X_\ell(E). \qquad (3B\text{-}III\text{-}6)$$

Here $X_u(E)$ is an unlinked part and can be identified as the self-energy of two nucleons, whereas $X_\ell(E)$ is a linked part and is connected with N-N interaction. The structure of $X_u(E)$ and $X_\ell(E)$ will be more easily understood in Fig.

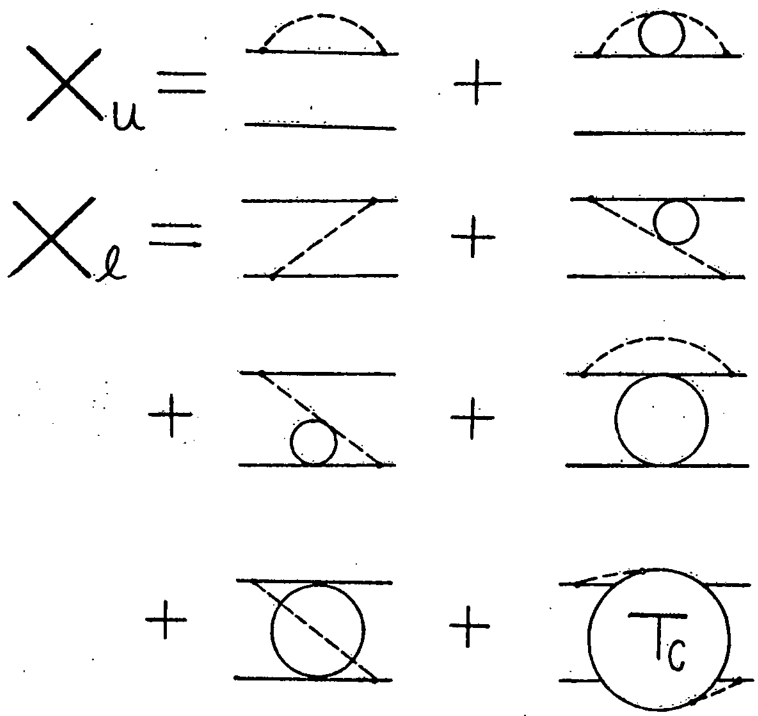


Fig. 3A-9

Diagrammatic structure of $X_u(E)$ and $X_\ell(E)$

3A-9. Since $X_\ell(E)$ is two-particle irreducible, it should be related to some part of the N-N potential, i.e. $\tilde{V}^{\pi}_{NN}(E)$. If we compare several lowest order diagrams for $\tilde{V}^{\pi}_{NN}(E)$ with those for $X_\ell(E)$, we find that they agree term by term. We may identify them as identical two-body operators, that is $\tilde{V}^{\pi}_{NN}(E) \equiv X_\ell(E)$, if we understand E now to be the energy in the two-body channel. This identification is reinforced by the fact, to be discussed later in this chapter, that $X_\ell(E)$ satisfies a discontinuity relation which is appropriate to the two-body potential, $\tilde{V}^{\pi}_{NN}(E)$. So adopting this identification, (3B-III-3) now is expressed as

$$[E - h_0 - V_{NN}(E) - X_u(E)]|\phi_0\rangle = R_{0,1}|\chi_i^+\rangle. \qquad \text{(3B-III-7)}$$

In (3B-III-7) $V_{NN}(E)$ also is understood to be in two-nucleon Hilbert space since it acts on the NN state. We may write $V_{NN}(E)$ for both NN and πNN Hilbert space with the same E. Strictly speaking the energy dependence should be differentiated for each case. But for simplicity we use the same E-dependence. We expect that there is no confusion. (The reason to keep the E-dependence is solely because we want to stress its dependence on energy.)

Writing $\tilde{G}_2(E) \equiv [E^+ - h_0 - X_u(E)]^{-1}$ which will turn out to be a propagator for two free, dressed nucleons (see section

C for the dispersion expression of $\tilde{G}_2(E)$), (3B-III-7) is rewritten: As will become clear later, the reason why we keep the self-energy operator $X_u(E)$ even after a supposed cancellation by mass counter term is to keep the πNN unitarity. Now equation (3B-III-7) becomes:

$$|\phi_0\rangle = [\tilde{G}_2^{-1}(E) - V_{NN}(E)]^{-1} R_{0,1} |\chi_i^+\rangle. \qquad (3B\text{-}III\text{-}8)$$

This is substituted back into (3B-III-2) and we find

$$|\phi_1\rangle = |\chi_i^+\rangle + [E^+ - h_1 - V_3(E)]^{-1} R_{0,1}^+ [\tilde{G}_2^{-1}(E) - V_{NN}(E)]^{-1} R_{0,1} |\chi_i^+\rangle. \qquad (3B\text{-}III\text{-}9)$$

This shows that the Faddeev state is distorted by explicit pion absorption and emission to produce $|\phi_1\rangle$; pion absorption-emission being expressed in terms of an operator $R_{0,1}^+[\tilde{G}_2^{-1}(E)-V_{NN}(E)]^{-1}R_{0,1}$. Also (3B-III-8) and (3B-III-9) show that we have only to solve an integral equation for $|\chi_i^+\rangle$ and use it as an input to obtain both $|\phi_0\rangle$ and $|\phi_1\rangle$ by integrations.

(ii) Wave functions for 2→3 and 2→2 processes

The boundary condition here is the one in which two nucleons are incident in a plane wave state. The wave functions for final NN and πNN states are denoted as $|\phi_0'\rangle$ and $|\phi_1'\rangle$ respectively which satisfy (3B-II-11). The boundary condition requires that $|\phi_1'\rangle$ is of the form:

$$|\phi_1'\rangle = [E^+ - h_1 - V_3(E)]^{-1} R_{0,1}^{\dagger} |\phi_0'\rangle , \qquad (3B\text{-}III\text{-}10)$$

whereas

$$|\phi_0'\rangle = |\tilde{i}_2\rangle + (E^+ - \tilde{H}_0)^{-1} R_{0,1} |\phi_1'\rangle, \qquad (3B\text{-}III\text{-}11)$$

where $|\tilde{i}_2\rangle$ is a solution of

$$(E^+ - \tilde{H}_0)|\phi\rangle = 0 .$$

with an outgoing wave boundary condition.

Combining (3B-III-10) and (3B-III-11) leads to

$$|\phi_0'\rangle = |\tilde{i}_2\rangle + (E^+ - \tilde{H}_0)^{-1} X(E) |\phi_0'\rangle . \qquad (3B\text{-}III\text{-}12)$$

This is easily transformed into

$$|\phi_0'\rangle = |\tilde{i}_2\rangle + [E^+ - H_0 - X(E)]^{-1} X(E) |\tilde{i}_2\rangle . \qquad (3B\text{-}III\text{-}12')$$

We now remember that $|\tilde{i}_2\rangle$ is expressed in terms of free two-nucleon state $|i_2\rangle$ by

$$|\tilde{i}_2\rangle = |i_2\rangle + (E^+ - \tilde{H}_0)^{-1} \tilde{W} |i_2\rangle, \qquad (3B\text{-}III\text{-}13)$$

and also remember an identity

$$(E - \tilde{H}_0)^{-1} = (E - \tilde{H}_0 - X)^{-1} - (E - \tilde{H}_0 - X)^{-1} X (E - \tilde{H}_0)^{-1} . \qquad (3B\text{-}III\text{-}13')$$

When all these expressions are put into (3B-III-12'), we find:

$$|\phi_0'\rangle = |i_2\rangle + [E^+ - \tilde{H}_0 - X(E)]^{-1}(X(E) + \tilde{W})|i_2\rangle . \qquad (3B\text{-}III\text{-}14)$$

As has been mentioned, $X_u(E)$ in $X(E)$ is a nucleon self-energy operator. So when a suitable counter term is added and subtracted in H, it is expected that $X_u(E)$ vanishes on-energy-shell. Thus we require $X_u(E)|i_2\rangle$ to vanish; $X_u(E)|_{on-energy-shell}=0$ is of course a requirement in analogy with the similar situation in relativistic quantum field theory where the self-energy-part vanishes on-mass-shell.

Keeping in mind what we have discussed above and remembering that $V_{NN}(E)=X_\ell(E)+\tilde{W}$, we arrive at

$$\begin{aligned} |\phi_0'\rangle &= |i_2\rangle + [E^+ - h_0 - V_{NN}(E) - X_u(E)]^{-1} V_{NN}(E)|\phi_0'\rangle \\ &= |i_2\rangle + \tilde{G}_2(E) V_{NN}(E) |\phi_0'\rangle \end{aligned} \qquad (3B\text{-}III\text{-}14')$$

from (3B-III-14). This is of Lippmann-Schwinger type.

As for $|\phi_1'\rangle$ we obtain it using (3B-III-10) once (3B-III-14') is solved for $|\phi_0'\rangle$.

Looking at (3B-III-8), (3B-III-9), (3B-III-10) and (3B-III-14') we find that although we started with a coupled-equation, it has formally decoupled. By decoupling we mean that only Faddeev state $|\chi_i^+\rangle$ and two-nucleon state

$|\phi_0'\rangle$ should be obtained by solving integral equations. Once this is done, other necessary wave functions are gained by integrations. This however, does not mean the complete decoupling since $V_{NN}(E)$ in (3B-III-14') is implicitly related to the Faddeev scattering state. So the decoupling is true if we are able to specify $V_{NN}(E)$ by other means.

This may be compared with the relativistic expressions for various amplitudes for the πNN system shown in the last section. A detailed discussion will be given in the next chapter.

IV. t-matrix Equations

In this section we shall transform the wave function expressions into those in terms of t-matrices. There are several ways in defining the t-matrix using wave function equations, which after all turn out to be equivalent.[5] We shall adopt a definition in which the t-matrix element is identified as a residue of the free propagator which can be identified in the homogeneous part of the equation when the equation is multiplied by a suitable state vector from the left.

(i) $3\to 3$ process

(a) Faddeev state

From (3B-III-1) we can easily identify the t-matrix T^F as

$$\langle b_3 | T^F(E) | a_3 \rangle = \langle b_3 | V_3(E) | \chi_a^+ \rangle. \qquad \text{(3B-IV-1)}$$

In (3B-IV-1) subscript 3 indicates that the state is that of three particles. Note that $T^F(E)$ thus identified does satisfy (3B-III-4').

(b) t-matrix associated with $|\phi_1\rangle$ ($\equiv T_{33}(E)$)

Making use of (3B-III-1) and (3B-III-4) we find that

$$\begin{aligned}\langle f_3 | T_{33}(E) | i_3 \rangle \equiv & \langle f_3 | V_3(E) | \chi_i^+ \rangle + \langle f_3 | R_{0,1}^+ [\tilde{G}_2^{-1}(E) - V_{NN}(E)]^{-1} R_{0,1} | \chi_i^+ \rangle \\ & + \langle f_3 | T^F(E) [E^+ - h_1]^{-1} R_{0,1}^+ [\tilde{G}_2^{-1}(E) - V_{NN}(E)]^{-1} R_{0,1} | \chi_i^+ \rangle \\ = & \langle f_3 | V_3(E) | i_3 \rangle + \langle f_3 | N(E) | \chi_i^+ \rangle \\ & + \sum_\ell \langle f_3 | T^F(E) | \ell_3 \rangle (E^+ - E_\ell)^{-1} \langle \ell_3 | N(E) | \chi_i^+ \rangle. \end{aligned} \qquad \text{(3B-IV-2)}$$

In (3B-IV-2) we have used $N(E) \equiv R_{0,1}^+ [\tilde{G}_2^{-1}(E) - V_{NN}(E)]^{-1} R_{0,1}$. When $\tilde{t}_{NN}(E)$ is defined to satisfy

$$\tilde{t}_{NN}(E) = V_{NN}(E) + V_{NN}(E) \tilde{G}_2(E) \tilde{t}_{NN}(E), \qquad \text{(3B-IV-3)}$$

in analogy with usual N-N t-matrix equation;

$$t_{NN}(E) = V_{NN}(E) + V_{NN}(E) G_2(E) t_{NN}(E), \quad (G_2(E) \equiv (E^+ - h_0)^{-1}).$$

then N(E) becomes

$$N(E)=R_{0,1}^{\dagger}\left[\tilde{G}_{T_2}(E)+\tilde{G}_{T_2}(E)\tilde{t}_{NN}(E)\tilde{G}_{T_2}(E)\right]R_{0,1}\ . \qquad \text{(3B-IV-2')}$$

This and $[E^{+}-h_1]^{-1}\equiv G_3(E)$, $W\equiv R_{0,1}^{+}$ and $Y\equiv R_{0,1}$ let (3B-IV-2) become

$$\langle f_3|T_{33}(E)|i_3\rangle = \langle f_3|T^{\bar{H}}(E)|i_3\rangle + \langle f_3|N(E)|i_3\rangle$$

$$+\sum_{\ell}\langle f_3|T^{\bar{H}}(E)|\ell_3\rangle G_3(E,\ell)\langle \ell_3|N(E)|\chi_i^{+}\rangle$$

and

$$\langle f_3|N(E)|\chi_i^{+}\rangle = \sum_{b}\langle f_3|W|b_2\rangle \tilde{G}_{T_2}(E,b)\Big\{\langle b_2|Y|\chi_i^{+}\rangle + \sum_{c}\langle b_2|\tilde{t}_{NN}(E)|c_2\rangle \qquad \text{(3B-IV-4)}$$

$$\times\, \tilde{G}_{T_2}(E,c)\langle c_2|Y|\chi_i^{+}\rangle\Big\},$$

where $G_3(E,\ell)=\langle\ell|(E^{+}-h_1)^{-1}|\ell\rangle$ and $|b_2\rangle$ represents a two-nucleon plane wave state. A more transparent form will be given after we obtain t-matrices for 3→2 process.

(ii) t-matrix for 3→2 process

Noting that $[\tilde{G}_2^{-1}(E)-V_{NN}(E)]^{-1}=\tilde{G}_2(E)+\tilde{G}_2(E)\tilde{t}_{NN}(E)\tilde{G}_2(E)$ as in (3B-IV-2'), (3B-III-8) is reexpressed to be

$$|\phi_0\rangle = \left[\tilde{G}_2(E)+\tilde{G}_{T_2}(E)\tilde{t}_{NN}(E)\tilde{G}_{T_2}(E)\right]Y|\chi_i^{+}\rangle. \qquad \text{(3B-IV-5)}$$

From this expression the corresponding t-matrix is straightforwardly derived,

$$\langle f_2|T_{23}(E)|i_3\rangle \equiv \langle f_2|Y|\chi_i^+\rangle + \sum_{b} \langle f_2|\tilde{t}_{NN}(E)|b_2\rangle \tilde{G}_2(E,b)\langle b_2|Y|\chi_i^+\rangle. \tag{3B-IV-6}$$

If we express $|\chi_i^+\rangle$ in terms of $|i_3\rangle$ (see (3B-III-1)), it is clear that

$$\langle f_2|Y|\chi_i^+\rangle = \langle f_2|Y|i_3\rangle + \sum_{e}\langle f_2|Y|e_3\rangle G_3(E,e)\langle e_3|T^{A}(E)|i_3\rangle, \tag{3B-IV-7}$$

which is plugged into (3B-IV-6) and we find

$$\begin{aligned}\langle f_2|T_{23}(E)|i_3\rangle = {} & \langle f_2|Y|i_3\rangle + \sum_{e}\langle f_2|Y|e_3\rangle G_3(E,e)\langle e_3|T^{A}(E)|i_3\rangle \\ & + \sum_{b}\langle f_2|\tilde{t}_{NN}(E)|b_2\rangle \tilde{G}_2(E,b)\Big\{\langle b_2|Y|i_3\rangle \\ & + \sum_{e}\langle b_2|Y|e_3\rangle G_3(E,e)\langle e_3|T^{A}(E)|i_3\rangle\Big\}.\end{aligned} \tag{3B-IV-6'}$$

We symbolically write this as

$$T_{23}(E) = Y + YG_3T^{A} + \tilde{t}_{NN}G_2(Y + YG_3T^{A}). \tag{3B-IV-6''}$$

Furthermore, if we define

$$\tilde{T}_{23} \equiv Y + YG_3T^{A}, \tag{3B-IV-8}$$

the expression for T_{23} becomes

$$T_{23} = \tilde{T}_{23} + \tilde{t}_{NN} \tilde{G}_{T_2} \tilde{T}_{23} . \qquad (3B\text{-}IV\text{-}9)$$

Note that this expression is of the same form as that of $\hat{\Gamma}_{-}^{(1)}$ and also the expression for $\tilde{T}_{23}$ is the same as that of $\hat{\Gamma}_{-}^{(2)}$ in their forms.

(iii) $T_{33}(E)$ in more compact form

We can express $T_{33}(E)$ in a simpler way just using (3B-IV-4) and (3B-IV-6). The result is

$$T_{33}(E) = T^{\bar{A}}(E) + W\tilde{G}_{T_2} T_{23}(E) + T^{\bar{A}}(E) G_{T_3} W \tilde{G}_{T_2} T^{23}(E), \qquad (3B\text{-}IV\text{-}4')$$

which will be further simplified later using $\tilde{T}_{32}(E)$ and $\tilde{T}_{32}(E)$.

(iv) 2→2 process

The t-matrix for this process is easily found to be

$$\begin{aligned} \langle f_2 | T_2(E) | i_2 \rangle &\equiv \langle f_2 | V_{NN}(E) | \phi_0' \rangle \qquad (3B\text{-}IV\text{-}9) \\ &= \langle f_2 | V_{NN}(E) | i_2 \rangle + \sum_{b} \langle f_2 | V_{NN}(E) | b_2 \rangle \\ &\quad \times \tilde{G}_{T_2}(E, b) \langle b_2 | V_{NN}(E) | \phi_0' \rangle . \end{aligned}$$

This means that

$$T_2(E) \equiv \tilde{t}_{NN}(E). \qquad (3B\text{-}IV\text{-}9')$$

(v) t-matrix for 2→3 process

From (3B-III-10) we obtain

$$|\phi_1'\rangle = (E^+ - h_1)^{-1} R^+_{0,1} |\phi_0'\rangle + (E^+ - h_1)^{-1} T^{F}(E)(E^+ - h_1)^{-1} R^+_{0,1} |\phi_0'\rangle, \quad \text{(3B-IV-10)}$$

from which it is easy to find that

$$\langle f_3|T_{32}(E)|i_2\rangle \equiv \langle f_3|W|\phi_0'\rangle + \sum_{\ell} \langle f_3|T^{F}(E)|\ell_3\rangle G_3(E,\ell)\langle \ell_3|W|\phi_0'\rangle. \quad \text{(3B-IV-11)}$$

With the help of (3B-III-14') and (3B-IV-9), (3B-IV-11) is transformed into

$$\begin{aligned} \langle f_3|T_{32}(E)|i_2\rangle = {} & \langle f_3|W|i_2\rangle + \sum_a \langle f_3|W|a_2\rangle \tilde{G}_2(E,a)\langle a_2|\tilde{t}_{NN}(E)|i_2\rangle \\ & + \sum_c \langle f_3|T^{F}(E)|c_3\rangle G_3(E,c)\Big\{\langle c_3|W|i_2\rangle \\ & + \sum_a \langle c_3|W|a_2\rangle \tilde{G}_2(E,a)\langle a_2|\tilde{t}_{NN}(E)|i_2\rangle\Big\}, \end{aligned} \quad \text{(3B-IV-11')}$$

which is further simplified by introducing

$$\tilde{T}_{32} \equiv W + T^{F} G_3 W. \quad \text{(3B-IV-12)}$$

The result is

$$T_{32} = \tilde{T}_{32} + \tilde{T}_{32}\tilde{G}_2\tilde{t}_{NN}. \quad \text{(3B-IV-13)}$$

Note that there also is a correspondence between $(\tilde{T}_{32}, T_{32})$ and $(\hat{\Gamma}_{+}^{(2)}, \hat{\Gamma}_{+}^{(1)})$ as is the case with $(\tilde{T}_{23}, T_{23})$.

(vi) T_{33} in a simpler form

When we look at the expressions for $T_{23}, \tilde{T}_{23}$, etc. it is rather easy to see that T_{33} can be expressed in a more compact way. We just write down the result because it is easy to obtain it.

$$T_{33}(E) = T^{\pi}(E) + \tilde{T}_{32}(E)\tilde{G}_{T_2}(E)T_{23}(E) \tag{3B-IV-14}$$

$$= T^{\pi}(E) + T_{32}(E)\tilde{G}_{T_2}(E)\tilde{T}_{23}(E). \tag{3B-IV-14'}$$

Note that the form of T_{33} thus obtained does satisfy the required form imposed on the amplitude when pion absorption occurs in intermediate states as has been stated at the end of Chapter II.

Before closing this section it would be better to summarize the result. By means of projection technique applied to Schrödinger type equation, we have obtained a set of (infinite number of) coupled equations, each amplitude which is the solution of these coupled equations describes a state containing two physical nucleons and a certain number of pions. Through defining the effective interaction D_1, we have reduced the equations to the ones which couple only NN and πNN states. The equations were then converted into the ones for the t-matrices. As we started with a Hamiltonian which does not include any danger of pion overcounting, the resultant amplitudes are free from it. As we closely look at the equations for them, the overcounting-free nature should be easily observed.

As for the form of the derived relations or equations for the amplitudes, it should be mentioned that Thomas[17], in his thesis, obtained the expression for the non-relativistic NN → πd t-matrix which is essentially the same as our T_{32} in (3B-IV-13). His method was a diagram counting which may be regarded as a less stringent Taylor's method. As Thomas did not seem interested in the unitarity of the amplitude, the off-shell nucleon self-energy effect in the N-N propagators did not appear there.

C. Studies in $X(E)$, $V_{NN}(E)$, $\tilde{G}_2(E)$ and $t_{\pi N}(E)$

Before going to the next chapter, we have a close look at several points in our non-relativistic amplitudes. Here we use the discontinuity expression (1) to see that our assertion of $X_\ell(E) \equiv \tilde{V}^{\pi}_{NN}(E)$ (see the last chapter) is plausible, (2) to obtain a possible form of the N-N potential $V_{NN}(E)$, which is valid above single pion production threshold, (3) to obtain a simpler form for $\tilde{G}_2(E)$;

two-nucleon free propagator which contains an off-energy-shell nucleon self-energy effect and (4) to give explicit forms for the π-N t-matrix including the direct Born term.

I. X(E)

In the previous section we defined X(E) which consists of two distinct parts (see Appendix E for details); $X_u(E)$ and $X_\ell(E)$. It is easy to see that $X_u(E)$ should be identified as the self-energy of two nucleons. But as for $X_\ell(E)$ its identification with $\tilde{V}^{\pi}_{NN}(E)$ was not very obvious in the last section.

It now seems that there is a better way to establish the equality $X_\ell(E)=\tilde{V}^{\pi}_{NN}(E)$; we look at the unitarity (or discontinuity) structure of $X_\ell(E)$. According to (C2-21) of Appendix C, a relativistic two-particle irreducible $2\to2$ amplitude has an off-mass-shell three-particle discontinuity relation shown there. Since the two-particle irreducible amplitude should be regarded as a two-body potential, (C2-21) may be considered as a relation that a two-body potential is expected to satisfy. We think that a similar relation should be satisfied by the non-relativistic potential. Therefore we are trying to obtain the discontinuity $\Delta_3 X_\ell(E)$ across the πNN elastic unitarity cut. First, we may assume that the heavy meson exchange part of N-N potential does not contribute to $\Delta_3 V_{NN}(E)$. This seems to be a reasonable assumption; even if it has cuts, they would be far from πNN elastic threshold. So we may have $\Delta_3[\tilde{W}+X_\ell(E)]=\Delta_3 X_\ell(E)$. As X(E) is written in the following form

$$X(E) = Y(E^+-h_1)^{-1}W + Y(E^+-h_1)^{-1}T^F(E)(E^--h_1)^{-1}W, \qquad (3C\text{-}I\text{-}1)$$

and using the discontinuity relation for the Faddeev amplitude:

$$\Delta_3 T^F(E) = T^F(E^-) I_3 T^F(E), \qquad (3C\text{-}I\text{-}2)$$

where $I_3 \equiv -2\pi i\delta(E-h_1)$, it is straightforward to show that

$$\Delta_3 X(E) = \tilde{T}_{23}(\bar{E}) I_3 \tilde{T}_{32}(E). \qquad (3C\text{-}I\text{-}3)$$

On the other hand, using the expression for $X_u(E)$ in Appendix E (E-11), we find that

$$\Delta_3 X_u(E) = \sum_{i=1,2} \Lambda_i(E^-) I_3 \Gamma_i(E). \qquad (3C\text{-}I\text{-}4)$$

In (3C-I-4) we adopted the definition in Appendix E;

$$\left.\begin{aligned} \Lambda_i(E) &\equiv Y_i + Y_i G_3 t^i_{\pi N}(E) \\ \Gamma_i(E) &\equiv W_i + t^i_{\pi N} G_3 W_i \end{aligned}\right\}, \qquad (3C\text{-}I\text{-}5)$$

and their discontinuity relations

$$\left.\begin{aligned} \Delta_3 \Lambda_i(E) &= \Lambda_i(E^-) I_3 t^i_{\pi N}(E) \\ \Delta_3 \Gamma_i(E) &= t^i_{\pi N}(E^-) I_3 \Gamma_i(E) \end{aligned}\right\}. \qquad (3C\text{-}I\text{-}6)$$

Note that (Λ_i, Γ_i) corresponds exactly to relativistic (γ_-^i, γ_+^i). From (3C-I-3) and (3C-I-4) we readily find:

$$\begin{aligned}\Delta_3 X_\ell(E) &= \tilde{T}_{23}(E^-) I_3 \tilde{T}_{32}(E) - \sum_{i=1,2} \Lambda_i(E^-) I_3 \Gamma_i(E) \\ &\equiv \left[\tilde{T}_{23}(E^-) I_3 \tilde{T}_{32}(E)\right]_\ell ,\end{aligned} \qquad (3C\text{-}I\text{-}7)$$

which exactly corresponds to (C2-21). So we can more confidently take $X_\ell(E)=\tilde{V}^\pi_{NN}(E)$ to be relevant.

As will be mentioned in the next subsection, $X_\ell(E)$ has other cuts than that from πNN elastic unitarity, one of which is due to πd elastic scattering that starts slightly below the branch point of πNN cut.

II. N-N Potential

As has been mentioned in Chapter III, section A, generalized potentials, which are defined in terms of particle irreducibility, are consistent with those which we actually encounter, e.g. realistic nuclear potentials, etc. The former are more general than the latter in that they are energy dependent and in general off-shell (see, for example, Feshbach[(23-iii)]). This means that they may well have some singularity structure which is associated with many-particle unitarity cuts, etc. The notion of off-shell (off-energy or off-mass) potentials may not be new but this seems necessary especially for N-N potentials used in pi-nucleus scattering problems at relatively high energy. The reason is that, as has been repeatedly pointed out, there is no distinction in principle between "scattered" and "exchanged" pions. Thus the "off-shell-ness" of N-N potentials is a natural direction. This off-shell nature of N-N potential allows in a natural way the introduction of inelasticity due to the virtual pion production.

In our non-relativistic model N-N potential is identified as

$$V_{NN}(E) = \tilde{W} + \tilde{V}^{\pi}_{NN}(E), \qquad (3C\text{-}II\text{-}1)$$

where $\tilde{V}^{\pi}_{NN}(E) = X_{\ell}(E)$ (see the last subsection). Here $\tilde{W}$ is a pure heavy meson part. As $\tilde{V}^{\pi}_{NN}(E)$ is off-energy shell, so is $V_{NN}(E)$.

We may construct $\tilde{W}$ on a semi-empirical basis, for example, by adopting models of heavy mesons. But the difficulty is that we should fit $V_{NN}(E)$ but not $\tilde{W}$ to known physical quantities associated with N-N. Also we should keep in mind that $V_{NN}(E)$ is an off-energy-shell potential. Therefore the determination of $V_{NN}(E)$ seems rather difficult.

As shown in (3C-I-3), there should be a definite discontinuity relation for off-energy-shell $\tilde{V}^{\pi}_{NN}(E)$ (or $X_{\ell}(E)$) across the πNN elastic cut;

$$\Delta_3 \tilde{V}^{\pi}_{NN}(E) = [\tilde{T}_{23}(E^-) I_3 \tilde{T}_{32}(E)]_{\ell} \qquad (3C\text{-}II\text{-}2)$$

valid at least below the second pion production threshold. As is discussed in Appendix C, general $\Delta\tilde{V}^{\pi}_{NN}(E)$ below the second pion production threshold has another discontinuity coming from π-d elastic cut. This is written as

$$\Delta_d \tilde{V}^{\pi}_{NN}(E) \equiv \tilde{\mathcal{J}}_{2d}(E^-) I_d \tilde{\mathcal{J}}_{d2}(E), \tag{3C-II-3}$$

where $\tilde{\mathcal{J}}_{d2}$ ($\tilde{\mathcal{J}}_{2d}$) is a two-particle irreducible NN→πd (πd→NN) t-matrix and I_d is a delta-function factor for an elastic πd process (see Appendix C).

Remembering our assumption on $\tilde{W}$, we can write $\Delta V_{NN}(E)=\Delta\tilde{V}^{\pi}_{NN}(E)$. $\Delta\tilde{V}^{\pi}_{NN}(E)$ is the quantity that we can calculate in terms of $T^F(E)$ and πNN vertices. So we may obtain $V_{NN}(E)$ by dispersing it with respect to off-shell-energy E; using once subtracted form we obtain

$$V_{NN}(E) = V_{NN}(R) + \frac{E-R}{2\pi i}\int_{\text{threshold}}^{\infty} \frac{\Delta V_{NN}(z)\,dz}{(z-R)(z-E^+)} + P(E) - P(R), \tag{3C-II-4}$$

where R is a subtraction point and P(E) is due to possible "left hand cuts" and states with more than three particles. When the left hand cuts are rather far from πd threshold, P(E)-P(R) is expected to be small as long as R is not far from physical region. This may actually be made possible by choosing R to be real, not far from but below the threshold. Then we could regard $V_{NN}(R)$ to be some phenomenological static potential. This is why once subtracted form is used. So finally we have obtained as a possible form of off-energy-shell N-N potential

$$V_{NN}(E) \cong v_{NN}^{ph} + \frac{E-R}{2\pi i}\int_{\text{threshold}}^{\infty} \frac{\Delta V_{NN}(z)\,dz}{(z-R)(z-E^{+})} \quad . \tag{3C-II-5}$$

In the above expression v_{NN}^{ph} is some phenomenological potential.

What we have described is just one possible construction of off-energy-shell potential and this will not be carried out further in our study here in the thesis. It may be worth while to find some other possible construction. There have been several works on the construction of N-N potential via Bethe-Salpeter equation with some reductions(24) like using Blankenbecler-Sugar kernel(25) in the propagator part. This may be one possible direction that we should pursue.

III. $\tilde{G}_2(E)$

Our aim here is to obtain the expression for $\tilde{G}_2(E)$ without containing $X_u(E)$ as it is often convenient that we eliminate its explicit appearance.

We first note that we can write that, since $\tilde{G}_2(E)=[E^{+}-h_0-X_u(E)]^{-1}$,

$$\begin{aligned} \tilde{G}_{2}(E) &= G_{2}(E) + G_{2}(E)X_u(E)\tilde{G}_{2}(E) \\ &= G_{2}(E) + \tilde{G}_{2}(E)X_u(E)G_{2}(E) \ , \end{aligned} \tag{3C-III-1}$$

where $G_2(E)=[E^+-h_0]^{-1}$. After some algebra it is not difficult to show that

$$\Delta\tilde{G}_2(E) = \tilde{G}_2(E^-)\Delta X_u(E)\tilde{G}_2(E) + \{1+\tilde{G}_2(E^-)X_u(E^-)\}\Delta G_2(E) \times\{1+X_u(E)\tilde{G}_2(E)\}. \qquad \text{(3C-III-2)}$$

In (3C-III-2) Δ may mean a general discontinuity, but in our present case Δ means $\Delta=\Delta_2+\Delta_3$; the discontinuity across NN and πNN elastic cut.

Since $\Delta G_2(E)=-2\pi i\delta(E-h_0)\equiv I_2$, $X_u(E^{\pm})$ in (3C-II-2) should be evaluated on-energy-shell, which means that there is no contribution from them (see the discussion between (3B-III-14) and (3B-III-14') in section B). So we arrive at the expression

$$\Delta\tilde{G}_2(E) = I_2 + \tilde{G}_2(E^-)\sum_{i=1,2}\Lambda_i(E^-)I_3\Gamma_i(E)\tilde{G}_2(E). \qquad \text{(3C-III-3)}$$

Again we find that (3C-II-3) corresponds to the discontinuity relation of d(s); relativistic dressed single-particle propagator. (See (C4-4) of Appendix C.)

We have noticed that $\Delta\tilde{G}_2(E)$ does not explicitly contain $X_u(E)$. Therefore it may be used to obtain $\tilde{G}_2(E)$ which is free from $X_u(E)$. In a proper expression where the matrix element of (3C-III-3) is taken (in two-nucleon center of mass system), (3C-III-3) becomes

$$\Delta\tilde{G}_{T_2}(E,\vec{P}) = -2\pi i\,\delta(E-\tfrac{P^2}{m}-2m)+\tilde{G}_2(E^-,\vec{P})\int d^3q\sum_i \Lambda_i(E^-,\vec{P},\vec{q})$$
$$\times\Big[-2\pi i\,\delta\big(E-2m-\mu-\tfrac{P^2}{2m}-\tfrac{P^2}{2(m+\mu)}-\tfrac{q^2}{2\mu'}\big)\Big] \tag{3C-III-4}$$
$$\times\,\Gamma_i(E,\vec{P},\vec{q})\,\tilde{G}_2(E,\vec{P}),$$

where m,μ are nucleon and pion masses respectively, μ' pi-nucleon reduced mass and Γ_i (and Λ_i) is abbreviated in its momentum dependence; properly speaking $\Gamma_i(E,\vec{P},\vec{q})$ should be replaced by $\Gamma_i(E-\frac{P^2}{2(m+\mu)}-\frac{P^2}{2m}-2m,\vec{q})$ etc. Note that we include rest masses in total energy.

When we assume that the "Hermitian analyticity" or "reality" holds for $\Lambda_i(E)$ and $\Gamma_i(E)$, which is quite reasonable (see (C1-11) of Appendix C and the discussion immediately before it, see also Appendix F), we have the relation $\Lambda_i(E^-,\vec{P},\vec{q})=\Lambda_i^*(E,\vec{P},\vec{q})$ and therefore $\tilde{G}_2(E^-,\vec{P}) = \tilde{G}_2^*(E,\vec{P})$ through $X_u(E)$ (since $X_u(E^-)=X_u^*(E^-)$). Then (3C-II-4) is now

$$\Delta\tilde{G}_2(E) = -2\pi i\,\delta(\epsilon-\tfrac{P^2}{m})-2\pi i\,|\tilde{G}_{T_2}(E,\vec{P})|^2 \tag{3C-III-5}$$
$$\times\int d^3q\,\delta\big(\epsilon-\mu-\tfrac{P^2}{2(m+\mu)}-\tfrac{P^2}{2m}-\tfrac{q^2}{2\mu'}\big)\sum_i|\Gamma_i(E,\vec{P},\vec{q})|^2.$$

In (3C-III-5) we write $\epsilon\equiv E-2m$.

We next apply dispersion relation to (3C-III-5) and obtain $\tilde{G}_2(E,\vec{P})$. First we set

$$\tilde{H}_2(E,\vec{P}) \equiv \tilde{G}_2(E,\vec{P})^{-1}(\varepsilon - \frac{P^2}{m})^{-1}, \qquad (3C\text{-}III\text{-}6)$$

Then as $\tilde{G}_2(E,\vec{P})$ has a simple pole at $E=2m+\frac{P^2}{m}$ with unit residue, we have a condition on $\tilde{H}_2$; $\tilde{H}_2(2m+\frac{P^2}{m},\vec{P})=1$. Now the discontinuity of $\tilde{H}_2$ becomes

$$\Delta \tilde{H}_2(E,\vec{P}) = (\varepsilon - \frac{P^2}{m})^{-1}(2\pi i)\int d^3q \sum_i |\Gamma_i(E,\vec{P},\vec{q})|^2 \times \delta(\varepsilon - \mu - \frac{P^2}{2m} - \frac{P^2}{2(m+\mu)} - \frac{q^2}{2\mu'}), \qquad (3C\text{-}III\text{-}7)$$

and we have solved this to find

$$\tilde{H}_2(E,\vec{P}) = 1 + (\varepsilon - \frac{P^2}{m})\int d^3q \sum_i |\Gamma_i(\hat{E},\vec{P},\vec{q})|^2 \Big/ (\hat{E}-E^+)(\hat{E}-2m-\frac{P^2}{m})^2, \qquad (3C\text{-}III\text{-}8)$$

where

$$\hat{E} = 2m+\mu+\frac{P^2}{2m}+\frac{P^2}{2(m+\mu)}+\frac{q^2}{2\mu'}.$$

Finally (3C-III-6) is used to obtain $\tilde{G}_2(E,\vec{P})$.

IV. πN t-matrix

In solving the Faddeev equation for $T^F(E)$, we put into the equation either two-particle πN amplitudes (i.e. πN potential) or more practically one-particle irreducible πN amplitudes (i.e. πN t-matrix) as known functions together with similar quantities for N-N interaction.

On the other hand there also exists single-particle reducible πN amplitude because of pion absorption and re-emission through πNN vertices. This part may be called the generalized Born term as may have been termed elsewhere. We write this contribution as $B\pi N$ and see what it looks like.

Experimental πN scattering information can be related to the total πN t-matrix which is

$$\Theta_{\pi N}(E) \equiv B_{\pi N}(E) + t_{\pi N}(E). \tag{3C-IV-1}$$

$t_{\pi N}(E)$ in the above expression is a one-particle irreducible t-matrix that satisfies Lippmann-Schwinger equation; $t_{\pi N}(E)=\tilde{v}_{\pi N}(E)+\tilde{v}_{\pi N}(E)G_2 t_{\pi N}(E)$ and is an imput to the equation for $T^F(E)$.

The method of determining the form of $B\pi N(E)$; the generalized direct Born term, is to make use of unitarity (or discontinuity) as has been adopted to study $X_\ell(E)$ etc. in the previous subsection. First we assume the form of $B\pi N(E)$ to be

$$B_{\pi N}(E) \equiv L(E)\Pi(E)K(E), \tag{3C-IV-2}$$

as it should be similar to the bare Born term; the amplitude should have the form corresponding to the process: pi-absorption → nucleon propagation → pi-emission.

Two-particle unitarity requires

$$\Delta_2 \Theta_{\pi N}(E) = \Theta_{\pi N}(E^-) I_2 \Theta_{\pi N}(E), \tag{3C-IV-3}$$

and as

$$\Delta_2 t_{\pi N}(E) = t_{\pi N}(E^-) I_2 t_{\pi N}(E), \tag{3C-IV-4}$$

which is easy to see, we obtain the following relations from (3C-IV-3)

$$\left.\begin{aligned} \Delta_2 L(E) &= t_{\pi N}(E^-) I_2 L(E) \\ \Delta_2 K(E) &= K(E^-) I_2 t_{\pi N}(E) \\ \Delta_2 \pi(E) &= \pi(E^-) K(E^-) I_2 L(E) \pi(E) \end{aligned}\right\}. \tag{3C-IV-5}$$

As is easily observed, the first two relations are satisfied by $\Gamma(E)$ and $\Lambda(E)$ respectively (see (3C-I-6)) when these vertices are restricted to N and πN Hilbert space.

In our present theory we assume that we may simply identify:

$$\left.\begin{aligned} L(E) &\equiv \Gamma(E) \\ \text{and} \quad K(E) &\equiv \Lambda(E) \end{aligned}\right\} \tag{3C-IV-6}$$

Now we are left with only the third relation in (3C-IV-5). We should remember that $\pi(E)$ has a pole at the physical nucleon mass in πN center of mass system. The method of obtaining $\pi(E)$ from these pieces of information is just the same as used to find $\tilde{G}_2(E,\vec{P})$. So in πN center of mass system we obtain

$$\Pi(E) = (E-m)^{-1} H_1(E)^{-1}, \tag{3C-IV-7}$$

where

$$H_1(E) = 1 + (E-m)\int \frac{d^3q}{(\tilde{E}-E^+)(\tilde{E}-m)^2} \left|\Gamma(\tilde{E}, \vec{p}=0, \vec{q})\right|^2, \tag{3C-IV-8}$$

and $\tilde{E} \equiv m+\mu + q^2/2\mu'$.

CHAPTER IV

SUMMARY OF OUR FORMAL STUDIES

In this chapter we first make a comparison between our relativistic and non-relativistic formulations. Then we summarize our studies of πNN interaction by answering the questions appearing in Chapter II. Lastly we extend the conclusions obtained for πNN scattering to general pi-nucleus scattering.

A. Comparison of Two Formulations

When we look at those relativistic and non-relativistic amplitudes of section A and B in Chapter 3, together with the equations that the amplitudes do satisfy, there seems to be a formal one to one correspondence between relativistic and non-relativistic amplitudes. Below, we list the correspondence among them (the bracketed numbers following the amplitudes are their addresses in Chapter 3).

$$\hat{m}^{(1)}: \text{(3A-IV-15)} \leftrightarrow T_{33}: \text{(3B-IV-14)}$$

$$\hat{\Gamma}^{(1)}_{\pm}: \text{(3A-IV-12,13)} \leftrightarrow \begin{cases} T_{32}: \text{(3B-IV-13)} \\ T_{23}: \text{(3B-IV-9)} \end{cases}$$

$\hat{\Gamma}_{\pm}^{(2)}$: (3A-IV-11,11') $\leftrightarrow$ $\begin{cases}\tilde{T}_{32}: \text{(3B-IV-12)} \\ \tilde{T}_{23}: \text{(3B-IV-8)}\end{cases}$

$\hat{m}^{(2)}$: (3A-IV-3')-(3A-IV-10) $\leftrightarrow$ T^F (Appendix B)

$\tilde{M}^{(1)}$: (3A-IV-1') $\leftrightarrow$ $\begin{cases}t_{\pi N} \\ t_{NN}\end{cases}$ (Obtained from $\tilde{v}_{\pi N}$ and V_{NN} by L-S equation)

$\tilde{M}^{(2)}$: (3A-IV-1') $\leftrightarrow$ $\begin{cases}\tilde{v}_{\pi N} \\ V_{NN}\end{cases}$ [defined in $V_3(E)$,(3B-II-10)]

$g_{\pm}$: (3A-III-1",3A-IV-1") $\leftrightarrow$ $\begin{cases}W \\ Y\end{cases}$

$\hat{m}^{(3)}$: (3A-IV-3') $\leftrightarrow$ C: (3B-II-10) see also Appendix B.

etc.

In Appendix C we also show unitarity relations for relativistic and non-relativistic amplitudes. Those relations also show a formal one-to-one correspondence between the two formulations. Thus, although two formulations are different in kinematics (relativistic, vs. non-relativistic) and in propagators (single-particle and global), they look similar in a formal sense. This is just like the formal similarity between Bethe-Salpeter and non-relativistic Lippmann-Schwinger equations.

When we closely look at the correspondence listed above and the expressions in Chapter 3, we notice one point which needs some examination. In our relativistic formulation there are terms $\mathcal{L}_{\pm}$ [(see (3A-IV-11) and the definition of $L_{\pm}$ above it] which are three-particle irreducible 3↔2 amplitudes. However we do not have such amplitudes in our non-relativistic formulation. This is due to the fact that in our non-relativistic model (1) we have assumed a static heavy meson exchange N-N potential and (2) the global propagators are used, which always allows us to find a cut intersecting less than four particle lines in any 3↔2 graphs. Note that this solely comes from our choice of a specific Hamiltonian.

In connection with the formal similarity between two formulations, we find that neither of them show explicit coupled structure among the amplitudes. Since our non-relativistic model started with a set of coupled equations we shall trace it and see how it has ended up with the uncoupled result. The coupled-equation (3B-II-6) was first reduced in the number of equations by pushing all the information due to higher number pion states into the effective two- and three-particle irreducible operator $D_1(E)$ acting in πNN Hilbert space (this procedure may be called "nesting"). Then the coupled equations are made to connect NN and πNN states only. This "reduced" coupled-equation generates X(E) in which we can identify nucleon self-energy part; $X_u(E)$, and N-N potential without pure heavy meson exchanges; $X_\ell(E)$. The identification of these functions has made the coupled-equation effectively decoupled, since they are assumed to be given.

In our relativistic formulation we take the opposite path to the non-relativistic reduction mentioned above. Up to the exposure of three-particle states, the amplitudes under consideration are decoupled. When we expose higher particle sectors as we partly did in the proof of unitarity (see Appendix C), we find that the amplitudes are coupled in a unitary manner.

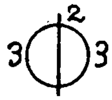
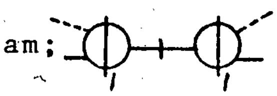
It is necessary to have coupled equations to solve the problem from the first principle, but that is practically impossible. Therefore for most practical purposes, we may leave the coupled nature of the amplitudes untouched. However, it may be necessary to take into account some aspects of the coupling when one wants, for example, to study the effect of pi-production in N-N collision. Especially in connection with general nuclear pion production problems, we may have to start thinking about such coupling aspects.

When we consider a practical application of the derived amplitudes, and their equations, of course the formal similarity between relativistic and non-relativistic amplitudes is not very important. In the non-relativistic case, the integration involved is of three dimensions whereas it is of four dimensions in the relativistic case. In addition, although there are several approximations involved in changing the product of single-particle propagators to a relativistic (Lorentz invariant) global propagator (see the discussion in Chapter 5), there still are several difficult problems remaining for practically solving the problem in the relativistic formulation. So as we shall see later, our practical application is worked out in non-relativistic form. But due to the formal correspondence between relativistic and non-relativistic amplitudes as we have seen, our understanding of the structure of πNN scattering is more transparent by adopting several useful concepts in the relativistic analysis of section A, Chapter 3.

B. Answers to the Questions in Chapter II

We now return to the questions raised in Chapter 2 with the results of Chapter 3. The description here may sometimes be either in terms of the relativistic or non-relativistic formulation we have in Chapter 3 because one formulation is better than the other in some cases for the purpose of easier presentation. But as we have seen, there is no formal difference between them. So we can guess, by looking at some conclusions using one of the formulations, similar (or same) conclusions using the other. We believe that there will not be any confusion in this description. Also we should point out that although the conclusions are for πNN process, they are also true with processes with πd (the problem of bound states is discussed in the next chapter).

(i) Potential Description of Elastic πNN Scattering Without Pi-absorption

The process is to be described by the connected "Faddeev" amplitude;  (3A-III-4) or T^{c} (Appendix B). If we regard an n-body irreducible amplitude for n→n process as an n-body potential, then the heuristic method of assuming πN potential in some Hamiltonian to calculate πNN amplitude does not seem to be wide of the mark. Note that here the potentials (πN and NN) are, properly speaking, energy dependent and off-energy-shell. In addition πN t-matrix used in the Faddeev equation is one-particle irreducible and thus should not contain "generalized" direct Born term; represented by the diagram;  which is equal to $\Gamma(E)\pi(E)\Lambda(E)$ in non-relativistic case.

(ii) Description of Pion Production and Absorption processes by Two Nucleons

Diagrammatic representation in (3A-III-3) or expressions (3A-IV-12') and (3B-IV-9) give a correct t-matrix for NN ← NNπ processes. In order to see the structure more transparently, we take expression (3B-IV-6) which is rewritten as

$$\langle f_2 | T_{23}(E) | i_3 \rangle = \langle \tilde{f}_2^{(-)} | Y | \chi_i^{+} \rangle, \qquad (4B\text{-}1)$$

where

$$|f_2^{(-)}\rangle = |f_2\rangle + \tilde{G}_{T2}(E^-)\tilde{t}_{NN}(E^-)|f_2\rangle$$
$$= |f_2\rangle + \tilde{G}_{T2}(E^-)V_{NN}(E^-)|f_2^{(-)}\rangle. \quad (4B-2)$$

In (4B-2) $|\tilde{f}_2^{(-)}\rangle$ is a wave function with possible nucleon self-energy in $\tilde{G}_2(E^-)$ and inelasticity due to pion production which is included in $V_{NN}(E^-)$. $|\chi_i^+\rangle$ is a Faddeev state in πNN scattering. We obtain a similar expression to (4B-1) for the relativistic case.

Expression (4B-1) is of the "distorted wave" form for a pion absorption process. This shows that a "naïve" DWBA expression (2B-5) is correct if one uses the one-particle irreducible $t_{\pi N}$ discussed above that is the πN t-matrix without generalized direct Born term for the pion distortion included in $|\chi_i^+\rangle$. If this state were to contain πN Born terms, it would lead to the pion overcounting. On the other hand, as has been mentioned above, the final two-nucleon state should include the effect of virtual pion emission because the energy is above the single pion production threshold.

Similar conclusions may be drawn for NN → πNN amplitudes.

(iii) Pion Absorption Effect on the Elastic πNN Scattering

The contribution to the total πNN → πNN amplitude from intermediate pion absorption process is expressed as

$$\hat{\Gamma}_{+}^{(2)} G_{T_2} \hat{\Gamma}_{-}^{(1)} \quad \text{in (3A-IV-14)} \qquad \text{(relativistic)}$$

or

$$\tilde{T}_{32}(E)\,\tilde{G}_{T_2}(E)\,T_{23}(E) \quad \text{in (3B-IV-14')} \qquad \text{(non-relativistic)}$$

The expression (3B-IV-14') can be re-expressed using wave functions:

$$\langle \chi_f^{(-)} | W \hat{G}_{T_2}(E) Y | \chi_i^{+} \rangle , \tag{4B-3}$$

where

$$\hat{G}_{T_2}(E) \equiv \tilde{G}_{T_2}(E) + \tilde{G}_{T_2}(E)\,\tilde{t}_{NN}(E)\,\tilde{G}_{T_2}(E) \tag{4B-4}$$

is a "complete" N-N propagator with N-N interactions included. Similar expression can be obtained for (3B-IV-14) and there, instead of $\hat{G}_2(E)$ we have $G_2^{NN}(s)$ defined as the Fourier transform of

$$-\langle 0 | T(\psi(x_1)\psi(x_2)\bar{\psi}(y_2)\bar{\psi}(y_1)) | 0 \rangle , \tag{4B-5}$$

which is a two-nucleon Green's function in Heisenberg picture. Formally this is more familiar but not easier to calculate.

In (4B-3), $|\chi_i^{+}\rangle$ and $|\chi_f^{(-)}\rangle$ are Faddeev states without the pion absorption effect as defined in (3B-III-1)

($|\chi_f^{(-)}\rangle$ is a time-reversed solution of $|\chi_f^{+}\rangle$).

As may be obvious, the total $\pi NN \to \pi NN$ amplitude is then

$$T_{33}(E) = T^{F}(E) + \langle \chi_f^{(-)} | W \hat{G}_2(E) Y | \chi_i^{+} \rangle . \qquad (4B-6)$$

Incidentally the answer to question (4) is to be found in the last section.

C. Possible Forms of General Pi-nucleus Amplitudes

Here we try to see possible forms of pi-nucleus amplitudes in analogy with what we have learned in πNN scattering. It is obvious that Taylor's cut structure analysis can be applied in general pion-nucleus amplitudes although it will require more care and patience. For a system with a pion and N-1 nucleons (or a nucleus with N-1 nucleons) we may expect to have the following t-matrix expression.

(i) π-nucleus Elastic Scattering Amplitude with no Intermediate pi absorption

The t-matrix for this process is a solution of generalized Faddeev equation for N particles, which is (N-1) irreducible. We write this as $\hat{m}_N^{(N-1)}$ in relativistic approach (or T_N^F in non-relativistic formulation) the connected part of which may be written as $m_N^{(N-1)}$.

We then introduce

$$H_j^{(N)} \equiv \sum_{\ell} m_{j(\ell)}^{j} G_{N-j(\ell)}^{-1}, \qquad (4C\text{-}1)$$

where (a) $m_{j(\ell)}^{j}$ is a j-irreducible amplitude for j→j process and ℓ represents a label attached to a group of j particles selected arbitrary out of N particles.

(b) $G_{N-j(\ell)}$ is a product of N-j single particle propagators which do not belong to group "ℓ" ($G_{o(\ell)} \equiv 1$).

We may write the "potential part" as

$$\hat{m}_N^{(N)} = \sum_{j=1}^{N} H_j^{(N)}, \qquad (4C\text{-}2)$$

and the equation for $\hat{m}_N^{(N-1)}$ will become

$$\hat{m}_N^{(N-1)} = \hat{m}_N^{(N)} + \hat{m}_N^{(N)} G_N \hat{m}_N^{(N-1)}, \qquad (4C\text{-}3)$$

which is a generalization of (3A-IV-3'). When the integral equation is made to have a compact kernel, only connected part, $m_N^{(N-1)}$, should appear in the equation. In that equation the input amplitudes are $m_j^{(j-1)}$ (j=2, ..N-1), in addition to $m_N^{(N)}$, for all possible combinations of j particles out of the total system. So the procedure is to start with solving the two-body scattering and go step by step to three-, four-, ... up to N-particle scattering problem. The important thing to note in our pi-(N-1 nucleon) problem here is that no pion absorption process appears in any input amplitudes for subsystems. A similar result will be reached in the non-relativistic approach.

(ii) Single Pion Absorption (or emission) Process and Elastic Pi-nucleus Scattering with Intermediate Pion Absorption

The amplitudes corresponding to the processes mentioned above are expected to have the following forms:

$$\left.\begin{aligned} \hat{\Gamma}_{N+}^{(N-2)} &= \hat{\Gamma}_{N+}^{(N-1)} + \hat{\Gamma}_{N+}^{(N-1)} G_{N-1} \hat{\underline{m}}_{N-1}^{(N-2)} \\ \hat{\Gamma}_{N-}^{(N-2)} &= \hat{\Gamma}_{N-}^{(N-1)} + \hat{\underline{m}}_{N-1}^{(N-2)} G_{N-1} \hat{\Gamma}_{N-}^{(N-1)} \\ \hat{m}_{N}^{(N-2)} &= \hat{m}_{N}^{(N-1)} + \hat{\Gamma}_{N+}^{(N-2)} G_{N-1} \hat{\Gamma}_{N-}^{(N-1)} \end{aligned}\right\} , \qquad (4C-4)$$

where, for example,

$$\left.\begin{aligned} \hat{\Gamma}_{N+}^{(N-1)} &= L_{+}^{(N)} + \hat{m}_{N}^{(N-1)} G_N L_{+}^{(N)} \\ L_{+}^{(N)} &= \sum_{j=2}^{N} \sum_{\ell} \Gamma_{j+(\ell)}^{(j)} G_{N-j(\ell)}^{-1} \end{aligned}\right\} , \qquad (4C-5)$$

and $\hat{m}_{N-1}^{(N-2)}$ is an amplitude for the scattering of nucleons only. The meaning of $\Gamma_{j+(\ell)}^{(j)}$ should be clear in analogy with the three-particle case ($\Gamma_{2+(i)}^{(2)} = \tilde{g}_{+}^{j}$ etc.). A similar form will be obtained in the non-relativistic approach. Again as may be clear, $\hat{m}_N^{(N-1)}$ in (4C-5) should not contain the effect of intermediate pion absorption. In the "distorted wave" language this is equivalent to saying that the optical potential describing the pion distortion in nuclear pi production or absorption problems should not contain the intermediate pion absorption. The expression (4C-5) can also be written using wave functions and hence in distorted wave forms.

There are several articles on the formal aspect of many-particle scattering (non-relativistic potential scattering) using the multi-particle version of the Faddeev equation.[26] They may be referred to in connnection with what we studied in this subsection.

The final remark in this subsection is on the difference between the Watson and Faddeev type of approaches[36-i]. We shall here restrict our discussion within non-relativistic potential scattering, for simplicity. In the Watson formalism, degrees of freedom associated with interactions among nucleons in the nucleus are made implicit by adopting propagators of the form $[E^+-\hat{H}_o]^{-1}$, where $\hat{H}_o$ contains N-N potentials in the nuclear system. On the other hand, the Faddeev type formalism uses the free propagator $[E^+-H_o]^{-1}$ such that $\hat{H}_o=H_o+\sum_{i>j} V_{NN}^{ij}$. In other words πN t-matrix in the Watson series does not express a free scattering but contains some nuclear information (remember that πN t-matrix in the Waston theory takes the form $t_{\pi N}=V_{\pi N}+V_{\pi N}[E^{+}-\hat{H}_{o}]^{-1}t_{\pi N}$). Of course the final $T_{\pi\text{-nucleus}}$ contains the same information independent of which formalism one may choose. It is less convenient to adopt the Watson type approach to derive the pi-nucleus amplitudes since we have the pion absorption effect and the pion exchange nature of nucleon-nucleon potentials to be taken into account. They seem naturally treated in the Faddeev picture where the propagators are free from the nuclear information. Once the expressions like (4C-4) etc. are derived, then we may regard the amplitudes appearing there to be re-expressed in the Watson form.

CHAPTER V

REDUCTION OF πNN AMPLITUDES

The purpose of this chapter is to obtain more practical forms of our πNN amplitudes.

In Section A we shall first describe how to treat bound states in many-body scattering. This is because physically interesting many-body scattering is always of bound state plus particle type. Especially we are concerned with bound-particle scattering in three-body systems here.

Next we introduce an approximation which considerably simplifies the solution of many-body scattering problem; the isobar approximation to two-body sub-amplitudes. Because of clarity and simplicity, our description will be given mostly in terms of what is used in non-relativistic potential scattering. But almost parallel argument is true, at least formally, in the relativistic treatment of the problem. Actually the notion of isobars has been most frequently used in relativistic field theory as well as in S-matrix theory.[30]

In section B we shall reduce our πNN amplitudes to the form suitable for actual computations. The reduction includes (1) the isobar approximation (or separable approximation) to input two-body t-matrices, (2) antisymmetrization of two nucleons and (3) the angular momentum-isospin decomposition. A detailed produce for (2) and (3) is shown in Appendix G.

A. Bound States and the Isobar Approximation

I. Bound State-Particle Scattering

The amplitudes $M^{(1)}$, $\Gamma_{\pm}^{(1)}$, T_{33}, T_{23}, etc. in Chapter 3 are those describing processes: three-body plane wave state $\longrightarrow$ three-body plane wave state, and three-body plane wave state $\longleftrightarrow$ two-body plane wave state. What we should have in order to describe physical processes involving three particles are those amplitudes which treat the scattering between a bound state and a particle. Our purpose is to see how to obtain such bound-particle amplitudes from amplitudes describing free three-particle scattering.

As has been mentioned, we are mainly concerned with non-relativistic potential scattering. For the description of relativisitc bound state problems see articles on the Bethe-Salpeter equation.(19)

First consider a two-body problem. We remember that bound states can be regarded as real poles (below the unitarity cut in the complex energy plane) of the s-matrix or T-matrix. Note that resonances and virtual states also can be identified as poles of these matrices. It is well known that at such a pole (bound state or resonance) the t-matrix factors. Let us assume that the pole is at $E = E_n$. Then what is meant by the above statement is that

$$T(\vec{q},E,\vec{p}) \sim g_n(\vec{q})\, g_n^*(\vec{p}) / (E-E_n)\,, \qquad (5A\text{-}I\text{-}1)$$

when $E \sim E_n$.

$g_n(\vec{q})$ is called a form factor and in the case of a bound state it is related to the bound state wave function; $\psi_n(\vec{q})$ through

$$g_n(\vec{q}) = (E_n - E_q)\,\psi_n(\vec{q}) \qquad (5A\text{-}I\text{-}2)$$

Next let us consider a bound-particle scattering among three particles, two of which are bound in the entrance as well as in the exit channels. Here we shall describe how to obtain a bound-particle scattering t-matrix from the t-matrix for free three-body scattering. We consider a process

$$(a+b)_n + C \longrightarrow a + (b+C)_m \;.$$

By bracketed pairs we mean bound states formed by those pairs, and m, n, etc. designate the bound state levels. The process is shown schematically in Fig. 5-1.

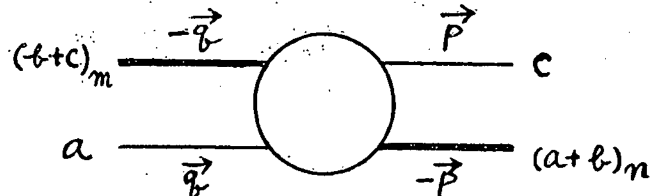


Fig. 5-1

The derivation of the t-matrix for the process under consideration is as follows:

Consider a connected Faddeev amplitude for three-body scattering in the three-particle center of mass system;

$$\langle \vec{q}, \vec{s} \,|\, T^{c}_{ac}(E) \,|\, \vec{p}, \vec{r} \rangle , \qquad \text{(5A-I-3)}$$

where $\vec{q}$: relative momentum of a and (b+c) in the exit channel

$\vec{p}$: relative momentum of c and (a+b) in the entrance channel

$\vec{s}$: relative momentum of b and c in the exit channel

$\vec{r}$: relative momentum of a and b in the entrance channel

and as usual subscripts in T^c indicate spectator particles in the entrance and exit channels. Then we can write (5A-I-3) in terms of two-body t-matrices;

$$\begin{aligned} \langle \vec{q}, \vec{s} \,|\, T^{c}_{ac}(E) \,|\, \vec{p}, \vec{r} \rangle = \iint d^3e\, d^3f\, t_a(\vec{s}, E - \tfrac{q^2}{2\tilde{\mu}_a}, \vec{e})\, G_3(E, \vec{q}) \\ \times \sum_{\alpha\beta} \langle \vec{q}, \vec{e} \,|\, \mathcal{F}_{\alpha\beta}(E) \,|\, \vec{p}, \vec{f} \rangle\, G_3(E, \vec{p})\, \bar{\delta}_{a\alpha} \bar{\delta}_{c\beta} \\ \times\, t_c(\vec{f}, E - \tfrac{p^2}{2\tilde{\mu}_c}, \vec{r}) , \end{aligned} \qquad \text{(5A-I-3')}$$

where $1/\tilde{\mu}_a \equiv 1/m_a + 1/m_b + m_c$, etc. and $\{\mathcal{F}_{\alpha\beta}\}$ is a different set of three-body t-matrices introduced by Alt, Grassberger and Sandhas[32] (AGS), which satisfies

$$\mathcal{F}_{\alpha\beta} = G_3^{-1}\bar{\delta}_{\alpha\beta} + t_\alpha G_3 \sum_\gamma \bar{\delta}_{\alpha\gamma}\mathcal{F}_{\gamma\beta} \quad , \qquad (5A\text{-}I\text{-}4)$$

and it is rather easy to show that $\mathcal{F}_{\alpha\beta}$ is related to the Faddeev amplitude $T^{A}_{\alpha\beta}$ (see Appendix B) through

$$\mathcal{F}_{\alpha\beta} = G_3^{-1}\bar{\delta}_{\alpha\beta} + \sum_{\gamma\varepsilon} \bar{\delta}_{\alpha\gamma} T^{A}_{\gamma\varepsilon}\bar{\delta}_{\varepsilon\beta} . \qquad (5A\text{-}I\text{-}4')$$

In (5A-I-3') ∿ (5A-I-4') $\delta_{\alpha\beta}$ stands for $\delta_{\alpha\beta} \equiv 1-\delta_{\alpha\beta}$.

In (5a-I-3') we let $E - (q^2/2\tilde{\mu}_a) \rightarrow E_m{}^{bc}$ and $E - (p^2/2\tilde{\mu}_c) \rightarrow E_n{}^{ab}$ where $E_b{}^{ab}$, etc. are negative bound state energies of a+b system, etc. With this limiting procedure, t_a and t_c factor, as in (5A-I-1), respectively. Therefore obtain as the residue

$$g_a^m(\vec{s})\iint d^3e\, d^3f\, g_a^{m*}(\vec{e})\, G_3(E,\vec{q}) \sum_{\alpha\beta} \bar{\delta}_{a\alpha} \langle \vec{q},\vec{e}\,|\,\mathcal{F}_{\alpha\beta}(E)\,|\,\vec{p},\vec{f}\rangle$$
$$\times\, \bar{\delta}_{\beta c}\, G_3(E,\vec{p})\, g_c^n(\vec{f}) \cdot g_c^{n*}(\vec{r}) .$$

(5A-I-5)

And the bound-particle scattering t-matrix is obtained by eliminating $g_a{}^m(\vec{s})$ and $g_c{}^{n*}(\vec{r})$ from (5A-I-5). Note that this can be written also as

$$T_{ac}^{bound}(\vec{q},E,\vec{p}) = \iint d^3e\, d^3f\, \psi_a^{n*}(\vec{e}) \sum_{\alpha\beta} \bar{\delta}_{a\alpha} \langle \vec{q},\vec{e} | \mathcal{T}_{\alpha\beta}(E) | \vec{p},\vec{f} \rangle \bar{\delta}_{\beta c}\, \psi_c^{n}(\vec{f})$$

(5A-I-6)

In (5A-I-6), $\psi_c{}^n(\vec{f})$, etc. are momentum space bound state wave functions.

Using $T_{ac}{}^{bound}(\vec{q},E,\vec{p})$, an elastic bound-particle scattering amplitude is given as

$$f_{aa}^{bound}(\vec{q},E,\vec{P}) = -(2\pi)^2 \mu_a T_{aa}^{bound}(\vec{q},E,\vec{P}), \qquad (5A\text{-}I\text{-}6')$$

where μ_a is the reduced mass of bound plus particle system. Similar procedure can be applied, in principle, to obtain a bound-particle amplitude from general many-body amplitude but we shall not discuss that subject here.

II. The Isobar Approximation

When we try to study a two-body elastic scattering problem through Lippmann-Schwinger equation, we have to solve an integral equation involving three-dimensional integral. After decomposing the amplitude in angular momentum eigenstates, the integral involved becomes one-dimensional. In three-body scattering problems integral equations are two-dimensional even after the angular momentum decomposition. This tendency continues as the number of particles involved in the scattering gets larger. Practically solving many-body scattering problem thus becomes difficult. In addition, increase in the number of particles (1) increases the number of channels to be coupled among themselves and (2) makes the kernels of integral equations non-compact; the compactness is essential for integral equations to be soluble. Some techniques like Faddev's procedure may be available to make non-compact kernels compact but even if this is performed, to solve many-body scattering problems analytically is practically impossible. Even numerically that is very difficult. So some reasonable approximation should be adopted by all means.

In sub-section I we mentioned that near a bound state or a resonance pole the t-matrix factors. Because of this factorization it is expected that the scattering integral equations are reduced in the dimension of integrations. Actually adopting factorized form of two-body t-matrix makes the dimension of integration in two-body scattering equation down to zero. What this approximation means is that a two-body system is regarded as an effective one-body system and thus makes problems easier.

This effective one-body system; either bound or resonance state is called an isobar or a quasi-particle. Of course an isobar also can be a correlated many-body system like a nucleus. Also a set of particles can have many "isobar" states.

When sub-t-matrices in the multi-particle scattering equations, for example t_α in (5A-I-4), are approximated by a summation of factorized terms like in (5A-I-1); the isobar approximation is made, then the equations may be simplified considerably. Most calculations in three-body scattering do adopt this approximation.[20] We also shall use it.

III. Unitarization of Isobar Amplitudes

Adopting the isobar approximation a two-body t-matrix is expressed as

$$t(\vec{q},E,\vec{p}) = \sum_n g_n(\vec{q})\, g_n(\vec{p}) \Big/ (E-E_n), \tag{5A-III-1}$$

where the summation is over some finite number of bound states and resonances. Better approximation is possible when the number of terms in (5A-III-1) is increased. Moreover we can improve the approximation by unitarizing the expression because (5A-III-1) does not satisfy two-body unitarity. We shall show how to unitarize a factorized form of t-matrix in a simplest example.

For simplicity we assume that one angular momentum state has at most one bound state or resonance, and we pick up one specific angular momentum state in which we have, for example, a bound state; angular momentum = J, third components m, m' etc. and E_J; bound state pole. Then the isobar approximation is

$$t^J_{m',m}(\vec{q},E,\vec{p}) \simeq g_J^{m'}(\vec{q})\, g_J^{m}(\vec{p}) / (E-E_J). \tag{5A-III-2}$$

As we can write $g_J{}^m(\vec{q}) = h_J(q) Y_J{}^m(\hat{q})$ where $Y_J{}^m(\hat{q})$ is an eigenstate of the total angular momentum J($h_J(q)$ is usually real), (5A-III-2) is reduced and becomes

$$t^J(q,E,p) \simeq h_J(q) h_J(p) / (E-E_J) \tag{5A-III-2'}$$

which is free from angular variables. We then assume the form of the unitarized amplitude as

$$t^J(q,E,p) = h_J(q)\, \tau_J(E)\, h_J(p). \tag{5A-III-3}$$

Two-body unitarity sets a restriction on $\tau_J(E)$; it should satisfy

$$\mathrm{Im}\,\tau^{-1}(E) = \pi \int S^2 dS\, \delta(E - S^2/2M)\, h_J^2(S), \qquad (5A\text{-}III\text{-}4)$$

where M is a two-body reduced mass. Taking into account the existence of the pole at $E = E_J (E_J<0)$ and assuming $\tau_J(E)$ to decrease sufficiently rapidly as $|E|\to\infty$, we can use a dispersion relation to find:

$$\tau_J(E) = \frac{1}{E-E_J}\left[\int_0^\infty \frac{S^2 dS\, h_J^2(S)}{(S^2/2M - E^+)(S^2/2M - E_J)}\right]^{-1}. \qquad (5A\text{-}III\text{-}5)$$

A similar procedure can be applied when the isobar is a resonance. Note that we have not used anything associated with "potentials". Also note that unitarized two-body isobar t-matrices used in three-body problems guarantee three-particle unitarity as well as bound-particle unitarity of the three-body amplitudes.

To close this sub-section it may be adequate to remark that (5A-III-5) can also be derived from a separable potential of the type

$$\mathcal{V}_J(q,p) = \lambda_J h_J(q) h_J(p). \qquad (5A\text{-}III\text{-}6)$$

Through Lippmann-Schwinger equation $\mathcal{V}_J(q,p)$ gives

$\tau_J(E)$ which can be made to have a bound state or a resonance pole by a suitable choice of the strength λ_J.

B. Reduction of the Amplitudes

Our practical reduction of the πNN scattering amplitudes is given for the non-relativistic case. Nevertheless it seems necessary to say some words on the reduction of relativistic amplitudes. So in I we shall discuss it briefly. In II we present our practical reduction of non-relativistic amplitudes.

I. On the Reduction of Relativistic Three-Body Amplitudes

It is practically very difficult to solve off-mass-shell three-body scattering equations in relativistic field theory. This could be understood through the fact that even the simplest two-body scattering is exceedingly difficult to solve; the Bethe-Salpeter equation can be solved approximately only in the case of scalar particles with the ladder approximation to the integral kernel (= two-particle irreducible amplitude).[19] In the case of spinor particles it has not yet been proven whether the Wick rotation is possible or not.

The first thing to be done in order to solve three-body equations is, as in the case of non-relativistic problems, to adopt the isobar approximation. This will reduce the integration from twelve to eight dimensional. Relativistically covariant angular momentum reduction is necessary to reduce the dimension of integration further down. Normally one then expects to use helicity decomposition. But helicity is well-defined only for on-mass-shell particles. To overcome this difficulty one may introduce three-dimensional solid harmonics to expand the amplitudes. This may be formally interesting but not practical. Instead, if the equtions can be made on-mass-shell by some technique, then one can use helicity expansion. This is actually possible when the products of single-particle propagators are replaced by Blankenbecler-Sugar type Green's function.[25] This replacement reduces the dimension of integration by one, since particles are now on-mass-shell. The price we have to pay, however, in adopting this technique is that the dynamical left-hand cut structure is missing. Also all the amplitudes now become off-energy-shell, which is not convenient because they are not directly connected with what we have in S-matrix theory nor in conventional quantum field theory. Nevertheless, there have been some attempts along this line. Detailed discussions and reductions are to be found in some articles.[21-ii),(33] We shall try to reduce our relativistic amplitudes in the near future to apply them to πNN problem.

II. Non-Relativistic πNN Equations in Isobar Approximations

(a) isobar approximation

(i) The connected Faddeev amplitudes $T_{jk}{}^{c}(E)$ (see Appendix B) satisfy the equation of the same form that the relativistic three-body (connected) amplitudes do; equations (3A-IV-9) and (3A-IV-10) with G_3 to be interpreted as a non-relativistic global propagator. Conventions with regard to the subscripts in T^c are standard ones; they specify initial and final spectator particles or label interacting pairs in the initial and final states and subscript zero indicates an interaction due to three-body potentials.

We adopt separable forms (isobar approximation) for our two-body input t-matrices, which means

$$t_j = \sum_\alpha g^j_\alpha \tau^j_\alpha g^{j*}_\alpha \quad (j=1,2,3) , \qquad (5B\text{-}II\text{-}1)$$

where j and α specify a pair and its quantum state respectively. Then utilizing (5A-I-4') ∿ (5A-I-5) together with the expression for the Faddeev amplitudes, (see Appendix B) we obtain:

$$X_{j\alpha,k\beta} = \tilde{U}_{j\alpha,k\beta} + \sum_{\ell\gamma} \tilde{U}_{j\alpha,\ell\gamma} \tau^{\ell}_{\gamma} X_{\ell\gamma,k\beta} \quad (j,k=1,2,3),$$

(5B-II-2)

where

$$\left.\begin{aligned} T^{c}_{jk} &\equiv \sum_{\alpha\beta} g^{j}_{\alpha} \tau^{j}_{\alpha} X_{j\alpha,k\beta} \tau^{k}_{\beta} g^{k*}_{\beta} \\ \tilde{U}_{j\alpha,k\beta} &\equiv Z_{j\alpha,k\beta} + Z'_{j\alpha,k\beta} \\ Z_{j\alpha,k\beta} &\equiv g^{j}_{\alpha} G_3 g^{k*}_{\beta} \\ Z'_{j\alpha,k\beta} &\equiv g^{j}_{\alpha} G_3 M_0 G_3 g^{k*}_{\beta} \end{aligned}\right\} .$$

(5B-II-3)

When labels α and β signify bound states of certain pairs, $X_{j\alpha,k\beta}$ is equal to the bound-particle amplitude in (5A-I-6); $T^{bound}_{j\alpha,k\beta} \equiv X_{j\alpha,k\beta}$, and (5B-II-2) is the equation for that amplitude. Since $T_{oj}{}^{c}$, $T_{ko}{}^{c}$ and $T_{oo}{}^{c}$ can be obtained easily from $T_{jk}{}^{c}(j,k=1,1,2,3)$ which are obtained through (5B-II-2) and (5B-II-3), we shall not write down their expressions in isobar approximated forms.

The equation (5B-II-2) is of the Amado-Lovelace type[(20)(31)] with $\tilde{U}_{j\alpha,k\beta}$ as a driving term. For our πNN problem, the solution of this equation is the central problem in that the amplitudes T_{23}, T_{32} and T_{33} may all be obtained directly from the resulting functions $X_{j\alpha,k\beta}$, and the πNN vertices, Γ and Λ (or W and Y) when $V_{NN}(E)$ is assumed to be known. However, if $V_{NN}(E)$ is also to be calculated, then one needs $\tilde{T}_{23}$ and $\tilde{T}_{32}$ for it. Then $\tilde{t}_{NN}(E)$ is obtained from $V_{NN}(E)$ through Lippmann-Schwinger equation, and it is used to calculate T_{23}, T_{32} and thus T_{33}.

Before going on to the next step, we present the connected part of T_{33}, T_{23}, $\tilde{T}_{23}$, etc. (defined as $X_{33}{}^{(1)}$, $X_{N3}{}^{(1)}$, $X_{N3}{}^{(2)}$, etc. respectively) below. i, k — are used as labels for spectators (for two-nucleon case those suffices mean nucleons which do not take part in the emission-absorption processes). The expressions are in terms of Λ^j, Γ^i, $X_{i\alpha,j\beta}$, etc.

(i) $Z_{i\alpha,jN} \equiv g^i_\alpha G_3 \Gamma^j \quad (Z_{iN,j\beta} \equiv \Lambda^i G_3 g^j_\beta)$

(ii) $X^{(2)}_{iN,j\beta} \equiv Z_{iN,j\beta} + \sum Z_{iN,k\alpha} \tau^k_\alpha X_{k\alpha,j\beta}$
(similar for $X^{(2)k\alpha}_{i\alpha,jN}$)

(iii) $X^{(2)}_{iN,k3}$ (= connected part of $\tilde{T}_{23}$) $= \sum_\beta X^{(2)}_{iN,k\beta} \tau^k_\beta g^k_\beta$
(similar for $X^{(2)}_{k3,jN}$)

(iv) $X^{(1)}_{iN,k3}$ (= connected part of T_{23}) $= X^{(2)}_{iN,k3} + \sum_{N'} \tilde{t}_{NN}(N') \tilde{G}_2 X^{(2)}_{iN',k3} + \tilde{t}_{NN}(N) \tilde{G}_2 \Lambda^k \delta_{ki}$
(similar for $X^{(1)}_{i3,kN}$)

(v) $X^{(1)}_{i3,j3}$ (= connected part of T_{33}) $= T^c_{ij} + \sum_{kN'} X^{(1)}_{i3,kN'} \tilde{G}_2 X^{(2)}_{kN',j3}$

$$+ \sum_k \Gamma^i \tilde{G}_2 X^{(2)}_{kN,j3} + \sum_k X^{(1)}_{i3,kN} \tilde{G}_2 \Lambda^j . \qquad (5B\text{-}II\text{-}3')$$

With these expressions we obtain, for example, the differential cross section for $NN \to \pi d$ (note that nucleons are treated non identical at this stage)

$$\frac{d\sigma^{(NN\to\pi d)}}{d\Omega} = (2\pi)^4 \mu_{\pi d}\,\mu_{NN} \frac{P_{\pi d}}{P_{NN}} \left| \sum_i X^{(1)}_{\pi d, iN} \right|^2 \qquad (5B\text{-}II\text{-}3')$$

where

$$X^{(1)}_{\pi d, iN} = X^{(2)}_{\pi d, iN} + \sum_{N'} X_{\pi d, iN'}\, \tilde{G}_2\, \tilde{t}_{NN}(N')$$

and $\mu_{\pi d}$, $P_{\pi d}$ are the reduced mass and relative momentum of π and d, etc.

(b) antisymmetrization and angular momentum reductions

It is rather tedious to present here all the details of the antisymmetrization and angular momentum decomposition. Instead we shall put that part in Appendix G, so it is suggested to have a glance at it. We therefore present only the final results ready for practical computations. Some explanations are due which are mostly associated with the notations used in expressing the reduced forms.

Let us choose an example; consider a reduced amplitude $B_{a,b}^{A(j)J\tau}$.

Here i) J,τ: total angular momentum and total isospin

ii) a,b: $\begin{cases} \alpha,\beta \text{ etc.} = \text{specifying } (N+\pi)+N \text{ states} \\ m,n \text{ etc.} = \text{specifying } (N+N)+\pi \text{ states} \\ N,N' \text{ etc.} = \text{specifying two-nucleon states} \\ 3,3' \text{ etc.} = \text{specifying } \pi NN \text{ uncorrelated states} \end{cases}$

These labels also stand for all quantum numbers like sub-angular momentum, channel spin, etc.

iii) A: this means that the amplitude is antisymmetric with respect to two nucleons. In this connection we also use a notation defined through

$$B^{A,J\tau}_{a,b} \equiv \{1-(-1)^{S_N+\ell_N+\tau}\} \hat{B}^{J\tau}_{a,b}$$

This occurs when at least one of a and b is in two-nucleon state. S_N, ℓ_N are the total spin and orbital angular momentum of that channel. Note that when both a and b are in two-nucleon states, $S_a = S_b = S_N$ and $\ell_a \equiv \ell_b$ (mod 2), and thus $\{1-(-1)^{S_a+\ell_a+\tau}\} = \{1-(-1)^{S_b+\ell_b+\tau}\}$.

iv) (j) which is next to A indicates that the amplitude is j-irreducible. (This may be omitted when it is perfectly clear).

v) Particle masses are specified as m_i ($i=1,2,3$) and reduced masses are defined as

$$\frac{1}{\mu_i} \equiv \frac{1}{m_j} + \frac{1}{m_k} \quad (i,j,k \text{ cyclic})$$

The pion will be considered as particle "3".

vi) Normalization of single-particle state.

$$\langle \vec{k}' | \vec{k} \rangle = \delta^3(k'-k) \longrightarrow \langle \vec{x}' | \vec{k} \rangle = \frac{1}{\sqrt{(2\pi)^3}} e^{i\vec{k}\cdot\vec{x}}.$$

vii) Rest masses are included in the energy.

Now we can write down the reduced equations.

(1) The Faddeev (Amado-Lovelace) equation. For our later purpose we drop the contribution coming from three-body potentials. Then an isobar-particle t-matrix is given:

$$X^{J\tau}_{a,b}(p_a,p_b,E) = Z^{J\tau}_{a,b}(p_a,p_b,E) + \sum_c \int p_c^2 dp_c \, Z^{J\tau}_{a,c}(p_a,p_c,E) \times \tau^c\left(E - \sum_i m_i - \frac{p_c^2}{2m_c} - \frac{p_c^2}{2\mu_c}\right) X^{J\tau}_{c,b}(p_c,p_b,E),$$

(5B-II-4)

where a, b, c are α,β, m,n, etc. Note that although we do not show explicitly by putting "A"s, nucleons in Z and X above are already antisymmetrized.

(2) Exchange terms for a(α or n) $\rightleftarrows$ NN

$$Z^{A,J\tau}_{a,N}(p_a,p_N,E) = \{1-(-1)^{S_N+l_N+\tau}\}\,\hat{Z}^{J\tau}_{a,N}(p_a,p_N,E),$$

(5B-II-5)

where

$$\hat{Z}^{J\tau}_{a,N} = (\sqrt{2})^{\delta_{an}} Z^{J\tau}_{a,N}.$$

$Z_{a,n}$ is similar to $Z_{a,\alpha}$, the difference being the form factor of α replaced by πNN vertex. We have similar expressions for $Z_{N,a}^{A,J\tau}$ and $\hat{Z}_{N,a}^{J\tau}$

(3) $X_{a,N}^{A(2),J\tau}$ and $X_{N,a}^{A(2),J\tau}$ $(a=\alpha,n)$

These are the connected part of $\tilde{T}_{32}$ and $\tilde{T}_{23}$ in isobar approximation.

$$\hat{X}^{(2),J\tau}_{a,N}(p_a,p_N,E) = (\sqrt{2})^{\delta_{aN}}\Big\{Z^{J\tau}_{a,N}(p_a,p_N,E) + \int p_b^2 dp_b \sum_b X^{J\tau}_{a,b}(p_a,p_b,E)$$
$$\times \tau^b\Big(E^{+}-\sum_i m_i - \frac{p_b^2}{2m_b} - \frac{p_b^2}{2\mu_b}\Big) Z^{J\tau}_{b,N}(p_b,p_N,E)\Big\}. \quad (5B\text{-}II\text{-}6)$$

$X_{a,N}^{A(2)J\tau}$ is related to $\hat{X}_{a,N}^{(2)J}$ in a manner shown in (5B-II-5). A similar expression is obtainable for $\hat{X}_{N,a}^{(2)J\tau}$.

(4) $X_{3,a}^{A(2),J\tau}(P_1,P_2,S,P_a,E)$ $(a = \alpha,n)$

This is diagrammatically shown as in Fig. 5-2.

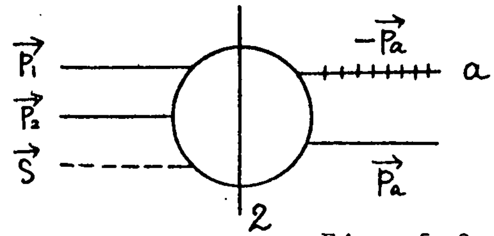


Fig. 5-2

The resultant t-matrix is

$$
\begin{aligned}
X^{A(2),J\tau}_{3,a}(P_1,P_2,S;P_a,E) = (\sqrt{2})^{\delta_{an}}\Big\{ & \sum_\beta g_\beta(u_1)\tau^\beta\Big(E-\Sigma m_i-\frac{P_1^2}{2m_1}-\frac{P_1^2}{2\mu_1}\Big) \\
& \times X^{J\tau}_{\beta,a}(P_1,P_a,E) - \sum_\beta g_\beta(u_2)\tau^\beta\Big(E-\Sigma m_i-\frac{P_2^2}{2m_2}-\frac{P_2^2}{2\mu_2}\Big) \\
& \times X^{J\tau}_{\beta,a}(P_2,P_a,E) + \frac{1}{\sqrt{2}}\sum_m g_m(v)\tau^m\Big(E-\Sigma m_i-\frac{S^2}{2m_3}-\frac{S^2}{2\mu_3}\Big) \\
& \times X^{J\tau}_{m,a}(S,P_a,E)\Big\}. \qquad \text{(5B-II-7)}
\end{aligned}
$$

Here $(\vec{u}_1, \vec{u}_2, \vec{v})$ are relative momenta between (nucleon 1 and pi, nucleon 2 and pi, two nucleons) in the exit channel. g_β etc. are form factors in the separable form of two-body t-matrices. We obtain similar expression for $X_{a,3}^{A(2),J\tau}$.

(5) $X_{3,N}^{A(2),J\tau}$ and $X_{N,3}^{A(2),J\tau}$

Diagrammatically $X_{3,N}^{(A(2),J\tau}$ is depicted as (Fig. 5-3)

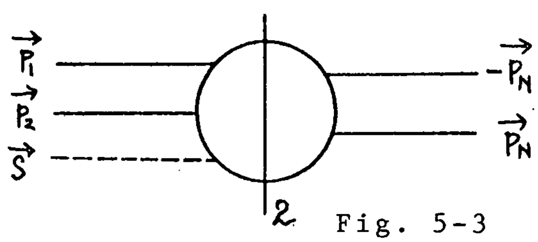


Fig. 5-3

The expression goes as

$$\hat{X}^{(2),J\tau}_{3,N}(P_1,P_2,S,P_N,E) = \sum_{\alpha}\Big\{g_\alpha(u_1)\tau^\alpha(E-\Sigma m_i-\frac{P_1^2}{2m_1}-\frac{P_1^2}{2\mu_1})\hat{X}^{(2),J\tau}_{\alpha,N}(P_1,P_N,E)$$
$$-g_\alpha(u_2)\tau^\alpha(E-\Sigma m_i-\frac{P_2^2}{2m_2}-\frac{P_2^2}{2\mu_2})\hat{X}^{(2),J\tau}_{\alpha,N}(P_2,P_N,E)\Big\}$$
$$+\frac{1}{2}\sum_n g_n(v)\tau^n(E-\Sigma m_i-\frac{S^2}{2m_3}-\frac{S^2}{2\mu_3})\hat{X}^{(2),J\tau}_{n,N}(S,P_N,E) \quad .$$

(5B-II-7)

where u_1, u_2, v are defined in a similar way as in (4). Similar expression is obtained for $\hat{X}_{N,3}^{(2)J\tau}$. We can obtain $X_{N,3}^{A(2),J\tau}$ and $\hat{X}_{2,N}^{A(2),J\tau}$ from $\hat{X}_{N,3}^{(2),J\tau}$ and $\hat{X}_{3,N}^{(2),J\tau}$ respectively.

(6) $\tilde{t}_{NN,\ell',\ell}^{AJ\tau S}$

Defining
$$\begin{cases} \hat{t}^{J\tau S}_{NN,\ell'\ell}(P,q,E) \equiv \frac{1}{2}[1+(-1)^{\ell'+\ell}]\,\tilde{t}_{NN,\ell'\ell} \\ \hat{V}^{J\tau S}_{NN,\ell'\ell}(P,q,E) \equiv \frac{1}{2}[1+(-1)^{\ell'+\ell}]\;V^{J\tau S}_{NN,\ell'\ell} \end{cases},$$

where $\hat{t}_{NN,\ell',\ell}^{J\tau S}$ satisfies

$$\hat{t}^{J\tau S_N}_{NN,\ell'\ell}(P,q,E) = \hat{V}^{J\tau S_N}_{NN,\ell',\ell}(P,q,E) + \sum_{\ell''}\int r^2 dr\,\hat{V}^{J\tau S_N}_{NN,\ell',\ell''}(P,r,E)\tilde{G}_2(E,r)$$
$$\times\,\hat{t}^{J\tau S_N}_{NN,\ell'',\ell}(r,q,E),$$

(5B-II-8)

and $\tilde{t}_{NN,\ell',\ell}^{A,J\tau S} = \{1-(-1)^{\ell+S+\tau}\}\,\hat{t}_{NN,\ell',\ell}^{J\tau S}$. The factor $\frac{1}{2}[1+(-1)^{\ell'+\ell}]$ just serves for the parity conservation. Note that the total spin S_N is conserved.

(7) $\hat{X}_{a,N}^{(1),J\tau}$ and $\hat{X}_{N,a}^{(1),J\tau}$ $(a = \alpha, n)$

$$\hat{X}^{(1),J\tau}_{a,N}(p_a,p_N,E) = \hat{X}^{(2),J\tau}_{a,N}(p_a,p_N,E) + \sum_{N'}\int r^2 dr\, \hat{X}^{(2),J\tau}_{a,N}(p_a,r,E) \times \tilde{G}_2(E,r)\, \hat{t}^{J\tau S_N}_{NN,\ell_N',\ell_N}(r,p_N,E)\, \delta_{S_N',S_N} . \quad (5B\text{-}II\text{-}9)$$

$\hat{X}_{N,a}{}^{(1),J\tau}$ is obtained in a similar manner.

(8) $\hat{X}_{3,N}{}^{(1),J\tau}(P_1,P_2,S,P_N;E)$ and $\hat{X}_{N,3}{}^{(1),J\tau}(P_N,P_1,P_2,S;E)$

$$\begin{aligned}\hat{X}^{(1),J\tau}_{3,N}(p_1,p_2,S,p_N;E) &= \hat{X}^{(2),J\tau}_{3,N}(p_1,p_2,S,p_N;E) + \sum_{N'}\int r^2 dr\, \hat{X}^{(2),J\tau}_{3,N'}(p_1,p_2,S,r,E) \\ &\times \tilde{G}_2(E,r)\, \hat{t}^{J\tau S}_{NN,\ell_N',\ell_N}(r,p_N,E) \\ &+ \hat{\Gamma}\left(u_1,E-\sum m_i-\frac{p_1^2}{2m_1}-\frac{p_1^2}{2\mu_1}\right)\tilde{G}_2(E,p_1)\sum_{\ell_N'}\hat{t}^{J\tau S_N}_{NN,\ell_N',\ell_N}(p_1,p_N,E) \\ &+ (\text{similar term with } 1\to 2) ,\end{aligned} \quad (5B\text{-}II\text{-}10)$$

and we have similar expression for $\hat{X}_{N,3}{}^{(1),\tau N}$.

(9) $X_{a,b}{}^{A(1),J\tau}$ $(a,b = \alpha,n)$

$$X^{A(1),J\tau}_{a,b}(p_a,p_b,E) = X^{J\tau}_{a,b}(p_a,p_b,E) + \sum_{N}\int r^2 dr\, \hat{X}^{(1),J\tau}_{a,N}(p_a,r,E)\,\tilde{G}_2(E,r) \times \hat{X}^{(2),J\tau}_{N,b}(r,p_b,E)\left\{1-(-1)^{\ell_N+S_N+\tau}\right\} . \quad (5B\text{-}II\text{-}11)$$

We shall not present $X_{3,3}{}^{A(1)J\tau}$ because the expression is lengthy. But it is quite easy to obtain it from $X_{a,b}{}^{A(1),j\tau}$. To clarify the normalization of our reduced amplitude, we give an expression for an elastic scattering amplitude;

$$f^{A(1),J\tau}_{a,a}(p,q,E) = -\pi\mu_a X^{A(1),J\tau}_{a,a}(p,q,E)\;, \qquad (5B\text{-}II\text{-}12)$$

where μ_a is the reduced mass of isobar "a" and the spectator. In Appendix F, we present a reduced form of the total πN t-matrix in P_{11} partial wave. It is easy to fit the t-matrix to the experimental phase shift in angular momentum decomposed form. In this case, upon choosing some suitable form factor or πNN vertices we hope to reproduce the correct phase shift while taking into account the effect of the nucleon pole.

CHAPTER VI

PION ABSORPTION EFFECT ON ELASTIC π-d SCATTERING

From this chapter on until Chapter 8 we shall consider some practical use of the formulation that we have developed so far. There are several processes associated with the πNN system. We shall briefly sketch some features of these processes and later concentrate on the problem of the effect of pi-absorption on the s-wave π-d scattering length.

A. Processes Associated with the πNN System

Physically meaningful processes associated with the πNN system are as follows:

$$
\begin{aligned}
&\text{(i)} && \pi d \rightarrow \pi d \\
&\text{(ii)} && \pi d \rightarrow \pi NN \\
&\text{(iii)} && \pi d \rightleftarrows NN \\
&\text{(iv)} && NN \rightarrow \pi NN \\
&\text{(v)} && NN \rightarrow NN
\end{aligned}
\qquad (6A\text{-}1)
$$

All the processes above are related to one another through unitarity. So learning one specific process above will help us understand other processes there. For example, as will be mentioned later, a study of π-d scattering may give us some important information about N-N interactions.

The main feature of the scattering problems associated with the πNN system, from a point of view of their theoretical description, is that various kind of approaches are possible. This is not always true with general pi-nucleus problem in which we have more particles to be taken care of. Let us take π-d elastic scattering as an example. Methods which have been used include: (a) the Watson multiple scattering method[34] which usually is calculated up to the impulse (or single scattering) term or double scattering term at the most, (b) the Glauber theory calculations including other fixed scatterer approximations,[35] (c) the Faddeev three-body approach,[27][35] (d) the dispersion relation approach based upon unitarity-analyticity,[7] etc. There is another example: In process (iv) we can find a calculation using current algebra technique.[8] So in this respect the πNN system may serve for studying the interrelation among various theoretical approaches as well as clarifying their range of applicability. However, it is not our subject to look into that problem, so we shall not discuss it in the thesis.

With regard to the theoretical calculation, it seems that there are a couple of interesting features: (1) In the calculation of processes involving the pion, i.e. (i)∿(iv) of (6A-1), an approximation including the single plus double pion scattering gives quantitative agreement with experiment in many cases, which is rather independent of energy and the theoretical method adopted. For some processes, the single scattering approximation gives an adequate qualitative explanation (for example, π-d elastic differential cross section). But this is not always true as we shall see, for example, in the calculation of the pion absorption contribution to πd scattering length that we will present in the later chapters. (2) Processes involving the deuteron require an inclusion of the deuteron D-state in many cases for good quantitative predictions. This is because at high momentum, the deuteron D-state wave function becomes comparable in magnitude with the S-state component.

B. Pion Absorption Effect on π-d Scattering Length

I. Introduction

We now choose to concentrate on the problem of π-d elastic scattering at very low energy. Specifically we study π-d scattering length; $a_{\pi d} \equiv \lim_{E\to 0} f_{\pi d}(E)$. Since $\pi d \to NN$ is an exothermic reaction, it always occurs even at

πd threshold. So in πd elastic scattering, there always is a contribution from the process: $\pi d \to NN \to \pi d$ even at very low energy. In this process the intermediate N-N state has at least $\sim$136 MeV in the center of mass system (as is mentioned above), and thus has a corresponding high momentum component. This means that the threshold $\pi d \to \pi d$ process may give some short distance information in nucleon-nucleon interactions. In potential scattering theory, the threshold scattering gives a real quantity; the real scattering length. But as the N-N intermediate state in π-d scattering is far above its elastic threshold, it develops an imaginary part and therefore the π-d scattering length $a_{\pi d}$ becomes complex. Note that $\mathrm{Im}\, a_{\pi d}$ results solely from the two-nucleon intermediate state.

Due to unitarity, we can relate $\mathrm{Im}\, a_{\pi d}$ to the integrated cross section $\sigma(\pi d \leftarrow NN)$ or $\sigma(NN \leftarrow \pi d)$. A simple calculation shows that $\mathrm{Im}\, a_{\pi d}$ is related to the threshold π-d production cross section through

$$\sigma(\pi d \leftarrow NN)\Big|_{th.} = 6\pi\, \mathrm{Im}\, a_{\pi d}/m, \qquad (6B\text{-}I\text{-}1)$$

where m is the nucleon mass, as before.

As for the real part of the scattering length, we write

$$\mathrm{Re}\, a_{\pi d} \equiv \tilde{a}_{\pi d} + a'_{\pi d}, \qquad (6B\text{-}I\text{-}2)$$

where $\tilde{a}_{\pi d}$ comes from the pion scattering part; coming from the Faddeev part of $X_{\pi d,\pi d}{}^{A(1)}$, and $a'_{\pi d}$ is the contribution from the intermediate pion absorption; from the residual part of $X_{\pi d,\pi d}{}^{A(1)}$ after the Faddeev part is subtracted out (see Eq. (5B-II-11)) for $a = b \equiv (\pi d)$). In contrast to $\mathrm{Im}\, a_{\pi d}$, $a'_{\pi d}$ does not have any especially simple relation with other quantities in other channels. It can be expressed in terms of $\sigma(\pi d \leftarrow NN)$ by dispersion relation on the grounds of the analyticity of the scattering amplitudes.[37] However, we would have to have $\sigma(\pi d \leftarrow NN)$ at all physical energies and also in some unphysical energy range where we should have to assume its value, for example, by extrapolations. So it seems that the direct calculation of $a'_{\pi d}$ through T_{33} [see (3B-IV-14)] (or $X_{3,3}{}^{(1)}$ in Chapter 5) of our non-relativistic formulation is easier than the dispersion approach.

II. A Review of Theory and Experiment on $a'_{\pi d}$ and $\mathrm{Im}\, a_{\pi d}$

(i) $\mathrm{Im}\, a_{\pi d}$

Most studies on this quantity so far are for the equivalent $\sigma(\pi d \leftarrow NN)|_{th}$ or its inverse $\sigma(NN \leftarrow \pi d)|_{th}$ both in theory and experiment. So our review will be on $\sigma(\pi d \leftarrow NN)|_{th}$.

Usually low energy $NN \to \pi d$ reactions are fitted to phenomenological expression by Gell-Mann et. al.[6].

$$\sigma(\pi d \leftarrow NN) = \alpha \eta + \beta \eta^3, \qquad \text{(6B-II-1)}$$

where α and β are constants and η is the center-of-mass system pion momentum in pion mass unit. It is usually assumed that the first term is due to the s-wave production, and the second term being due to the p-wave production. The form of (6B-II-1) essentially is a result from the combination of threshold behaviour of the production t-matrices and the phase space factor. What we are concerned with is "α".

There has been an argument that we can relate the following three processes, all being near threshold, by detailed balance, charge independence and the technique of extrapolation to zero energy.[42]

$$\left.\begin{aligned} &a)\ \gamma + p \to \pi^+ + n \\ &b)\ \pi^{\pm} + p \to \pi^{\pm} + p \\ &c)\ p + p \to \pi^+ + d \end{aligned}\right\} \qquad \text{(6B-II-2)}$$

Reactions (a) and (b) are found to be consistent within experimental error. The results of (a) and (b) suggest that the threshold reaction should give $\alpha = 250\ \mu b$. But as will be mentioned soon, most experiments as well as model calculations indicated smaller values of α. So it is interesting to know the "correct" value of α both experimentally and theoretically to see whether those assumptions leading to α = 250 μb are plausible or not, which may be important in low energy particle physics. Also once a "reliable" α can be determined from experiments, it will be compared with those values obtained in model calculations using various N-N interactions (potentials), and thus hopefully we will be able to determine which N-N interaction (potential) is the best.

There have been several calculations and experiments to determine α and β. Woodruff[39] improved Lichtenberg's[38] calculations and was able to get β consistent with experiments but was not for α. Koltun and Reitan[40] tried similar calculation and obtained α which was in good agreement with the value deduced from experiment at that time[41] ($\alpha \sim 138$ μb). They adopted some Hamiltonian and performed a perturbation calculation. The pion-nucleon interaction used in the Hamiltonian consists of two parts; (i) a modified static πNN vertex which approximately satisfies Galilean invariance and (ii) the s-wave direct πN interaction which is a kind of scattering length approximation to the low energy πN scattering (this corresponds, in our modern language, to ρ and σ meson exchanges in πN interaction). The Deuteron wave function and the initial state N-N scattering wave function were obtained from the Hamada-Johnson potential. What they showed in the calculation is that (1) the direct production term is small compared with the term containing the pion rescattering after production and (2) the direct term gets smaller due to the cancellation between contributions from deuteron S-state and D-state.

Later Rose performed an experiment[42] and obtained $\alpha \cong 240$ μb rather different from old values but is close to what is suggested from reactions (a) and (b) in (6B-II-2). Reitan[43] recalculated α using Koltun-Reitan (KR) method with several refinements and found $\alpha \sim 201$ μb which is close to Rose's value. Reitan also found that α is not a constant but varies with energy. However the energy dependence was observed to be weak.

KR method was further applied later by Thomas and Afnan [44] and by Pradhan and Singh.[45] The former group used several different deuteron models with different D-state probability as well as different short range behavior. They found α to be quite sensitive to the dueteron models. With supposedly most realistic deuteron, obtained from the Reid soft-core potential, and also most recent πN scattering lengths,[46] they obtained $\alpha \sim 114$ μb, which is amazingly small. The latter group also adopted several different deuteron models as well as different N-N potentials for the initial N-N distortion. They then performed several phase-equivalent transformations on the potentials which modified the off-shell behavior of N-N interactions. The result was that local N-N potentials seem to be rather rigid to the phase-equivalent transformations whereas separable ones are sensitive to the transformations, judging from the resultant α values. When the Reid potential is used (without any transformation) they obtained $\alpha \sim 150$ μb.

Lazard, Ballot and Becker[47] took a similar but different approach to evaluate $\sigma(\pi d \leftarrow NN)$ from threshold up to about 300 MeV in pion energies and found $\alpha \sim 300$ μb different from any result obtained from KR method. Afnan and Thomas[28] used the imput informations used by Lazard et. al. in their KR calculation and found $\alpha \sim 203$ μb. Those theoretical results mentioned so far seem to indicate that α may be sensitive to every piece of input function as well as the approximations made in the propagators, etc.

Since KR method shows that the direct production term is far smaller than the rescattering term, Afnan and Thomas[28] questioned the convergence of the perturbation treatment. As KR method would introduce pion overcounting in going to higher order calculation, they set up a three-body Faddeev type model to calculate $\pi d \rightarrow \pi d$, $NN \rightarrow \pi d$, etc. This model with the Reid soft-core deuteron gives $\alpha \sim 220$μb consistent with Rose's value, but different from more recent value by Richard-Serre et. al.[48] of $\alpha \sim 180$ μb. Afnan and Thomas then observed the difference in experimental values together with Reitan's result suggesting the energy dependence of α. They noticed that older experiments giving smaller values of α were performed at higher energies than where newer experiments were done. So they suggested α to be a decreasing function of the energy; the dependence being not so weak. Their model three-body calculation actually showed this tendency. Recently Spuller and Measday[48] reanalyzed the data used by Richard-Serre et. al. not based upon constant α,β assumption but adopting some possible energy dependence together with a resonance like behavior in some part of the parametrization in the cross section; the form is

$$\alpha = \alpha_0 + \alpha_1 \eta + \alpha_2 \eta^2 + \alpha_3 \eta^3 + \mathrm{Res}, \qquad \text{(6B-II-3)}$$

where Res is of the Lorentzian type resonant form representing the p-wave contribution from 3-3 resonance. The analysis shows that α_0 (α at zero pion energy) may stay somewhere between 200 μb and 300 μb, which is consistent with Rose's value as well as the value suggested from the consistency among (a), (b) and (c) in (6B-II-2) at very low energy.

(ii) $a'_{\pi d}$

Experimentally this quantity cannot be measured directly. The same is true of $\tilde{a}_{\pi d}$ which comes from pure πd scattering without pion absorption: The only measurable quantity is the total $\mathrm{Re}\,a_{\pi d}$ ($\equiv \tilde{a}_{\pi d} + a'_{\pi d}$). The most recent measurement[50] of Re $a_{\pi d}$ gives

$$Re\, a_{\pi d} = -0.052 \left(\begin{matrix}+0.022\\-0.017\end{matrix}\right) \mu^{-1}.$$

The negative sign here corresponds to weak, repulsive interaction. As is easily seen, the uncertainty in the above value is about 50%.

With regard to the value of $\tilde{a}_{\pi d}$, there have been a number of calculations(28),(51) along the multiple scattering approach; from the impulse type to the three-body Faddeev type calculations. It seems that from those calculations the inclusion of single and double pion scattering is sufficient quantitatively; the multiple scattering effect higher than the second order is rather small. However, the calculated value of $\tilde{a}_{\pi d}$ is sensitive to the πN scattering length. The S_{31} scattering length; a_3, is less well known than a_1; the S_{11} scattering length. The lowest order contribution to $a_{\pi d}$ comes from the combination of a_1+2a_3, and although both a_1 and a_3 are of the same order, they have opposite signs and a_1+2a_3 tends to get very small. Thus $a_{\pi d}$ is known to the same extent as a_1 and a_3. According to Afnan and Thomas,(28) the calculated value of $a_{\pi d}$ using recent a_1 and a_3 and taking into account several corrections like $\pi^{\pm}-\pi^{\circ}$ mass difference, etc. is $-0.037 \pm 0.005\ \mu^{-1}$.

As for $a'_{\pi d}$ there have been few calculations. More than ten years ago, Brueckner[52] estimated the effect of pion absorption on the level shifts of pionic atoms. Using a simple Born type approximation, he found that the level shifts are negative (as in a weak repulsion) and are of the same order of magnitude as the level width (in terms of $a'_{\pi d}$ and $\mathrm{Im}a_{\pi d}$ it says that $a'_{\pi d}<0$ and $a'_{\pi d} = 0\ (\mathrm{Im}a_{\pi d})$). Thouless[53] included the effect of crossed absorption in the Brueckner's calculation, but the essential feature was not changed. Using the available values of α (see the discussion in $\mathrm{Im}a_{\pi d}$ in the last sub-section) for $\mathrm{Im}a_{\pi d}$ their estimate gives that $a'_{\pi d} \approx -0.005 \sim -0.003\ (\mu^{-1})$.

Later Beder[37] evaluated $a'_{\pi d}$ using the unsubtracted dispersion relation applied to $\sigma(NN\rightarrow\pi d)$ data. He obtained a large positive value for $a'_{\pi d}$, contrary to Brueckner's result, but his method appears to be much less reliable. A recent calculation by Afnan and Thomas using their Faddeev approach gives $a'_{\pi d} = -0.005\ \mu^{-1}$, which is consistent with Brueckner's value. However, we have observed that the accuracy of their calculation is limited by their own model. Therefore we shall use our formulation in Chap. 3∿5 to calculate $a'_{\pi d}$, which is considerably more general.

CHAPTER VII

METHOD OF CALCULATIONS

In this chapter we present our method of calculating $a'_{\pi d}$ and $\mathrm{Im}\, a_{\pi d}$; the real and imaginary parts of the πd scattering length contributed from the intermediate pion absorption.

A. Equations and Approximations

The total $\pi d \to \pi d$ amplitude is given in (5B-II-11); $X^{A(1),J\tau}_{a,b}$ where "a" and "b" should stand for πd. When we write

$$X^{A(1),J\tau}_{\pi d,\pi d} = X^{J\tau}_{\pi d,\pi d} + J^{A(1),J\tau}_{\pi d,\pi d}, \qquad (7A\text{-}1)$$

$X^{J\tau}_{\pi d,\pi d}$ is the Faddeev part which describes πd elastic scattering without intermediate pion absorption. At threshold this gives $\tilde{a}_{\pi d}$ (see Chapter VI). Our present interest is in $J^{A(1),J\tau}_{\pi d,\pi d}$ which comes from the contribution from the pion absorption in intermediate states. Explicitly:

$$J^{A(1),J\pi}_{\pi d,\pi d}(p',p,E) = \sum_N \int_0^\infty r^2 dr\, \hat{X}^{(1),J\tau}_{\pi d,N}(p',r,E)\,\tilde{G}_{T_2}(E,r) \qquad (7A\text{-}1')$$

$$\times\ \hat{X}^{(2),J\tau}_{N,\pi d}(r,p,E)\left\{1-(-1)^{\ell_N+S_N+\tau}\right\},$$

where

(1) $\hat{X}^{(1),J\tau}_{\pi d,N}(P',P,E)$ is an amplitude for NN→πd with the initial N-N interaction

(2) $\hat{X}^{(2),J\tau}_{N,\pi d}(P',P,E)$ is for πd→NN without the final N-N interaction

(3) $\tilde{G}_2(E,r)$ is a two-nucleon propagator with off-shell nucleon self-energy effect.

Then the part of the πd scattering length under consideration is expressed:

$$a'_{\pi d} + i\,\mathrm{Im}\,a_{\pi d} = -\frac{\pi m_d \mu}{m_d+\mu} \lim_{p\to 0} J^{A(1),J\tau}_{\pi d,\pi d}(P,P,\varepsilon_P). \quad (7A-2)$$

In the above expression, $J=\tau=1$ and $\varepsilon_p = md+\mu+\sigma_p$ where md; deuteron mass, μ; pion mass and σ_p; the kinetic energy of the pion and deuteron in their center of mass system.

As for $\hat{X}^{(1),J\tau}_{\pi d,N}$ and $\hat{X}^{(1),J\tau}_{N,\pi d}$, they can both be obtained from the solution of Faddeev equation; $X^{J\tau}_{a,b}$, πNN vertices $\hat{\Lambda}$ and $\hat{\Gamma}$, and (for $\hat{X}^{(1),J\tau}_{\pi d,N}$) $\tilde{t}^{J\tau S}_{NN}$. This is easily understood through (5B-II-6) and (5B-II-9). We should just remember that all the amplitudes appearing in (7A-1') have been antysymmetrized and decomposed into angular momentum and isospin eigenstates. This means that subscripts πd, N etc. in the amplitudes stand for, for example, πd≡(nd,ldsdjd ndtdjdtd). For details of this notation, see Chap. V and Appendix G. We have adopted here a normalization convention:

$\langle\vec{k}|\vec{k}'\rangle = \delta^3(k-k')$ (this is equivalent to saying that $\sum_{\vec{k}} \Rightarrow \int d^3k$).

In our application of the non-relativistic πNN scattering formulation we do not solve the exact equations but make the following approximations.

(i) $\tilde{G}_2(E,r)$ in $\hat{X}^{(1),J\tau}_{\pi d,N}$ is replaced by $G_2(E,r)$, the free two-nucleon propagator without off-shell nucleon self-energy effect. This is a reasonable approximation at πd threshold as real πNN process does not occur.

(ii) $\tilde{t}^{J\tau}_{NN}$ (also appearing in $\hat{X}^{(1),J\tau}_{\pi d,N}$), this is a t-matrix for N-N scattering, including the inelastic effect coming from the virtual production of a pion. For threshold πd scattering this may be replaced by $t^{J\tau S}_{NN}$, which only contains the elastic information.

(iii) The Faddeev amplitude is approximated by its first term; $Z^{J\tau}_{a,b}$. This makes both $\hat{X}^{(1),J\tau}_{\pi d,N}$ and $X^{(2),J\tau}_{N,\pi d}$ include one scattering of the pion before absorption or after emission. Since πN scattering at low energy is weak, we expect that this approximation works reasonably.

The forms we get after these approximations are:

$$\hat{X}^{(2),J\tau}_{\pi d,N}(p,q,E) \simeq \sqrt{2}\Big\{ Z^{J\tau}_{\pi d,N}(p,q,E) + \sum_{\alpha}\int r^2 dr \times Z^{J\tau}_{\pi d,\alpha}(p,r,E)\,\tau^{\alpha}\Big(E-\sum_i m_i - \frac{r^2}{2m_\alpha} - \frac{r^2}{2\mu_\alpha}\Big) Z^{J\tau}_{\alpha,N}(r,q,E)\Big. , \qquad (7A\text{-}3)$$

and

$$\hat{X}^{(1),J\tau}_{\pi d,N}(p,q,E) \simeq \hat{X}^{(2),J\tau}_{\pi d,N}(p,q,E) + \sum_{N'}\int_0^\infty r^2 dr\, \hat{X}^{(2),J\tau}_{\pi d,N'}(p,r,E) \times G_2(E,r)\, t^{J\tau S_N}_{NN,\ell_N'\ell_N}(r,q,E) \qquad (7A\text{-}3')$$

In (7A-3) α stands for a state of correlated πN pair. We also have a similar expression for $\hat{X}^{(2),J\tau}_{N,\pi d}$.

We shall not calculate $\hat{X}^{(1),J\tau}_{\pi d,N}$ independently for $\sigma(\pi d \leftarrow NN)$ but calculate (7A-1) directly under the approximation discussed so far. Therefore, α is obtained from Im$a_{\pi d}$ through (6B-I-1). The terms included in our calculation are shown diagrammatically in Fig. (7-1). In this figure diagrams A and B represent the processes through pion absorption-re-emission without any πN scattering. On the other hand diagrams C to F contain at least one πN scattering before or after pi-absorption. For each diagram shown we also include the process where the pion is emitted by the nucleon which has not absorbed the pion; for example, diagram A is considered to include A'. Also diagrams C and E are meant to include their conjugate diagrams in which the order of the direct pion absorption (or emission) and the pion absorption (or emission) after (or before) single πN scattering is interchanged (C includes $\bar{C}$, for example).

It may be relevant here to make some comparison between our method of calculation and that of Koltun and

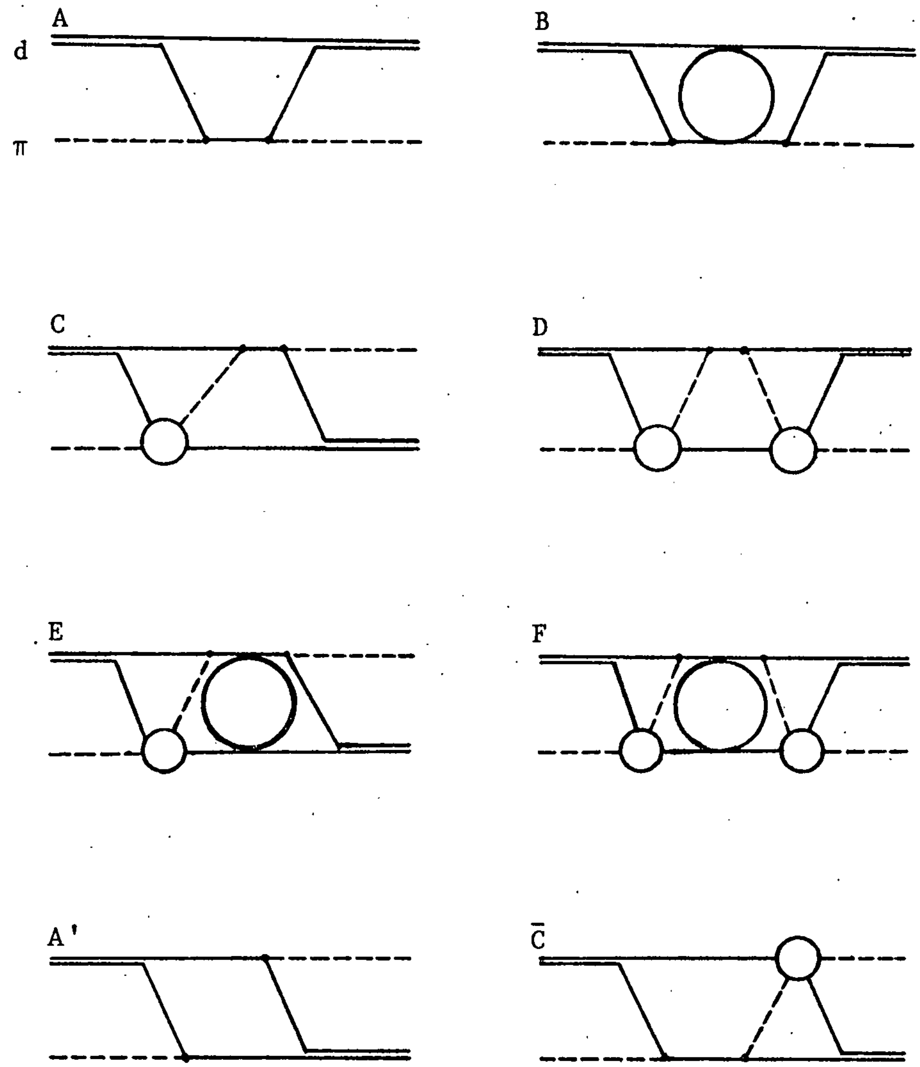


Fig. 7-1

Diagrams included in our calculation. For each diagram shown, we also include the process where the pion is emitted by the nucleon which has not absorbed the pion (diagram A includes A', for example). Also diagrams C and E are meant to include their conjugate diagrams in which the order of single and double pion scattering is interchanged (conjugate of C; $\bar{C}$ is shown above).

Reitan (KR) for α. The first point is that of course we shall not calculate $\sigma(NN\to\pi d)$ directly as KR did. The bonus of our method, however, is that we obtain $a'_{\pi d}$ at the same time, which is not possible in the KR method. Second, within the approximation we have adopted, the calculation includes up to pion rescattering, which is the same as the KR formulation does offer. There is a difference, however, in the treatment of the rescattering; KR adopted the S-wave rescattering only, which is an extension of the scattering length approximation, whereas we have used separable πN interaction in S_{11}, S_{31}. We also include P_{33} partial wave. Third, within our non-relativistic forms we do not make any approximation to the intermediate πNN propagation; KR adopted a certain kind of static closure approximation for that. Fourth, there is no backward propagation of pions in our method, which would correspond to a four-body state. On the other hand it appears in the KR rescattering term, which makes the analytic expression simpler.

B. Input Functions

(i) Deuteron Form Factor (wave function)

Since previous calculations have shown that the results are sensitive to the deuteron structure, especially to short range behavior, we try to see that tendency reflected in Im$a_{\pi d}$ and $a'_{\pi d}$. For that purpose we have chosen three different models;

(a) The S-wave Hulthén deuteron which results from a simple Yamaguchi type S-wave N-N separable potential. This choice is merely for its simple analytic form.

(b) The Reid deuteron; obtained from Reid soft-core N-N potential. We have used a separable form expression to the Reid potential obtained from Pieper's application of the Ernst-Shakin-Thaler procedure.[55] The Reid potential has been considered to be one of the best realistic N-N potentials as it is fitted to several observed nucleon-nucleon and nuclear quantities. Therefore this deuteron comes from N-N dynamics.

(c) The McGee deuteron.[56] This is similar to those constructed by Gourdin et al.,[56] which were obtained from analytic properties of dpn vertex (the fit was made to electron scattering, deuteron photo disintegration etc.). So although this deuteron is very similar to Reid deuteron in its momentum space behavior, its construction is not through N-N dynamics.

As for the D-state probability; P_D, the Reid deuteron shows P_D=6.49% whereas the McGee deuteron has P_D=7.0%.

(ii) N-N Potential for Two-nucleon Intermediate State

For calculational simplicity, we have adopted separable potentials of Mongan.[57] The potentials are of second rank; with repulsive and attractive parts. In the case of s-wave πd elastic scattering the resultant N-N intermediate state stays in $^3P_1$ partial wave (the notation here stands for $^{2S+1}L_J$). Among four different models of Mongan we have chosen type 1 and 2. Type 2 potential in $^3P_1$ happens to reduce to a rank 1 separable potential. Type 1 reproduces the experimental phase shift better in this partial wave.

(iii) Pion-nucleon t-matrices

In our calculation it is necessary to have πN t-matrices in several partial waves, which are found to be important at low energies. We pick up S_{11}, S_{31} and P_{33} partial waves which contribute to the rescattering term (other πN P-waves are small at low energy). For the purpose of later comparison with the Afnam-Thomas result,[28] we have adopted their separable t-matrices for those partial waves. Those t-matrices are adjusted to reproduce scattering length (or volume) and to the threshold behavior. For S_{11} and S_{31}, these t-matrices fit well to the low energy phase shifts. For P_{33} wave, the t-matrix is fitted to the 3-3 resonance pole. The forms of the interactions or wave functions in (i)-(iii) are listed in Appendix H.

(iv) πNN Vertices

The most general forms of πNN vertices are $\Lambda(E)$ and $\Gamma(E)$ with Hermitian analyticity $\Lambda^*(E) = \Gamma(E)$, which satisfy

$$\left.\begin{aligned} \Lambda(E) &= Y(E) + Y(E) G_2 t_{\pi N}(E) \\ \text{and} \qquad \Gamma(E) &= W(E) + t_{\pi N}(E) G_2 W(E) \end{aligned}\right\} , \tag{7B-1}$$

(see Appendix E and Chap. III, Section C), where Y(E) and W(E) are two-particle irreducible πNN vertices. However, not much is known about the form of Y(E) and W(E) for non-relativistic applications (in principle they can be determined by P_{11} total πN scattering phase shift through $\theta_{\pi N}(E)$ of Chap. III, section C). The only information available for Y(E) (and W(E)) is that it has the form $\sim f\vec{\sigma}\cdot\vec{k}$, where the coupling constant f may be determined by the πN Born term (through the residue of πN P_{11} scattering amplitude) or the long range one-pion-exchange (OPE) contribution to N-N scattering. With this in mind and also taking into account the fact that $t_{\pi N}(E)$ in (7B-1) comes only from P_{11} π-N scattering (which we have neglected), we consider $\Lambda(E) \equiv Y(E)$ (and $\Gamma(E) \equiv W(E)$) to be given from the following πNN coupling.

$$H_I = i(4\pi)^{\frac{1}{2}} \frac{f}{\mu} \vec{\sigma} \left\{ \vec{\nabla}_\pi [\underline{\tau}\cdot\underline{\phi}(x)] + \frac{1}{2m} [\overleftarrow{P}\cdot\underline{\tau}\cdot\underline{\pi}(x) + \underline{\tau}\cdot\underline{\pi}(x)\vec{P}] \right\}, \tag{7B-2}$$

In the above expression $\underline{\phi}(x)$ and $\underline{\pi}(x)$ are the pion field and its conjugate momentum respectively. Both of them are isospin vectors, and their representations are:

$$\langle 0|\underline{\phi}(x)|\vec{q}\xi\rangle = [2\omega_q(2\pi)^3]^{-\frac{1}{2}} e^{-iqx}\chi_\xi \qquad (7B\text{-}3)$$

and

$$\langle 0|\underline{\pi}(x)|\vec{q}\xi\rangle = -i[\omega_q/2(2\pi)^3]^{\frac{1}{2}} e^{-iqx}\chi_\xi, \qquad (7B\text{-}3')$$

where χ_ξ is an isospin eigenstate of the pion. The form (7B-2) was used for example in KR and we shall call it the "standard" Galilean invariant form since it approximately keeps the Galilean invariance in the process of reducing it from the relativistic πN interaction Hamiltonian.

Recently there has been a series of discussions on the non-relativistic reduction of relativistic πNN vertices.[58] Especially, it has been shown that the nature of the interactions of a nucleon, which emits or absorbs the pion, with the rest of the system seems to affect the reduced form of πNN vertices. With this in mind, we introduce another form of πNN coupling (or vertex) which depends upon the relative momentum of the pion and the nucleon before absorption (or after emission) of the pion. Its form for pion absorption is:

$$K(\vec{q}) = i(4\pi)^{\frac{1}{2}} \frac{f}{\mu} \frac{1}{\sqrt{\omega_r (2\pi)^3}} \underline{\tau} \cdot \underline{a}_r \vec{\sigma} \cdot \vec{q}, \qquad (7B\text{-}4)$$

where ω_r is the pion energy (with momentum $\vec{r}$) and $\underline{a}_r$ is an isovector pion annihilation operator. We call (7B-4) the "modified" Galilean invariant coupling. Both (7B-2) and (7B-4) give the same static limit of Chew-Low type. For a more transparent understanding of both πNN vertices we have here, let us consider an absorption process in which an incoming pion and a nucleon have momenta $\vec{r}$ and $\vec{p}$, respectively (see Fig. 7-2).

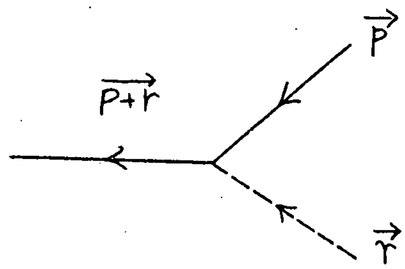


Fig. 7-2

Then we have, as an absorption vertex:

$$\Lambda(\vec{S}) = i(4\pi)^{\frac{1}{2}} \frac{f}{\mu} \frac{1}{\sqrt{(2\pi)^3 \omega_r}} \underline{\tau} \cdot \underline{a}_r \vec{\sigma} \cdot \vec{S}, \qquad (7B\text{-}5)$$

where, for the "standard" form

$$\vec{S} = \left(1 - \frac{\omega_r}{2m}\right) \vec{r} - \frac{\omega_r}{m} \vec{p} \qquad (7B\text{-}6)$$

and, for the "modified" form

$$\vec{S} = \frac{m}{m+\omega_r}\vec{r} - \frac{\omega_r}{m+\omega_r}\vec{p} \quad . \qquad (7B-7)$$

The difference between (7B-2) and (7B-4) coming from the coefficients of $\vec{r}$ and $\vec{p}$ is of the order of ω_r/m, which is small (~0.14) for a threshold pion but may not be negligible for a pion carrying a large momentum which actually occurs in pion rescattering terms. There might be some difference in the values of f for (7B-2) and (7B-4) when it is determined from the residue of the π-N Born term. However, the difference in f for the two vertices turns out to be $O(\mu^2/m^2)$. So we may use the same value of f for both.

In terms of $\Lambda(\vec{s})$ above, we obtain for example, the driving term for πd→NN:

$$Z_{N,\pi d}(\vec{p},\vec{q},E) = \frac{\Lambda(\vec{t})\, g^d(\vec{k})}{E^+ - \omega_q - \frac{p^2}{2m} - \frac{(\vec{p}+\vec{q})^2}{2m}} \quad , \qquad (7B-8)$$

where $\vec{k}=\vec{p}+\frac{\vec{q}}{2}$ and $\vec{t}$ is given in (7B-6) or (7B-7) with the replacement; $\vec{r}\to\vec{q}$ and $\vec{p}\to-(\vec{p}+\vec{q})$. When we calculate (7B-8) in its angular momentum reduced form, $\Lambda(\vec{t})$ reduced in angular momentum-isospin takes the form:

$$\hat{\Lambda}(t) = 4\pi i \frac{\sqrt{3}\; t}{\sqrt{\omega_q 2(2\pi)^3}} \frac{f}{\mu} \quad . \qquad (7B-9)$$

Our numerical procedure is as follows. First the driving terms, $\{Z^{J\tau}_{a,b}\}$ are evaluated for all channels concerned, using the formula (G2-3) of Appendix G. Then the double scattering term is calculated by multiplying two Z's and isobar propagators, and then we sum and integrate over all possible intermediate states in that product. Lastly, we form products of the double scattering terms, N-N t-matrix, N-N propagator, etc., and momentum integrations are carried out. Whenever a principal integration appears, we use the identity

$$P\int_0^\infty \frac{f(a^2)}{x^2-a^2}\,dx \equiv 0$$

and make a subtraction so that the integration becomes smooth. We divide the region of integration $[0,\infty]$ into two parts. In both regions we use Gauss-Legendre quadrature of order 38 which turns out to be accurate enough for our purpose. (The final numbers are expected to have 2~3% numerical uncertainty at the worst.) In the integration of higher momentum we transform the integration variable from p to x through $p=\tan[\frac{\pi}{4}(1+x)]$ to integrate with respect to x.

Lastly, typical physical constants to be used in the calculation are

$$m = \frac{M_p + M_n}{2} = 938.9264 \text{ MeV}$$

$$\mu = (m_{\pi^+} + m_{\pi^-} + m_{\pi^o})/3 = 138.034 \text{ MeV}$$

$$f^2 = 0.0822$$

ε_d (dueteron binding energy) = 2.2246 MeV.

CHAPTER VIII

RESULTS AND DISCUSSION ON OUR CALCULATION

A. General Feature

The numerical calculation has been performed according to the details discussed in the last chapter. The results; $a'\pi d$ and Im$a\pi d$, which come from those processes are shown in diagrams A to F of Fig. 7-1 and in Table 1 ~ Table 6. For a later discussion we have also calculated $a'\pi d$ and Im$a\pi d$ resulting from the Afnan-Thomas effective πNN vertex[28] which is obtained from their separable P_{11} πN t-matrix. These numbers are shown in Table 7.

In the tables we find entries SS, DS and DD. They stand for the following; (1) SS: comes from the process in which deuteron's S-state contributes to both initial and final states of the scattering, (2) DS: the S-state contributes in the initial state while the D-state contributes in the final state and its reversed process and (3) DD: same as SS except that the D-state replaces the S-state. (A+B) may be identified as the "impulse" term (or direct term) which includes intermediate two-nucleon interactions and (C+D+...+F) is identified as the "rescattering" term.

Looking at the tables, we find that with realistic deuterons total SS contribution dominates over DS and DD. But this should not be interpreted as due to the fact that the deuteron D-state probability is small. If we look closely at the tables, we see that it is due to internal cancellation of large numbers (cancellation between impulse and rescattering contributions). Another noticeable thing is that the "rescattering" term is far larger than the "impulse" contribution independent of the deuteron model used. In DW language this is equivalent to saying that the pion distortion has determined the main feature of the result. This result comes from the pseudoscalar nature of the pion, which is reflected in the p-wave dominance of the low energy pion absorption (or emission) by the nucleon. We can understand this feature by observing the πNN vertex in (7B-1).[2] There it is easy to see that the s-wave pion absorption (or emission) contribution at low energy is about μ/m of the similar contribution coming from the p-wave. This tendency determines the impulse term. However, in the resulting term the pion can have the p-wave component as well as large momentum contribution to the s-wave part before its emission or absorption. This is because the process is going off-energy-shell. Thus the "μ/m reduction" disappears there.

When realistic deuterons are used, there is some factor which further reduces the impulse contribution.

That is, in $A_{,}$(SS) for Imπd is amazingly small, and in (A+B) we can observe that the deuteron S and D states tend to cancel each other's contribution. This feature can be understood partly in the following observation. Let us look at the radial part of the deuteron wave function in momentum space. It is normalized as follows

$$\int_0^\infty q^2 dq \{\phi_S^2(q) + \phi_D^2(q)\} = 1 \qquad (8A-1)$$

where ϕ_S and ϕ_D are S and D components respectively. When we define the sign of $\phi_S(q)$ by requiring: $\phi_S(q)>0$, it turns out that $\phi_D(q)$ stays negative for all values of $q(>0)$. On the other hand $\phi_S(q)$ becomes negative for $q \gtrsim 2.0\ f_m^{-1}$; it has a zero at $q \sim 2.0\ f_m^{-1}$. For large q both $\phi_S(q)$ and $\phi_D(q)$ behave in a similar manner. So the difference in their signs at low momentum seems to bring about the S-D cancellation in (A+B). As for the smallness of the SS contribution to Imπd in A, we can understand its feature as follows. In diagram A the contribution to the imaginary part is totally determined by the quantity proportional to $\phi_j^2(q)$ (j=S,D) at the pole of the two-nucleon propagator. For πd threshold energy the pole is at $q^2 \simeq m_\mu \simeq (2.6\mu)^2$. So since the zero of $\phi_S(q)$ is around $q=2.0\ f_m^{-1} \simeq 2.85\mu$, $\phi_S^2(q)$ at the pole becomes very small and thus a small contribution to Imπd results. Since the Hulthén deuteron does not have any D-state nor any zero in the wave function, it is free from any reduction of the kind mentioned above.

In rescattering terms as a whole, what has been observed in the impulse term does not appear; the overall structure of $\phi_S(q)$ and $\phi_D(q)$ (their integration over q) is reflected more here. Notice, however, that some features of the impulse term appear in the contributions C and E as "half" of these diagrams is of impulse nature.

Another interesting tendency that has been observed is that for a given deuteron and a πNN coupling models, $a'_{\pi d}$ becomes larger and $\mathrm{Im}\,a_{\pi d}$ becomes smaller when Mongan 1 potential is used than the corresponding values obtained from the use of Mongan 2 potential. The difference, however, is generally small. When we look at $a'_{\pi d}$ and $\mathrm{Im}\,a_{\pi d}$ together as a complex number, the difference seems to be in the rotation angle in the complex $a_{\pi d}$ plane; Mongan 1 result has a smaller rotation angle measured from the real axis. Actually it is easy to see that the norms of the two complex numbers are pretty much the same.

We have checked that the relative difference in the phase shift values between the one obtained from Mongan 1 potential and the one from Mongan 2 potential is ~1/27. We also have compared the rotation angles mentioned above obtained from the two potentials. The relative difference turned out to be ≃1/8 ~ 1/10, depending upon the deuteron models (excluding Hulthén). So the difference in the rotation angles does not dominantly come from the on-shell difference in $^3P_{11}$ potentials.

In this respect it would be worth having some more calculations to be done with various types of N-N potentials for $^3P_1$ two-nucleon state to see whether the "rotation" nature persists or not.

B. Results

We now look at our results coming from different input functions individually and make a comparison within them as well as with other results, and draw some conclusions.

First we find that for a given πNN vertex and $^3P_1$ N-N potential, the Pieper-Reid and McGee deuterons give rather similar results as a whole. This is because both deuterons behave in pretty much the same way with slight deviations from each other; for example, the zero of $\phi_S(q)$ in the McGee deuteron is at a slightly larger momentum than for the Pieper-Reid deuteron. On the other hand, the Hulthén results are very different from both of them. Of course we have used the Hulthén deuteron not to obtain the result which may be compared with experimental values but to see the sensitivity of the calculation upon the input data.

(i) $a'\pi d$

The main contribution comes from SS in diagram C. This has a negative sign and its magnitude is somewhere between 4 and 8 (in $10^{-3}\mu^{-1}$) depending upon the deuteron model adopted. This supports the result of Brueckner in his very simple model. The dependence of this quantity upon the types of deuteron model, $^3P_1$ N-N potential and πNN coupling seems to be rather sensitive like Im$a\pi d$. But if we compare values of Im$a\pi d$ in different dueteron models with those of $a'\pi d$ including the Hulthén deuteron, $a'\pi d$ is less sensitive to deuteron models than Im$a\pi d$. This may be the reflection that the overall structure of the deuteron etc.; the integrated value but not their local properties, determines $a'\pi d$. When the P_{33} πN interaction is turned off in the rescattering term, it is found that $a'\pi d$ becomes larger in magnitude by 15~20%. As for the difference in $a'\pi d$ resulting from different πNN vertices, it is not large, about 10% for realistic deuterons.

In view of the fact that Hulthén is not a realistic deuteron we have obtained that $-7.3 < a'\pi d < -5.2 (10^{-3}\mu^{-1})$. This is not much different from the Afnan-Thomas value[28] of -4.9 $(10^{-3}\mu^{-1})$ obtained from their three-body model. So taking Re$a\pi d$ of Bailey et al.[50] the contribution from the pion absorption process to Re$a\pi d$ is estimated to be about 10%.

(ii) Imaπd

The main contribution comes from two rescattering diagrams without intermediate N-N interaction; D and C. Diagrams E and F stand for the rescattering with N-N interaction. Each of them separately is non-negligible but (E+F) becomes rather small and even close to the contribution from the impulse term.

Except for the case in which the Hulthén deuteron and standard Galilean invariant (G.I.) coupling are adopted, the value of Imaπd stays within 4 and 11 in $10^{-3}\mu^{-1}$. Together with Imaπd we have calculated α for each set of input functions. The values of α are listed in Table 8.

We have observed that if the P_{33} πN scattering contribution is turned off, Imaπd decreases about 20% in every case with different input functions. Also Imaπd is larger when the standard G.I. coupling is used as πNN vertices.

When we ignore those values associated with the Hulthén deuteron as being unrealistic, the range of reliable values is shown as $4.25 \leqslant \text{Im}a_{\pi d} \leqslant 6.06$ in $10^{-3}\mu^{-1}$. When we use realistic deuterons the difference due to deuteron models is observed to be about 10%. If the Pieper-Reid[52] deuteron is taken to be the most reliable, we find that $4.25 \leqslant \text{Im}a_{\pi d} \leqslant 5.61$ ($10^{-3}\mu^{-1}$), which corresponds to $241 \leqslant \alpha \leqslant 318$ (μb). This is consistent with Rose's[42] and Spuller-Measday's analysis.[50] Note that our result is obtained exactly at the πd threshold. As for the difference in Im$a_{\pi d}$ due to different πNN vertices, we have found that for realistic deuterons it is about 10%, just like $a'_{\pi d}$. The big difference observed in the Hulthén result may be interpreted as follows: The Hulthén deuteron wave function in momentum space remains large at high momentum compared with the S-wave Pieper-Reid and McGee wave functions. This could enhance the difference of $O(\frac{\omega_q}{m})$ between the "standard" and "modified" πNN vertices and hence comes the difference. For all the deuteron models the difference due to a different choice of πNN vertices appears as a multiplicative factor $[m/(m+\mu)]^2$ in the case of impulse terms. This is easily understood from (7B-5) and (7B-6) when ω_r is set equal to μ. However, for the rescattering the above arguement does not apply.

At this stage we are not sure which is better, the standard or the modified G.I. πNN coupling. The standard G.I. has been used most frequently so far, but as was mentioned in the last chapter, the non-relativistic limit of the Lorentz invariant πNN vertex seems subject to an environment for the pion absorbing or emitting nucleon. So the "standard" G.I. vertex does not have to be the right one.

It may be worth while to compare our result with the values obtained from the Koltun-Reitan (KR) method. For that purpose we have evaluated the values of I_j; (j=1,...6) from our calculation. I_j's appearing in the KR type approach are related to α; $\alpha \propto |\sum_j I_j|^2$. Our I_j values are shown in Table 9. Note that I_3 and I_5, as well as I_4 and I_6 are inseparable in our calculation. When we compare our I_j values with those in one of the standard KR type calculations,(40,45) it is found that our I_3+I_5 and I_4+I_6 are generally larger even when they are from the combination of the Pieper-Reid deuteron and the modified G.I. πNN vertex, which has produced the smallest Imaπd (or α) in our calculation. (We have also found some difference in the value of I_2 which comes from the deuteron D-state contribution to the impulse term.) Both of these numbers are associated with the contribution to α from the rescattering diagrams. The difference between our method and that of KR is therefore mainly in the rescattering part. As has been remarked, we obtained smaller Imaπd when the P_{33} πN scattering contribution was switched off in the rescattering term. In the usual KR type calculation, only S_{11} and S_{31} contributions are included in the rescattering part. Therefore the fact that our I_3+I_5 and I_4+I_6 are large compared with those of the standard KR result, may be attributed to our inclusion of P_{33} partial wave. Actually in Reitan's calculation(43) within KR formalism, α has become larger (α ~ 201μ) due to some improvements among which was the inclusion of the πN p-wave in the pion rescattering term. However, things do not seem so simple, as an improved KR type calculation by Thomas and Afnan,(44) in which more recent πN S_{11} and S_{31} scattering lengths are used, showed a substantial decrease in α ($\alpha \sim 114\mu b$). This decrease is in the rescattering term! So we may have to worry about the sensitivity of the result on every piece of input functions.

C. Remarks on the πNN Vertices

As has been discussed in B of this chapter, the "standard" and "modified" Galilean invariant vertices give rather similar results (the difference is ~10%). This means that when integrated over the internal momentum, the difference of $O(\omega_q/m)$ between the two vertices never becomes large in the rescattering term. This may be the case when other part of the integrand function decreases sufficiently rapidly for large momentum. If we freeze ω_q dependence in our πNN vertices at $\omega_q=\mu$, as has been adopted in some nuclear pi-production problems, what would happen? It is easily guessed that the relative difference in the calculated results (using "standard" and "modified" vertices) will be of $O(\mu^2/m^2)$. The point of interest then is rather in the resultant $a'\pi d$ and $\text{Im}a\pi d$ in this "frozen" limit. Therefore we have calculated these quantities using the "modified" Galilean invariant vertex with $\omega_q \equiv \mu$ using the Pieper-Reid deuteron and Mongan 1 N-N potential. The impulse terms are the same as what appear in Table 4 as they should be. This is because the outer pion is in zero energy. On the other hand the contributions to $a'\pi d$ and Im$a\pi d$ from the rescattering terms are approximately doubled. This result can be understood as follows; When ω_q dependence is kept, the factor $1/\sqrt{\omega_q}$ coming from the relativistic normalization at each πNN vertex obtains a major contribution at the pole of the N-N propagator ($p^2=m\mu$) in the momentum integration and this gives $1/\sqrt{\omega_q} \sim 1/\sqrt{2\mu}$. This makes the values from the rescattering contribution about one-half of the ones from the "frozen" ω_q. Thus we know that the dependence of $1/\sqrt{\omega_q}$ should be kept in Galilean invariant πNN vertices in order to find Im$a\pi d$ consistent with experiments. This is rather annoying because the factor $1/\sqrt{\omega_q}$ comes from the normalization of the relativistic field operators. But as particle emission-absorption processes can be most naturally understood in terms of relativistic kinematics, some relativistic feature may well appear in non-relativistic πNN vertices.

Motivated by the above result we also have adopted another πNN vertex in our calculation. The vertex is obtained from the separable Afnan-Thomas πN P_{11} t-matrix. This t-matrix has a pole at $\varepsilon=-\mu$ (the energy is measured non-relativistically excluding the rest masses) and therefore the effective πNN vertex can be identified as the square-root of the numerator function in the P_{11} t-matrix when the denominator is identified as $(\varepsilon+\mu)$. The effective πNN vertex thus obtained gives $f^2_{eff}=0.080$ at $\varepsilon=-\mu$, which is very close to our f^2 value (=0.082). Also the vertex turns out to be a very slowly decreasing function of the relative momentum. The resultant a'πd and Imaπd are shown in Table 7 using the Pieper-Reid deuteron. Because weak momentum dependence remains in the impulse terms, they are slightly smaller than the modified Galilean result. But in the rescattering terms that weak momentum dependence is not enough to reduce the results as the $1/\sqrt{\omega_q}$ factor does in our Galilean invariant vertices. Thus the total Imaπd becomes about twice as large as our results from the Galilean invariant vertices. As for a'πd obtained from the Afnan-Thomas πNN vertex, it does not become so large. This may be due to the overall nature of the principal value integration of the input functions. Incidentally, we have found that in the Afnan-Thomas calculation, the amplitude T(πd←NN) is smaller than the one obtained from the field theory by a factor of $\sqrt{2}$ (we have found that this comes from their antisymmetrization procedure which we think is incorrect). The combination of the square of this factor and the Afnan-Thomas effective πNN vertex just used in our model might make the Afnan-Thomas value of α in their Faddeev type calculation not very different from ours. In conclusion it seems necessary to find reliable πNN vertices for nuclear pion production and absorption problems.

D. Summary

We have obtained, in this chapter, the values of $a'_{\pi d}$ and $\mathrm{Im}\,a_{\pi d}$ using the equations we have developed in earlier chapters. These values have turned out to be in agreement with experimentally available values (at least for $\mathrm{Im}\,a_{\pi d}$) when realistic deuteron models are used. As has been remarked in Chapter VI, there are uncertainties in experimental values. Therefore theoretical results in very good agreement with experiments do not mean much at this stage.

It should be adequate to discuss some possible uncertainties in our calculated values (as well as in most of the other theoretical results). The first factor is a numerical uncertainty. As has been mentioned in the last chapter, this is estimated to be at most 2~3%. This uncertainty can be eliminated rather easily. Second, we used non-relativistic kinematics which brings in the non-Galilean invariance. As our calculation is at the πd threshold, this would cause an uncertainty of $O(\mu/m)$ (this uncertainty may be called the one due to the non-relativistic treatment). The third factor is due to the approximation to the multiple scattering. We have taken up to the pion rescattering terms but the intermediate N-N interactions are exactly included. This treatment may bring in an uncertainty of about 10%.

One may tend to think that if we use relativistic formulations and solve the multiple scattering equation exactly (if technically possible), we would get a reliable result. But the thing is not that simple; there is the fourth and probably the most serious factor. At present we do not know well the off-shell behavior of the input functions.

Input functions; potentials, sub-t-matrices and πNN vertices, are thought to be realistic if they reproduce available on-shell data well. For example, nuclear potentials can use several on-shell data for their realistic fits; N-N phase shifts, deuteron binding energy, nuclear matter binding energy, certain quadrupole moments etc. Each on-shell information reduces the off-shell ambiguities one by one. So in this respect we may say that deuteron wave functions in our calculation can be less ambiguous; the Reid deuteron can reproduce many observed quantities and so it may be thought to be realistic. As for $^3P_1$ N-N potentials, our Mongan potentials are fitted to the phase shift only. Therefore they are more ambiguous than the deuteron wave functions. It may not be so helpful to use nuclear matter calculation for reducing their ambiguity. Phase shifts are the only information at present for πN t-matrices we have used. So their off-shell ambiguities are not easily resolved. Lastly, there does not seem to be any direct information available to determine the πNN vertices. The only way to determine their form seems to use an assumed form to calculate some physical quantities like α as we have done! Therefore unless these ambiguities are removed by some means, it is not so easy to have improved calculations.

CHAPTER IX

CONCLUSION

The aim of our study was to clarify the structure of the pion-nucleus scattering amplitudes with a proper account of the pions in scattering and in nucleon-nucleon interactions.

We chose the πNN system for our studies in pi-nucleus scattering because of its simple structure. But it has turned out that it gives us several important aspects which are common to the general pi-nucleus scattering problem. For this πNN system, the basic processes to be considered are $\pi NN \rightarrow \pi NN$ and $\pi NN \leftrightarrow NN$. In our relativistic approach to the problem, we made use of the method due to Taylor. This allowed us to decompose the πNN amplitudes in a unique way so that we could identify the sub-amplitudes appearing in the total amplitude unambiguously. On the other hand, we adopted an alternative approach; a Hamiltonian (Schrödinger wave function) method, for the non-relativistic treatment of the same problem. Using the projection technque, we obtained an effective, finite set of coupled equations for πNN and NN states. Both relativistic and non-relativistic approaches gave us a set of t-matrix equations for processes; $\pi NN \to \pi NN$, $\pi NN \leftrightarrow NN$ and $NN \to NN$, respectively, and we observed a formal correspondence between the results of relativistic and non-relativistic formulations.

With regard to the structure of $\pi NN \to \pi NN$ and $\pi NN \leftrightarrow NN$ amplitudes, our two formal approaches have clarified the following points: (1) as long as the intermediate pion absorption process is not taken into account, the application of Watson type multiple scattering approach to the elastic πNN scattering, where πN potentials are assumed, may be justified. The important point is that two-body πN t-matrices in the multiple scattering series do not include the (generalized) direct Born terms. (2) For $\pi NN \leftrightarrow NN$ reactions, the distorted wave (DW) expression which is familiar in nuclear reaction theory seems consistent with the exact result that we have obtained, as long as the part describing the pion scattering (or pion distortion) does not contain the effect of intermediate pion absorption. (In other words: the πN t-matrices in the pion distortion part do not contain the direct Born term mentioned above.) (3) The total elastic πNN scattering contains the part describing the intermediate pion absorption once and only once. These results are just the reflection of correct pion counting and the unitary structure of the amplitudes. We then observed that the same argument may apply to the structure of general pi-nucleus scattering.

We observed that for a suitable set of input functions, the equations for the πNN amplitudes are effectively decoupled. With this in mind the non-relativistic equations for πNN amplitudes were reduced to a practical form ready for applications.

We applied this reduced set of equations to study the effect of pion absorption on the real and imaginary parts of the pion-deuteron scattering length ; a'πd and Imaπd. The calculation was done with some suitable approximations and we have obtained the result consistent with experiments when realistic deuterons and Galilean invariant πNN vertices commonly adopted were used. However we could observe possible ambiguities in the calculated results due to the different off-shell behavior of the input functions.

It may be relevant to discuss some possible problems to which our πNN studies can now be applied. First, as we discussed in Chap. III, section C, we should have an off-energy-shell N-N potential; $V_{NN}(E)$, expecially for energies above the threshold of pion production. This seems necessary in order to (i) treat two kinds of pions (in scattering and in N-N potential) impartially and (ii) consistently keep the πNN unitarity even in NN→NN and πNN↔NN amplitudes. We may evaluate $V_{NN}(E)$ using the method described in that chapter to know how important the off-shell effect would be. Second, we can improve pi-nucleus optical potentials used in many pionic-atom as well as pi production absorption problems. As may be clear by now through our approach with correct pion countings, the optical potentials for pion production and absorption problems should not contain the effect of intermediate pion absorption. This means that the P_{11} πN t-matrices appearing in the optical potentials should be one-particle irreducible; $t_{\pi N}$ but not $\theta_{\pi N}$, in Chap. III, section C, should be used there. On the other hand the optical potentials for pionic-atom problems should have the pion absorption effect in them (but not through $\theta_{\pi N}$). As shown in Appendix F, we have a convenient representation for P_{11} πN t-matrix. This contains one particle irreducible part; $t_{\pi N}$ and the generalized direct Born term; $B_{\pi N}$ separately yet satisfies the two-particle unitarity. So we can fit this t-matrix to the phase shift and make use of it to correctly modify the pi-nucleus optical potentials. The modification is to be in the p-wave part of the potentials which, in conventionally used model optical potentials, contains the gradient of the nuclear density distribution; $\nabla\rho$. The change there would affect the result obtained from the potentials non-negligibly as long as the energy of the pion: E (in the Lab. system) is $0<E<300$ MeV. Third, we could improve our calculation of $a'_{\pi d}$ and Im$a_{\pi d}$ by solving the complete Faddeev equation with an improved $V_{NN}(E)$ to be used.

To end this chapter we should mention here the problems which still remain unsolved in our studies. As far as our formal development of the πNN problem is concerned, there would not be many to be solved or clarified further. We discuss one major point concerned with the formal part. The equations we have obtained do not show explicit crossing symmetry. Therefore the amplitudes satisfying those equations are not explicitly crossing symmetric either although they implicitly contain all the crossed contributions. As has been remarked in Chap. III, section A, the explicit crossing symmetric amplitudes may be obtained by using the cutting procedure in every possible channel, but this makes it impossible for a many-particle amplitude to be related to other amplitudes in a transparent manner. It has been reported[64] that the inclusion of the π-nucleus crossing would be important at low energies, but it seems inappropriate at the present stage to conclude that it really is the case. If this turns out to be true, then we may have to think about the crossing problem more seriously.

There is the problem of Galilean invariance in the non-relativistic approach to pi-nucleus scattering and this seems to be more important in practical applications of the formulation. The reason is simply because it is almost impossible to perform a complete relativistic calculation in pi-nucleus scattering problems. Therefore we need to have a non-relativistic or semi-relativistic approach, which violates Galilean invariance. This would not be so serious as long as the scattering that one is considering is at low energy, but would become non-negligible in the higher energy region. It seems that we should study this problem more seriously.

In connection with this Galilean non-invariance problem, there is a problem concerned with the ambiguities in the forms of the non-relativistic limit of the πNN vertices. As we saw in our calculations in Chap. VIII, this affects the calculation rather seriously. So until this problem is clarified all the calculated results are not very convincing. In practical calculations, another annoying factor is the off-shell ambiguity in the input potentials or t-matrices. What we can do is to use the maximum amount of information available (not merely all observed quantities to be fitted) to reduce the degrees of the ambiguity there.

REFERENCES

1. Particle Data Group ed. "Review of Particle Properties" April 1974.

For an extensive review of theoretical methods in pion-nucleus scattering, the following [(2)-(4)] articles are recommended.

2. D. S. Koltun, in "Advances in Nuclear Physics" Vol. 3, 71 (1969).

3. J. M. Eisenberg, in Proc. Int. Conf. on Few Particle Problems in the Nuclear Interaction, ed. I. Šlaus et al., P796 (1973).

4. Reviews mostly on the multiple scattering approach are found in; W. R. Gibbs and B. F. Gibson V. ed. Proc. LAMPF Summer School on the Theory of Pion-Nucleus Scattering, Los Alamos Conf. Proc. LA-5443-C (1973).

5. The Watson multiple scattering theory is described in detail in; M. L. Goldberger and K. M. Watson, "Collision Theory", Wiley (1964).

6. R. J. Glauber, in "Lectures in Theoretical Physics" Vol. 1, P315 Interscience (1959).

7. There are several attempts along this line. See, for example, D. Schiff and J. Tran Thanh Van, Nucl. Phys. B3, 671 (1967).

8. An example of the application of current algebra technique is found in: M. K. Banerjee, C. A. Levinson, M. D. Shuster and D. A. Zollman, Phys. Rev. C3, 509 (1971).

9. The distorted wave method in nuclear reaction is discussed in detail in; N.Austern "Direct Nuclear Reaction Theory" Wiley (1970).

10. J. G. Taylor, Nuovo Cimento Suppl. 1, 857 (1963).

11. A good review of Taylor's method in connection with the Bethe-Salpeter equation is found in; M. M. Broido,"Reports on Progress in Physics" Vol. 32, Part 2, 493 (1963).

12. J. G. Taylor, Phys. Rev. 150, 1321 (1966a).

13. K. Symanzik, J. Math. Phys. 1, 249 (1960). Symanzik's analysis is based upon the structure study of many-particle retarded Green's functions.

14. H. Lehmann, K. Symanzik and W. Zimmermann, Nuovo Cimento 1, 205 (1955).

15. G. t'Hooft and M. Veltman, "Diagrammar" CERN Report, Sept. (1973).

16. F. Zachariasen, Phys. Rev. 101, 371 (1956).

17. A. W. Thomas, Ph.D. Thesis, Flinders Univ. (unpublished) (1973).

18. B. Lippmann and J. Schwinger, Phys. Rev. 79, 469 (1950).

19. (i) E. E. Salpeter and H. A. Bethe, Phys. Rev. 84, 1232 (1951).
(ii) A good review is found in; D. Lurié "Particles and Fields" Wiley (1968). See also references cited there.

20. There are numerous articles on three-particle Faddeev equation.
(i) L. D. Faddeev, "Mathematical Aspects of the Three-body Problem in Quantum Scattering Theory" Davey, N.Y. (1965).
(ii) Another article often referred to is; C. Lovelace, Phys. Rev. 135B, 1225 (1964). In this paper, the separable approximation to the potentials are fully utilized.

21. Relativistic three-body equations are suggested by summing the perturbation graphs in analogy with non-relativistic Faddeev equation. See, for example,
(i) V. A. Alessandrini and R. L. Omnés, Phys. Rev. 139B, 167 (1965).
(ii) D. Freedman, C. Lovelace and M. Namyslowski, Nuovo Cimento 43, 259 (1966).

22. K. L. Kowalski, Phys. Rev. D7, 1806 (1973).

23. The projection operator method we adopted is common in nuclear physics. For the application to the theory of effective interactions, see
(i) C. Bloch and J. Horowitz, Nucl. Phys. 8, 91 (1958).
(ii) B. H. Brandow, Rev. Mod. Phys. 39, 771 (1967)
For the formulation of nuclear reaction theory by projection techniques, see
(iii) H. Feshbach, Ann. Phys. 5, 357 (1958); 19, 287 (1962).

24. (i) M. H. Partovi and E. L. Lomon, Phys. Rev. D2, 1999 (1970).
(ii) W. T. Nutt and L. Wilets, Phys. Rev. D1, 2303 (1973).

25. R. Blankenbecler and R. Sugar, Phys. Rev. 142, 1051 (1966).

26. (i) I. Sloan, Phys. Rev. C6, 1944 (1972).
(ii) Gy. Bencze, Nucl. Phys. A210, 568 (1973).
(iii) E. Redish, Phys. Rev. C10, 67 (1974).

27. A readable article is:
J-M. Lévy-Leblond, Comm. Math. Phys. 4, 157 (1967).
This is on the Galilean invariant Lee model.

28. (i) I. R. Afnan and A. W. Thomas, Phys. Rev. C10, 109 (1974).
(ii) A. W. Thomas, in Proc. Int. Conf. on Few-Body Problems in Nuclear and Particles, Quebec (1974) (in press).

29. R. Amado, Phys. Rev. 132, 485 (1963).

30. See for example;
L. F. Cook and B. W. Lee, Phys. Rev. 127, 283 (1962); 297 (1962). In these papers, the isobar assumption is taken and is combined with N/D equations to study the inelastic effect in π-N scattering.

31. C. Lovelace, in "Strong Interaction and High Energy Physics", ed. R. G. Moorehouse, Oliver and Boyd, (1964).

32. E. O. Alt, P. Grassberger and W. Sandhas, Nucl. Phys. B2, 167 (1967). The three-body equation shown in this paper is especially convenient for the formulation of bound-particle scattering problems.

33. J. L. Basdevant and R. Kreps, Phys.Rev. 141, 1398, 1404, 1409 (1966). They are for an application of the Alessandrini-Omnés method.

34. C. Carlson, Phys. Rev. C2, 1224 (1970). In this article the double scattering is treated on-energy-shell.

35. (i) C. Michael and C. Wilkin, Nucl. Phys. B11, 99 (1969).
(ii) K. Gabathuler and C. Wilkin, Nucl. Phys. B70, 215 (1974). Here the authors used a formalism which is a little different from the Glauber theory. The application was made for πd elastic scattering near 3-3 resonance region.

36. (i) F. Myhrer, Ph.D. Thesis, Univ. of Rochester (unpublished) (1972).
(ii) F. Myhrer and D. S. Koltun, Phys. Lett. 46B, 322 (1973).
(iii) F. Myhrer and D. S. Koltun, Nucl. Phys. B86, 441 (1975).
(iv) D. D. Brayshaw, Phys. Rev. C11, 1196 (1975).

37. D. Beder, Nucl. Phys. B14, 586 (1969).

38. See, for example;
D. B. Lichtenberg, Phys. Rev. 100, 303 (1955); 105, 1084 (1957).

39. A. E. Woodruff, Phys. Rev. 117, 1113 (1960).

40. D. S. Koltun and A. Reitan, Phys. Rev. 141, 1413 (1966).

41. F. S. Crawford Jr. and M. L. Stevenson, Phys. Rev. 97, 1305 (1955).

42. C. M. Rose, Phys. Rev. 154, 1305 (1967).

43. A. Reitan, Nucl. Phys. B11, 170 (1969).

44. A. W. Thomas and I. R. Afnan, Phys. Rev. Lett. 26, 906 (1971).

45. H. C. Pradhan and Y. Singh, Can. J. Phys. 51, 343 (1973).

46. V. K. Samarayake and W. S. Woolcock, Nucl. Phys. B48, 205 (1972).

47. C. Lazard, J. L. Ballot and F. Becker, Nuovo Cimento 65B, 117 (1970).

48. C. Richard-Serre et.al., Nucl. Phys. B20, 413 (1970).

49. J. Spuller and D. F. Measday, Univ. British Columbia preprint (to appear in Phys. Rev. D).

50. J. Bailey et.al., Phys. Lett. 50B, 403 (1974).

51. (i) V. M. Kolybasov and A. E. Kudryavtsev, Nucl. Phys. B41, 510 (1972).
(ii) P. Bendix and D. Beder, Phys. Lett. 49B, 140 (1974).
(iii) F. Myhrer, SIN preprint (1974).

52. K. A. Brueckner, Phys. Rev. 98, 769 (1955).

53. D. J. Thouless, Proc. Phys. Soc. Lond. 69, 280 (1956).

54. (i) R. Reid, Ann. Phys. 50, 411 (1968).
(ii) S. C. Pieper, Phys. Rev. C9, 883 (1974).

55. D. J. Ernst, C. M. Shakin and R. M. Thaler, Phys. Rev. C9, 1780 (1974).

56. Original approach is in:
(i) M. Gourdin, M. LeBellac, F. M. Renard and J. Tran Thanh Van, Nuovo Cimento 37, 524 (1965).
McGee constructed a similar deuteron in;
(ii) I. J. McGee, Phys. Rev. 151, 772 (1966).

57. T. R. Mongan, Phys. Rev. 178, 1597 (1969).

58. (i) M. V. Barnhill III, Nucl. Phys. A131, 106 (1969).
(ii) M. Bolsterli, W. R. Gibbs, B.F. Gibson and G. J. Stephenson Jr., Phys. Rev. C10, 1225 (1974).

59. D. I. Olive, Nuovo Cimento 26, 73 (1962); 29, 326 (1963).

60. R. J. Eden, P. V. Landshoff, D. I. Olive and J. C. Polkinghorne, "The Analytic S-matrix", Cambridge Univ. Press (1966).

61. R. Cutkosky, J. Math. Phys. 1, 429 (1960).

62. General method adopted here may be found, for example, in G. Barton "Dispersion Techniques in Field Theory", Benjamin (1965).

63. I. R. Sloan and J. C. Aarons, Nucl. Phys. A198, 321 (1972).

64. (i) J. B. Cammarata and M. J. Banerjee, Phys. Rev. Lett. 31, 610 (1973).
(ii) J. B. Cammarata, Ph.D. Thesis, Univ. of Maryland (unpublished) (1974).

TABLE 1

$\acute{a}_{\pi d}$ and Im$a_{\pi d}$ using Pieper-Reid deuteron. πNN coupling is of standard Galilean invariant type. All numbers are in $10^{-3} m_\pi^{-1}$. As for the types of diagrams see Fig. 7-1.

$^3P_1$ N-N pot Diagram		Mongan 1		Mongan 2	
		$\acute{a}_{\pi d}$	Im$a_{\pi d}$	$\acute{a}_{\pi d}$	Im$a_{\pi d}$
A	SS	-2.202	0.071	same as Mongan 1	
	DS	2.725	-0.559		
	DD	0.235	1.108		
	TOT	0.758	0.620		
B	SS	-0.363	0.384	-0.289	0.339
	DS	-0.780	-0.314	-0.641	-0.351
	DD	0.021	-0.343	0.087	-0.287
	TOT	-1.122	-0.274	-0.843	-0.299
A+B	SS	-2.565	0.455	-2.491	0.410
	DS	1.945	-1.005	2.084	-0.910
	DD	0.257	0.762	0.322	0.821
	TOT	-0.364	0.212	-0.086	0.321
C	SS	-6.011	0.987	same as Mongan 1	
	DS	-3.292	-2.905		
	DD	-0.495	-2.488		
	TOT	-9.798	-4.406		
D	SS	-0.300	3.460	same as Mongan 1	
	DS	1.003	4.797		
	DD	0.786	1.662		
	TOT	1.489	9.919		
E	SS	1.316	0.600	1.042	0.691
	DS	0.732	1.091	0.395	1.067
	DD	-0.177	0.710	-0.251	0.626
	TOT	1.860	2.401	1.186	2.384
F	SS	-0.005	-1.017	0.394	-0.785
	DS	-0.301	-1.610	0.326	-1.305
	DD	-0.234	-0.613	0.013	-0.515
	TOT	-0.541	-3.240	0.733	-2.605
Σ C to F	SS	-5.001	4.031	-4.875	4.354
	DS	-1.859	1.373	-1.568	1.654
	DD	-0.120	-0.729	0.053	-0.715
	TOT	-6.980	4.675	-6.390	5.293
Σ all	SS	-7.566	4.486	-7.366	4.764
	DS	0.085	0.368	0.516	0.744
	DD	0.137	0.033	0.375	0.106
	TOT	-7.344	4.888	-6.942	5.614

SS denotes the contribution in which the deuteron state is S state both in initial and final states of scattering. Similar convention for DS and DD. TOT is the total contribution.

TABLE 2

$\vec{a}_{\pi d}$ and Im$a_{\pi d}$ using McGee deuteron. πNN coupling is of standard Galilean invariant type. Unit is the same as in Table 1.

$^3P_1$ N-N pot / Diagram		Mongan 1		Mongan 2	
		$\vec{a}_{\pi d}$	Im$a_{\pi d}$	$\vec{a}_{\pi d}$	Im$a_{\pi d}$
A	SS	-2.543	0.218	same as Mongan 1	
	DS	2.271	-1.010		
	DD	0.305	1.117		
	TOT	0.033	0.325		
B	SS	-0.102	0.307	-0.067	0.282
	DS	-0.695	-0.049	-0.606	-0.131
	DD	-0.097	-0.401	0.063	-0.325
	TOT	-0.894	-0.143	-0.610	-0.174
A+B	SS	-2.645	0.525	-2.610	0.500
	DS	1.575	-1.059	1.664	-1.141
	DD	0.208	0.716	0.369	0.792
	TOT	-0.861	0.183	-0.577	0.151
C	SS	-4.932	1.646	same as Mongan 1	
	DS	-3.647	-2.375		
	DD	-1.649	-3.337		
	TOT	-10.228	-4.066		
D	SS	-0.198	3.111	same as Mongan 1	
	DS	2.031	5.442		
	DD	1.921	2.380		
	TOT	3.754	10.933		
E	SS	1.099	0.111	0.927	0.292
	DS	1.522	1.065	0.727	0.985
	DD	0.686	1.337	0.034	1.088
	TOT	3.307	2.513	1.688	2.365
F	SS	-0.229	-1.032	0.300	-0.757
	DS	-1.076	-2.161	0.103	-1.704
	DD	-0.887	-1.040	-0.226	-0.865
	TOT	-2.192	-4.233	0.177	-3.326
ΣC to F					
	SS	-4.260	3.836	-3.903	4.292
	DS	-1.169	1.971	-0.786	2.348
	DD	+0.071	-0.661	0.080	-0.735
	TOT	-5.359	5.146	-4.608	5.905
Σ all	SS	-6.905	4.361	-6.513	4.792
	DS	0.406	0.913	0.879	1.207
	DD	0.279	0.055	0.449	0.057
	TOT	-6.220	5.329	-5.185	6.057

TABLE 3

$\acute{a}_{\pi d}$ and Im$a_{\pi d}$ with Hulthén S-wave deuteron. πNN vertex is of standard Galilean invariant type. Because of no D-state, contribution DS and DD are missing.

$^3P_1$ N-N pot Diagram		Mongan 1		Mongan 2	
		$\acute{a}_{\pi d}$	Im$a_{\pi d}$	$\acute{a}_{\pi d}$	Im$a_{\pi d}$
A	SS	-3.367	1.771	same as Mongan 1	
B	SS	0.304	-0.162	0.424	-0.030
A+B	SS	-3.063	1.608	-2.943	1.740
C	SS	-6.312	11.067	same as Mongan 1	
D	SS	2.266	17.294	same as Mongan 1	
E	SS	1.084	-2.753	2.186	-1.817
F	SS	-1.656	-5.974	1.201	-4.603
ΣC to F	SS	-4.617	19.635	-0.659	21.942
Σ all	SS	-7.680	21.243	-3.602	23.682

TABLE 4

$a_{\pi d}$ and Im $a_{\pi d}$. Pieper-Reid deuteron and modified Galilean invariant πNN coupling are used.

$^3P_1$ N-N pot Diagram		Mongan 1		Mongan 2	
		$a_{\pi d}$	Im $a_{\pi d}$	$a_{\pi d}$	Im $a_{\pi d}$
A	SS	-1.674	0.054	same as Mongan 1 case	
	DS	2.071	-0.425		
	DD	0.179	0.842		
	TOT	0.576	0.471		
B	SS	-0.276	0.292	-0.220	0.258
	DS	-0.593	-0.239	-0.487	-0.267
	DD	0.016	-0.261	0.066	-0.218
	TOT	-0.853	-0.208	-0.641	-0.227
A+B	SS	-1.950	0.346	-1.894	0.312
	DS	1.478	-0.764	1.584	-0.692
	DD	0.195	0.581	0.245	0.624
	TOT	-0.277	0.163	-0.065	0.244
C	SS	-4.608	0.794	same as Mongan 1 case	
	DS	-1.461	-2.477		
	DD	-0.121	-1.487		
	TOT	-6.190	-3.170		
D	SS	-1.562	2.942	same as Mongan 1 case	
	DS	-0.555	3.031		
	DD	0.166	0.781		
	TOT	-1.951	6.754		
E	SS	0.675	0.772	0.581	0.761
	DS	0.116	0.614	-0.061	0.646
	DD	-0.134	0.384	-0.179	0.336
	TOT	0.657	1.769	0.340	1.743
F	SS	0.493	-0.399	0.613	-0.288
	DS	0.273	-0.768	0.485	-0.591
	DD	-0.050	-0.260	0.517	-0.210
	TOT	0.716	-1.427	1.149	-1.089
Σ C to F	SS	-5.002	4.109	-4.976	4.209
	DS	-1.627	0.400	-1.593	0.609
	DD	-0.140	-0.582	-0.083	-0.580
	TOT	-6.768	3.926	-6.652	4.238
Σ all	SS	-6.952	4.455	-6.870	4.521
	DS	-0.149	-0.364	-0.009	-0.083
	DD	0.055	-0.001	0.162	0.044
	TOT	-7.045	4.252	-6.717	4.482

TABLE 5

$\acute{a}_{\pi d}$ and Im$a_{\pi d}$. McGee deuteron and modified Galilean invariant πNN coupling are adopted.

$^3P_1$ N-N pot diagram		Mongan 1		Mongan 2	
		$\acute{a}_{\pi d}$	Im$a_{\pi d}$	$\acute{a}_{\pi d}$	Im$a_{\pi d}$
A	SS	-1.933	0.166	same as Mongan 1	
	DS	1.726	-0.767		
	DD	0.232	0.894		
	TOT	0.025	0.292		
B	SS	-0.077	0.233	-0.051	0.215
	DS	-0.532	-0.037	-0.461	-0.099
	DD	-0.074	-0.304	0.048	-0.247
	TOT	-0.614	-0.108	-0.463	-0.132
A+B	SS	-2.010	0.399	-1.984	0.380
	DS	1.262	-0.805	1.265	-0.867
	DD	0.158	0.589	0.280	0.647
	TOT	-0.590	0.184	-0.439	0.160
C	SS	-4.258	1.470	same as Mongan 1 case	
	DS	-1.687	-2.564		
	DD	-0.657	-1.953		
	TOT	-6.603	-3.047		
D	SS	-1.303	3.263	same as Mongan 1	
	DS	-0.079	3.742		
	DD	0.454	1.073		
	TOT	-0.928	8.078		
E	SS	0.751	0.387	0.689	0.439
	DS	0.483	0.937	0.082	0.815
	DD	0.248	0.710	-0.063	0.575
	TOT	1.482	2.033	0.709	1.829
F	SS	0.392	-0.636	0.594	-0.469
	DS	0.057	-1.159	0.449	-0.897
	DD	-0.190	-0.409	0.866	-0.332
	TOT	0.258	-2.204	1.051	-1.698
Σ C to H					
	SS	-4.419	4.483	-4.278	4.702
	DS	-1.226	0.956	-1.235	1.097
	DD	-0.145	-0.580	-0.258	-0.638
	TOT	-5.579	4.859	-5.771	5.162
Σ all	SS	-6.429	4.882	-6.262	5.083
	DS	0.037	0.151	0.030	0.230
	DD	0.013	-0.009	0.022	-0.009
	TOT	-6.379	5.043	-6.209	5.332

TABLE 6

$\acute{a}_{\pi d}$ and Im$a_{\pi d}$. Hulthén deuteron and modified Galilean invariant πNN coupling are used.

$^3P_1$ N-N pot diagram	Mongan 1		Mongan 2	
	$\acute{a}_{\pi d}$	Im$a_{\pi d}$	$\acute{a}_{\pi d}$	Im$a_{\pi d}$
A	-2.559	1.346	same as Mongan 1	
B	0.231	-0.123	0.322	-0.023
A+B	-2.328	1.222	-2.237	1.323
C	-3.581	5.162	same as Mongan 1	
D	-0.429	4.950	same as Mongan 1	
E	0.610	-1.094	1.062	-0.697
F	-0.029	-1.469	0.579	-1.108
Σ C to F	-3.429	7.550	-2.369	8.307
Σ all	-5.757	8.772	-4.606	9.630

TABLE 7

aπd and Imaπd. Pieper-Reid deuteron and Afnan-Thomas effective πNN vertex are used. Unit and notations are the same as Table 1~6.

$3p_1$ N-N pot diagram		Mongan 1		Mongan 2	
		aπd	Imaπd	aπd	Imaπd
A	SS	-1.483	0.043	same as Mongan 1	
	DS	1.666	-0.348		
	DD	0.091	0.705		
	TOT	0.274	0.400		
B	SS	-0.199	0.216	-0.164	0.197
	DS	-0.439	-0.201	-0.371	-0.224
	DD	0.009	-0.205	0.070	-0.170
	TOT	-0.629	-0.190	-0.465	-0.197
A+B	SS	-1.682	0.259	-1.647	0.240
	DS	1.227	-0.549	1.295	-0.572
	DD	0.100	0.500	0.161	0.535
	TOT	-0.355	0.210	-0.191	0.248
C	SS	-6.164	1.144	same as Mongan 1	
	DS	-2.222	-3.595		
	DD	-0.268	-2.053		
	TOT	-8.650	-4.503		
D	SS	-2.114	7.622	same as Mongan 1	
	DS	0.201	7.524		
	DD	0.804	1.857		
	TOT	-1.109	17.003		
E	SS	1.095	0.928	0.955	0.950
	DS	0.375	1.015	0.019	0.961
	DD	-0.037	0.632	-0.202	0.518
	TOT	1.433	2.574	0.735	2.429
F	SS	0.977	-1.448	1.358	-1.140
	DS	0.210	-2.275	0.882	-1.811
	DD	-0.289	-0.694	0.019	-0.572
	TOT	0.898	-4.417	2.259	-3.523
Σ C to F					
	SS	-6.207	8.247	-5.965	8.577
	DS	-1.431	2.669	-1.117	3.079
	DD	0.210	-0.259	0.354	-0.249
	TOT	-7.428	10.657	-6.729	11.407
Σ all	SS	-7.889	8.506	-7.612	8.817
	DS	-0.204	2.120	0.178	2.507
	DD	0.310	0.241	0.515	0.286
	TOT	-7.783	10.867	-6.920	11.655

TABLE 8

The values of α (in unit of μb)

deuteron	type of πNN coupling	N-N pot MONGAN 1	N-N pot MONGAN 2
P - R	S.G.I.	277	318
P - R	M.G.I.	241	254
McGee	S.G.I.	302	343
McGee	M.G.I.	286	302
Hulthén	S.G.I.	1203	1341
Hulthén	M.G.I.	497	545
P - R	E.A-T	615	660

P - R = Pieper-Reid

S.G.I. = Standard Galilean invariant

M.G.I. = Modified Galilean invariant

E.A-T = Effective Afnan-Thomas

TABLE 9

Values of I_j

πNN coupling	deuteron	$3p_1$ NN pot	I_1	I_2	I_3+I_5	I_4+I_6
S.G.I.	P-R	M1	-0.071	0.096	-0.168	-0.108
S.G.I.	P-R	M2	-0.068	0.102	-0.176	-0.114
S.G.I.	M-G	M1	-0.077	0.093	-0.153	-0.121
S.G.I.	M-G	M2	-0.076	0.098	-0.164	-0.133
S.G.I.	HUL	M1	-0.138	--	-0.367	--
S.G.I.	HUL	M2	-0.143	--	-0.388	--
M.G.I.	P-R	M1	-0.063	0.084	-0.171	-0.077
M.G.I.	P-R	M2	-0.059	0.088	-0.175	-0.082
M.G.I.	M-G	M1	-0.067	0.083	-0.176	-0.088
M.G.I.	M-G	M2	-0.066	0.086	-0.182	-0.093
M.G.I.	HUL	M1	-0.120	--	-0.203	--
M.G.I.	HUL	M2	-0.125	--	-0.214	--
A-PT	P-R	M1	-0.054	0.078	-0.270	-0.115
A-PT	P-R	M2	-0.052	0.081	-0.274	-0.122
S.G.I.*	RSC	RSC	-0.070	0.078	-0.120	-0.035

Abbreviation

- S.G.I. - Standard Galilean invariant type
- M.G.I. - modified Galilean invariant type
- A-T - Afnan-Thomas effective vertex
- P-R - Pieper-Reid
- M-G - McGee
- HUL - HULTHÉN
- M1 - Mongan 1
- M2 - Mongan 2
- RSC - Reid soft core

*The last line in the table is from Koltun-Reitan type calculation by Pradhan and Singh (45).

TABLE 10

$\acute{a}_{\pi d}$ and Im$a_{\pi d}$ obtained from the combination of Pieper-Reid deuteron and modified Galilean invariant πNN coupling with pion energy factor $\frac{1}{\sqrt{\omega k}}$ set equal to $\frac{1}{\sqrt{\mu}}$. Mongan type 1 $3p_1$ N-N potential is used. (unit is the same as in Table 1~7).

diagram		$\acute{a}_{\pi d}$	Im$a_{\pi d}$
A+B	SS*	-1.950	0.346
	DS	1.478	-0.764
	DD	0.195	0.581
	TOT	-0.277	0.163
Σ C to F			
	SS	-9.056	9.631
	DS	-3.118	0.860
	DD	-0.185	-1.129
	TOT	-12.359	9.363
Σ all	SS	-11.006	9.977
	DS	-1.640	0.096
	DD	0.010	-0.548
	TOT	-12.585	9.526

*The numbers here are the same as those appearing in Table 4.

APPENDIX F

REDUCED FORM OF THE P_{11} πN t-MATRIX

In Section C of Chap. 3, we show that taking into account the pion absorption in P_{11}, the πN t-matrix takes the form [see (3C-I-III-3)∿(3C-III-8)]:

$$\Theta_{\pi N}(E) = t_{\pi N}(E) + \Gamma(E)\Pi(E)\Lambda(E). \qquad \text{(F-1)}$$

In this appendix we decompose it into explicit angular momentum eigenstates and pick up the P_{11} partial wave. Then we study it a little more in detail from a practical viewpoint.

(i) P_{11} πN t-matrix

As shown in (3C-I-5), we define one-particle irreducible πNN vertices (in πN Hilbert space):

$$\left.\begin{aligned} \Lambda &\equiv Y + Y G_{T_2} t_{\pi N} \\ \Gamma &\equiv W + t_{\pi N} G_{T_2} W \end{aligned}\right\} \qquad \text{(F-2)}$$

We then put angular momentum reduced from of every quantity in (F-2) to find:

$$\hat{\Lambda}(\varepsilon,q) = \hat{Y}(\varepsilon,q) + \int_0^\infty k^2 dk\, \hat{Y}(\varepsilon,k) \frac{1}{\varepsilon^+ - k^2/2\mu'}\, t^{1/2\,1/2}_{1,1/2}(k,q,\varepsilon)$$

$$\hat{\Gamma}(\varepsilon,q) = \hat{W}(\varepsilon,q) + \int_0^\infty k^2 dk\, t^{1/2\,1/2}_{1,1/2}(q,k,\varepsilon) \frac{1}{\varepsilon^+ - k^2/2\mu'}\, \hat{W}(\varepsilon,k) \qquad \text{(F-3)}$$

where $t_{1,\frac{1}{2}}^{\frac{1}{2}\frac{1}{2}}$ is the P_{11} πN t-matrix ($t_{LS}^{J\tau}$ is the general form of the angular momentum decomposed $t\pi N$) and $\varepsilon \equiv E-m-\mu$.

We are required to have the Hermitian analyticity or reality of $\hat{\Lambda}$ and $\hat{\Gamma}$; $\hat{\Lambda}^*(\varepsilon,q) = \hat{\Gamma}(\varepsilon^-,q)$. From (F-3) it is easy to see that this condition is satisfied provided (1) $\hat{Y}$ and $\hat{W}$ satisfy the reality condition and (2) $t_{1,\frac{1}{2}}^{\frac{1}{2}\frac{1}{2}}$ is a real function of ε (this is the Hermitian analyticity for $t_{1,\frac{1}{2}}^{\frac{1}{2}\frac{1}{2}}$). Usually when a two-body t-matrix satisfies Lippmann-Schwinger (L.S.) equation with a real (or Hermitian) potential, then it can be shown to be Hermitian analytic. As $t_{1,\frac{1}{2}}^{\frac{1}{2}\frac{1}{2}}$ satisfies L.S. equation with 2-particle irreducible πN amplitude, which is Hermitian below the pion production threshold, it satisfies Hermitian analyticity. As for $\hat{Y}$ and $\hat{W}$, they are defined to be two-particle irreducible. Therefore in the πN elastic region, we could ignore its energy dependence and this makes them satisfy Hermitian analyticity; $\hat{Y}$ and $\hat{W}$ in this case should be complex conjugate with each other, which is consistent with the field theoretic πNN coupling.

The total P_{11} πN t-matrix then takes the form:

$$\hat{\Theta}(q,p,\varepsilon) = t_{1,\frac{1}{2}}^{\frac{1}{2}\frac{1}{2}}(q,p,\varepsilon) + \hat{\Gamma}(\varepsilon,q)\pi(\varepsilon)\hat{\Lambda}(\varepsilon,p), \quad \text{(F-4)}$$

where

$$\pi(\varepsilon) = (\varepsilon+\mu)^{-1} H_1^{-1}(\varepsilon) \tag{F-5}$$

and

$$H_1(\varepsilon) = 1 - (\varepsilon+\mu)\int_0^\infty \frac{k^2 dk\, |\hat{\Lambda}(k^2/2\mu', k)|^2}{(\varepsilon^+ - k^2/2\mu')(\mu + \frac{k^2}{2\mu'})^2} \tag{F-6}$$

in πN center of mass system.

There is a requirement imposed on $\pi(\varepsilon)$ and hence on $H_1(\varepsilon)$; $\pi(\varepsilon)$ should not have any other negative poles than the one corresponding to the nucleon. This means that $H_1(\varepsilon)$ should not have any zero for $\varepsilon<0$. We know that $H_1(\varepsilon)$ is an analytic function in ε with a cut along the positive real axis. In addition it has the following properties.

(1) $H_1(\varepsilon)$ is real, for $\varepsilon \leq 0$.

(2) $H_1(-\mu) = 1$.

(3) $\frac{dH_1(\varepsilon)}{d\varepsilon} = \int_0^\infty |\hat{\Lambda}|^2 k^2 dk \Big/ (\varepsilon^+ - \frac{k^2}{2\mu'})^2(\mu + \frac{k^2}{2\mu'}) > 0$, for $\varepsilon \leq 0$.

Since $H_1(\varepsilon)$ is a monotonically increasing function for $\varepsilon \leq 0$ [from (3)], it will have one zero if and only if $H_1(-\infty) < 0$. So the condition to be satisfied is

$$1 - \int_0^\infty \frac{k^2 dk\, |\hat{\Lambda}(k^2/2\mu', k)|^2}{(\mu + k^2/2\mu')^2} > 0, \tag{F-7}$$

It seems that this condition may be related to the one which restricts the magnitude of the wave function renormalization constant of the nucleon[29] (in the non-relativistic sense). So if (F-7) is not satisfied the particle corresponding to the zero of $H_1(\varepsilon)$ may show its "ghost" nature. This actually can be confirmed as follows: Suppose there is a zero of $H_1(\varepsilon)$ at $\varepsilon = \eta(\eta<-\mu)$; $H_1(\eta) = 0$. Then

$$H_1(\varepsilon)=H_1(\varepsilon)-H_1(\eta) = (\varepsilon-\eta)\int_0^\infty \frac{k^2dk\,|\hat{\Lambda}(k^2/2\mu', k)|^2}{(\mu+k^2/2\mu')(\eta-k^2/2\mu')(\varepsilon^+-k^2/2\mu')}$$

(F-6)

So the residue of $\pi(\varepsilon)$ at $\varepsilon=\eta$ is found to be

$$\frac{1}{\eta+\mu}\left[\int_0^\infty \frac{k^2dk\,|\hat{\Lambda}(k^2/2\mu', k)|^2}{(\mu+k^2/2\mu')(\eta-k^2/2\mu')^2}\right]^{-1} < 0,$$

(F-7)

which corresponds to the non-Hermitian nature of the basic Hamiltonian and thus the zero at $\varepsilon=\eta$ is a ghost.

(ii) Separable approximation to $t_{1,\frac{1}{2}}^{\frac{1}{2}\frac{1}{2}}(q,P,\varepsilon)$

This is aimed for more practical purposes. Let us suppose that $t_{1,\frac{1}{2}}^{\frac{1}{2}\frac{1}{2}}$ is separable;

$$t_{1,1/2}^{1/2\,1/2}(q,P,\varepsilon) \equiv h(q)\nu(\varepsilon^+)h(q) \quad (h(q);\text{real}), \qquad \text{(F-8)}$$

where $$\nu(\varepsilon)^{-1} = \beta - \int_0^\infty \frac{s^2ds\,h^2(s)}{\varepsilon^+ - s^2/2\mu'} .$$

(F-8')

In (H-8') β is a real constant. Then from (F-3) we obtain

$$\hat{\Lambda}(\varepsilon,q) = \hat{Y}(q) + \int_0^\infty \frac{k^2dk\,\hat{Y}(k)\,h(k)}{\varepsilon^+ - k^2/2\mu'}\,h(q)\,\nu(\varepsilon), \qquad \text{(F-9)}$$

and

$$\hat{\Gamma}(\varepsilon,q) = \hat{W}(q) + \int_0^\infty \frac{k^2dk\,h(k)\,\hat{W}(k)}{\varepsilon^+ - k^2/2\mu'}\,h(q)\,\nu(\varepsilon). \qquad \text{(F-9')}$$

Since $\hat{W}(q)$, $\hat{Y}(q)$ and $h(q)$ should show a similar threshold behavior; $\propto q$ for small q, we may take another step to assume that

$$\hat{Y}(q) = C\,h(q), \quad [\hat{W}(q) = C^*h(q)], \qquad \text{(F-10)}$$

where C is a complex constant to be determined (either real or pure imaginary. Then we can easily find that

$$\left.\begin{aligned} \hat{\Lambda}(\varepsilon,q) &= \beta C\,\nu(\varepsilon)\,h(q) \\ \hat{\Gamma}(\varepsilon,q) &= \beta C^*\,\nu(\varepsilon)\,h(q) \end{aligned}\right\}. \qquad \text{(F-11)}$$

Note that in this expression Hermitian analyticity is explicitly shown to be satisfied.

Now we obtain

$$\hat{\Theta}(q,p,\varepsilon) = h(q)\,K(\varepsilon^+)\,h(p), \qquad \text{(F-12)}$$

and

$$K(\varepsilon^+) = \nu(\varepsilon) + \beta^2 |c|^2 \pi(\varepsilon)\, \nu^2(\varepsilon). \qquad \text{(F-13)}$$

In (F-13) the second term comes from the generalized direct Born term. Note that for the calculation of the Faddeev amplitude T^H for πNN, we should use $\nu(\varepsilon^+)$ but not the whole $K(\varepsilon^+)$.

For a practical calculation we may, for example, take $h(q) \equiv q/(q^2 \pm \gamma^2)$ and then $\gamma(\varepsilon)$, $\hat{\Lambda}(\varepsilon, q)$ and $\pi(\varepsilon)$ can be calculated. To fit $\hat{\theta}(q,P,\varepsilon)$ to the experimental P_{11} πN phase shift, there are three parameters to be varied; γ, β and C. The data to be used here may be the P_{11} scattering volume, the phase shift at an arbitrary energy and possibly the position of the "Roper resonance." Though the assumption in (F-10) may not be effective, we think it worth while to carry out a numerical fit using (F-12) and (F-13). In our calculation of the pion absorption effect on πd scattering length in Chap. 6 8, we do not use the result discussed in this appendix.

APPENDIX G

REDUCTION OF πNN AMPLITUDES

1. Antisymmetrization

Physically interesting amplitudes are the connected parts of those in (3B-IV-8) ~ (3B-IV-14'). So the antisymmetrization of two nucleons are considered for those connected amplitudes taking into account the isobar (or separable) approximation to our two-body t-matrices; as in (5B-II-1) two-body t-matrix is expressed as $t_j = \sum_\alpha g^j_\alpha \tau^j_\alpha g^j_\alpha$, where j is a pair label and α specifies states like angular momentum. τ^j_α may be considered as an α-isobar propagator of j-th pair. Together with τ^j_α we introduce particle exchange type driving term: $Z_{j\alpha,k\beta} \equiv g^j_\alpha G_3 g^{k*}_\beta$ (see 5B-II-3). As has been mentioned in Section B, Chap. 5, we neglect possible πNN three-body forces hoping that their contribution is small.

We follow conventions used by Afnan and Thomas [28]. These are:

a. nucleon labels are "1" and "2" while particle "3" means the pion.

b. α, β, γ...etc. represent pairwise interacting states between the pion and nucleon while m,n... etc. denote pairwise interacting states between two nucleons. An example is that α_1 means the 1st πN pair in the α-state (with nucleon "1" staying as a spectator). For pure two-nucleon states, N_2 is a state where nucleon "2" has absorbed or is going to emit a pion. Of course after two-nucleon antisymmetrization labels "1", "2" and "3" disappear. Momentum and other quantum numbers are explicitly shown in the amplitudes whenever necessary.

(i) Faddeev Amplitudes

Because of the particle identity between "1" and "2" and knowing the fact that two nucleons ("1" and "2") have been antisymmetrized in the form factors (form factors $\propto$ wave functions), we first get the following relations. (see (5B-II-3) for the definition of Z and X used below).

$$\left.\begin{aligned}
Z_{\alpha,m} &\equiv Z_{\alpha_1,m_3} = -Z_{\alpha_2,m_3} \quad \left(=\tfrac{1}{2}[Z_{\alpha_1,m_3} - Z_{\alpha_2,m_3}]\right) \\
Z_{n,\alpha} &\equiv Z_{n_3,\alpha_1} = -Z_{n_3,\alpha_2} \quad \left(=\tfrac{1}{2}[Z_{n_3,\alpha_1} - Z_{n_3,\alpha_2}]\right) \\
Z_{\alpha,\beta} &\equiv -Z_{\alpha_1,\beta_2} = -Z_{\alpha_2,\beta_1} \\
\tau_\alpha &\equiv \tau_{\alpha_1} = \tau_{\alpha_2} \\
\tau_n &\equiv 2\tau_{n_3}
\end{aligned}\right\} \qquad \text{(G1-1)}$$

We shall make symmetric and antisymmetric combinations out of $X_{a,b}$ $(a,b=\alpha_j,n_i)$ utilizing (G1-1).

(a) Amplitudes
$$\left.\begin{aligned} X_{\alpha,m} &\equiv (X_{\alpha_1,m_3} - X_{\alpha_2,m_3})/2 \\ X_{n,m} &\equiv X_{n_3,m_3}/2 \end{aligned}\right\}$$
satisfy

$$\left.\begin{aligned} X_{\alpha,m} &= Z_{\alpha,m} + \sum_\beta Z_{\alpha,\beta}\tau_\beta X_{\beta,m} + \sum_n Z_{\alpha,n}\tau_n X_{n,m} \\ X_{n,m} &= \sum_\beta Z_{n,\beta}\tau_\beta X_{\beta,m} \end{aligned}\right\}. \qquad \text{(G1-2)}$$

(G1-2) is closed by itself.

(b) $Y_{\alpha,m} \equiv (X_{\alpha_1,m_3} + X_{\alpha_2,m_3})/2$ satisfies

$$Y_{\alpha,m} = \sum_\beta Z_{\alpha,\beta}\tau_\beta Y_{\beta,m}, \qquad \text{(G1-3)}$$

which also is closed by itself. As (G1-3) is homogeneous, it will not correspond to any scattering phenomena.

(c) Amplitudes
$$\left.\begin{aligned} Y_{n,\alpha} &\equiv (X_{n_3,\alpha_1} + X_{n_3,\alpha_2})/2 \\ T_{\alpha,\beta} &\equiv (X_{\alpha_1,\beta_2} - X_{\alpha_2,\beta_1})/2 \\ U_{\alpha,\beta} &\equiv (X_{\alpha_1,\beta_1} - X_{\alpha_2,\beta_2})/2 \end{aligned}\right\}$$

show the equations they satisfy,

$$\left.\begin{aligned} T_{\alpha,\beta} &= \sum_\gamma Z_{\alpha,\gamma}\tau_\gamma U_{\gamma,\beta} + \sum_n Z_{\alpha,n}\tau_n Y_{n,\beta}/2 \\ U_{\alpha,\beta} &= \sum_\gamma Z_{\alpha,\gamma}\tau_\gamma T_{\gamma,\beta} + \sum_n Z_{\alpha,n}\tau_n Y_{n,\beta}/2 \\ Y_{n,\alpha} &= \sum_\beta Z_{n,\beta}\tau_\beta (U_{\alpha,\beta} + T_{\alpha,\beta}) \end{aligned}\right\}. \qquad \text{(G1-4)}$$

Forming $K_{\alpha,\beta} \equiv T_{\alpha,\beta} + U_{\alpha,\beta}$ shows that (G1-4) becomes

$$\left.\begin{aligned} K_{\alpha,\beta} &= \sum_\gamma Z_{\alpha,\gamma} \tau_\gamma K_{\gamma,\beta} + \sum_n Z_{\alpha,n} \tau_n Y_{n,\beta} \\ Y_{n,\beta} &= \sum_\alpha Z_{n,\alpha} \tau_\alpha K_{\alpha,\beta} \end{aligned}\right\} . \qquad \text{(G1-4')}$$

This (G1-4') does not correspond to a set of scattering equations as upon eliminating $Y_{n,\beta}$, it becomes a set of coupled homogeneous equations for $\{K_{\alpha,\beta}\}$.

(d) Amplitudes $\left.\begin{aligned} V_{\alpha,\beta} &\equiv -(X_{\alpha_1,\beta_2} + X_{\alpha_2,\beta_1})/2 \\ W_{\alpha,\beta} &\equiv (X_{\alpha_1,\beta_1} + X_{\alpha_2,\beta_2})/2 \end{aligned}\right\}$ lead to

$$\left.\begin{aligned} V_{\alpha,\beta} &= Z_{\alpha,\beta} + \sum_\gamma Z_{\alpha,\gamma} \tau_\gamma W_{\gamma,\beta} + \sum_n Z_{\alpha,n} \tau_n X_{n,\beta}/2 \\ W_{\alpha,\beta} &= \sum_\gamma Z_{\alpha,\gamma} \tau_\gamma V_{\gamma,\beta} + \sum_n Z_{\alpha,n} \tau_n X_{n,\beta}/2 \end{aligned}\right\} , \qquad \text{(G1-5)}$$

where $X_{n,\beta} \equiv (X_{n_3,\beta_1} - X_{n_3,\beta_2})/2$, which turns out to satisfy

$$X_{n,\beta} = Z_{n,\beta} + \sum_\alpha Z_{n,\alpha} \tau_\alpha (W_{\alpha,\beta} + V_{\alpha,\beta}) . \qquad \text{(G1-5')}$$

So $X_{\alpha,\beta} \equiv V_{\alpha,\beta} + W_{\alpha,\beta}$ makes (G1-5) and (G1-5') to be rewritten as

$$\left.\begin{aligned} X_{\alpha,\beta} &= Z_{\alpha,\beta} + \sum_\gamma Z_{\alpha,\gamma} \tau_\gamma X_{\gamma,\beta} + \sum_n Z_{\alpha,n} \tau_n X_{n,\beta} \\ X_{n,\beta} &= Z_{n,\beta} + \sum_\alpha Z_{n,\alpha} \tau_\alpha X_{\alpha,\beta} \end{aligned}\right\} , \qquad \text{(G1-6)}$$

which is closed.

Our physical amplitudes turn out to be

$$\left.\begin{aligned} T(\alpha \leftarrow m) &= \sqrt{2}\, X_{\alpha,m} \\ T(n \leftarrow m) &= 2\, X_{n,m} \\ T(n \leftarrow \alpha) &= \sqrt{2}\, X_{n,\alpha} \\ T(\alpha \leftarrow \beta) &= X_{\alpha,\beta} \end{aligned}\right\} \quad . \qquad \text{(G1-7)}$$

So we have only to solve (G1-2) and (G1-6), and other combinations like $Y_{n,\alpha}$ correspond to physically uninteresting solutions.

(ii) Two-particle Irreducible Amplitudes Including N-N States

We work in C.M. system of three or bound-particle system. So for example, $Z_{n_3,N_1}(\vec{q},\vec{p})$ corresponds to the diagram shown in Fig. G-1. With respect to the momentum in two-nucleon states, we regard that it implicitly carries other quantum numbers as well (spin, isospin, etc.).

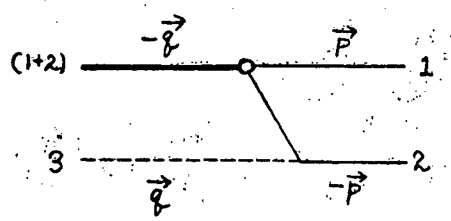


Fig. G-1

(a) Amplitudes for (n or α ⇄ NN) processes

With the definition ($a_j = \alpha_j$ or n_j)

$$\left.\begin{aligned} Z^{ex}_{a_j,N_1}(\vec{q},\vec{p}) &\equiv Z_{a_j,N_1}(\vec{q},-\vec{p}) \\ Z^{ex}_{N_1,a_j}(\vec{q},\vec{p}) &\equiv Z_{N_1,a_j}(-\vec{q},\vec{p}) \end{aligned}\right\} \quad , \qquad \text{(G1-8)}$$

where $\vec{p}\to-\vec{p}$ also implies the interchange of the third component of spin and isospin between two nucleons, the antisymmetric driving term for $NN\to n$ becomes (noting the relations in (G1-1)),

$$\begin{aligned} Z^{A}_{n,N}(\vec{q},\vec{p}) &\equiv \frac{1}{\sqrt{2}}\left[Z_{n_3,N_1}(\vec{q},\vec{p}) - Z_{n_3,N_1}(\vec{q},-\vec{p}) + Z_{n_3,N_2}(\vec{q},-\vec{p}) - Z_{n_3,N_2}(\vec{q},\vec{p})\right] \\ &= \sqrt{2}\left[Z_{n,N} - Z^{ex}_{n,N}\right](\vec{q},\vec{p}), \end{aligned} \qquad \text{(G1-9)}$$

where $Z_{n,N} \equiv Z_{n_3,N_1} = -Z_{n_3,N_2}$ and superscript "A" means antisymmetrized. Similar results are found for $Z^{A}_{N,n}(\vec{q},\vec{p})$. Also it is easy to see that

$$\left.\begin{aligned} Z^{A}_{\alpha,N}(\vec{q},\vec{p}) &= \left[Z_{\alpha,N} - Z^{ex}_{\alpha,N}\right](\vec{q},\vec{p}) \\ Z^{A}_{N,\alpha}(\vec{q},\vec{p}) &= \left[Z_{N,\alpha} - Z^{ex}_{N,\alpha}\right](\vec{q},\vec{p}) \end{aligned}\right\} . \qquad \text{(G1-10)}$$

Here we also define:

$$\left.\begin{aligned} Z_{\alpha,N} &\equiv -Z_{\alpha_1,N_2} = -Z_{\alpha_2,N_1} \\ Z_{N,\alpha} &\equiv -Z_{N_1,\alpha_2} = -Z_{N_2,\alpha_1} \end{aligned}\right\} . \qquad \text{(G1-10')}$$

With those in (G1-8) - (G1-10') in mind, we find the physical (antisymmetrized) amplitudes for $a\rightleftarrows NN$ ($a=\alpha,n$),

$$\left.\begin{aligned} X^{A(1)}_{a,N} &= (\sqrt{2})^{\delta_{an}}\Big[\left(Z_{a,N} - Z^{ex}_{a,N}\right) + \sum_{y=\beta,m} X_{a,y}\,\tau_y\left(Z_{y,N} - Z^{ex}_{y,N}\right)\Big] \\ X^{A(2)}_{N,a} &= (\sqrt{2})^{\delta_{an}}\Big[\left(Z_{N,a} - Z^{ex}_{N,a}\right) + \sum_{y=\beta,m}\left(Z_{N,y} - Z^{ex}_{N,y}\right)\tau_y\, X_{y,a}\Big] \end{aligned}\right\} , \qquad \text{(G1-11)}$$

where superscript A means that the amplitudes are anti-symmetrized and superscript "2" in the bracket indicates that the amplitude is two-particle irreducible.

(b) (n, α or $NN \to NN\pi$) process

The processes are typically expressed diagrammatically (Fig. G-2) as follows (in the case of ($n \to NN\pi$)),

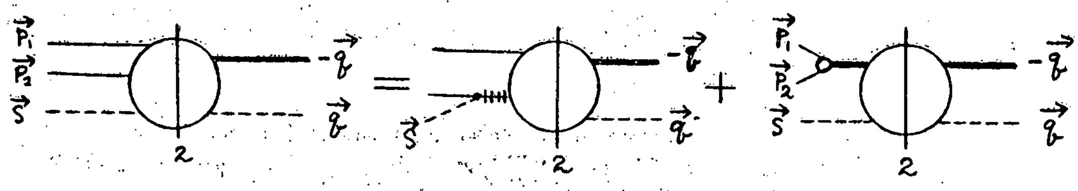


Fig. G-2

For simplicity form factors at the dissociation of isobars (or correlated pairs) and isobar propagators are labelled by the momentum of the spectator particles. The amplitudes for $a = \alpha$ or n are,

$$X^{A(2)}_{3,a}(\vec{P}_1,\vec{P}_2,\vec{S};\vec{q}) = (\sqrt{2})^{\delta_{an}}\Big[\sum_m \frac{1}{\sqrt{2}} g^*_m(\vec{S})\tau_m(\vec{S})X_{m,a}(\vec{S},\vec{q}) + \sum_\beta\{g^*_\beta(\vec{P}_1)\tau_\beta(\vec{P}_1)X_{\beta,a}(\vec{P}_1,\vec{q}) - g^*_\beta(\vec{P}_2)\tau_\beta(\vec{P}_2)X_{\beta,a}(\vec{P}_2,\vec{q})\}\Big] \tag{G1-12}$$

Also it is easy to show that for $NN \to NN\pi$

$$X^{A(2)}_{3,N}(\vec{P}_1,\vec{P}_2,\vec{S};\vec{q}) = \sum_\alpha\Big[g^*_\alpha(\vec{P}_1)\tau_\alpha(\vec{P}_1)X^{A(2)}_{\alpha,N}(\vec{P}_1,\vec{q}) - g^*_\alpha(\vec{P}_2)\tau_\alpha(\vec{P}_2)X^{A(2)}_{\alpha,N}(\vec{P}_2,\vec{q})\Big] + \sum_n g^*_n(\vec{S})\tau_n(\vec{S})X^{A(2)}_{n,N}(\vec{S},\vec{q})/2 \tag{G1-13}$$

We can write down the amplitude for NNπ → α,n or NN in a similar manner. Since these processes are not of physical interest to us, we shall not write the results.

(iii) One-particle irreducible amplitudes

(a) (NN → α or n) process

It is straightforward to show that for a=α or n

$$X^{A(1)}_{a,N} = X^{A(2)}_{a,N} + X^{A(2)}_{a,N}\,\tilde{G}_{T_2}\,\tilde{t}^{A}_{NN}/2\,, \tag{G1-14}$$

where $\tilde{t}^{A}_{NN}$ is an antisymmetrized N-N t-matrix with inelasticity. In a similar fashion we obtain

$$X^{A(1)}_{N,a} = X^{A(2)}_{N,a} + \tilde{t}^{A}_{NN}\,\tilde{G}_{T_2}\,X^{A(2)}_{N,a}/2. \tag{G1-14'}$$

(b) (a→b) process for a,b=α,n

The result just obtained above is used to obtain

$$\begin{aligned} X^{A(1)}_{a,b} &= X_{a,b} + X^{A(1)}_{a,N}\,\tilde{G}_{T_2}\,X^{A(2)}_{N,b}/2 \\ &= X_{a,b} + X^{A(2)}_{a,N}\,\tilde{G}_{T_2}\,X^{A(1)}_{N,b}/2\,, \end{aligned} \tag{G1-15}$$

where $X_{a,b}$ is a Faddeev amplitude appearing in (i).

(c) NN → πNN amplitude

The process is understood in Fig. G-3. The resultant amplitude is

$$X^{A(1)}_{3,N}(\vec{p}_1,\vec{p}_2,\vec{s};\vec{q}) = X^{A(2)}_{3,N}(\vec{p}_1,\vec{p}_2,\vec{s};\vec{q}) + \sum_{\vec{r}} X^{A(2)}_{3,N}(\vec{p}_1,\vec{p}_2,\vec{s};\vec{r})\,\tilde{G}_{T_2}(\vec{r})\,\tilde{t}_{NN}(\vec{r},\vec{q})/2$$

$$+\left[\Gamma(\vec{P}_1)\tilde{G}_{T_2}(\vec{P}_1)\tilde{t}^{A}_{NN}(\vec{P}_1,\vec{q}) - \Gamma(\vec{P}_2)\tilde{G}_{T_2}(\vec{P}_2)\tilde{t}^{A}_{NN}(\vec{P}_2,\vec{q})\right].$$

(G1-16)

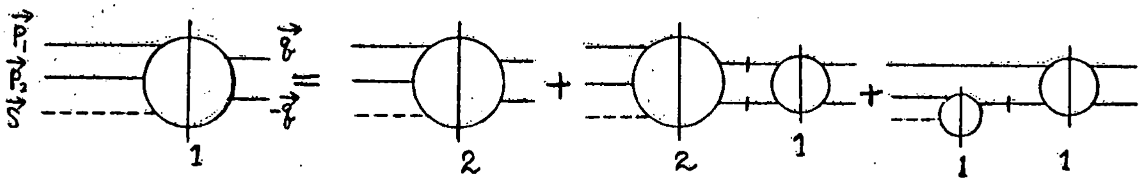


Fig. G-3

A similar result is obtained for one-particle irreducible $\pi NN \to NN$ amplitude.

(d) (α or $n \to \pi NN$) amplitude

The situation is similar to (c). The result is (for $a=\alpha,n$)

$$X^{A(1)}_{3,a} = X^{A(2)}_{3,a} + \left[\Gamma(\vec{P}_1)\tilde{G}_{T_2}(\vec{P}_1)X^{A(2)}_{N,a}(\vec{P}_1,\vec{q}) - \Gamma(\vec{P}_2)\tilde{G}_{T_2}(\vec{P}_2)X^{A(2)}_{N,a}(\vec{P}_2,\vec{q})\right]$$
$$+\sum_{\vec{u}} X^{A(1)}_{3,N}(\vec{P}_1,\vec{P}_2,\vec{S};\vec{u})\,\tilde{G}_{T_2}(\vec{u})\,X^{A(2)}_{N,a}(\vec{u},\vec{q})/2 \,. \qquad \text{(G1-17)}$$

Similar expression is for $X^{A(1)}_{a,3}$.

(e) $\pi NN \rightarrow \pi NN$ amplitude

The structure is shown in Fig. G-4.

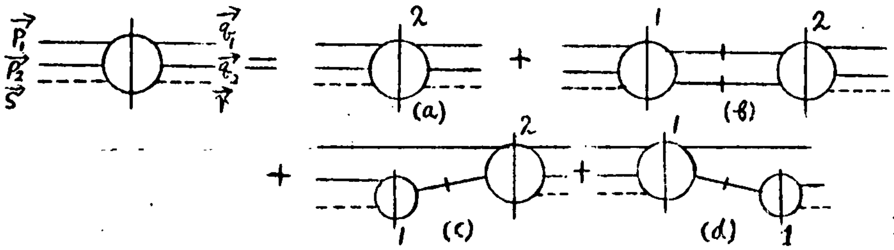


Fig. G-4

We shall then obtain contributions from each diagram separately by writing $X_{3,3}^{A(1)} \equiv \sum_{j=1}^{4} \mathcal{F}_j^{A(1)}$

$$\text{(a)}\quad \mathcal{F}_1^{A(1)} = \sum_\alpha \left[X_{3,\alpha}^{A(1)}(\vec{p}_1,\vec{p}_2,\vec{s};\vec{q}_1)\,\tau_\alpha(\vec{q}_1)\,g_\alpha(\vec{q}_1) - X_{3,\alpha}^{A(1)}(\vec{p}_1,\vec{p}_2,\vec{s};\vec{q}_2)\,\tau_\alpha(\vec{q}_2)\,g_\alpha(\vec{q}_2)\right] + \frac{1}{\sqrt{2}}\sum_n X_{3,n}^{A(2)}(\vec{p}_1,\vec{p}_2,\vec{s};\vec{r})\,\tau_n(\vec{r})\,g_n(\vec{r}) \tag{G1-18}$$

$$\text{(b)}\quad \mathcal{F}_2^{A(1)} = \sum_u X_{3,N}^{A(1)}(\vec{p}_1,\vec{p}_2,\vec{s};\vec{u})\,\tilde{G}_{T_2}(\vec{u})\,X_{N,3}^{A(2)}(\vec{u};\vec{q}_1,\vec{q}_2,\vec{r}) \tag{G1-19}$$

$$\text{(c)}\quad \mathcal{F}_3^{A(1)} = \Gamma(\vec{p}_1)\,\tilde{G}_{T_2}(\vec{p}_1)\,X_{N,3}^{A(2)}(\vec{p}_1;\vec{q}_1,\vec{q}_2,\vec{r}) - \Gamma'(\vec{p}_2)\,\tilde{G}_{T_2}(\vec{p}_2)\,X_{N,3}^{A(2)}(\vec{p}_2,\vec{q}_1,\vec{q}_2,\vec{r}) \tag{G1-20}$$

$$\text{(d)}\quad \mathcal{F}_4^{A(1)} = X_{3,N}^{A(1)}(\vec{p}_1,\vec{p}_2,\vec{s};\vec{q}_1)\,\tilde{G}_{T_2}(\vec{q}_1)\,\Lambda(\vec{q}_1) - X_{3,N}^{A(1)}(\vec{p}_1,\vec{p}_2,\vec{s};\vec{q}_2)\,\tilde{G}_{T_2}(\vec{q}_2)\,\Lambda(\vec{q}_2). \tag{G1-21}$$

2. The Angular Momentum Reduction

We shall decompose our antisymmetrized amplitudes into total angular momentum-isospin eigenstates. As we are working in non-relativistic kinematics and we have couplings to two-nucleon states where total spin is a good quentum number, it is most convenient to adopt channel spin coupling scheme.

(i) Faddeev part and two-particle irreducible amplitudes

The central part of the angular momentum reduction in our problem is the reduction of driving term; $Z_{a,b}$ (a,b = α, n or N). So we shall start with it. There are several works on this problem. We here adopt the method due to Sloan and Aarons[63] used in n-d scattering problem. Since the whole process of derivation can be seen in their paper (for three spin 1/2 particles) we just give here the result for the general case; three particles with different spins and isospins.

(a) Kinematics and notations

The kinematics involved in $Z_{a,b}$ is shown in Fig.G-5, where we consider an exchange process of $i+(j,k)\rightarrow k+(i,j)$; (j,k) etc. represent correlated pairs and the label of pair (j,k) is i([i,j,k] is made cyclic) as is commonly accepted. Momenta are assigned as in the figure.

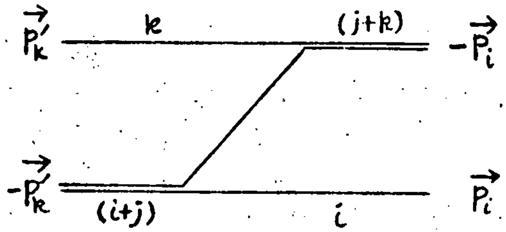


Fig. G-5

Relative momenta at the vertices are

$$\left.\begin{aligned}\vec{q}_i &= -\rho_{ki}\vec{P}_i - \vec{P}'_k \quad \left(\rho_{ki} \equiv \frac{m_k}{m_j+m_k}\right)\\ \vec{q}'_k &= \rho_{ik}\vec{P}'_k + \vec{P}_i \quad \left(\rho_{ik} \equiv \frac{m_i}{m_i+m_j}\right)\end{aligned}\right\} \qquad \text{(G2-1)}$$

Angular momenta and isospin are expressed as follows:

j: spin of a particle,

t: isospin of a particle,

η: total spin of a pair system,

$\bar{t}$: total isospin of a pair system,

λ: orbital angular momentum of a pair,

$\bar{j}$: total angular momentum of a pair,

$S \equiv |\bar{j}+j|$: channel spin

ℓ: orbital angular momentum of a pair-spectator system

J: total angular momentum of the whole system

τ: total isospin of the whole system

With the above convention the form factor appearing in the driving term is written as

$$g_\alpha(\vec{q}) = \sum_\lambda g_\lambda^{\bar{j}\eta\bar{t}}(q)\, \mathcal{Y}_{\bar{j}\lambda\eta}^{\bar{m}}(\hat{q})\, |\bar{t}\rangle, \qquad \text{(G2-2)}$$

where

m: third component of $\bar{j}$

α: label specifying the quantum state of the pair associated with the form factor

$y^{\bar{m}}_{j\lambda\eta}(\hat{q})$: angular momentum eigenstate of the pair associated with the form factor

$|\bar{t}>$: pair isospin eigenstate

$g^{\bar{j}\eta\bar{t}}_{\lambda}(q)$: scalar part of the form factor, which is usually real.

For the $N \leftrightarrow \pi N$ vertex, we have

$$\left.\begin{aligned} \Lambda^{\bar{m}}(\vec{q},\varepsilon) &\equiv \hat{\Lambda}(q,\varepsilon)\,\mathcal{Y}^{\bar{m}\,*}_{\bar{j}\lambda\eta}(\hat{q})\langle\bar{t}| \\ \Gamma^{\bar{m}}(\vec{q},\varepsilon) &\equiv \hat{\Gamma}(q,\varepsilon)\,\mathcal{Y}^{\bar{m}}_{\bar{j}\lambda\eta}(\hat{q})|\bar{t}\rangle \end{aligned}\right\}, \qquad \text{(G2-2')}$$

where $\eta=\bar{j}=\bar{t}=1/2$ and $\lambda=1$. Also $\hat{\Gamma}(q,\varepsilon^*) = \hat{\Lambda}^*(q,\varepsilon)$ from reality.

(b) The driving term and Faddeev equation

An angular-momentum-reduced driving term (not anti-symmetrized) is given as follows,

$$\begin{aligned} Z^{J\tau}_{\alpha_k',\beta_i}(p_k',p_i,E) &= (-1)^{c}\left[\bar{t}_i\bar{t}_k'\eta_i\eta_k'\bar{j}_i\bar{j}_k'S_iS_k'\ell_i\ell_k'\right]^{\frac{1}{2}}\begin{Bmatrix} t_i & t_j & \bar{t}_k' \\ t_k & \tau & \bar{t}_i \end{Bmatrix} \\ &\times\sum_{\lambda_i\lambda_k'}(-1)^{\lambda_k'}[\lambda_i\lambda_k']^{\frac{1}{2}}\sum_{\nu,\nu'}^{\lambda_i,\lambda_k'}\left\{\frac{(2\lambda_i+1)!\,(2\lambda_k'+1)!}{(2\nu)!\,(2\lambda_i-2\nu)!\,(2\nu')!\,(2\lambda_k'-2\nu')!}\right\}^{\frac{1}{2}}(\rho_{ik})^{\nu'}(\rho_{ki})^{\nu} \\ &\times(p_k')^{\lambda_i+\nu'-\nu}(p_i)^{\lambda_k'+\nu-\nu'}\sum_{\sigma}(-1)^{\sigma}[\sigma]\,\mathcal{F}^{\bar{j}_k'\eta_k'\bar{t}_k'\bar{j}_i\eta_i\bar{t}_i}_{\sigma\lambda_k'\lambda_i}(p_k',p_i,E)\sum_{abc}[abc] \\ &\times\begin{pmatrix} \nu' & \lambda_i-\nu & a \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \nu & \lambda_k'-\nu' & b \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \ell_k' & a & \sigma \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \ell_i & b & \sigma \\ 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} a & c & b \\ \ell_i & \sigma & \ell_k' \end{Bmatrix} \\ &\times\begin{Bmatrix} S_k' & c & S_i \\ \ell_i & J & \ell_k' \end{Bmatrix}\begin{Bmatrix} \lambda_k' & \lambda_i & c \\ \nu' & \lambda_i-\nu & a \\ \lambda_k'-\nu' & \nu & b \end{Bmatrix}\sum_{\Omega}(-1)^{\Omega}\begin{Bmatrix} c & \lambda_k' & \lambda_i \\ \Omega & S_i & S_k' \end{Bmatrix} \end{aligned}$$

$$\times\begin{Bmatrix} j_k' & \eta_k' & \Omega \\ \lambda_k' & S_k' & \bar{j}_k' \end{Bmatrix}\begin{Bmatrix} j_i & \eta_i & \Omega \\ \lambda_i & S_i & \bar{j}_i \end{Bmatrix}\begin{Bmatrix} j_k' & \eta_k' & \Omega \\ j_i & \eta_i & j_j \end{Bmatrix},$$

(G2-3)

where

$$\alpha_k' \equiv (\ell_k' S_k' \bar{j}_k' \eta_k' \bar{t}_k' j_k' t_k')$$

$$\beta_i \equiv (\ell_i S_i \bar{j}_i \eta_i \bar{t}_i j_i t_i)$$

$$[a\, b\, c \cdots\cdots] \equiv (2a+1)(2b+1)(2c+1) \cdots\cdots$$

$$C \equiv t_i + t_j + 2t_k' + \bar{t}_k' + 2j_i + j_j + 3j_k + 2\eta_i + 3\eta_k' - \bar{j}_i - \bar{j}_k' + S_i + S_k' + \ell_i + \ell_k' + 3J$$

$$\mathcal{F}_{\sigma\lambda_k'\lambda_i}^{\bar{j}_k'\eta_k'\bar{t}_k',\bar{j}_i\eta_i\bar{t}_i}(P_k', P_i, E) \equiv \frac{1}{2}\int_{-1}^{1}\frac{P_\sigma(x)(q_k')^{-\lambda_k'} g_{\lambda_k'}^{\bar{j}_k'\eta_k'\bar{t}_k'}(q_k')(q_i)^{-\lambda_i} g_{\lambda_i}^{\bar{j}_i\eta_i\bar{t}_i}(q_i)}{E - 2m - \mu - \frac{P_i^2}{2m_i} - \frac{P_k'^2}{2m_k} - \frac{(\vec{P_i} + \vec{P_k'})^2}{2m_j} + i\epsilon}\, dx$$

with $x = \cos(P_i \widehat{\ } P_k')$. Also standard angular momentum coupling coefficients are used in (G2-3).

In order to get an angular momentum decomposition of $Z_{\alpha,\beta}$ etc. in connection with their definition in (G1-1), we need to depart from the cyclic notation. The change in the ordering of i,j and k to meet our purpose will introduce some additional phase factor in (G2-3) which comes from angular momentum coupling coefficients and spherical harmonics; $Y_\ell^m(\hat{q})$. It then turns out [keeping an abbreviation $\alpha \equiv (\ell S \bar{j} \eta \bar{t} j t$ + other quantum numbers)] that

$Z_{\alpha,\beta}^{J\tau}$ is obtained from $Z_{\alpha_1,\beta_2}^{j\tau}$ by multiplying

$-(-1)^{\lambda_i+j_j+j_k-\eta_i+t_j+t_k-\bar{t}_i}$ before $\sum_{\lambda i}$ is taken.

(The same is true when α or β is replaced by N)

$Z^{J\tau}_{n,\alpha}$ multiply $Z^{J\tau}_{n_3,\alpha_1}$ by $(-1)^{i^+}$,

$$i^+ = \lambda_i+\lambda_k'+j_i+2j_j+j_k'-\eta_i-\eta_k'+t_i+2t_j+t_k-\bar{t}_i-\bar{t}_k$$

$Z^{J\tau}_{\alpha,n}$ this does not require any change in $Z^{J\tau}_{\alpha_1,n_3}$.

With the above prescriptions in mind a set of reduced anti-symmetrized Faddeev equations is now

$$X^{J\tau}_{a,b}(q,p,E) = Z^{J\tau}_{a,b}(q,p,E) + \sum_e \int s^2 ds\, Z^{J\tau}_{a,e}(q,s,E) \times \tau_e\left(E-2m-\mu-\frac{s^2}{2m_e}-\frac{s^2}{2\mu_e}\right) X^{J\tau}_{e,b}(s,p,E), \qquad \text{(G2-4)}$$

where a,b,e=n,α and μ_e is the reduced mass of a pair associated with state "e" whereas m_e is the mass of the spectator particle. Note that when both a and b are the states of the nucleon pair, there is no inhomogeneous term in (G2-4).

(c) Amplitudes including N-N states

These are $X^{A(2)}_{a,N}$, $X^{A(2)}_{N,a}$, $X^{A(2)}_{3,a}$ etc., in (ii) of the previous section. In order to obtain "J-τ" representation of them, the decomposition of $Z^{A}_{a,N}$ and $Z^{A}_{N,a}$ into angular momentum eigenstate is necessary, [see (G1-9) and (G1-10)].

This is done as follows. Remembering that

$$Z_{a,N}(\vec{q},\vec{p},E)=\sum_{\substack{J\tau\tilde{a}\tilde{N}\\ m_a m_N}} Z^{J\tau}_{\tilde{a},\tilde{N}}(q,p,E)\Pi(C-G)\,Y^{m_a}_{\ell_a}(\hat{q})\,Y^{m_N *}_{\ell_N}(\hat{p}), \tag{G2-5}$$

where $\tilde{a}\equiv(\ell_a S_a \bar{j}_a \eta_a t_a)$ etc. and $\Pi(C-G)$ means the products of several Clebsch-Gordan coefficients. When we decompose $Z^{ex}_{a,N}(\vec{q},\vec{p},E)\equiv Z_{a,N}(\vec{q},-\vec{p},E)$ into J-τ representation, it is easy to see that changes are in (1) some 6-j symbols in $Z^{J\tau}_{\tilde{a},\tilde{N}}(q,p,E)$, (2) some Clebsch-Gordan coefficients in $\Pi(C-G)$ and (3) $Y^{*mN}_{\ell N}(\hat{p})\to Y^{*mN}_{\ell N}(-\hat{p})$, of the original $Z_{a,N}(\vec{q},\vec{p},E)$.

It turns out after some observation, taking into account the symmetry properties of 6-j symbols, that there is no change in $Z^{J\tau}_{\tilde{a}\tilde{N}}(q,p,E)$. As for the change in $\Pi(C-G)$, we get an extra factor $(-1)^{S_N+\tau}$ coming from $\langle\bar{j}_i j_i \bar{m}_i m_i|S_N M_N\rangle$ $\to\langle\bar{j}_i j_i -\bar{m}_i -m_i|S_N -M_N\rangle$ and a similar change in the Clebsch-Gordan coefficient for isospin coupling (note that $j_i=\bar{j}_i=1/2$ here). The third factor; $Y^{*mN}_{\ell N}(-\hat{p})$ is equal to $(-1)^{\ell N}\times Y^{*mN}_{\ell N}(\hat{q})$. Collecting all these results, we obtain

$$Z^{A,J\tau}_{a,N}(q,p,E)=(\sqrt{2})^{\delta_{aN}}\{1-(-1)^{S_N+\ell_N+\tau}\}\,Z^{J\tau}_{a,N}(q,p,E). \tag{G2-6}$$

Note that the factor $[1-(-1)^{S_N+\ell_N+\tau}]$ is exactly the one appearing in the angular momentum reduction of an amplitude for two-fermion scattering (with isospin). A similar result is obtained for $Z_{N,a}^{AJ\tau}$.

We thus go on to decompose $X_{a,N}^{A(2)}$ and $X_{N,a}^{A(2)}$. If we define

$$X_{a,N}^{A(2),J\tau} \equiv \{1-(-1)^{\ell_N+S_N+\tau}\}\hat{X}_{a,N}^{(2),J\tau}$$ and also for $X_{N,a}^{A(2)J\tau}$

we easily get from (G1-11),

$$\hat{X}_{a,N}^{(2),J\tau}(q,p,E) = (\sqrt{2})^{\delta_{an}}\Big\{Z_{a,N}^{J\tau}(q,p,E) + \sum_b\int_0^\infty s^2 ds\, X_{a,b}^{J\tau}(q,s,E) \times \tau_b(E^+-2m-\mu-\frac{s^2}{2m_b}-\frac{s^2}{2\mu_b})Z_{b,N}^{J\tau}(s,p,E)\Big\}, \quad \text{(G2-7)}$$

and similar expression for $X_{N,a}^{(2)J\tau}$. We then decompose (G1-12) and (G1-13) into angular momentum, which result

$$X_{3,a}^{A(2),J\tau}(P_1,P_2,S,q,E) = (\sqrt{2})^{\delta_{an}}\Big\{\sum_\alpha\Big[g_\alpha(u_1)\tau_\alpha(E^+-2m-\mu-\frac{P_1^2}{2m_\alpha}-\frac{P_1^2}{2\mu_\alpha})X_{\alpha,a}^{J\tau}(P_1,q,E) - g_\alpha(u_2)\tau_\alpha(E^+-2m-\mu-\frac{P_2^2}{2m_\alpha}-\frac{P_2^2}{2\mu_\alpha})X_{\alpha,a}^{J\tau}(P_2,q,E)\Big] + \sum_m g_m(v)\tau_m(E^+-2m-\mu-\frac{S^2}{2m_m}-\frac{S^2}{2\mu_m})X_{m,a}^{J\tau}(S,q,E)\Big\},$$

and (G2-8)

$$\hat{X}_{3,N}^{(2),J\tau}(P_1,P_2,S,q,E) = \sum_\alpha\Big\{g_\alpha(u_1)\tau_\alpha(E^+-2m-\mu-\frac{P_1^2}{2m_\alpha}-\frac{P_1^2}{2\mu_\alpha})\hat{X}_{\alpha,N}^{(2),J\tau}(P_1,q,E) - g_\alpha(u_2)\tau_\alpha(E^+-2m-\mu-\frac{P_2^2}{2m_\alpha}-\frac{P_2^2}{2\mu_\alpha})\hat{X}_{\alpha,N}^{(2),J\tau}(P_2,q,E)\Big\} + \sum_m g_m(v)\tau_m(E^+-2m-\mu-\frac{S^2}{2m_m}-\frac{S^2}{2\mu_m})\hat{X}_{m,N}^{(2),J\tau}(S,q,E),$$

(G2-8')

where

$$\begin{pmatrix} \vec{u}_1 \\ \vec{u}_3 \\ \vec{v} \end{pmatrix} \text{ is a relative momentum between } \begin{pmatrix} \text{nucleon with momentum } P_1 \text{ and the pion} \\ \text{nucleon with momentum } P_2 \text{ and the pion} \\ \text{two nucleons} \end{pmatrix}$$

(ii) One-particle irreducible amplitudes

(a) Amplitude for NN → NN

Note that in two-nucleon scattering the total spin is conserved. So we obtain from Lippmann-Schwinger equation:

$$\tilde{t}^{A,J\tau S}_{NN\,\ell'\ell}(p,q,E) = \frac{1}{2}\left\{1+(-1)^{\ell'+\ell}\right\}\left\{1-(-1)^{\ell+\tau+S}\right\}\tilde{t}^{J\tau S}_{NN\,\ell'\ell}(p,q,E), \quad \text{(G2-9)}$$

where the first factor in the right-hand side of (G2- 9) serves for parity conservation and $t^{J\tau S}_{NN\ell'\ell}$ satisfies

$$\tilde{t}^{J\tau S}_{NN\,\ell'\ell}(p,q,E) = V^{J\tau S}_{NN\,\ell'\ell}(p,q,E) + \sum_{\ell''}\int_0^\infty r^2 dr\, V^{J\tau S}_{NN\,\ell'\ell''}(p,r,E)\,\tilde{G}_2(E,r)\,\tilde{t}^{J\tau S}_{NN\,\ell''\ell}(r,q,E).$$

(G2-10)

As we have done in (i), we define

$$\hat{t}^{A,J\tau S}_{NN\,\ell'\ell} \equiv \left\{1-(-1)^{\ell+S+\tau}\right\}\hat{t}^{J\tau S}_{NN\,\ell'\ell}\,.$$

(b) $X^{A(1)}_{a,N}$, $X^{A(1)}_{3,N}$ and $X^{A(1)}_{a,b}$

Using $\hat{t}^{J\tau S}_{NN\ell'\ell}$ we readily obtain

$$\hat{X}^{(1),J\tau}_{a,N}(p,q,E) = \hat{X}^{(2),J\tau}_{a,N}(p,q,E) + \sum_{N'}\int_0^\infty r^2 dr\, \hat{X}^{(2),J\tau}_{a,N'}(p,r,E)\, \tilde{G}_2(E,r)\, \hat{t}^{J\tau S_N}_{NN\,\ell'_N\ell_N}(r,q,E), \tag{G2-11}$$

$$\begin{aligned}\hat{X}^{(1),J\tau}_{3,N}(p_1,p_2,s,q,E) &= \hat{X}^{(2),J\tau}_{3,N}(p_1,p_2,s,q,E) + \sum_{N'}\int_0^\infty r^2 dr\, \hat{X}^{(2),J\tau}_{3,N'}(p_1,p_2,s,r,E) \\ &\times \tilde{G}_2(E,r)\, \hat{t}^{J\tau S_N}_{NN\,\ell'_N\ell_N}(r,q,E) \\ &+ \hat{\Gamma}\left(u_1, E-2m-\mu-\frac{p_2^2}{2(m+\mu)}-\frac{p_1^2}{2m}\right)\tilde{G}_2(E,p_1) \\ &\times \sum_{\ell'_N}\hat{t}^{J\tau S_N}_{NN\,\ell'_N\ell_N}(p_1,q,E) \\ &+ (\text{similar term with } u_1\rightarrow u_2 \text{ and } p_1\rightarrow p_2),\end{aligned} \tag{G2-12}$$

and

$$\begin{aligned}X^{A(1),J\tau}_{a,b}(p,q,E) &= X^{J\tau}_{a,b}(p,q,E) + \sum_{N}\int_0^\infty r^2 dr\, \hat{X}^{(1),J\tau}_{a,N}(p,r,E)\, \tilde{G}_2(E,r)\, \hat{X}^{(2),J\tau}_{N,b}(r,q,E) \\ &\times \left\{1-(-1)^{\ell_N+S_N+\tau}\right\}.\end{aligned} \tag{G2-13}$$

As the expressions get lengthy we shall not write down $X^{A(1)J\tau}_{3,a}$ nor $X^{A(1)J\tau}_{3,3}$. But these are just obtained straightforwardly.

APPENDIX H

INPUT FUNCTIONS FOR OUR CALCULATION

(i) Deuteron Wave Function

(a) The Hulthen S-Wave Deuteron

The momentum space Hulthén wave function is of the form,

$$\phi(p) = N \frac{1}{(p^2+\alpha^2)(p^2+\beta^2)}, \tag{H-1}$$

where $\alpha^2 = \varepsilon_d m$, $\beta = 6.255\ \alpha$ and $N = [\alpha\beta(\alpha+\beta)^3]^{\frac{1}{2}}/\pi$ is the normalization constant. This wave function can be obtained from a S-wave separable potential of the Yamaguchi type in which $1/(P^2+\beta^2)$ is a form factor.

(b) The Pieper-Reid Deuteron[54]

This is constructed from Pieper's separable potentials which are obtained by applying the procedure of Ernst et. al.[53] to the Reid soft core potential. The deuteron wave function thus obtained is the same as what we get by numerically solving Schrodinger equation using the Reid potential up to several digits. We use Pieper's rank 1 potential and obtained (angular parts have been taken out)

$$\phi_\ell(p) = -C \sum_{n=1}^{8} b_{\ell n}\, w_{\ell n}(p)/(p^2+\alpha^2), \tag{H-2}$$

where $W_{\ell n}(q) = q^{\ell}(q^2+\alpha^2 n)^{-2-\ell/2}$, $\alpha_n = 3/2\ n$ in f_m^{-1}, $b_{\ell n}$'s are dimensionless constants (see Pieper's article for a detail) and C is the normalization constant. All the deuteron properties are in good agreement of what we can get from Reid potential. For example the D-state probability is 6.49%.

(c) The McGee Deuteron(56)

The form of this wave function comes from the study of analytic properties of the dpn vertex function. The general forms of S and D state wave functions are of the form (angular parts have been eliminated)

$$\phi_S(p)/N = \frac{1}{p^2+\alpha^2} + \int_{\alpha+\lambda}^{\infty} \frac{\hat{\sigma}_S(z)dz}{p^2+z^2}, \qquad \text{(H-3)}$$

and

$$\phi_D(p)/N\rho = -\frac{p^2}{p^2+\alpha^2} - \int_{\alpha+\lambda}^{\infty} \frac{\hat{\sigma}_D(z)dz\, p^2}{p^2+z^2}, \qquad \text{(H-3')}$$

where N: normalization constant

ρ: asymptotic D/S ratio

$\hat{\sigma}_s(z)$, $\hat{\sigma}_p(z)$: spectral functions

λ: minimum decay constant

In McGee's approximation, spectral functions are replaced by a finite summation of delta functions and the result is

$$\phi_S(p) = N(2/\pi)^{1/2}\left(\frac{1}{p^2+\alpha^2} + \sum_{j=1}^{4} C_j \frac{1}{p^2+\epsilon_j^2}\right), \quad \text{(H-4)}$$

and

$$\phi_D(p) = -\rho N(2/\pi)^{1/2}\left(\frac{p^2}{\alpha^2(p^2+\alpha^2)} + \sum_{j=1}^{5} C_j' \frac{p^2}{\epsilon_j'^2(p^2+\epsilon_j'^2)}\right). \quad \text{(H-4')}$$

Here $N = 0.8896\ \text{fm}^{-1/2}$ and $\rho = 0.0269$. The D-state probability is 7%. C_j, ϵ_j etc. are found in McGee's paper.

(ii) $^3P_1$ N-N Separable Potentials of Mongan[57]

After angular momentum dependence has been eliminated, the potentials here take the form:

$$V_{\ell'\ell}(q,p) = (i)^{\ell'-\ell}\left[g_{\ell'}(q)g_\ell(p) - h_{\ell'}(q)h_\ell(p)\right], \quad \text{(H-5)}$$

where the first term in the expression comes from the repulsive part and the second term, from the attractive part.

For $^3P_{11}$ states $\ell'=\ell$ and the forms get a little simpler.

(a) Mongan type 1 potential

$$g_\ell(p) = C_R p^\ell/(p^2+a_R^2)^{(\ell+1)/2}, \quad h_\ell(p) = C_A p^\ell/(p^2+a_A^2)^{(\ell+1)/2}.$$

For $^3P_1$ $\quad C_R = 3.498, \quad C_A = 18.89 \quad$ [in $(\mathrm{MeV/fm})^{1/2}$]

$a_R = 0.697, \quad a_A = 2.322 \quad$ [in fm^{-1}]

(b) Mongan type 2 potential

$$g_\ell(p) = C_R p^\ell/(p^2+a_R^2)^{(\ell+2)/2}, \quad h_\ell(p) = C_A p^\ell/(p^2+a_A^2)^{(\ell+2)/2}.$$

For $^3P_1$ $\quad C_R = 45.63 \quad C_A = 0.0 \quad$ [in $(\mathrm{MeV/fm})^{1/2}$]

$a_R = 2.178 \quad a_A = 0.0 \quad$ [in fm^{-1}]

Mongan type 1 fits the experimental phase shift better in $^3P_1$ partial waves.

(iii) Pi-Nucleon Separable Potentials of Afnan and Thomas[28]

The general form of the potentials here is $v(q,p) = \lambda \times g(q)g(p)$.

(a) S_{11} partial wave

$\lambda=-1$, $g(p)=C/(p^2+\beta^2)$, where $C=31.02$ $(\mathrm{MeV/fm})^{1/2}$ and $\beta= 2.629$ fm^{-1}.

(b) S_{31} partial wave

$\lambda=1$, $g(p) = C(1- 0.5048\ e^{-p^2})/(p^2+\beta^2)$, where $C=100.6$ $(\mathrm{MeV/fm})^{1/2}$ and $\beta=3.045$ fm^{-1}.

(c) P_{33} Partial Wave

$$\lambda=1,\ g(p) = C_p/(p^2+\beta^2),\ \text{where}$$
$$C=32.7047(\text{MeV/fm})^{\frac{1}{2}}\ \text{and}\ \beta=5.3344\ \text{fm}^{-1}.$$

S_{11} and S_{31} potentials reproduce observed low energy phase shifts quite well. P_{33} potential is adjusted to have the correct 3-3 resonance pole and its low energy fit is not so well as S_{11} and S_{31} potentials.