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THE MAGNETIC PROPERTIES OF DEFORMED NUCLEI

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Lung-wen Chiao
(Thesis)

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ABSTRACT

It is shown that the magnetic moments of odd-A deformed nuclei can be interpreted in terms of the independent-particle model with interconfigurational mixing due to the very-short-range residual forces. The latter are implied by using the empirically reduced spin gyromagnetic ratios. The effects of these residual forces on the collective gyromagnetic ratios g_R are discussed in terms of pair correlation. The effect of particle-rotation interaction on the magnetic moment and the collective gyromagnetic ratio are shown. The g_R values are obtained from the magnetic moments and the matrix elements for M1 transitions in this band. It is found that these mechanisms give a satisfactory account of the collective gyromagnetic ratio of Dy^{161}, Ho^{165}, Er^{167}, and Hf^{179}.

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INTRODUCTION

In this paper are presented results of an investigation of the magnetic moments of deformed nuclei in the regions $151 < A \leq 191$ and $A > 222$ where the strong-coupling model of Bohr and Mottelson has been highly successful.[1,2] The objects of this investigation are (a) to determine whether there is evidence that the spin gyromagnetic ratios g_s when "quenched" or reduced in magnitude are more appropriate than the traditionally assumed free-nucleon values for nuclear magnetic-moment calculations, and (b) to see how the particle-rotation interaction will affect nuclear magnetic properties.

For the first purpose, the nuclei are grouped according to the asymptotic quantum numbers Σ (projection of intrinsic spin) in order best to display the contribution of the intrinsic spin to the magnetic-moment calculations. That is, $\langle \vec{s} \cdot \vec{I} \rangle$ is positive or negative accordingly as Σ is positive or negative. An examination of the compilation of calculated and experimental magnetic moments of odd-mass deformed nuclei given by Mottelson and Nilsson[3] shows that the discrepancies are of the same sign as that of Σ for odd-Z nuclei and of opposite sign for odd-N nuclei. The magnitude of deviation further suggests that the effective g_s factor for protons in nuclei is roughly 4 instead of the free value 5.585, while that for neutrons is -2.4 instead of -3.826. Using these values and Nilsson's wave function interpolated to proper deformation, we have calculated the magnetic moments of both odd-A and odd-odd nuclei. The reduction of g_s is attributed to configurational mixing due to the very-short-range inter-nucleon forces.

For the second purpose, we shall single out nuclear states of odd parity in the 50 to 82 shells ($h_{11/2}$), even parity in the 82 to 126 shells ($i_{13/2}$), and odd parity in the > 126 shell ($j_{15/2}$). This class of nuclei has particularly large j values for the odd nucleon and will therefore be especially susceptible to appreciable particle-rotation interaction. For these nuclei, there is an extra off-diagonal contribution to the magnetic-moment calculations. This interaction will also alter the relation between M1-transition matrix elements and the ground-state magnetic moments. Bernstein and de Boer have analyzed the experimental data in the strong-coupling approximation and get g_R and g_K values for odd-A deformed nuclei.[4] A reanalysis of these data taking into account the particle-rotation interaction gives different g_R values which are nearly equal to those of the adjacent even-even nuclei. It is then unfortunately not possible to detect with any certainty the effects of the other important perturbation, the pair-correlation, on collective motion of odd-A nuclei from the measurements of the collective gyromagnetic ratio since such effects will be obscured by the contributions from the particle-rotation interaction.[5]

II. DEFORMED NUCLEI AND THE STRONG-COUPLING MODEL

The nuclear shape depends upon the configuration of the nucleons. For nuclei in the regions of closed shells, the equilibrium shape of the nucleus is approximately spherical, and the nucleons are considered as moving independently in an essentially spherical potential which represents the interaction of any one nucleon with the remaining nucleons. In addition to this, there is also a strong spin-orbit coupling force. The states of an individual nucleon can then be classified, and this leads to the concept of shells. For nuclei in the regions far removed from closed shells, the nuclear equilibrium shape deviates strongly from spherical symmetry. There can then be collective oscillations about this equilibrium shape. These oscillations will modify the effective nuclear field and so be coupled to the motion of the nucleons. A description of nuclei in such regions has to be given in terms of collective coordinates specifying the shape of the distorted core and its spatial orientation.

Besides the vibrations, there is also a rotational type of motion; that is, the nuclei rotate as a whole, preserving both shape and internal structure. Associated with this rotational motion of a spheroidal nucleus is a sequence of rotational energy levels given by

$$E_{rot.} = \frac{\hbar^2}{2\Im} \left[I(I+1) - I_0(I_0+1)\right] , \qquad (1)$$

where I and I_0 are the spins of a given level and the ground state, respectively. Such levels have been well identified in the regions $150 < A < 192$ and $A > 222$. The individual nucleons in the spheroidal nuclei are coupled separately to the symmetry axis in states characterized by their component of angular momentum Ω_i along the symmetry axis. The total Ω is given by $\Sigma\Omega_i$. Because of the axial symmetry, the particle states Ω_i and $-\Omega_i$ are degenerate, and nucleons will fill pairwise into these states. The rotational motion is characterized by the quantum numbers I, M, and K where I is the total angular momentum of rotational and nucleonic motion, M its projection on a fixed axis in space, and K its projection on the nuclear symmetry axis. In the vibrational ground state, the collective rotational angular momentum R is perpendicular to the nuclear symmetry axis, and thus we have $K = \Omega$. The angular-momentum coupling scheme for deformed nuclei in the ground state is shown in Fig. 1.

The wave functions for these nuclei may then be written in the form given by Bohr and Mottelson:[2]

$$\langle \vec{r} \,|\Omega IMK\rangle = \left(\frac{2I+1}{16\pi^2}\right)^{1/2} \phi\, [\chi_\Omega \mathcal{D}^I_{MK}(\theta_i) + (-)^{I-\Sigma j_i} \chi_{-\Omega} \mathcal{D}^I_{M-K}(\theta_i)] \qquad (2)$$

where ϕ describes the vibrational motion, $\mathcal{D}$ the eigenfunction of a symmetric top-- describes the rotational motion, and θ_i refers to the Eulerian angles specifying a coordinate system fixed in the nucleus. The normalization is such that $\mathcal{D}$ gives the unitary transformation from the fixed coordinate system to the nuclear coordinate system. Here χ_Ω represents the motion of the particle with respect to the deformed nucleus.

Assuming a three-dimensional harmonic-oscillator-type potential with inclusion of spin-orbit coupling, Nilsson has calculated the eigenvalues of χ_Ω as a function of the axial deformation and expresses the eigenstate in terms of spherical components.[6] Several other quantum numbers are introduced to distinguish the different single-particle states in a nonspherical field. These are the "asymptotic" quantum numbers which should characterize these states in the limit where the nuclear potential becomes a very anisotropic, axially symmetric harmonic oscillator. Here N is the total number of nodes in the wave function, N_z is the number of nodal planes perpendicular to the symmetry axis, and Λ is the component of the particles orbital angular momentum along the symmetry

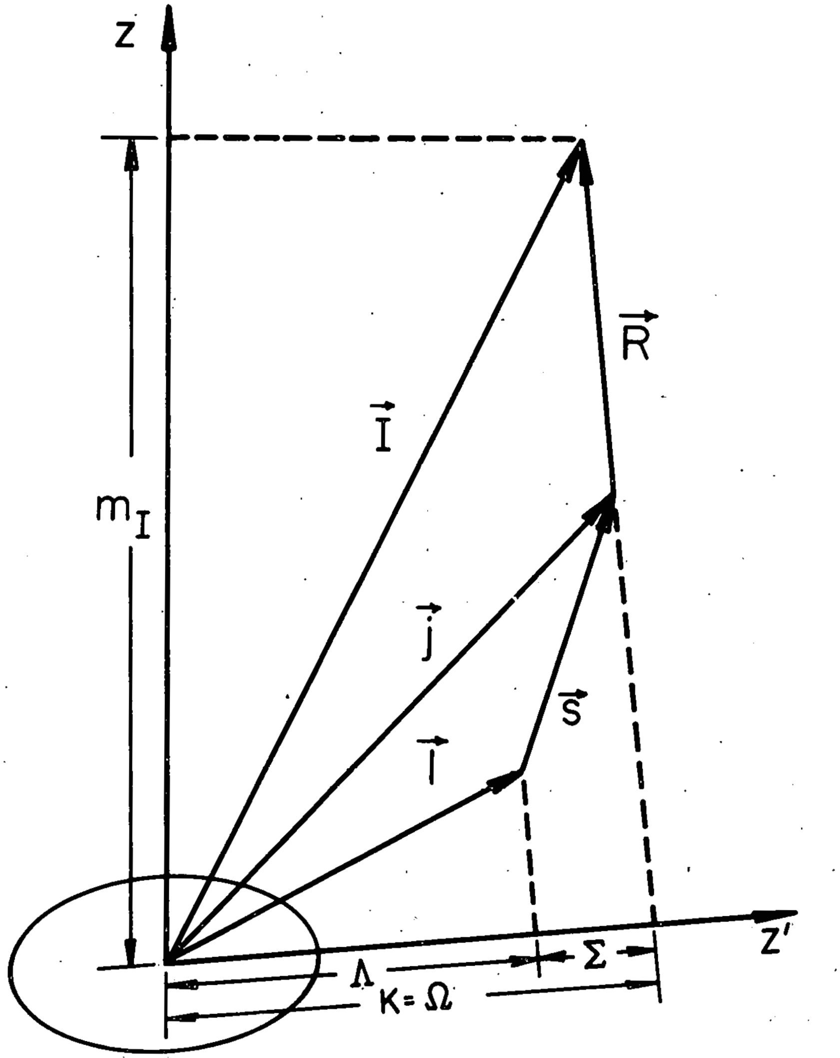

Fig. 1. Angular-momentum coupling scheme for deformed nuclei in the ground state.

axis, Σ is the spin component along the symmetry axis, and we have $\Omega = \Lambda + \Sigma$. In this representation, χ_Ω is given in terms of the state vectors $|N\ell\Lambda\Sigma\rangle$.

The parity of the state is determined by $(-1)^N$, and the values of ℓ are N, N-2, ... 1, or 0, depending on whether N is odd or even. The original wave-function tables have been recently supplemented.[3] In Figs. 2 through 5, these states are plotted as a function of deformation parameters δ and η, which characterize the eccentricity of the nuclear potential. The parameter δ is defined by

$$\omega_x^2 = \omega_o^2 (1 + 2/3\ \delta) = \omega_y^2 \tag{3a}$$

and

$$\omega_z^2 = \omega_o^2 (1- 4/3\ \delta) . \tag{3b}$$

Here δ and η are related by

$$\eta = \frac{\delta}{\kappa} [1- 4/3\ \delta^2 - 16/27\ \delta^3]^{-1/6} \tag{4}$$

where κ is 0.061 for $50 \leq z \leq 82$, and κ is 0.05 for other nucleonic states. Written in square brackets beside each orbital in Figs. 2 through 5 are the asymptotic quantum numbers $\Omega N \eta_z \Lambda$. In the region of very strong deformations, the nucleonic wave functions are more nearly pure states when expanded in terms of the eigenvectors of an anisotropic harmonic-oscillator potential characterized by the asymptotic quantum numbers.

With the availibility of these wave functions, it is appropriate to determine to what extent nuclear properties such as magnetic moments may be consistent with the general features of the strong-coupling model and to check the degree to which the separation of rotational and intrinsic nucleonic motion is justified.

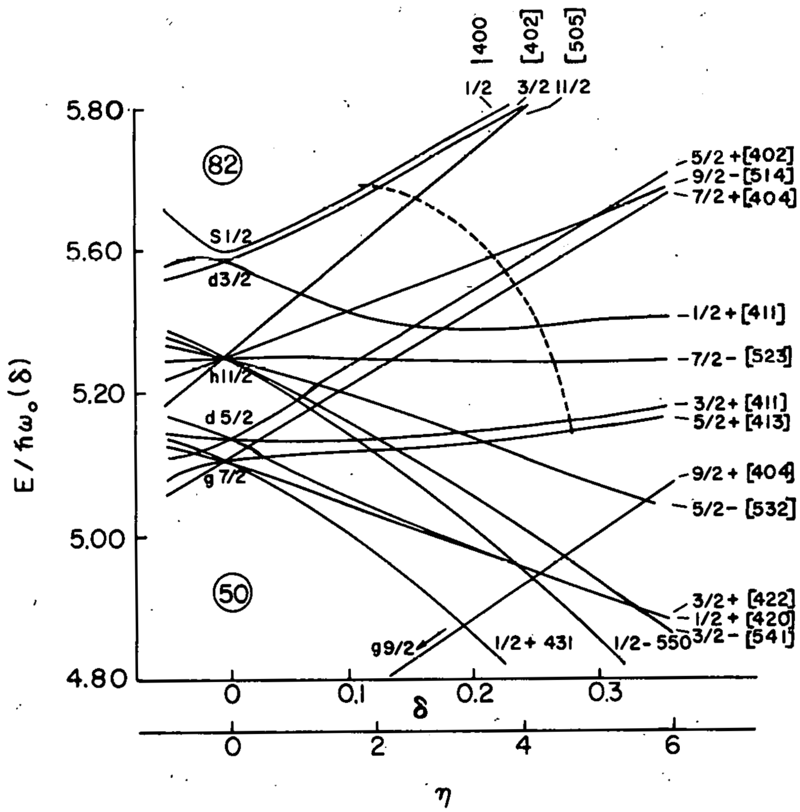

Fig. 2. Single-particle levels for odd-Z nuclei in the region $50 \leq Z \leq 82$. The dashed line indicates very roughly the deformations assumed to be appropriate to rare-earth isotopes.

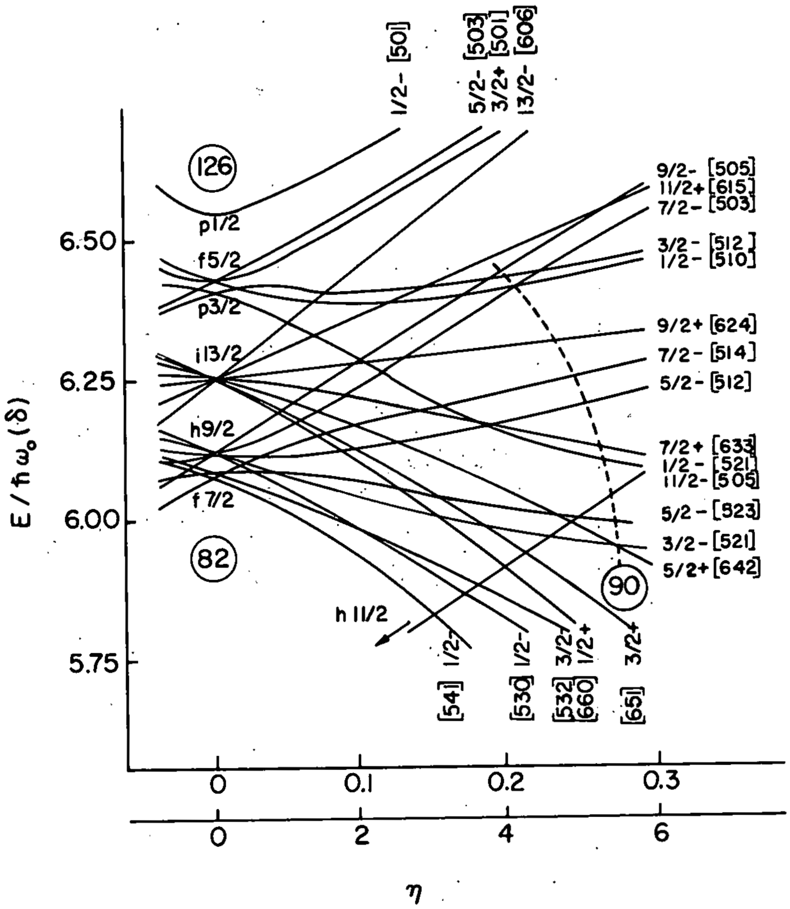

Fig. 3. Single-particle levels for odd-N nuclei in the region $82 \leq N \leq 126$. The dashed line indicates very roughly the deformations assumed to be appropriate to rare-earth isotopes.

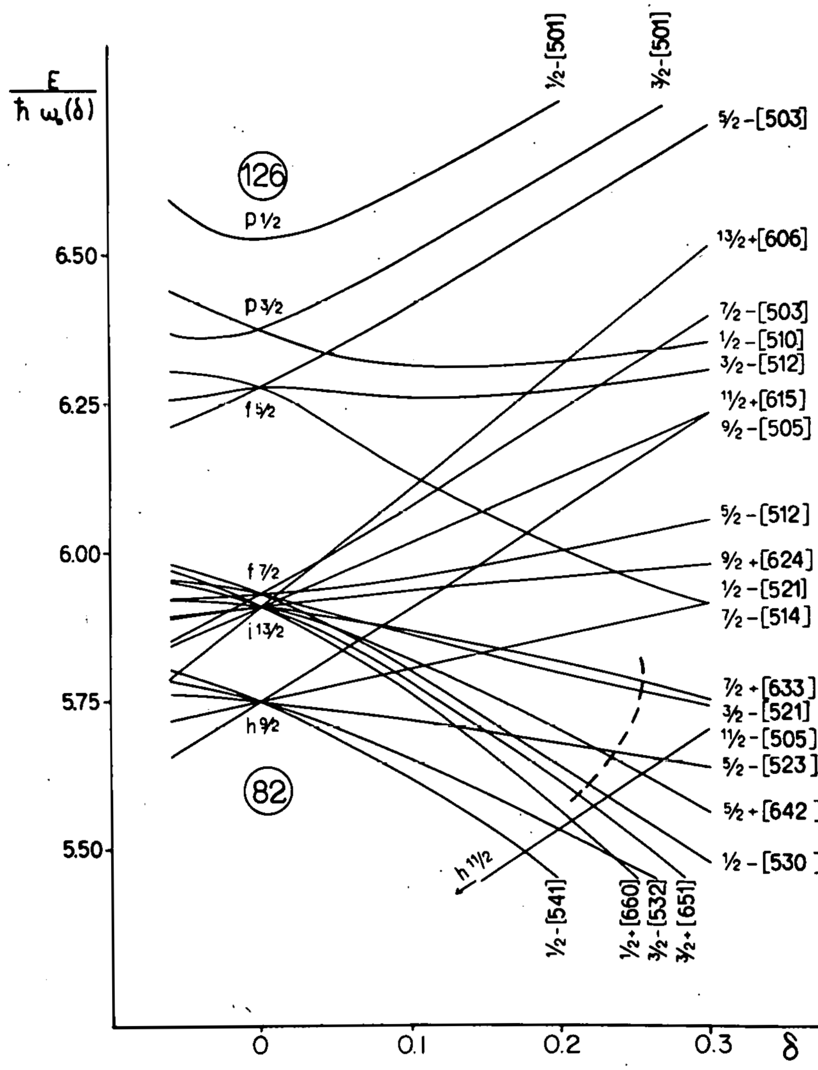

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Fig. 4. Single-particle levels for odd-Z nuclei in the heavy-element region Z > 82. The dashed line indicates very roughly the deformations assumed to be appropriate to actinide isotopes.

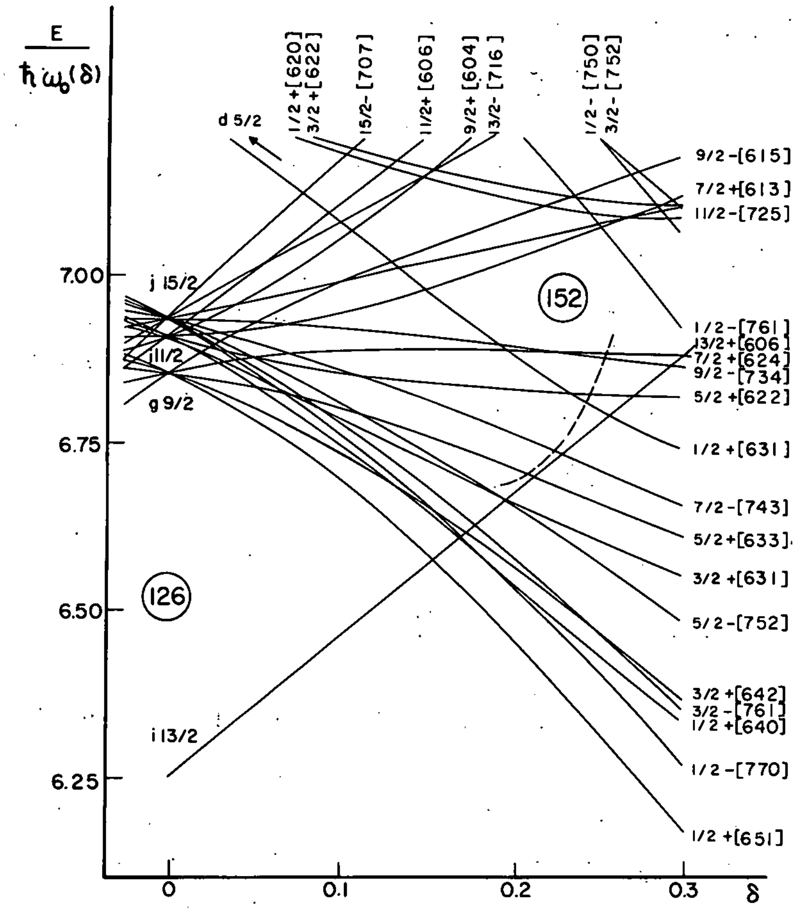

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Fig. 5. Single-particle levels for odd-N nuclei in the heavy-element region N > 126. The dashed line indicates very roughly the deformations assumed to be appropriate to actinide isotopes.

III. MAGNETIC-MOMENT CALCULATIONS

The nuclear-magnetic-moment operator is

$$\vec{\mu} = \frac{e}{2Mc} \sum_i (g_\ell \vec{\ell}_i + g_s \vec{s}_i) + \frac{e}{2Mc} g_R \vec{R} \tag{5}$$

where $\vec{\ell}_i$, $\vec{s}_i$ and $\vec{R}$ are, respectively, the orbital, spin, and collective angular momenta of nucleons. The corresponding gyromagnetic ratios are g_ℓ which is 1 for the proton and 0 for the neutron, g_s, which is customarily taken to be 5.585 for the proton and -3.826 for the neutron, and g_R, which equals Z/A for a uniformly charged nucleus and the hydrodynamic or the rigid-body model of collective rotation.

The magnetic moment is obtained from the expectation value of the z component of $\vec{\mu}$ for the nuclear substate in which the spin is along the z axis. Thus we have

$$\mu = \langle \mu_z \rangle_{M=I} \tag{6}$$

which in the strong coupling scheme of Fig. 1 is given by

$$\mu = \langle \mu_z \rangle_{M=I} = \frac{\langle \vec{\mu} \cdot \vec{I} \rangle}{I+1} . \tag{7}$$

Consider now the magnetic moment for an odd-A nucleus in which all the particles except the last one fill the different orbits in pairs. Then all the nucleons except the last one are paired with $\Sigma s_z=0$ and $\Sigma \ell_z=0$ and will not contribute to the magnetic moment except via the collective rotational motion. The evaluation of the matrix element $\langle \vec{\mu} \cdot \vec{I} \rangle$ is then simply that of the last nucleon, and we can write the magnetic moment in units of the nuclear magneton (n.m.), $e\hbar/2mc$, as

$$\mu = \frac{1}{I+1} \left[g_s \langle \vec{s} \cdot \vec{I} \rangle + g_\ell \langle \vec{\ell} \cdot \vec{I} \rangle + g_R \langle \vec{R} \cdot \vec{I} \rangle \right], \tag{8}$$

where $\vec{s}$ and $\vec{\ell}$ are the spin angular momenta of the last nucleon.

Using $\vec{j} = \vec{\ell}+\vec{s}$ and $\vec{I} = \vec{j}+\vec{R}$, we may replace the last two terms by the more readily calculable terms $\langle \vec{j} \cdot \vec{I} \rangle$ and $\langle \vec{I}^2 \rangle$. Now we have

$$\mu = \frac{1}{I+1} \left[(g_s - g_\ell) \langle \vec{s} \cdot \vec{I} \rangle + (g_\ell - g_R) \langle \vec{j} \cdot \vec{I} \rangle + g_R \langle \vec{I}^2 \rangle \right] \tag{9}$$

It can also be written in the formally simple form

$$\mu = g_R \frac{I(I+1) - K^2}{I + 1} + g_K \frac{K^2}{I + 1} \qquad (10)$$

with

$$g_K = \frac{1}{K} \left[g_s \langle s_z \rangle + g_\ell \langle \ell_z \rangle \right] \qquad (11)$$

Here $\langle \vec{s} \cdot \vec{I} \rangle$ is given in terms of the Nilsson eigenvectors

$$\langle \vec{s} \cdot \vec{I} \rangle = \frac{K}{2} \sum_\ell \left(a^2_{\ell\Omega-1/2} - a^2_{\ell\Omega+1/2} \right) + 1/2(I+1/2) \; (-)^{I-1/2+\ell} \sum a^2_{\ell 0} \, \delta_{\Omega,1/2} \delta_{K,1/2} \qquad (12)$$

and $\langle \vec{j} \cdot \vec{I} \rangle$ is given in the j- Ω representation of Bohr and Mottelson

$$\langle \vec{j} \cdot \vec{I} \rangle = \Omega K + 1/2(I+1/2) \, a \, (-)^{I-1/2} \, \delta_{\Omega,1/2} \delta_{K,1/2} \qquad (13)$$

The magnetic moment can thus be evaluated. Mottelson and Nilsson have compiled the calculated and experimental magnetic moments of odd-A deformed nuclei.[3] The discrepancies are appreciable and systematic. On grouping the nuclei according to the asymptotic quantum number Σ (projection of intrinsic spin) in order best to display the contributions of the intrinsic spin to the magnetic-moment calculations (i.e. $\langle \vec{s} \cdot \vec{I} \rangle$ is positive or negative accordingly as Σ is positive or negative), we note that the deviations of calculated values from experimental values are of the same sign within each group, and the sense of each deviation indicates that the effective g_s factors are reduced in magnitude from the free-space values.

The choice of reduced values to minimize deviations has been discussed in an earlier paper of Rasmussen and Chiao.[7] Taking the values $g_s=4$ for the proton and $g_s= -2.4$ for the neutron, we have recalculated the nuclear magnetic moments of the deformed odd-mass nuclei for which measured magnetic moments exist. The matrix elements $\langle \vec{s} \cdot \vec{I} \rangle$ are evaluated from the Mottelson and Nilsson wave functions at η=2, 4, and 6 and interpolated by a three-point Lagrangian interpolation to the assumed deformations, δ, which are obtained from measured electric-quadrupole transition probabilities.[3,8]

If we assume that the nuclear potential and charge distribution have the same shape, the values of δ may be estimated from the measured electric-quadrupole moment or the electric-quadrupole transition probability between two states in a rotational band. These quantities may be expressed in terms of the intrinsic quadrupole moment, Q_0, which is approximately

$$Q_0 = 4/5\ Z\ R_0^{\ 2}\ \delta\ (1+1/2\ \delta + \ldots.) \tag{14}$$

where $R_0 \cong 1.2 \times 10^{-13}\ A^{1/3}$ cm is the mean charge radius of the nucleus. Some have chosen to express the nuclear deformation by the parameter β connected with Q_0 by the relation

$$Q_0 = \frac{3}{\sqrt{5\pi}}\ Z\ R_0^{\ 2}\ \beta\ (1+ 0.16\ \beta)\ . \tag{15}$$

For reasons that will be discussed later, the g_R values are somewhat arbitrarily taken to be Z/A for odd-Z nuclei and Z/2A for odd-N nuclei. The uncertainty in the calculated value associated with the uncertainty in the assumed value of g_R is small. In Tables I and II, the calculated values are compared with empirical values. Both are also plotted against I in Figs. 6 and 7. The reference lines in these diagrams are the limiting "asymptotic" values.[5]

It is noted that among the cases showing deviations are spin-1/2 nuclei, which need a special analysis, and nuclei that have particularly large j values for the odd nucleon, thereby being especially susceptible to appreciable Coriolis mixing of different K quantum numbers. This mechanism and its effect on the magnetic moment are discussed. Apart from these cases, the agreement between the calculated and experimental values are generally good for odd-Z nuclei, showing that the effective g_s factor for a proton in most odd-Z nuclei is around 4. In the case of odd-N nuclei, the effective g_s factors seem to scatter more. A value more reduced in magnitude should be used for some nuclei, e.g. Er^{167} and Hf^{179}, while a value much less reduced should be used for U^{233} and U^{235}. (See Section VI-c.)

Table I. Magnetic moments of odd-Z nuclei

Nuclei	Energy of Isomeric state (kev)	$I_{exp.}$	Assigned orbital $(N,\eta_z,\Lambda,\Sigma)$	Assumed deformation	$\langle\vec{s}\cdot\vec{I}\rangle$	$g_R = \frac{Z}{A}$	μ_{theo} (n.m.)	$\mu_{exp.}$ (n.m.)	a values[a]
${}_{63}Eu^{153}$		5/2	413 ↓	0.29	-0.946	0.412	1.3	1.5[b]	
${}_{65}Tb^{159}$		3/2	411 ↑	0.29	0.609	0.409	1.9	±1.94[c,d]	
${}_{67}Ho^{165}$		7/2	523 ↑	0.28	1.405	0.406	4.0	4.18[c,d]	
${}_{69}Tm^{169}$		1/2	411 ↓	0.27	-0.125	0.408	-0.2	-0.21[b]	a=-0.76
${}_{69}Tm^{169}$	118	5/2	411 ↓	0.27	0.058	0.408	0.7	0.55±0.15[e]	a=0.76
${}_{71}Lu^{175}$		7/2	404 ↓	0.26	-1.565	0.406	2.0	2.0 ±0.2[b]	
${}_{71}Lu^{175}$	114	9/2	404 ↓	0.26	-1.565	0.406	2.5	2.25±0.9[f]	
${}_{73}Ta^{181}$		7/2	404 ↓	0.23	-1.552	0.403	2.0	2.34[g]	
${}_{73}Ta^{181}$	482	5/2	402 ↑	0.23	-1.202	0.403	3.1	3.0-3.3[h]	
${}_{75}Re^{185}$		5/2	402 ↑	0.19	1.198	0.405	3.1	3.14[h]	
${}_{75}Re^{187}$		5/2	402 ↑	0.19	1.198	0.401	3.1	3.18[h]	
${}_{77}Ir^{191}$		3/2	402 ↓	0.14	-0.622	0.403	0.4	0.2[h]	
${}_{77}Ir^{193}$		3/2	402 ↓	0.12	-0.613	0.399	0.4	0.2[h]	
${}_{89}Ac^{227}$		1/2	530 ↑	0.2	0.904	0.392	1.1	1.1[h]	a^+=-1.66
${}_{91}Pa^{233}$		3/2	530 ↑	0.3	1.000	0.391	2.1	2.1[j]	a=-1.31
${}_{93}Np^{237}$		5/2	642 ↑	0.25	0.734	0.392	2.7	>2.7[c,k]	
${}_{93}Np^{237}$	60	5/2	523 ↓	0.25	-0.805	0.392	1.35	2.0±0.5[h]	
${}_{95}Am^{241}$		5/2	523 ↓	0.27	-0.826	0.394	1.4	1.4[g]	
${}_{95}Am^{243}$		5/2	523 ↓	0.27	-0.826	0.391	1.4	1.4[f]	

a. Values of a involved in the magnetic-moment calculations (taken from ref.3) or the theoretical a^+ values calculated fron Nilsson's wave function.[6]

b. See reference 4.

c. These magnetic moments were measured by paramagnetic resonance and are subject to error arising from uncertainty in the values of $\langle 1/r^3 \rangle$,[9] which were calculated by the use of hydrogenic wave functions. The values listed have been corrected by using $\langle 1/r^3 \rangle$ values obtained from self-consistent field eigenfunctions.[10,11]

d. J. M. Baker and B. Bleaney, Proc. Roy. Soc. (London) 245, 156 (1958).

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k. C.A. Hutchison, Jr., and B. Weinstock, J.Chem. Phys. 32, 56 (1960).

Table II. Magnetic Moments of Odd Neutron Nuclei

Nuclei	Energy of Isomeric state (kev)	I_{exp}	Assigned orbital $(N, \eta_z, \Lambda, \Sigma)$	Assumed deformation	$\langle \vec{s} \cdot \vec{I} \rangle$	$g_R = \frac{Z}{2A}$	$\mu_{theo.}$ (n.m.)	$\mu_{exp.}$ (n.m.)	a values[a]
$_{64}Gd^{155}$		3/2	521 ↑	0.27	0.483	0.206	-0.37	-0.28[b]	
$_{64}Gd^{157}$		3/2	521 ↑	0.27	0.483	0.204	-0.37	-0.37[b]	
$_{66}Dy^{161}$		5/2	642 ↑	0.28	0.777	0.205	-0.39	-0.48[c,d]	
$_{66}Dy^{163}$		5/2	523 ↓	0.28	-0.757	0.202	0.67	0.67[c,d]	
$_{68}Er^{167}$		7/2	633 ↑	0.29	1.302	0.204	-0.54	-0.59[c,e]	
$_{70}Yb^{171}$		1/2	521 ↓	0.28	-0.250	0.205	0.40	0.46[c,f]	a=0.87
$_{70}Yb^{173}$		5/2	512 ↑	0.27	1.008	0.202	-0.55	-0.65[f]	
$_{72}Hf^{177}$		7/2	514 ↓	0.25	-1.162	0.203	0.80	0.61[b]	
$_{72}Hf^{177}$	114	9/2	514 ↓	0.25	-1.162	0.203	0.97	0.99±0.27[g]	
$_{72}Hf^{179}$		9/2	624 ↑	0.25	-1.812	0.201	-0.63	-0.47[b]	
$_{74}W^{183}$		1/2	510 ↑	0.21	-0.250	0.202	0.48	0.12[h]	a=0.19
$_{76}Os^{187}$		1/2	510 ↑	0.18	-0.250	0.203	0.48	0.12[h]	a=0.19
$_{76}Os^{189}$		3/2	512 ↓	0.15	-0.423	0.201	0.50	0.65[h]	
$_{92}U^{233}$		5/2	633 ↓	0.23	-0.408	0.197	0.42	0.97[c,j]	
$_{92}U^{235}$		7/2	743 ↑	0.24	1.100	0.196	-0.43	-0.66±0.07[c,j]	
$_{94}Pu^{239}$		1/2	631 ↓	0.26	0.119	0.197	-0.06	±0.02[k]	$a^{+}=-0.96$
$_{94}Pu^{241}$		5/2	622 ↑	0.27	0.783	0.195	-0.40	>|0.1|[ℓ]	

a. See footnote a, Table I.
b. See footnote b, Table I.
c. See footnote c, Table I.
d. J.G. Park, Proc. Roy. Soc. (London) A245, 118 (1958).
e. R.J. Elliott and K.W.H. Stevens, Proc. Roy. Soc. (London) 219A, 387 (1953).
f. A.H. Cooke and J.G. Park, Proc. Phys. Soc. (London) A69, 282 (1956).
g. See footnote f, Table I.
h. See footnote h, Table I.
j. P.B. Dorain, C.A. Hutchison, Jr., and E. Wong, Phys. Rev. 105, 1307 (1957).
k. See reference 11.
ℓ. B. Bleaney, P.M. Llewellyn, M.H.L. Pryce, and G.R. Hall, Phil. Mag. 45, 991 (1954).

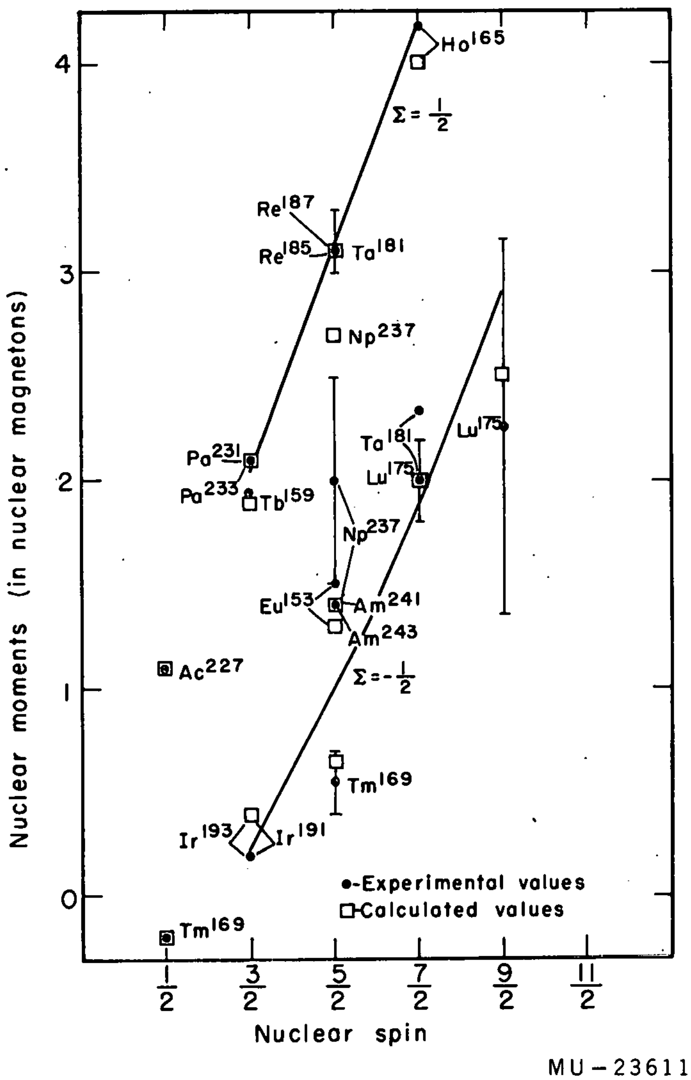

Fig. 6. Magnetic moments of odd-proton nuclei.

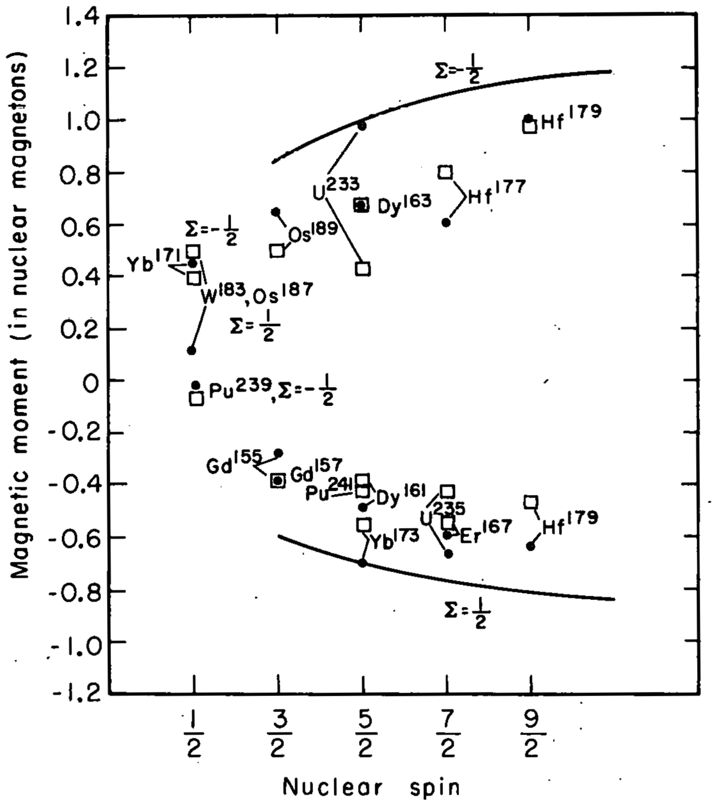

Fig. 7. Magnetic moments of odd-neutron nuclei.

We further extend such calculations to odd-odd nuclei. The wave functions of such nuclei can be written as products of the Nilsson wave functions for the last proton and for the last neutron:

$$\psi = \left(\frac{2\pi+1}{16\pi^2}\right)^{1/2} \left[\chi_{\Omega_p} \chi_{\pm\Omega_n} \mathcal{D}^I_{MK} + (-)^{I-j_p-j_n} \chi_{-\Omega_p} \chi_{\pm\Omega_n} \mathcal{D}^I_{M-K}\right] \tag{16}$$

where the sign of Ω_n is determined from the coupling of the angular momenta of the proton and the neutron. To calculate the magnetic moments calculations of these nuclei, the following expression for the magnetic moment is used:

$$\mu = \frac{1}{I+1} \left\{ K \left[g_s \langle s_z \rangle + g_\ell \langle \ell_z \rangle\right]_p + K\left[g_s \langle s_z \rangle\right]_n + \left[I(I+1) - K\Omega\right] g_R \right\} . \tag{17}$$

Most ground states of odd-odd nuclei obey the Gallagher-Moszkowski coupling rule that the intrinsic spins of the proton and neutron are parallel.[12,13] In this state, the proposed reduction of g_s for the proton and neutron will nearly cancel. Nuclear states with proton and neutron spins antiparallel to each other, such as the 6+ state of Np^{236} and 5- state of Am^{242}, should show magnetic moments shifted from values calculated with free space g_s factors. Not many magnetic moments have been measured for odd-odd nuclei. In Table III we have listed cases for which measurements would be possible with those for which magnetic moments are known. The values of μ^* and μ calculated with free space and reduced g_s factors are very close to each other and also close to the asymptotic values.

Table III. Magnetic moments of odd-odd nuclei

Nucleus	Spin	Configuration[a]		$\langle s_z \rangle_p$	$\langle \ell_z \rangle_p$	$\langle s_z \rangle_n$	$g_R = \frac{3}{4}\frac{Z}{A}$	μ^*_{Theo} (n.m.)	μ_{Theo} (n.m.)	$\mu_{asym.}$ (n.m.)	μ_{exp}
		Proton	Neutron								
$_{63}Eu^{152}$	3-	[411↑	+521↑]	0.400	1.100	0.296	0.311	1.88	1.73	1.58	1.9[b,c]
$_{63}Eu^{154}$	3-	[411↑	+521↑]	0.400	1.100	0.296	0.307	1.88	1.72	1.58	2.0[b,c]
$_{65}Tb^{160}$	3-	[411↑	+521↑]	0.400	1.100	0.296	0.305	1.88	1.72	1.58	1.60±0.25[d]
$_{67}Ho^{162}$	6-	[523↑	+642↑]	0.398	3.102	0.304	0.310	3.57	3.66	3.52	
$_{67}Ho^{166}$	7-	[523↑	+633↑]	0.398	3.102	0.366	0.303	3.70	3.60	3.59	3.3±0.5[e]
$_{69}Tm^{170}$	1-	[411↓	+521↓]	-0.356	0.856	-0.263	0.304	0.09	0.13	0.25	0.24[f]
$_{71}Lu^{176}$	7-	[404↓	+514↓]	-0.449	3.949	-0.362	0.334	2.77	2.94	3.09	3.27[g]
$_{71}Lu^{176m}$	1+	[404↓	-624↑]	-0.449	3.949	-0.417	0.334	1.74	1.74	1.77	
$_{73}Ta^{180m}$	1+	[404↓	-624↑]	-0.449	3.949	-0.417	0.304	1.72	1.73	1.75	
$_{93}Np^{236}$	1+	[523↑	-743↓]	-0.312	2.812	-0.326	0.295	1.31	1.32	1.60	
$_{93}Np^{236}$	6+	[523↑	-743↓]	-0.312	2.812	0.326	0.295	0.10	0.70	0.08	
$_{93}Np^{238}$	2+	[642↑	-631↓]	0.283	2.217	0.150	0.293	2.18	2.05	2.06	
$_{95}Am^{242}$	1-	[523↓	-622↑]	-0.312	2.812	-0.792	0.294	0.29	0.29	0.29	0.33[h]
$_{95}Am^{242}$	5-	[523↓	+622↑]	-0.312	2.812	0.792	0.294	-1.39	-0.04	0.08	

a. See references 12 and 13.
b. See footnote c, Table I.
c. M. Abraham, R. Kedzie, and C. D. Jeffries, Phys. Rev. 108, 58 (1957).
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e. H. Postma, A. R. Miedema, and M. C. Eversdizk Smulders, Physica 25, 671 (1959).
f. A. Y. Cabezas and I. Lindgren, Phys. Rev. 120, 920 (1960).
g. See footnote h, Table I.
h. See footnote j, Table I.

IV. INTERCONFIGURATIONAL MIXING

Our relatively good success in calculating magnetic moments with empirically reduced g_s factors suggests that the apparent moment quenching throughout the deformed region arises from small admixtures of many different configurations. It is most likely that the quenching of the g_s factors is not primarily caused by alterations of the virtual meson contribution to the nucleonic magnetic moment for nucleons in nuclear matter. Drell and Walecka show this effect to be about 7%.[14]

In calculating the single-particle wave functions, a spheroidal potential has been used to represent the potential due to interactions of the very-short-range forces and other exchange forces. It is therefore expected that the different nucleon configurations should inevitably get mixed with each other to a considerable extent in the stationary states of nuclei. Their modifications of magnetic moments can be discussed in terms of the variation principle in the limit imposed by the exclusion principle. In zero-order approximation, the core of paired nucleons does not modify the magnetic moment. The interactions between nucleons may be considered to consist of the very-short-range Wigner force and exchange forces whose potential operator when applied to nuclear wave functions "exchanges" either the space coordinates of interacting nucleons (Majorana force) or their spin coordinates (Bartlett force) or both (Heisenberg force). These scalar forces between nucleons within the core can not polarize the core itself, but the core can be polarized by its interaction with the unpaired nucleon.

The paired nucleons in the odd group are polarized antiparallel to the odd nucleon so that they can approach each other, thus increasing the interaction energy of the very-short-range forces. The spin-dependent forces give rise to an opposite but smaller polarization effect. Paired nucleons of the even group are polarized parallel by these spin-dependent forces, while the spin-independent forces have no effect. The polarizations of both groups decrease the magnitude of the magnetic moment in the same way, and result in the quenching of the g_s factors used in our calculations with these zero-order single-particle wave functions.

Arima and Horie[15] as well as Blin-Stoyle[16] have calculated the magnetic-moments for spherical nuclei, showing that a small amount of mixing of certain kinds of configurations can produce changes in magnetic moments of nuclei without changing appreciably the pure shell-model configurations upon which they base their calculations. The nuclear wave function of mixed configurations can be written as $|(j_1^{n})_0 \; j \; jm \rangle + \sum_J \alpha_J \; |(j_1^{n-1}) \; j_1 \; (j_2 j)_J jm \rangle$ where $|(j_1^{n})_0 j \; jm \rangle$ represents a simple shell-model configuration and the $|(j_1^{n-1}) \; j_1 \; (j_2 j)_J jm \rangle$ represent the admixed configurations. The magnetic moment of the nucleus is then obtained from this wave function by calculating the expectation value of the particle magnetic-moment operator. For small values of α_J, the most important contributions to the magnetic moment are those linear in α_J. Contributions of this kind can occur only if $|(j_1^{n})_0 j \; jm \rangle$ and $|(j_1^{n-1}) j_1 (j_2 j)_J jm \rangle$ differ at most by one single state and the orbital state is the same. Thus the most important configurations for mixing are those in which a nucleon pair is broken by promotion of a nucleon to the unoccupied orbital of its spin-orbit conjugate. Using simple perturbation theory, one can write

$$\alpha_J = - \langle (j_1^{n})_0 j \; jm \; | V \; | (j_1^{n-1}) j_1 (j_2 j)_J jm \rangle / \Delta E_J \tag{18}$$

where V is the internucleonic interaction, and ΔE_J is the zeroth-order energy difference between the mixed states. Using a delta-function interaction, Arima and Horie as well as Blin-Stoyle have been able to account for the deviations of magnetic moments from the Schmidt limits.

The important point is that nuclear magnetic moments are extremely sensitive to admixtures to the zero-order shell-model wave function which contribute in first order to the magnetic moment. For example, a 5% admixture of the $h_{11/2}$ state into the $h_{9/2}$ changes the Bi^{209} moment by 1.4 n.m. In a few cases, however, the distribution of nucleons among the various shell-model states is such that no type of admixture can lead to a contribution to the magnetic moment which is linear in its amplitude of mixing. Under these circumstances the magnetic moment should lie close to the Schmidt value. The proton configuration in ${}^{59}_{82}Pr^{141}$ is $(g_{7/2}{}^{6})$, $(d_{5/2}{}^{3})$, which does not offer much possibility for configuration mixing. The experimental magnetic moment has been recorrected to the value 5.1 n.m.,[17] which is very near the upper Schmidt limit of 4.8 n.m. appropriate to a proton.

Throughout the deformed region there are always many ways of forming the broken pairs just mentioned. Gauvin has made similar magnetic-moment calculations for deformed nuclei.[18] The two-nucleon potential is assumed to be

$$V_{ij} = - V_0 \delta(r_i - r_j) \, (0.8 + 0.4 \, P_\sigma) \qquad (19)$$

where P_σ is the exchange operator for the nucleon spins, and V_0 is a positive constant. To first order, only those admixtures which differ from the original state by one particle excitation contribute to the magnetic moment. Furthermore, such single-particle excitation can go only to a state which has the same parity and the same value of the quantum number, Ω, as the state that is left vacant. For each such possible excitation there is a corresponding one between the same states having the opposite sign of Ω. If we denote the admixture amplitudes for such a pair of excitations by α and β, for states with positive and negative Ω, respectively, the contribution to the magnetic moment from this pair is

$$\Delta\mu = \frac{I}{I+1} \quad 2(\alpha-\beta) \, \langle i | \, \mu_z \, | f \rangle \, . \qquad (20)$$

One can evaluate α-β by using perturbation theory and Nilsson's wave functions. The foregoing formalism has been applied to the ground-state configurations of Lu^{175} and Ta^{181}; both have an unpaired proton in the $g_j = 7/2, \Omega = 7/2$ level. The corrected values for the particle magnetic moments are 2.17 n.m. for Lu^{175}, and 2.38 n.m., 2.24 n.m., or 2.29 n.m. for Ta^{181}, depending on the actual proton configurations of Ta^{181}. These values correspond to $g_s = 2.57$ and $g_s = 2.01$, 2.44, or 2.31, while the experimental g_s values are 3.74 and 2.90 from g_K values given by Bernstein and de Boer.[4] It seems that the results could be improved if more mixing configurations were considered by allowing excitations to take place between states with different quantum numbers Ω.

The very-short-range spin-independent forces are much larger than the spin-dependent forces and correspond to the most important feature of the residual interparticle forces not incorporated into the static self-consistent nuclear field used for single-particle calculations. These very-short-range components of interparticle forces have been treated as pair-correlation added to the nuclear field, and the total Hamiltonian has been solved exactly.[19] Therefore, an alternative way to calculate the particle magnetic-moment correction, $\Delta\mu$, is to use these wave functions as zero-order and consider the rest of the forces as perturbation. However, it is doubtful that the corrections obtained would differ from the present results.

The effect of both pair-correlation and mixing of the quantum number on the magnetic moment will be further investigated in the following sections, especially with regards to their effects on the collective gyromagnetic ratio, g_R.

V. PAIR-CORRELATION

A. Introduction

Many have considered the effect of pair correlations on nuclear properties.[19,21] It can explain the wide energy gap encountered in the collective spectra of deformed even-even nuclei.

Expressed in terms of the single-particle states of the average nuclear potential, the pair-correlation interaction scatters pairs of particles from the originally filled, doubly degenerate single-particle orbitals into the higher-lying levels which are left unoccupied. The new total intrinsic wave function that most effectively utilizes this additional type of interaction and forms the ground state is then a state with a diffuse Fermi surface. Any excited state is therefore associated with an excitation energy of about at least the width of the diffuseness of the Fermi surface.

Let the Hamiltonian of the static self-consistent nuclear field be denoted H_s and the corresponding single-particle states $|\nu\rangle$, where ν represents the quantum numbers. One then defines the particle-creation operators by

$$a_\nu^+ |0\rangle = |\nu\rangle \qquad (21)$$

$$a_{-\nu}^+|0\rangle = |-\nu\rangle = T|\nu\rangle, \qquad (22)$$

where $|-\nu\rangle$ is the conjugate state and is equaled to the time-reversed state $T|\nu\rangle$ by arbitrarily fixing the phase of T.

The total Hamiltonian then has the form

$$H' = \sum_\nu \varepsilon_\nu (a_\nu^+ a_\nu + a_{-\nu}^+ a_{-\nu}) - G \sum_{\nu\nu'} a_{\nu'}^+ a_{-\nu'}^+ a_{-\nu} a_\nu , \qquad (23)$$

where ε_ν denotes the eigenvalues of H_s, and the second term represents the pair-correlation interaction with the limiting assumption that the residual force acts only when two particles move in a J=0 state. Said force displays the main features of the δ force, although the latter has minor but nonnegligible effects on pairs of particles in states of nonvanishing but small angular momentum. The a_ν's obey the anticommutation relations to ensure the antisymmetry of the many-particle states.

The Hamiltonian (23) describes a system with a fixed number of particles, N. Bardeen et al. find an eigenfunction of this system by transforming to a system that is described by the Hamiltonian

$$H = H' - \lambda N, \tag{24}$$

where λ, the chemical potential, is treated as a Lagrangian multiplier,[22] thus represents the energy of the last added particle. The choice of λ determines the average value of N, which we shall denote by η.

The ground state can be expressed as an admixture of single-particle configurations with the admixture coefficients determined by the parameters U_ν and V_ν, subject to the normalization condition

$$U_\nu^2 + V_\nu^2 = 1 \tag{25}$$

and to the auxiliary condition

$$2 \sum_\nu V_\nu^2 = \eta . \tag{26}$$

A convenient way to obtain the ground-state energy and the corresponding wave function is to determine the so-called quasi-particle operators defined by

$$\alpha_\nu = U_\nu a_\nu - V_\nu a_{-\nu}^{+} \tag{27a}$$

and

$$\beta_{\alpha-\nu} \equiv \beta_\nu = U_\nu \;_{-\nu} + V_\nu \;_{\nu}^{+} . \tag{27b}$$

In terms of α_ν and β_ν, the transformed Hamiltonian is

$$H = U + H_{11} + H_{20} + H_{int.}, \tag{28}$$

where U is independent of the quasi-particle, H_{20} creates or annihilates two quasi-particles, $H_{int.}$ contains products of four quasi-particle creation or annihilation operators, and H_{11} is an operator of the single quasi-particle type. When H_{20} is set identically equal to zero, the

Hamiltonian describes a system of noninteracting quasi-particles in the approximation that $H_{int.}$ may be neglected. We are led to the equations:

$$U_\nu^{\ 2} = 1/2 \left(1 + \frac{\mathcal{E}_\nu - \lambda}{E_\nu}\right) \tag{29}$$

$$V_\nu^{\ 2} = 1/2 \ (1 - \left(\frac{\mathcal{E}_\nu - \lambda}{E_\nu}\right), \tag{30}$$

where

$$E_\nu = \left[(\mathcal{E}_\nu - \lambda)^2 + \Delta^2\right]^{1/2} \tag{31}$$

and

$$\Delta = G \sum_\nu U_\nu V_\nu \ . \tag{32}$$

The parameters λ and Δ can be determined from Eqs. (26) and (31). The interpretation of $V_\nu^{\ 2}$ as the probability of the state being populated by a pair is evident from Eq. (26). The wave functions are the quasi-particle creation operators operating on the quasi-particle vacuum, the state where all single particles are coupled in pairs to zero angular momentum. The ground state of an even-even nucleus is the quasi-vacuum state, which in terms of the single-particle model is

$$\psi_0 = \prod_\nu (U_\nu + V_\nu a_\nu^{\ +} a_{-\nu}^{\ +} | 0 \rangle . \tag{33}$$

For the ground state of an odd-A nucleus, the odd particle occupies, say, the orbital $\mathcal{E}_\nu{}'$. The particle is entirely unaffected by the pairing force, which only scatters pairs of particles. The trial function of the ground state of such an odd-A nucleus is

$$\psi = a_{\nu'}^{+} \prod_{\nu \neq \nu'} (U_\nu + V_\nu a_\nu^{\ +} a_{-\nu}^{\ +}) |0\rangle \ . \tag{34}$$

Now U_ν and V_ν are still given by (29) and (30), but the sums over states in determining λ and Δ exclude the "blocked" ν' state.

For the ground state of odd A, ν' lies near the Fermi surface and λ is not appreciably changed with respect to the "even" case of n/2 pairs. Therefore the wave functions are the quasi-particle creation operators operating on the quasi-particle vacuum.

B. Effect of Pair-Correlation on the Magnetic Moment

Consider the magnetic moment of an odd-A nucleus. The quasi-particle operator may be represented by a sum of single-particle operators.

$$\mu^{op.} = \sum_{\nu \nu'} \left(\langle \nu|\mu_z|\nu'\rangle a_\nu^+ a_{\nu'} + \langle -\nu|\mu_z|-\nu'\rangle a_{-\nu}^+ a_{-\nu'} \right). \qquad (35)$$

By the canonical transformations (27ab) and the phase convention (22), it is

$$\mu^{op.} = \sum_{\nu\nu'} \langle \nu|\mu_z|\nu'\rangle (U_\nu U_{\nu'} + V_\nu V_{\nu'})(\alpha_\nu^+ \alpha_{\nu'} - \beta_{\nu'}^+ \beta_\nu) + \sum_{\nu\nu'} \langle \nu|\mu_z|\nu'\rangle (U_\nu V_{\nu'} - V_\nu U_{\nu'}) (\alpha_\nu^+ \beta_{\nu'}^+ - \beta_\nu \alpha_{\nu'}). \qquad (36)$$

The first term corresponds to the magnetic moment of a quasi-particle and is the same as that of a single-particle. This is understood, since particles and corresponding holes have the same magnetic moments. The second term contains the two quasi-particle operators. It can therefore have no matrix elements with the ground state of an odd-A nucleus. Thus, with pairing interaction, the odd-A nuclei will exhibit single-particle values for their particle magnetic moments.

The effect of pairing interaction on the collective magnetic moment, $\vec{\mu}_{coll.} = g_R \vec{R}$, has been well investigated by Nilsson and Prior.[5] In the single-particle model, with the assumption that the self-consistent field determining the single-particle orbitals is cranked around externally, the collective gyromagnetic ratio of a nucleus is given by

$$g_R = \frac{\hbar^2}{\mathfrak{I}} \sum_f \frac{\langle i|\mu_x|f\rangle\langle f|J_x|i\rangle}{\mathcal{E}_f - \mathcal{E}_i} + \text{complex conjugate} \qquad (37)$$

and

$$\mathfrak{I} = \frac{\hbar^2}{2\mathfrak{I}} \sum_f \frac{|\langle i|J_x|f\rangle|^2}{\mathcal{E}_f - \mathcal{E}_i}, \qquad (38)$$

where $J_x = \sum_i j_x$ is the angular-momentum operator associated with the rotation and $\mu_x = \sum_i (g_\ell \ell_x + g_s S_x)$ is the collective magnetic-moment operator.

If we use asymptotic wave functions, these formulas give g_R values rather close to the ratio Z/A which show little variation,[23] while the experimental values are generally less than the ratio Z/A (Table IV) and vary as shown in Fig. 8. The low experimental values must signify a greater role of neutrons relative to protons in carrying the rotational angular momentum. The trend of variation is reversed when the holes play the parts of particles near the end of the shells, showing that the nucleons outside of closed shells are the main contributors to the rotational angular momentum.

Inclusion of pairing interaction gives

$$g_R = \frac{\Im_p}{\Im} + (g_s^{\,p} - 1)\,\frac{W_p}{\Im} + g_s^{\,n}\,\frac{W_n}{\Im} \tag{39}$$

where

$$\frac{1}{2\hbar^2}\, W = \sum_{\nu\nu'} \frac{\langle \nu' | j_x | \nu \rangle \langle \nu | s_x | \nu' \rangle}{E_\nu + E_{\nu'}} \left(U_\nu V_{\nu'} - V_\nu U_{\nu'}\right)^2 \tag{40}$$

and

$$\Im = 2\hbar^2 \sum_{\nu\nu'} \frac{|\langle \nu | j_x | \nu' \rangle|^2}{E_\nu + E_{\nu'}} \left(U_\nu V_{\nu'} - V_\nu U_{\nu'}\right)^2 . \tag{41}$$

Since the last two terms in Eq. (39) are small, g_R is approximately equal to the relative fraction contributed to the moment of inertia. Using a considerably larger pairing energy for protons than for neutrons, thus increasing the share of angular momentum carried by the neutrons, one gets g_R values smaller than the ratio Z/A. Nilsson and Prior have calculated g_R for even-even nuclei.[5] Their results are found to be in reasonable agreement with the experimental values.

The empirical g_R values for odd-A nuclei listed in Table IV are taken from the analysis made by Bernstein and de Boer.[4] The g_R values show the same trend of variation as those for even-even nuclei (Fig. 8). The g_R values for odd-Z nuclei are somewhat higher than for neighboring even-even nuclei, and the g_R values for odd-N nuclei, somewhat lower. A rationale for such a g_R shift may be provided by considerations of pair-correlation effects due to the blocking of an orbital and effects of the Coriolis force which couples the near-lying one-quasi-particle states.

If an odd neutron is added to an even-even nucleus, one orbital is blocked from the neutron-pair correlation, thereby reducing the effective energy gap for neutrons. This reduces g_R. The same argument gives an increased g_R for odd-Z nuclei. The effect of Coriolis force on magnetic moments will be discussed in detail in terms of single-particle states.

Table IV. Collective gyromagnetic ratios of deformed nuclei.

Odd-Z nuclei		Even-even nuclei		Odd-A nuclei	
Isotope	g_R [a]	Isotope	g_R	Isotope	g_R [a]
$_{63}Eu^{153}$	0.452±0.012	$_{62}Sm^{152}$	0.20±0.04 [b,c] 0.21±0.04 [d]	$_{64}Gd^{155}$	0.34 ±0.07
$_{65}Tb^{159}$	0.51 ±0.04 [b]	$_{62}Sm^{154}$	0.31±0.03 [e] 0.21±0.04 [d] 0.295±0.03 [e]	$_{64}Gd^{157}$	0.22 ±0.06
$_{67}Ho^{165}$	0.554±0.047 [b]	$_{64}Gd^{154}$	0.36±0.06 [f]	$_{66}Dy^{161}$	0.210±0.072 [b]
$_{69}Tm^{169}$	0.23 ±0.12	$_{60}Dy^{160}$	0.20±0.04 [b,c]	$_{66}Dy^{163}$	0.302±0.018
$_{71}Lu^{175}$	0.30 ±0.06	$_{68}Er^{166}$	0.322±0.024 [b,g]	$_{68}Er^{167}$	0.098±0.035 [b]
$_{73}Ta^{181}$	0.327±0.009	$_{72}Hf^{180}$	0.20±0.15 [e]	$_{70}Yb^{173}$	0.20 ±0.09
$_{75}Re^{185}$	0.413±0.042	$_{74}W^{184}$	0.38±0.05 [h]	$_{72}Hf^{177}$	0.215±0.014
$_{75}Re^{187}$	0.413±0.043			$_{72}Hf^{179}$	0.203±0.034

a. See reference 4.

b. These values are corrected using Judd and Lindgren's values of $\langle 1/r^3 \rangle$.[10]

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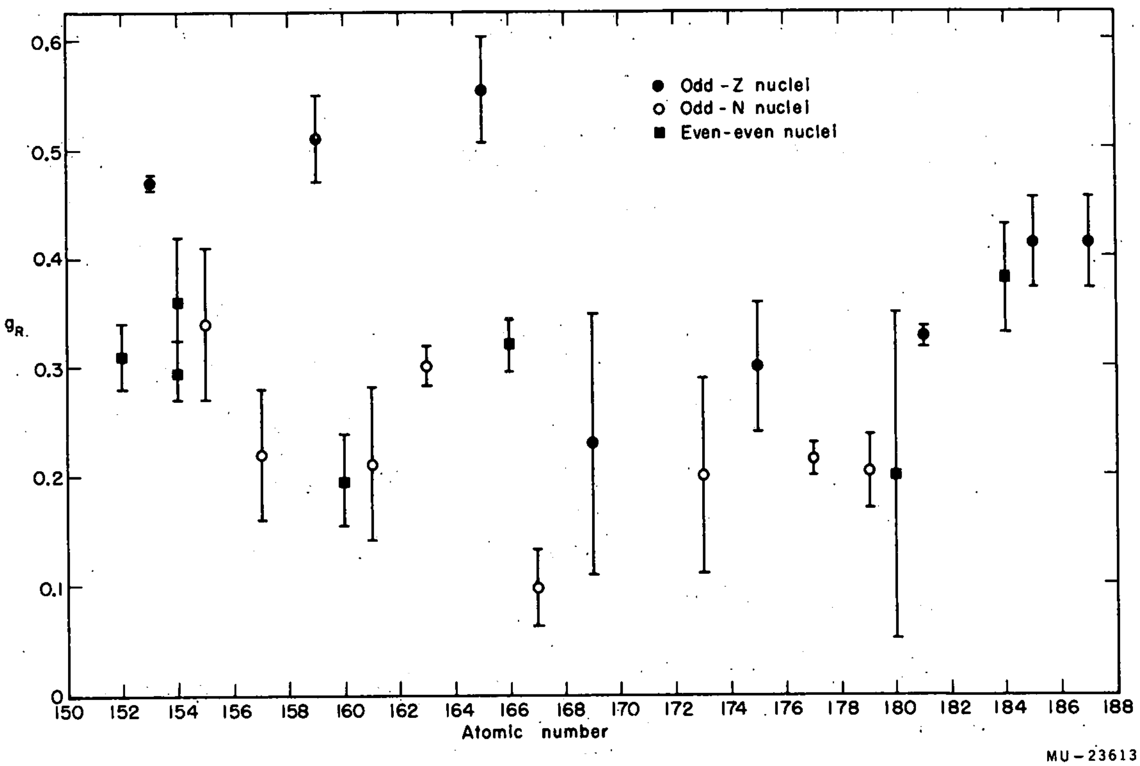

Fig. 8. The collective gyromagnetic ratios of deformed nuclei.

VI. PARTICLE-ROTATION INTERACTION

A. Introduction

The particle-rotation interaction arises from the Coriolis force which comes into play when a particle is moving in a rotating system. The nature of this interaction can be understood best by considering a model simpler than the actual nuclei, such as the system of a single particle coupled to a rigid top. The Hamiltonian of this system is

$$\mathcal{H} = \frac{\hbar^2}{2m} \Delta^2(\vec{r}) + V(\vec{r}) + \sum_{i=1}^{3} \frac{\hbar^2}{2\Im_i} \langle \vec{R}_i^{\,2} \rangle \tag{42}$$

where $\vec{r}$ is the position vector of the particle in the rotating system of coordinates corresponding to the instantaneous position of the principal axes of the top, $\Im_i$ and R_i are the principal moments of inertia and angular momenta of the top, respectively, and m is the reduced mass of the system. For the analogy with actual nuclei, we assume that both the top and the potential are axially symmetrical, that is, $\Im_1 = \Im_2 = \Im$. The top angular momentum $\vec{R}$ is coupled with Ω, the component of the particle momentum on the symmetry axis to form the angular momentum I with the component K on the symmetry axis.

The term $\langle \vec{R}^2 \rangle$ can then be written as

$$\begin{aligned} \langle \vec{R}^2 \rangle &= \langle (\vec{I}-\vec{j})^2 \rangle \\ &= I(I+1)+j(j+1)-2\langle \vec{I}\cdot\vec{j} \rangle \\ &= I(I+1)+j(j+1)-2K\Omega - 2I_1 j_1 - 2I_2 j_2 \,. \end{aligned} \tag{43}$$

We can also rewrite that part of the Hamiltonian in the form

$$\sum_{i=1}^{3} \frac{\hbar^2}{2\Im_i} \langle \vec{R}_i^{\,2} \rangle = \frac{\hbar^2}{2\Im_3} (K - \Omega)^2 + \frac{\hbar^2}{2\Im} \Big[I(I+1)+j(j+1)-K^2-\Omega^2- (I_- j_+ + I_+ j_-) \Big] \tag{44}$$

where $I_\pm = I_1 \pm i\, I_2$ and $j_\pm = j_1 \pm i\, j_2$. The last term $-\frac{\hbar^2}{2\Im_1} (I_+ j_- + I_- j_+)$ corresponds to the effect of the Coriolis force acting between the particle and the collective rotation. When this effect is neglected, it is clear that K and Ω will be good quantum numbers. The Coriolis operator makes no diagonal contribution unless Ω is 1/2, in which case there is a diagonal contribution.

$$\langle \Omega=1/2 | I_+ j_- + I_- j_+ | \Omega = 1/2 \rangle = (-)^{I+j}(j+1/2)(I+1/2) \qquad (45)$$

known as the "decoupling term" to the rotational spectrum.

Rotational spectra of the simple form given in Eq. (1) are obtained when the rotation is so slow that the particle motion can adjust adiabatically to the changing orientation of the potential field. The finite rotational frequency gives rise to small nonadiabatic excitations resulting from the Coriolis force acting on the particle. To lowest order in the rotational frequency, these virtual excitations imply an increase in the energy of the nucleus proportional to $I(I+1)$ and thus provide the moment of inertia associated with the rotational motion. To higher order, the nonadiabatic effects give rise to a coupling between the rotational and intrinsic motion, which implies deviation from the rotational spectrum as well as the geometrical relations for the nuclear moments.

In odd-A nuclei, the nonadiabatic excitations of the last odd nucleon will play a special role, since these do not involve the breaking of any pairs. Prior has correlated the increase in the moment of inertia in going from an even-even nucleus to the next heavier odd-mass nucleus with the effect of the Coriolis force on the odd nucleon.[24] In addition to the increase in the moment of inertia, the band may show an $I^2(I+1)^2$ energy term, which corresponds to relaxing the axial symmetry of the model we have been discussing. Small deviations from the $I(I+1)$ dependence may rise from the higher-order effects of the near-lying bands associated with the lowest states of the last odd particle. Such effects have been noted, and detailed analyses have been made for the spectra of Pa^{231}, Pa^{233}, and W^{183}.[25,26] Even when the deviations from the simple form are small, nuclear properties such as the electromagnetic transition probabilities may be appreciably affected. Kerman has given formulas for the electromagnetic transition probabilities between admixed states and related transition probabilities of W^{183} quantitatively to the observed energy perturbations.[26]

We will investigate the effect of particle-rotation interaction on magnetic moments and magnetic parameters g_R and g_K by treating this interaction as a perturbation. The matrix elements of the Coriolis-force operator are

$$H^I_{K',K} = H^I_{K,K'} = -\frac{\hbar^2}{2\Im}\langle I'K'\Omega'|I_+j_- + I_-j_+|IK\Omega\rangle$$
$$= -\frac{\hbar^2}{2\Im}\left[I(I+1) - K(K+1)\right]^{1/2}\delta_{K',K+1}\langle\Omega'|j_+|\Omega\rangle$$
$$-\frac{\hbar^2}{2\Im}\left[I(I+1) - K'(K+1)\right]^{1/2}\delta_{K',K+1}\langle\Omega'|j_-|\Omega\rangle \qquad (46)$$

By decomposing the operators j_+ and j_- as $j_\pm = l_\pm + s_\pm$, the matrix elements $\langle\Omega'|j_\pm|\Omega\rangle$ can be obtained by using Nilsson's wave function. They are

$$\langle\Omega'|j_-|\Omega\rangle = \sum_\ell \Big\{a_{\ell\Omega'-1/2}\,a_{\ell\Omega-1/2}\left[\ell(\ell+1) - (\Omega'-1/2)(\Omega'+1/2)\right]^{1/2}$$
$$+ a_{\ell\Omega'+1/2}\,a_{\ell\Omega+1/2}\left[\ell(\ell+1) - (\Omega'+1/2)(\Omega'+3/2)\right]^{1/2}$$
$$+ a_{\ell\Omega'+1/2}\,a_{\ell\Omega-1/2}\Big\} \qquad (47)$$

for $\Omega' = \Omega-1$, and

$$\langle\Omega'|j_+|\Omega\rangle = \sum_\ell \Big\{a_{\ell\Omega'-1/2}\,a_{\ell\Omega-1/2}\left[\ell(\ell+1) - (\Omega-1/2)(\Omega+1/2)\right]^{1/2}$$
$$+ a_{\ell\Omega'+1/2}\,a_{\ell\Omega+1/2}\left[\ell(\ell+1) - (\Omega+1/2)(\Omega+3/2)\right]^{1/2}$$
$$+ a_{\ell\Omega'-1/2}\,a_{\ell\Omega'+1/2}\Big\} \qquad (48)$$

for $\Omega'=\Omega+1$. Thus the Coriolis interaction couples states which have the same spin and parity but differ by one in K and Ω. The strength of the interaction increases with increasing j and increasing I.

In the case $K = \Omega = 1/2$, it is possible for the operators j+ and j- to connect states (with the same parity), both of which have $K = \Omega = 1/2$. The matrix is given by

$$\langle 1/2|j_-I_+ + j_+I_-|1/2\rangle = (-)^{I-j}(I+1/2)\sum_\ell\Big\{a_{\ell 0}a_{\ell 0} + \sqrt{\ell(\ell+1)}$$
$$(a_{\ell 0}'a_{\ell 1} + a_{\ell 1}'a_{\ell 0})\Big\}. \qquad (49)$$

A secular equation can then be written for levels of spin I. If we express it in determinantal form, the diagonal elements are $H_{KK} - W$, where H_{KK} represents the eigenvalues of interacting intrinsic states. The first off-diagonal elements are given by Eq. (46); all other off-diagonal elements are zero. For example, the secular equation for 1 = 7/2 and j $\geq$ 7/2 could be written as

$$\begin{vmatrix} H_{1/2,1/2} - E & H_{1/2,3/2} & 0 & 0 \\ H_{1/2,3/2} & H_{3/2,3/2}-E & H_{3/2,5/2} & 0 \\ 0 & H_{3/2,5/2} & H_{5/2,5/2}-E & H_{5/2,7/2} \\ 0 & 0 & H_{5/2,7/2} & H_{9/2,7/2}-E \end{vmatrix} = 0. \quad (50)$$

The roots of the secular equation give the energies of the perturbed level I. In order to find the eigenfunctions corresponding to root E, we substitute the value of E in the set of equations

$$(H_{1/2,1/2} - E)a_{1/2} + H_{1/2,3/2}a_{3/2} = 0 \quad (51)$$

and

$$H_{1/2,3/2}a_{1/2} + (H_{3/2,3/2} - E)a_{3/2} + H_{3/2,5/2}a_{5/2} = 0 \quad (52)$$

and solve for the ratios $a_{3/2}/a_{1/2}$, $a_{5/2}/a_{1/2}$, $a_{n/2}/a_{1/2}$, where n is the final numerator in the series. A knowledge of these ratios, plus the normalization condition

$$a_{1/2}^{*}a_{1/2} + a_{3/2}^{*}a_{3/2} + \dots = 1 \quad (53)$$

determines the a's. The wave function of these mixed levels may be written in the form

$$\psi_{mixed} = a_{1/2}|7/2,1/2\rangle + a_{3/2}|7/2,3/2\rangle + a_{5/2}|7/2,5/2\rangle + a_{7/2}|7/2,7/2\rangle \quad (54)$$

where the wave function of the pure level I in the rotational band K is written $|IK\rangle$.

B. Effect of Particle-Rotation Interaction on the Magnetic Moment and the Electromagnetic Transition Probability

The effect of this K admixture on the magnetic moment comes from off-diagonal contributions to the matrix elements $\langle \vec{s}\cdot\vec{I} \rangle$ and $\langle \vec{j}\cdot\vec{I} \rangle$ in the expression for the magnetic moment. We can write

$$\langle \vec{s}\cdot\vec{I} \rangle = \sum_K a_K^* a_K \langle K | \vec{s}\cdot\vec{I} | K \rangle + \sum_K a_K^* a_{K+1} \langle K | s_+ I_- | K+1 \rangle \qquad (55)$$

and

$$\langle \vec{j}\cdot\vec{I} \rangle = \sum_K a_K^* a_K \langle K | \vec{j}\cdot\vec{I} | K \rangle + \sum_K a_K^* a_{K+1} \langle K | j_+ I_- | K+1 \rangle . \qquad (56)$$

The expression of the diagonal contribution is given in Eqs. (12) and (13). The off-diagonal contributions are

$$\langle K | s_+ I_- | K+1 \rangle = \left[I(I+1) - K(K+1) \right]^{1/2} \sum_\ell a_{\ell\Omega+1/2}^{(K+1)} a_{\ell\Omega+1/2}^{(K)} \qquad (57)$$

and

$$\langle K | j_+ I_- | K+1 \rangle = \left[I(I+1) - K(K+1) \right]^{1/2} \sum_\ell (a_{\ell\Omega-1/2}^{(K)} a_{\ell\Omega+1/2}^{(K+1)} \left[\ell(\ell+1) - (\Omega-1/2)(\Omega+1/2) \right]^{1/2} + a_{\ell\Omega+1/2}^{(K)} a_{\ell\Omega+3/2}^{(K+1)} \left[(\ell+1) - (\Omega+1/2)(\Omega+3/2) \right]^{1/2} + a_{\ell\Omega+1/2}^{(K+1)} a_{\ell\Omega+1/2}^{(K)} . \qquad (58)$$

This effect on magnetic moment for some nuclei may be illustrated by the example of the 87-kev level of Pa^{233}. The calculation of Marshall fitting the even-parity levels of Pa^{233} gives a wave function for the 87-kev level (I = 5/2+) of

$$\psi_{mixed} = 0.94 \, |5/2\ 5/2\rangle + 0.34 \, |5/2\ 3/2\rangle + 0.09 \, |5/2\ 1/2\rangle .$$

The principal effect of this K admixture on the magnetic moment comes from an off-diagonal contribution between K=5/2 and 3/2 components. This contribution is about 1.1 n.m. The calculated moment for the pure K=5/2 state at deformation $\delta = 0.2$ using Nilsson's wave functions and g_s=4 is 2.6 n.m. Taking into account all components, we predict a magnetic moment of 3.8 n.m. for the 87-kev state in Pa^{233}. This state has a half life of 37mμsec, and the magnetic moment may be measured by external magnetic-field attenuation of the α- γ angular correlation with Np^{237}.

The electromagnetic-transition probabilities between the mixed states can be given in terms of the above mixing amplitudes and the usual gamma-ray transition matrix elements.[2,6]

The strength of a nuclear gamma-ray transition of multipole order, λ, between an initial state, i, and a final state, f, with magnetic quantum numbers, M', may be characterized by the reduced transition probability

$$B(\lambda, I \rightarrow I') = \sum_{\mu M'} |\langle I'M'| \mathfrak{M}(\lambda,\mu)| IM \rangle \qquad (59)$$

where $\mathfrak{M}(\lambda,\mu)$ is the μ component of the transition operator of multipole order λ. The transition probability per second is

$$T = \frac{8\pi(2\lambda+1)}{\lambda\left[(2\lambda+1)!!\right]^2} \frac{1}{\hbar} \left(\frac{\omega}{c}\right)^{2\lambda+1} B(\lambda). \qquad (60)$$

The factor B (Eλ) also enters the expression for the Eλ Coulomb-excitation cross sections.

The electric- and magnetic-multipole operators are given by

$$\mathfrak{M}_e(\lambda,\mu) = \sum_{p=1}^{A} e_p r_p^{\lambda} Y_{\lambda\mu}(\theta_p,\phi_p) \qquad (61)$$

and

$$\mathfrak{M}_m(\lambda,\mu) = \frac{e\hbar}{2Mc} \sum_{p=1}^{A} \left(g_s\vec{s} + \frac{2}{\lambda+1} g_\ell \vec{\ell}\right)_p \vec{\nabla}_p \left[r_p^{\lambda} Y_{\lambda\mu}(\theta,\phi)\right], \qquad (62)$$

respectively.

In the coordinates appropriate to the strong-coupling model, the multipole operators (61) and (62) take the form

$$\mathfrak{M}_e(\lambda,\mu) = \sum_p \left(e_p - \frac{Ze}{A^\lambda}\right) r_p^{\lambda} Y_{\lambda\mu}(\theta_p\phi_p) + \frac{3}{4\pi} Z e R_0^{\lambda} \alpha^*_{\lambda,\mu} \qquad (63)$$

and

$$\mathfrak{M}_m(\lambda,\mu) = \frac{e\hbar}{2Mc} \sum_p \left(g_s\vec{s} + \frac{2}{\lambda+1} g_\ell\vec{\ell}\right)_p \cdot \vec{\nabla}_p \left[r_p^{\lambda} Y_{\lambda\mu}(\theta_p,\phi_p)\right] + \frac{e\hbar}{Mc} \frac{1}{\lambda+1} g_R \int \vec{R}(\vec{r}) \cdot \vec{\nabla}\left[r_\lambda Y_{\lambda\mu}(\theta,\phi)\right] d\tau. \qquad (64)$$

The first terms in the expressions represent the transition moments of the most loosely bound particles which can be individually excited. The sum over p is to be taken over the transforming nucleons. The last terms represent the multipole moment generated by the collective motion of the nucleons, and $\alpha_{\lambda\mu}{}^{+}$ is the Hermitian conjugate of the coordinate describing the deformation of the nuclear surface defined by

$$R(\theta,\phi) = R_0 \left[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta,\phi) \right]. \tag{65}$$

Here R_0 is the nuclear radius, $\vec{R}(\vec{r})$ is the collective angular-momentum density, and one has $\int \vec{R}(\vec{r})\, d\tau = \vec{R}$.

It is convenient to express the multipole operators in the coordinate system fixed in the nucleus,

$$\mathcal{M}(\lambda,\mu) = \sum_{\nu} \mathcal{D}_{\mu\nu}{}^{\lambda}(\theta_i)\, \mathcal{M}'(\lambda,\nu), \tag{66}$$

where $\mathcal{M}'$ is of the same functional form as $\mathcal{M}$. The reduced transition probability then takes the form

$$B(\lambda, I \rightarrow I') = \sum_{\mu,M'} \left| \langle I'M' \,|\, \sum_{\nu} \mathcal{D}_{\mu\nu}^{\,I} \mathcal{M}'(\lambda,\nu) \,|\, IM \rangle \right|^2 . \tag{67}$$

Integration over the Eulerian angles, θ_i, gives

$$\int \mathcal{D}^{\,I'}_{M'K'} \mathcal{D}^{\lambda}_{\mu\nu} \mathcal{D}^{I}_{MK}\, d\Omega^3 = \frac{8\pi^2}{2I'+1} \langle I\,\lambda\, M\,\mu \,|\, I\,\lambda\, I'M' \rangle \langle I\,\lambda\, K\,\nu \,|\, I\,\lambda\, I'K' \rangle \tag{68}$$

where $\langle I\,\lambda\, M\,\mu \,|\, I\,\lambda\, I'M' \rangle$ and $\langle I\,\lambda\, K\,\nu \,|\, I\,\lambda\, I'K' \rangle$ are the vector addition coefficients. One then obtains

$$B(\lambda, I \rightarrow I') = \Big| \langle I\,\lambda\, K\, K'-K \,|\, I\,\lambda\, I'K' \rangle \int \chi'_{\Omega'}{}^{+} \mathcal{M}(\lambda, K'-K)\, \chi_{\Omega}\, d\tau \; + \langle I\,\lambda\, K\, -K'-K \,|\, I\,\lambda\, I'-K' \rangle \int \left[(-)^{I'-j'} \chi'_{-\Omega'} \right]^{+} \mathcal{M}(\lambda, -K'-K)\, \chi_{\Omega}\, d\tau \Big|^2 . \tag{69}$$

The second term contributes only for the unusual case $\lambda \geq K + K'$.

The operator $\vec{\ell} \cdot (\vec{\nabla} r^{\lambda} Y_{\lambda\nu})$ can be rewritten

$$\vec{\ell} \cdot (\vec{\nabla} r^{\lambda} Y_{\lambda\nu}) = \left(\frac{2\lambda+1}{2\lambda-1}\right)^{1/2} \left[(\lambda^2-\nu^2)\right]^{1/2} \ell_z r^{\lambda-1} Y_{(\lambda-1)\nu} + \frac{1}{2}\left[(\lambda-\nu)(\lambda-\nu-1)\right]^{1/2}$$

$$\ell_- r^{\lambda-1} Y_{(\lambda-1)(\nu+1)} - \frac{1}{2}\left[(\lambda+\nu)(\lambda+\nu-1)\right]^{1/2} \ell_+ r^{\lambda-1} Y_{(\lambda-1)(\nu-1)}, \quad (70)$$

where $\ell_+ = \ell_x + i\ \ell_y$, $\ell_- = \ell_x - i\ \ell_y$. The same formula holds for $\vec{s} \cdot (\vec{\nabla} r^{\lambda} Y_{\lambda\nu})$ with s- exchanged for l- and so on.

We are interested in the magnetic-dipole transition probabilities from a state IK to another state I'K'. We have

$$B(M1, I \to I') = \frac{3}{16\pi}\left(\frac{e\hbar}{2Mc}\right)^2 \left| \langle I\lambda KK'-K \ | I\lambda I'K' \rangle \right|^2 G_{KK'}^{\ 2} . \quad (71)$$

The expression for $G_{KK'}$ in terms of Nilsson's wave function is

$$G_{KK'} = \sum_{\ell} a'_{\ell\Lambda'} \, a_{\ell\Lambda}$$

$$(g_s - g_R)\, \delta_{\Lambda,\Lambda'} \begin{cases} \delta_{\Sigma,\Sigma'} (-)^{\Sigma-1/2} \\ \sqrt{2}\delta_{\Sigma',-1/2}\delta_{\Sigma,1/2} \\ -\sqrt{2}\delta_{\Sigma,1/2}\delta_{\Sigma,-1/2} \end{cases}$$

$$+(g_\ell - g_R)\delta_{\Sigma,\Sigma'} \begin{cases} 2\Lambda'\delta_{\Lambda,\Lambda'} \\ \sqrt{2}\ \left[\ell(\ell+1) - \Lambda'(\Lambda'+1)\right]^{1/2} \\ \qquad \delta_{\Lambda.\Lambda'+1} \end{cases}$$

$$-\sqrt{2}\ \left[\ell(\ell+1) - \Lambda(\Lambda+1)\right]^{1/2}$$

$$\delta_{\Lambda,\Lambda'-1} . \quad (72)$$

The magnitude of the magnetic-dipole transition probability between rotational states can thus be related to the gyromagnetic ratios g_s and g_R. A knowledge of both the magnetic-dipole transition probability and the ground-state magnetic moment can thus yield g_s and g_R separately (cf. K. Alder et al.[21]).

The reduced transition probabilities between the mixed states are

$$B(M1) = \frac{3}{16\pi} \left(\frac{e\hbar}{2Mc}\right)^2 \left[\sum_{K,K'} a_K^{I} a_{K'}^{I'} G_{KK'} \langle I\, 1\, K\, K'-K \,|\, I\, K\, I'\, K'\rangle\right]^2. \quad (73)$$

The sensitiveness of this transition probability to mixing of wave functions implies that one would get very different g_R values from the Coulomb-excitation studies and ground-state moments of nuclei when the particle-rotation interaction is considered.

Since the successive rotational states in an odd-A nucleus have $\Delta I=1$, the gamma radiation emitted in the decay of these states will in general be a mixture of magnetic-dipole M1 and electric-quadrupole E2 radiations. The absolute E2 transition probability can be determined from the cross section for Coulomb excitation. Thus, a determination of the M1 transition as compared with the E2 transition in the decay of the first excited state will also yield the absolute M1 transition probability. This information can be obtained from angular distributions or internal-conversion measurements of the emitted radiation (or from the lifetime of the excited state). The M1 transition probability in the cascade transition ($I_0+2\to I_0+1$) can be determined from the relative strength of M1 and E2 radiations in this transition together with the branching ratio between the mixed M1 and E2 cascade radiation and pure E2 cross-over ($I_0\to I_0+2$) decay of the second excited state. If one of these data is available, one may estimate the absolute M1 transition probability in the cascade radiation.

Bernstein and de Boer have combined available experimental data to obtain the reduced magnetic transition probabilities between rotational states of deformed odd-A rare earth nuclei.[4] The results are interpreted to yield g_K and g_R. In view of the possible particle-rotation interaction, we will renormalize the data for several nuclei.

C. Numerical Calculations of the Effect of Particle-Rotation Interaction on the Magnetic Moment

It has been mentioned that nuclei having particularly large j values for the odd nucleon would be especially susceptible to appreciable particle-rotation interactions. Somewhat arbitrarily, we single out nuclei belonging to states of odd parity in the 50 to 82 shell ($h_{11/2}$), even parity in 82 to 126 shell ($i_{13/2}$), and odd parity in the > 126 shell ($j_{15/2}$) to investigate the particle-rotation-interaction effects on magnetic moment and magnetic parameters.

The interacting rotational bands to be considered are all based on particle states in which the odd nucleon is in orbitals of the same $\vec{j}$ (but different Ω). If a nucleus has rotational bands based on particle states in which the odd nucleon is in orbitals of different $\vec{j}$, there will be no Coriolis interaction between them if they have opposite parity; if the bands have the same parity, there usually will be an interaction between them, but it will be relatively weak. Even when the interacting states are so closely lying that the mixing amplitude may be comparable to that of interaction between states of the same $\vec{j}$, the mixing effects on magnetic properties are still negligible because of the smallness of the matrix elements (S+ I-) and (j+ I-). For example, the ground state of Eu^{153} (413, 5/2+) has 1% mixing of (411, 3/2+). The effect on the magnetic moment is only 0.02 n.m., and g_R differs by 0.005 with and without mixing of wave functions.

In order to calculate the magnetic moment of nuclei subjected to particle-rotation interaction, the unperturbed moment of inertia $\Im_0$ is assumed to be the same for all interacting bands. The dependence of the off-diagonal elements on I, j, and K is given in Eq. (46). The magnitude of these matrix elements was allowed to vary with a variable parameter k replacing the $\hbar^2/2\Im_0$ term. Values of this term and of k are chosen in such a way as to give the best agreement with experimental rotational bands (within 1 kev). The k's are smaller than the $\hbar^2/2\Im_0$ values mainly because the $\langle j_+ \rangle$ matrix elements calculated by using single-particle wave functions are larger than the values one would get if one used the quasi-particle wave functions. The diagonal elements are taken from energy levels of Mottelson and Nilsson. To transfer the energy from the units $\hbar\omega_0$ to kev, we have used[6]

$$E \cong 41\,\kappa(A^{-1/3},$$

where E is in Mev and $41\,\kappa(A)^{-1/3}$ is in units of $\hbar\mathring{\omega}_0$.

In all cases, only the interaction between the state $|\Omega\,\rangle$ under consideration and its neighboring states $|\Omega\text{-}1\,\rangle$ and $|\Omega\,\rangle$ are taken into account. The other states could be coupled to the state $|\Omega\,\rangle$ only through their coupling with the states $|\Omega\text{-}1\,\rangle$ and $|\Omega+1\,\rangle$. Therefore their mixing amplitudes should be negligible. The secular equation will therefore be third-order.

Figures 9 through 14 show the rotational spectra of the ground state of these specified nuclei.[3,4] The assumed positions of interacting states are also shown in dotted lines labeled with the quantum numbers K, parity, and $(Nn_Z\Lambda)$. Table V lists their mixing coefficients and the calculated magnetic moments g_R and g_s with and without K mixing. The magnetic-dipole transition probabilities are taken from Bernstein et al.[4] The experimental magnetic moments are taken from Tables I and II. These values differ from those of Bernstein et al. because of the $\langle 1/r^3\rangle$ correction we have mentioned.[4]

The results show that the particle-rotation interaction does not have as significant an effect on the g_s values as on the g_R values. In the latter case, the rather large correction indicates that the M1-transition matrix elements are very sensitive to mixing of K quantum numbers. The magnetic moments are calculated with the assumed g_s factors, that is, using 4.0 for the proton and -2.4 for neutron. Therefore, the results show the effect of K mixing on the magnetic moment rather than the correction that would actually be caused by such kind of mixing.

Nilsson has pointed out that the corrected g_R values should be close to those of neighboring even-even nuclei, which we have also listed in Table V for comparison. They are close to each other within experimental uncertainty.

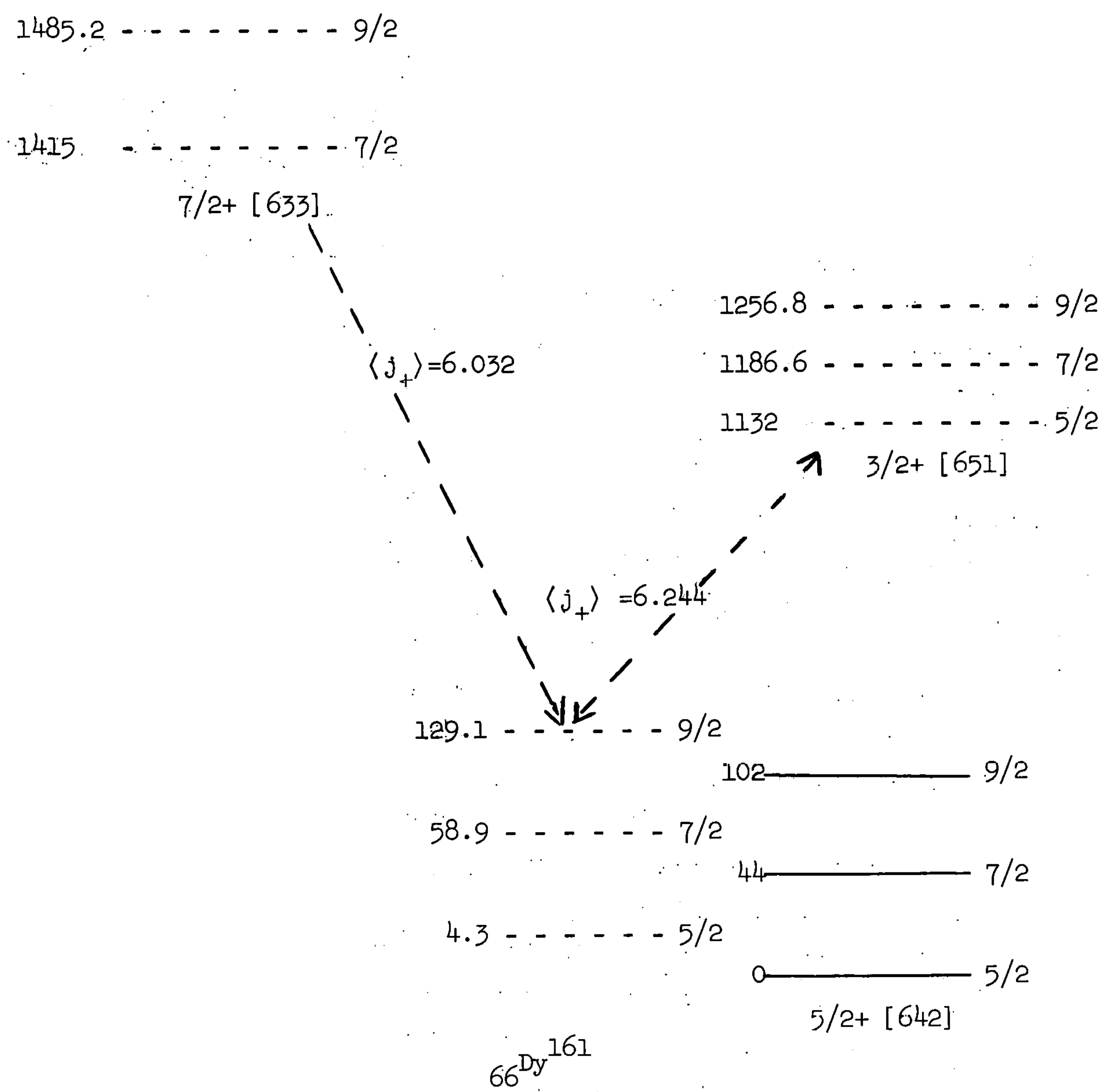

Fig. 9. Rotational spectra and assumed interacting bands of Dy^{161}, with $\hbar^2/2\Im_0 = 7.8$ and K=5.

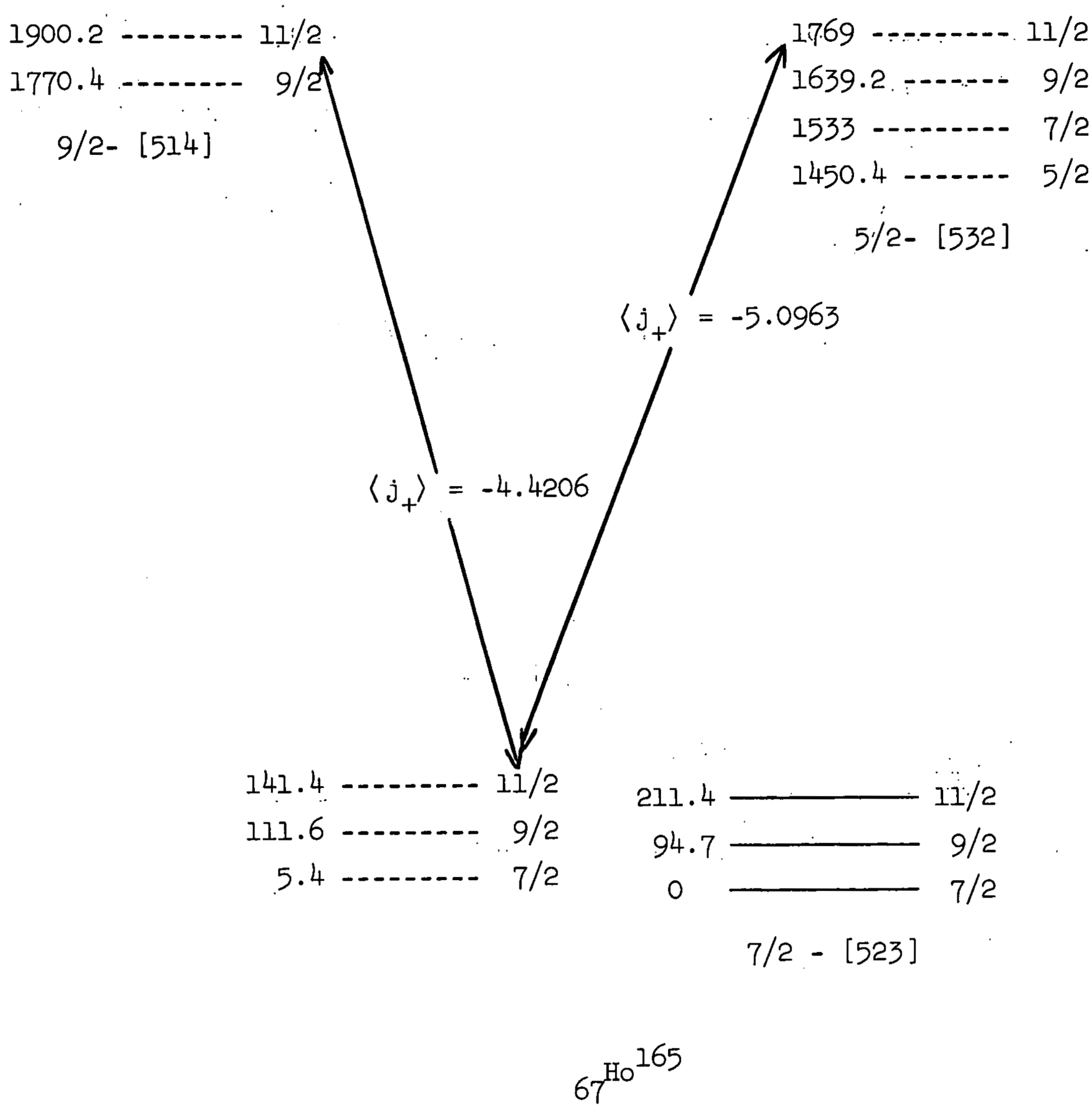

Fig. 10. Rotational spectra and the assumed interacting bands of Ho^{165}, with $\hbar^2/2\Im_0 = 11.8$ and $k = 6.7$.

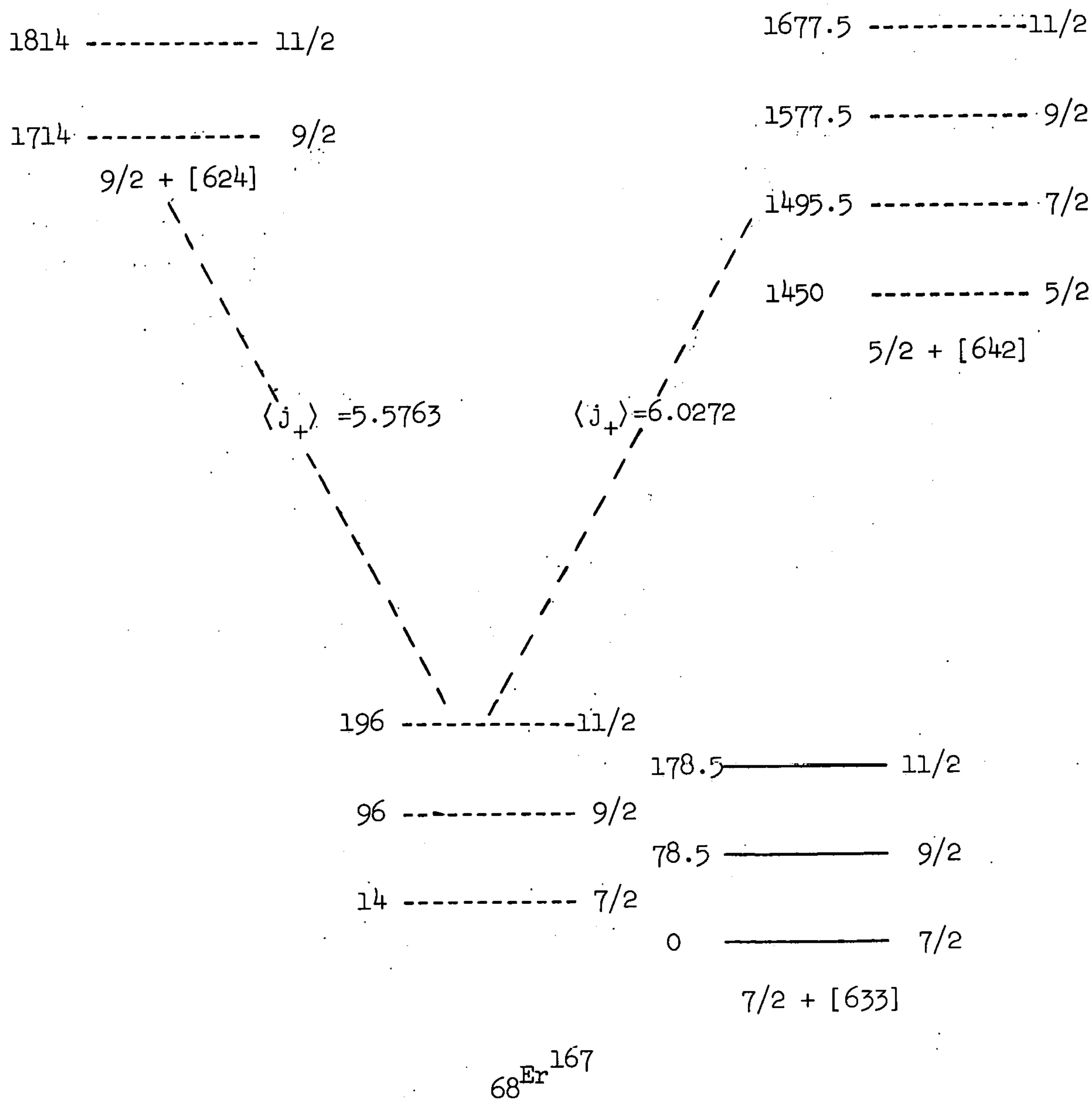

Fig. 11. Rotational spectra and the assumed interacting bands of $_{68}Er^{167}$, with $\hbar^2/2\Im_0 = k = 9.1$.

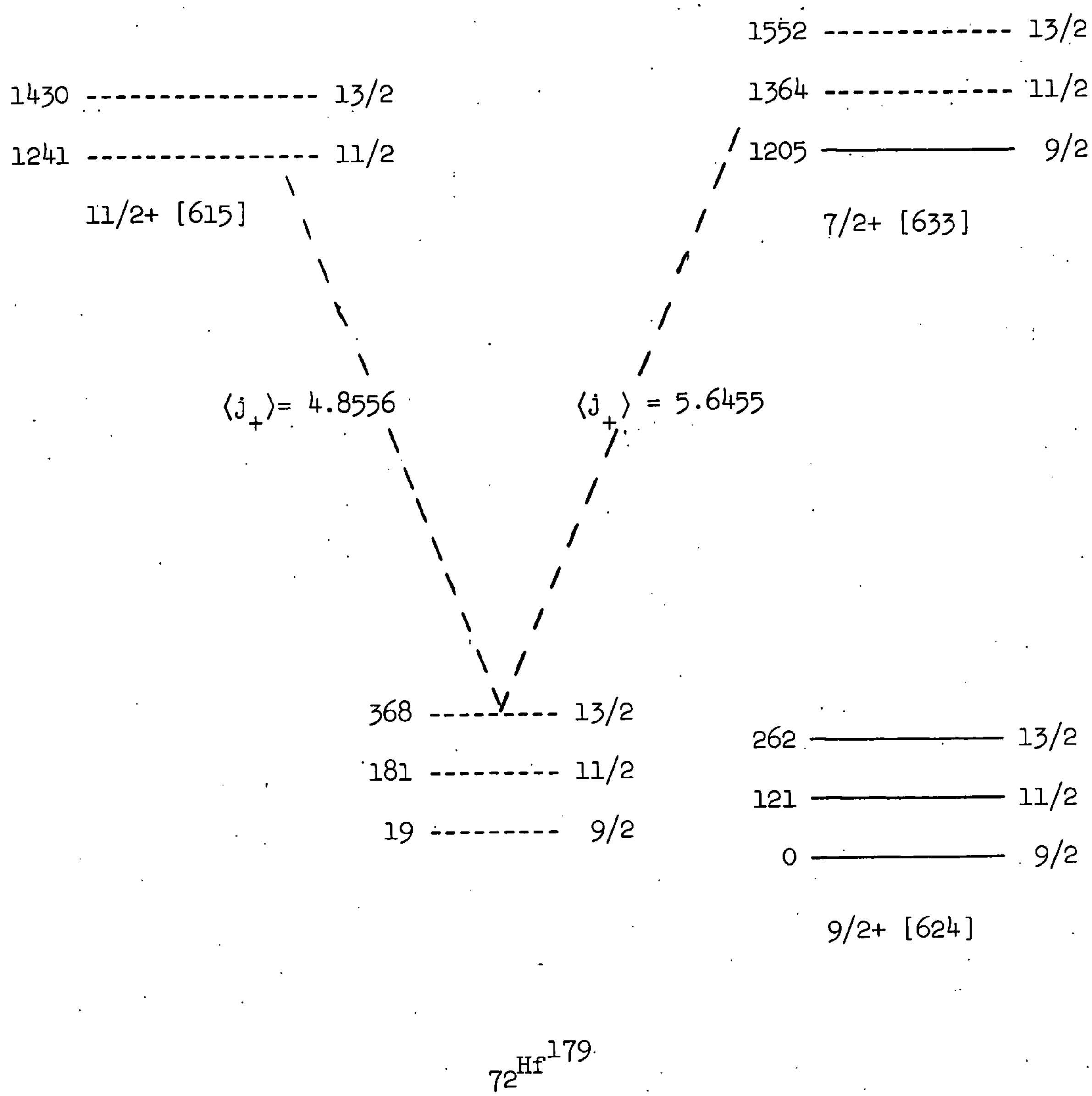

Fig. 12. Rotational spectra and assumed interacting bands of $_{72}Hf^{179}$, with $\hbar^2/2\Im_0 = 14.5$ and k= 9.

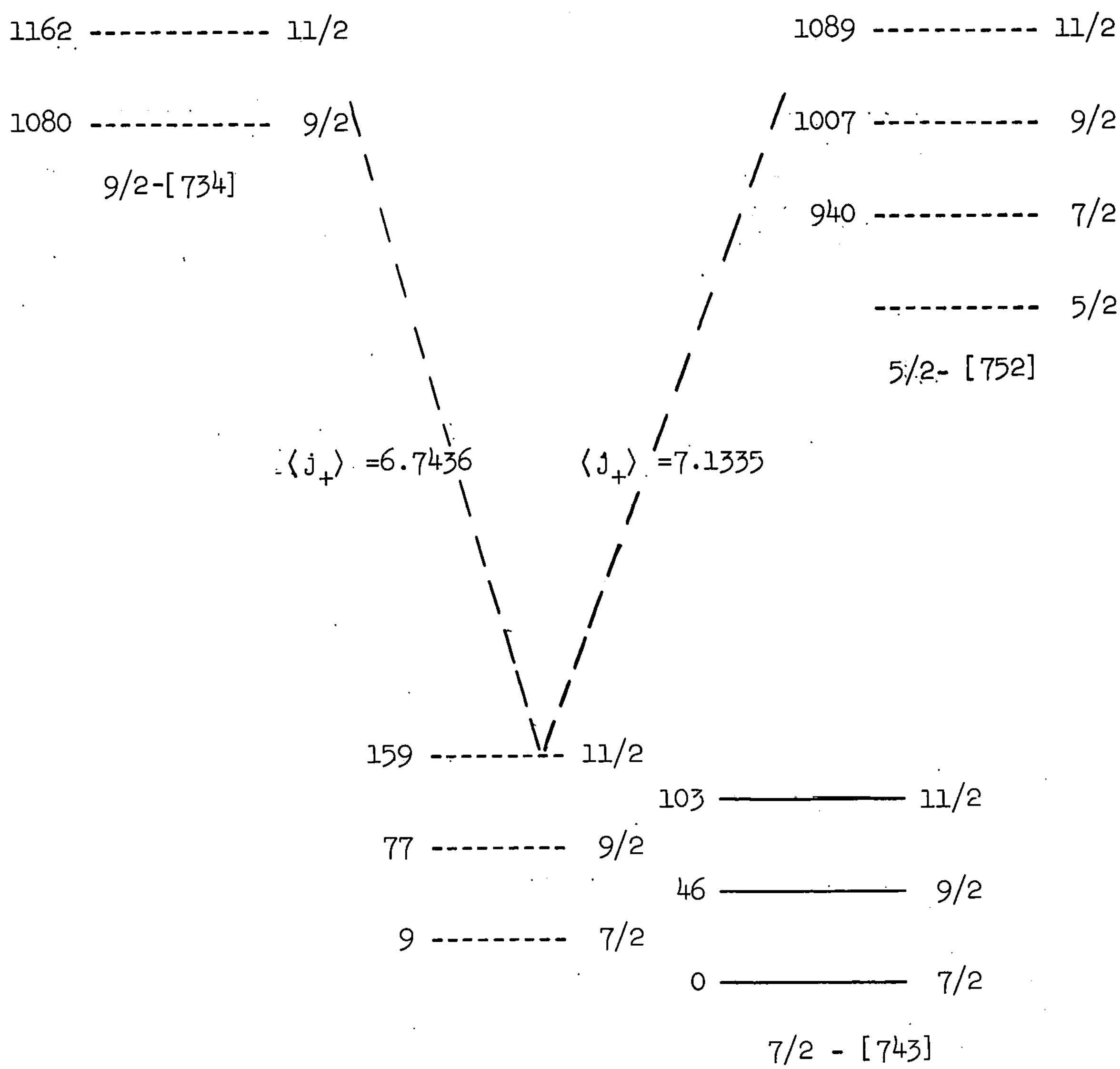

Fig. 13. Rotational spectra and assumed interacting bands of $_{92}U^{235}$, with $\hbar^2/2\Im_0 = 7.06$ and k=4.5.

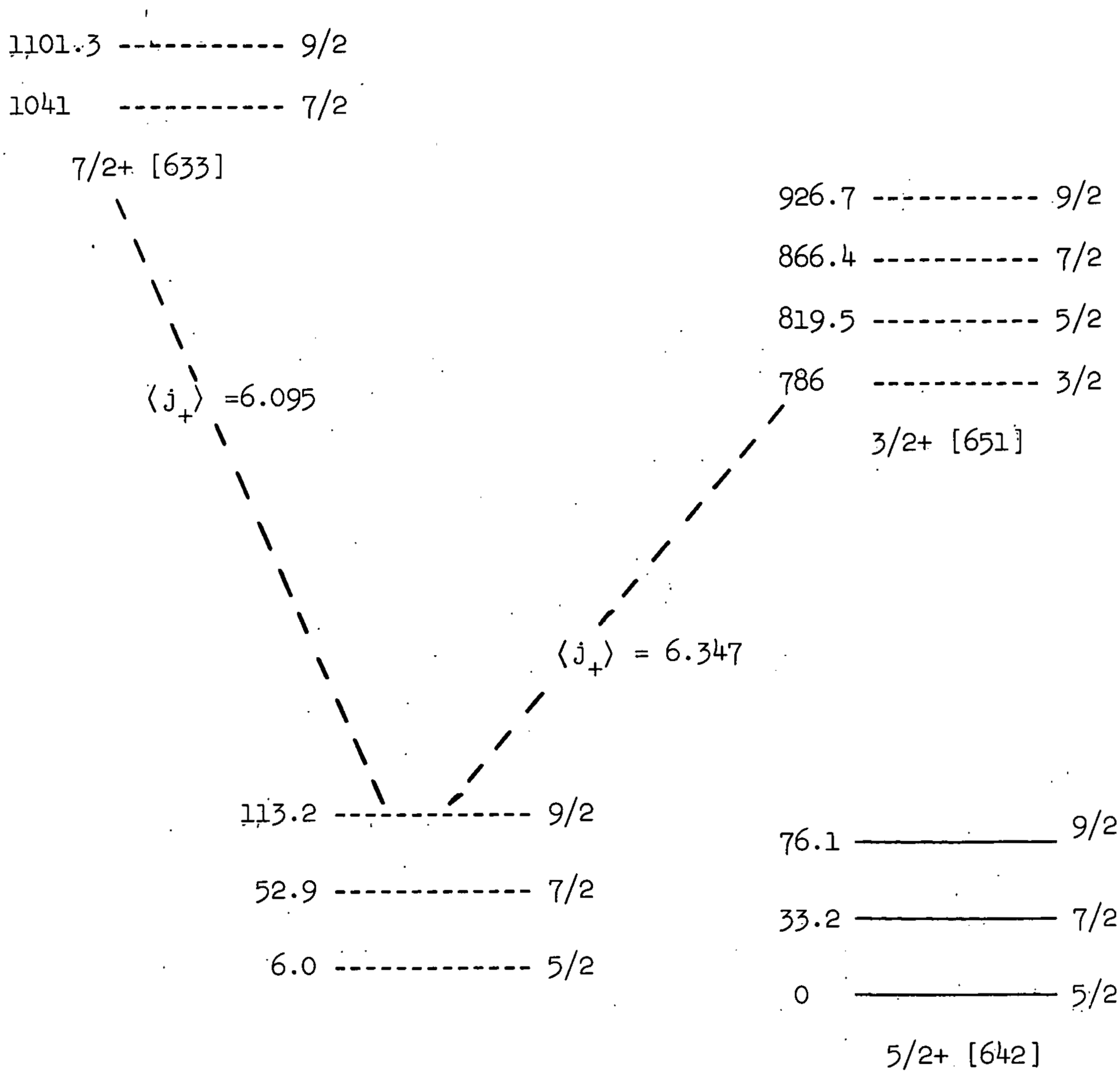

Fig. 14. Rotational spectra and assumed interacting bands of Np^{237}, with $\hbar^2/2\Im_0 = 6.7$ and $k = 4.85$.

Table V. Effect of particle-rotation interaction on the magnetic moment and gyromagnetic ratio of some deformed nuclei.

Nuclei	Spin Level	Mixing coefficients	B(M1; I'→ I)	Calculated values without K mixing			Calculated values with K mixing				g_R of neighboring even-even nuclei
				μ	g_R	g_s	μ	g_R	g_s	$\mu_{exp.}$	
$_{66}Dy^{161}$	5/2	$a_{3/2}$=0.062, $a_{5/2}$=0.998	0.101±0.050; 7/2→5/2	-0.37	0.210 (±0.072)	-2.83 (±0.23)	-0.48	0.295 (±0.072)	-2.57 (±0.22)	-0.48	0.20±0.04
	7/2	$a_{3/2}$=0.095, $a_{5/2}$=0.994, $a_{7/2}$=0.056									
	9/2	$a_{3/2}$=0.123, $a_{5/2}$=0.989, $a_{7/2}$=0.084									
$_{67}Ho^{165}$	7/2	$a_{5/2}$=-0.060, $a_{7/2}$=0.998	0.542±0.082; 11/2→9/2	4.0	0.554 (±0.047)	4.32 (±0.12)	4.1	0.391 (±0.060)	4.23 (±0.11)	4.18	0.320 (±0.024)
	9/2	$a_{5/2}$=-0.089, $a_{7/2}$=0.994, $a_{9/2}$=0.053									
	11/2	$a_{5/2}$=-0.114, $a_{7/2}$=0.990, $a_{9/2}$=-0.077									
$_{68}Er^{167}$	7/2	$a_{5/2}$=0.095, $a_{7/2}$=0.995	0.094±0.026; 11/2→9/2	-0.54	0.098 (±0.035)	-2.31 (±0.09)	-0.65	0.226 (±0.055)	-2.24 (±0.08)	-0.59	0.320 (±0.024)
	9/2	$a_{5/2}$=0.129, $a_{7/2}$=0.986, $a_{9/2}$=0.106									
	11/2	$a_{5/2}$=0.175, $a_{7/2}$=0.976, $a_{9/2}$=0.133									
$_{72}Hf^{179}$	9/2	$a_{7/2}$=0.125, $a_{9/2}$=0.992	0.165±0.031; 13/2→11/2	-0.63	0.203 (±0.034)	-1.93 (±0.14)	-0.69	0.379 (±0.052)	-2.20 (±0.17)	-0.47 (±0.03)	0.20 (±0.15)
	11/2	$a_{7/2}$=0.178, $a_{9/2}$=0.976, $a_{11/2}$=0.126									
	13/2	$a_{7/2}$=0.218, $a_{9/2}$=0.960, $a_{11/2}$=0.176									
$_{92}U^{235}$	7/2	$a_{5/2}$=0.090, $a_{7/2}$=0.995		-0.43			-0.49			-0.67 (±0.07)	
	9/2	$a_{5/2}$=0.133, $a_{7/2}$=0.987, $a_{9/2}$=0.087									
	11/2	$a_{5/2}$=0.265, $a_{7/2}$=0.957, $a_{9/2}$=0.123									
$_{93}Np^{237}$	5/2	$a_{3/2}$=0.087, $a_{5/2}$=0.996		2.7			3.0			> 2.7	
	7/2	$a_{3/2}$=0.126, $a_{5/2}$=0.989, $a_{7/2}$=0.077									

Nilsson and Prior have expressed the difference between odd-A and neighboring even-even nuclei as[5]

$$g_R = \frac{\delta\mathfrak{J}}{\mathfrak{J}}(g_\ell - g_R) + \frac{\delta W}{\mathfrak{J}}(g_s - g_\ell) \tag{74}$$

where $\delta\mathfrak{J}$ is the contribution of the odd particle to the moment of inertia connecting the one-quasi-particle state with other states of the same kind. Some of the difference may be due to the blocking effect. If the quasi-particle formulation is sufficiently accurate to estimate this difference, $\delta\mathfrak{J}$ should be very nearly equal to the odd-even difference in the moments of inertia. Similarly, δW is the contribution to the expression W of the odd particle. Nilsson and Prior find that if one inserts in this formula the empirical odd-even differences in the moment of inertia and estimate the somewhat smaller second term from its "asymptotic" expression, one usually finds too large corrections. The spin-matrix elements are much smaller than those calculated from the single-particle wave functions because of the spin polarization effect we have mentioned. However, even with a 50% reduction of the latter term, the correction still appears somewhat too large. It is likely that the second term should be negligible.

It is hard to distinguish between effects of the Coriolis force and pair-correlation effects due to blocking of an orbital, thereby reducing the effective energy gap for nucleons of the odd group. Both these mechanisms affect moment of inertia and g_R in the same way to the lowest order.

VII. CONCLUSION

In this paper an attempt has been made to discuss the interpretation of the available data on magnetic moments of deformed nuclei in terms of the collective and individual-particle aspects of nuclear motion. From the foregoing discussion, the following points can be made.

The particle magnetic moments are very sensitive to nuclear-wave-function mixing caused by the very-short-range residual forces, both spin-independent and spin-dependent. These effects are equivalent to using reduced g_s factors. The reduction is somewhat different for different nuclei. Admixtures due to the Coriolis force affect particle magnetic moments to a smaller extent.

The deviation of the collective gyromagnetic ratio from Z/A can be satisfactorily accounted for by pair-correlation for even-even nuclei. For odd-A nuclei, g_R is affected in addition by Coriolis force and blocking effect.

However, it should be pointed out that it remains to be shown whether the terms present in, for example, the delta forces but neglected in the pairing interaction will have any effect on the collective gyromagnetic ratio. It appears likely that such an effect should be small because g_R is essentially the ratio $\frac{\mathcal{J}_p}{\mathcal{J}_p + \mathcal{J}_n}$, and $\mathcal{J}_p$ and $\mathcal{J}_n$ should be affected in about the same way by these terms.

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