DP-873

AEC RESEARCH AND DEVELOPMENT REPORT

MEASUREMENTS OF BUCKLINGS AND VOID EFFECTS IN D_2O-MODERATED, ORGANIC- OR H_2O-COOLED LATTICES OF UO_2 ROD CLUSTERS

F. D. BENTON

Savannah River Laboratory

Aiken, South Carolina

DISCLAIMER

Printed in USA. Price $0.50
Available from the Office of Technical Services
U. S. Department of Commerce
Washington 25, D. C.

DP-873

Reactor Technology
(TID-4500, 30th Ed.)

MEASUREMENTS OF BUCKLINGS AND VOID EFFECTS IN D_2O - MODERATED, ORGANIC - OR H_2O - COOLED LATTICES OF UO_2 ROD CLUSTERS

by

Frederick D. Benton

Work done by

F. D. Benton
A. E. Dunklee
H. R. Fike
W. E. Graves
E. L. Holt
R. L. Hooker
T. J. Hurley
C. F. Rose
S. V. Topp

Approved by

J. L. Crandall, Research Manager
Experimental Physics Division

June 1964

E. I. DU PONT DE NEMOURS & COMPANY
EXPLOSIVES DEPARTMENT - ATOMIC ENERGY DIVISION
TECHNICAL DIVISION - SAVANNAH RIVER LABORATORY
AIKEN, SOUTH CAROLINA

CONTRACT AT (07-2) - 1 WITH THE
UNITED STATES ATOMIC ENERGY COMMISSION

ABSTRACT

Material bucklings for a variety of UO_2 rod clusters on a triangular 9.33-inch lattice pitch in D_2O were determined by substitution experiments in the Process Development Pile. Measurements were made with clusters of 19, 31, and 37 half-inch UO_2 rods in which the coolant media were varied to include D_2O, H_2O, organics, or air. The bucklings, which were determined both by a one-group perturbation method and by a two-group flux matching method, ranged from 0.80 to 5.60 m^{-2}.

CONTENTS

LIST OF TABLES AND FIGURES

Table

Figure

MEASUREMENTS OF BUCKLINGS AND VOID EFFECTS IN D_2O - MODERATED, ORGANIC - OR H_2O - COOLED LATTICES OF UO_2 ROD CLUSTERS

INTRODUCTION

Most of the past studies made by Du Pont on D_2O-moderated power reactors involved liquid D_2O as the coolant. Recent cost studies[1] indicate that H_2O-fog and organic coolants may compete economically with liquid D_2O. The experimental buckling studies described in this report cover a large variety of coolants for D_2O-moderated reactors to provide normalization points for reactivity calculations. The results of these studies were required to test lattice parameter calculations[2] and the physics portion of the Savannah River Laboratory's cost optimization code[3].

SUMMARY

Bucklings were measured by substitution techniques in the Process Development Pile (PDP) for fuel assemblies consisting of clusters of 19, 31, or 37 natural UO_2 rods with various rod spacings. The UO_2 rods were made up from 0.500-in.-diameter sintered pellets of 10.4 g/cm^3 average density stacked in aluminum tubes 0.020 in. thick and of 0.547-in. OD. The hexagonal clusters were placed in aluminum housing tubes, which in most cases were selected to allow only a small portion of the coolant between the outer cluster boundary and the housing wall. "Dowtherm" A* (a mixture of 26.5% diphenyl and 73.5% diphenyl oxide), polyethylene, and H_2O were used to simulate coolants. Air- and D_2O-filled assemblies were also studied.

The one-region reference loading consisted of 31-rod UO_2 clusters contained in D_2O-filled aluminum housing tubes of 5.00-in. OD[5]. Both reference and test lattices were at a 9.33-in. triangular pitch, which is in the range of practical interest.

Measurements were made by the successive substitution technique in which effective critical water heights were determined after the substitution of one, three, and seven test assemblies. Two techniques of analysis were used: a one-group perturbation method and a two-group flux matching method. In the perturbation method the lattice containing test assemblies was assumed to consist of a test region and a reference region, separated by an intermediate region of the immediately adjoining test and reference fuel assemblies. A criticality equation was then written for each successive

*"Dowtherm" is a trademark of the Dow Chemical Company.

substitution measurement as a function of the measured critical water height, the statistical weights and diffusion coefficient ratios of the different regions, and the buckling differences between the one- and three-region piles. These equations were then extrapolated to give the buckling for a one-region lattice of the test assemblies. The two-group method utilized only the seven assembly substitutions. In addition to the measured pile height and radius, this method required knowledge of fast and slow diffusion coefficients, thermal diffusion area, neutron age, and resonance escape probability for each of the two regions. Results of both methods of analysis, with a description of the test fuel assemblies, are summarized in Table VI. For this particular study there seems to be little to choose between the two-group and perturbation analyses.

DISCUSSION

DESCRIPTION OF FUEL ASSEMBLIES AND COOLANTS

The fuel assemblies (Figure 1) consisted of 19, 31, or 37 UO_2 rods in housing tubes filled with the appropriate coolant. The natural UO_2 rods were made up of 0.500-in.-diameter sintered pellets of 10.4 g/cm^3 average density stacked in Type 6063 aluminum tubes 0.020 in. thick and of 0.547-in. OD. These rods were assembled in triangular arrays at average center-to-center spacings of 0.650, 0.607, or 0.598 in. to form hexagonal clusters. The clusters were placed in Type 6063 aluminum housing tubes, which in most cases were selected to allow only a small portion of the coolant to lie between the outer boundary of the cluster and the housing wall. In general, the housing tubes were cylindrical. However for one assembly, a 19-rod cluster was placed in a tight-fitting housing of hexagonal cross section.

Three substances, "Dowtherm" A, low density polyethylene, and H_2O, were used to simulate hydrogenous coolants. "Dowtherm" A (a liquid mixture of 26.5% diphenyl and 73.5% diphenyl oxide with a density of 1.06 g/cm^3) mocked up a typical organic coolant. This commercial mixture was compared to a chemically pure mixture of ingredients by the danger coefficient technique in the Standard Pile (SP)[4]. Results indicated that the nuclear properties of the two mixtures were identical. The polyethylene mocked up H_2O fog coolant at about 0.1 normal H_2O density. It was wrapped loosely around the individual fuel rods in the cluster so as to fill about 80% of the space between rods. The average amount wrapped around each rod was 0.0618 gram per centimeter of length. H_2O was used as the third hydrogenous coolant primarily to extend the hydrogen density to a maximum.

Air-filled assemblies were used to mock up the other extreme of minimum hydrogen density and to permit a study of void effects. To provide another normalization point, measurements were also made for one assembly with D_2O as the coolant.

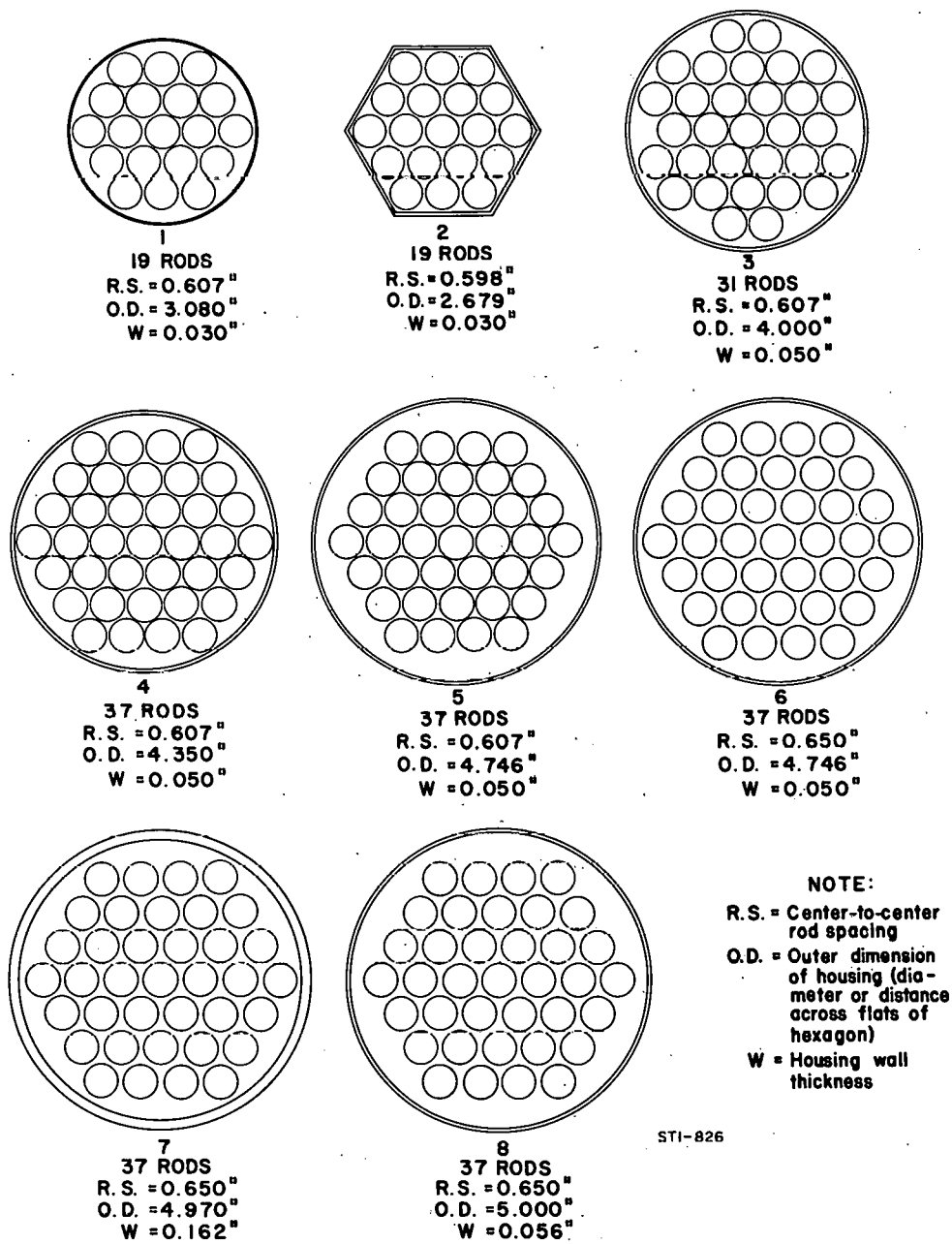

FIG. 1 CROSS SECTIONS OF UO_2 ROD CLUSTERS FOR BUCKLING MEASUREMENTS

MEASUREMENTS

Bucklings were determined by the substitution method, in which the fuel assemblies to be measured replaced standard fuel assemblies in a one-region lattice of known nuclear properties. The standard fuel assemblies consisted of the previously described UO_2 rods assembled at a 0.650-in. center-to-center spacing to form 31-rod clusters[5]. These clusters were contained in 5.00-in.-OD aluminum housing tubes of 0.056-in. wall thickness, open at the bottom to the moderator. One hundred-fifty-one such assemblies were placed in the center of the Process Development Pile (PDP)[6] at a 9.33-in. triangular pitch and were surrounded by several rings of lithium-aluminum poison rods spaced 4.67 in. apart. The loading pattern is shown in Figure 2.

Gold-pin flux traverses were made to determine the extrapolated pile radius and pile height, and consequently the material buckling, of the critical one-region lattice. Two quantities derived from these flux traverses, the extrapolated pile radius, R, and the differences between the extrapolated pile height and the critical moderator height, H-S, were assumed to remain constant and independent of substituted assemblies throughout the entire experiment, except as noted in a subsequent section. Values of these quantities are given in Table I.

TABLE I

Measured Constants of the One-Region Reference Lattice

Pile Radius:	R = 165.8 cm
Pile Height - Moderator Height:	H-S = 11.0 cm
Material Buckling:	$B_m^2 = 4.355\ m^{-2}$(a)

(a) Corrected to account for the presence of control rod guide tubes. Also corrected to temperature of 22.8°C and to moderator purity of 99.58 mol % D_2O.

For each type of test assembly, critical water height measurements were made of (1) the one-region reference pile, (2) the pile with one test assembly substituted in the center position, (3) the pile with three test assemblies substituted one in the center position and the other two in adjoining positions adjacent to one another, and (4) the pile with seven test assemblies substituted in the seven central positions. The substituted assemblies were at the same 9.33-in. lattice pitch as the reference assemblies.

ANALYSIS

Two analysis techniques[7] were used: a one-group perturbation method and a two-group flux matching method.

ONE-GROUP PERTURBATION METHOD

The one-group perturbation method, suggested and described by Persson[8], was a "successive substitution" technique which used the measurements performed with the three different sizes of test regions to extrapolate to the condition at which the test lattice extended throughout the pile. Since one-group theory does not properly take into account the interaction between the test and the reference regions, an intermediate region was introduced into the analysis. To define these three regions, the lattices were decomposed into unit cells consisting of rhomboids with the two vertices at the acute (60°) angles terminating on fuel assemblies. Rhomboids with both acute vertices terminating on test assemblies were assigned to the test region; those with one vertex on a test assembly and the other on a reference assembly were assigned to the intermediate region; and those with both vertices on reference assemblies were assigned to the reference region. These regions are shown in Figure 2 and 3, respectively, for one and seven test assemblies.

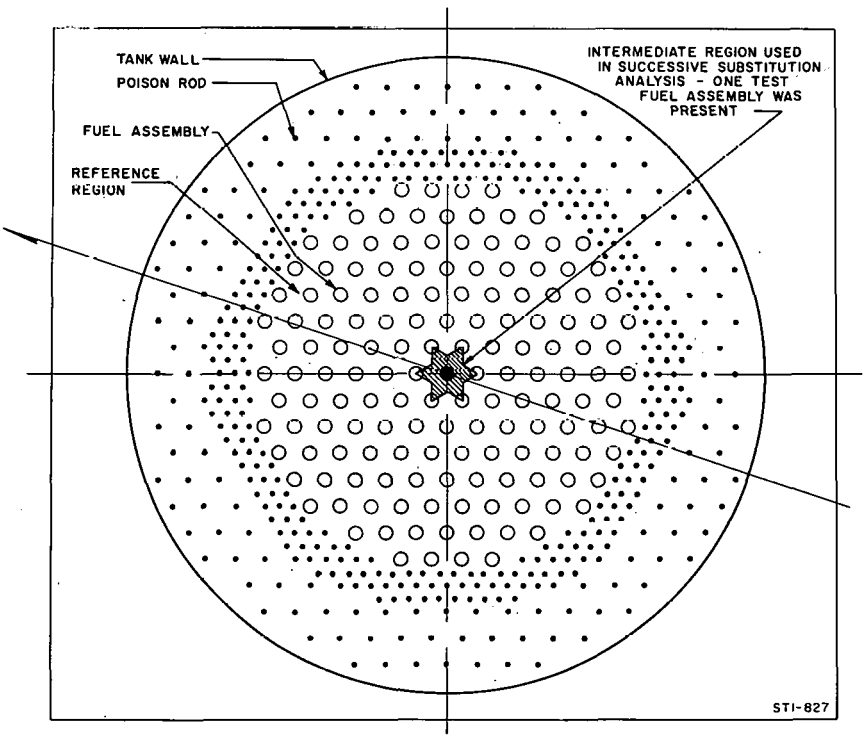

FIG. 2 PILE LOADING DIAGRAM - SINGLE SUBSTITUTION

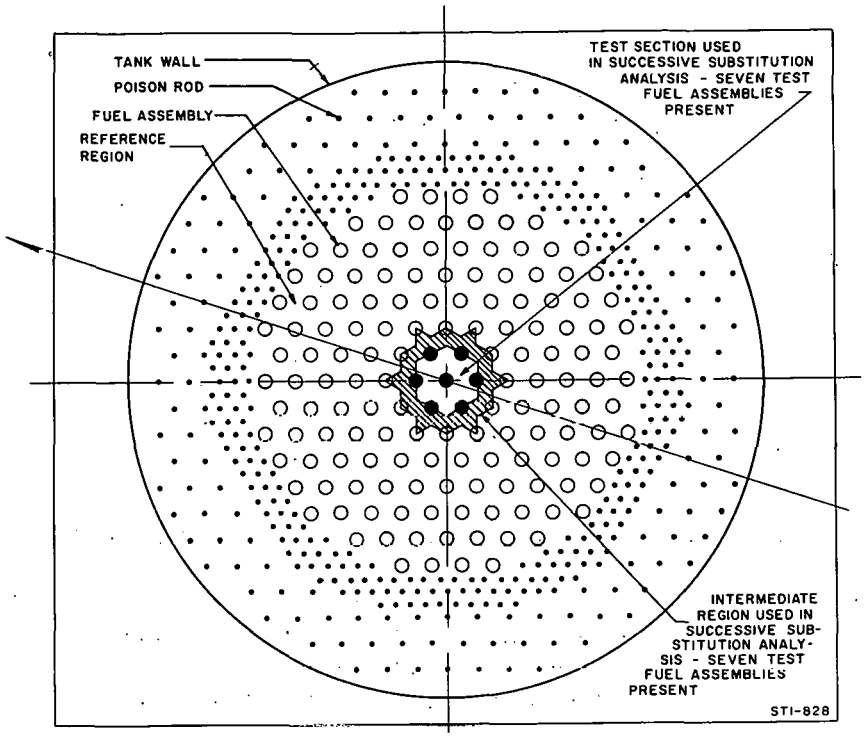

FIG. 3 PILE LOADING DIAGRAM - SEVEN SUBSTITUTIONS

The final equation used in the Persson analysis is given below.

$$\frac{(\alpha^2-\alpha_1^2)(1+\xi)}{\overline{W}_r} - \beta^2 \frac{\sum_i (W_{r_i} - U_{r_i})\delta D_{r_i}/D_{z_1}}{\overline{W}_r} = (\alpha_3^2-\alpha_1^2) + \delta\alpha_2^2 \frac{\overline{W}_{r_2}}{\overline{W}_r} \quad (1)$$

In equation (1), the subscript 1 refers to the reference region, 2 refers to the intermediate region, and 3 refers to the test region. The quantity α^2 is the vertical buckling of the pile with test assemblies in place; α_1^2 is the vertical buckling of the pile when all fuel assemblies are of the reference type; α_3^2 is the vertical buckling that would be obtained if all of the reference fuel were replaced by test fuel and if the radial buckling remained constant; $\delta\alpha_2^2 = \alpha_2^2 - (\alpha_3^2+\alpha_1^2)/2$; and β^2 is the radial buckling of the pile. The symbol D refers to the one-group diffusion coefficient, with the subscripts r and z referring to radial and vertical directions, respectively; and $\delta D_i = D_i - D_1$. The radial statistical weights W_{r_i} and U_{r_i} are given by

$$W_{r_i} = \frac{\int_i \phi_{r_i} \phi'_{r_i} \, r dr}{\sum_i \int_i \phi_{r_i} \phi'_{r_i} \, r dr} \tag{2}$$

and

$$U_{r_i} = \frac{1}{\beta\beta'_i} \frac{\int_i (\nabla\phi_{r_i})(\nabla\phi'_{r_i}) \, r dr}{\sum_i \int_i \phi_{r_i} \phi'_{r_i} \, r dr} \tag{3}$$

In equations (2) and (3), ϕ_{r_i} and ϕ'_{r_i} are respectively the radial components of the unperturbed flux and of the perturbed flux in the i^{th} region, and β'_i is the radial buckling in the i^{th} region. The perturbed fluxes were calculated by one-group theory. The remaining quantities in equation (1) are given by

$$\xi = \sum_i W_{r_i} \frac{\delta D_{z_i}}{D_{z_1}} + \sum_i U_{r_i} \frac{\delta D_{r_i}}{2 D_{z_1}} \tag{4}$$

$$\overline{W}_r = W_{r_3} \frac{D_{z_3}}{D_{z_1}} + \frac{1}{2} W_{r_2} \frac{D_{z_2}}{D_{z_1}} - \frac{1}{2} U_{r_3} \frac{\delta D_{r_3}}{D_{z_1}} - \frac{1}{4} U_{r_2} \frac{\delta D_{r_2}}{D_{z_1}} \tag{5}$$

$$\overline{W}_{r_2} = W_{r_2} \frac{D_{z_2}}{D_{z_1}} - \frac{1}{2} U_{r_2} \frac{\delta D_{r_2}}{D_{z_1}} \tag{6}$$

More recent reports[9] by Persson indicate that terms of the form $U\delta D$ in equations (4), (5), and (6) should be omitted. The numerical values of these terms are negligible, and their presence in this analysis has no effect on the experimental results. If the left-hand side of equation (1) is plotted against $\overline{W}_{r_2}/\overline{W}_r$ for increasing numbers of lattice substitutions, a straight line should result with an intercept of $\alpha_3^2-\alpha_1^2$ on the zero abscissa axis. If there were no anisotropy, the value of $\alpha_3^2-\alpha_1^2$ would also be the difference between the material bucklings of the test and reference lattices. Corrections necessary to allow for streaming effects and for extraneous materials in the lattice are discussed in a later section.

The following formulas were used to calculate $\delta D_{r_i}/D_{z_1}$ and $\delta D_{z_i}/D_{z_1}$:

$$\frac{\delta D_{r_i}}{D_{z_1}} = \frac{L_{r_i}^2}{L_{r_i}^2 + \tau_{r_i}} \frac{D_{s_{r_i}} - D_{s_{r_1}}}{D_{s_{z_1}}} + \frac{\tau_{r_i}}{L_{r_i}^2 + \tau_{r_i}} \frac{D_{f_{r_i}} - D_{f_{r_1}}}{D_{f_{z_1}}} \quad (7)$$

$$\frac{\delta D_{z_i}}{D_{z_1}} = \frac{L_{z_i}^2}{L_{z_i}^2 + \tau_{z_i}} \frac{D_{s_{z_i}} - D_{s_{z_1}}}{D_{s_{z_1}}} + \frac{\tau_{z_i}}{L_{z_i}^2 + \tau_{z_i}} \frac{D_{f_{z_i}} - D_{f_{z_1}}}{D_{f_{z_1}}} \quad (8)$$

These formulas have been experimentally verified for air-filled coolant channels in the test region[7] and were taken to be sufficiently accurate for organic and H_2O coolants in the test region.

The radial and vertical values of the slow and fast diffusion coefficients, D_{s_r}, D_{s_z}, D_{f_r}, and D_{f_z}, were calculated by the methods of Benoist[10,11]. The equations used were

$$\frac{D_{s_k}}{\frac{1}{3}\lambda_m} = 1 + \frac{\phi_m}{\phi_t}\left\{\frac{V_c}{V_t} + \frac{V_u}{V_t}\left(1 - \frac{\lambda_m}{\lambda_u}\right) + \frac{V_c \phi_c}{V_t \phi_m}\frac{r_c}{\lambda_m}\left(Q_k^*\right)\right.$$

$$+ \frac{V_c}{V_t}\frac{r_u}{\lambda_m}\left[\frac{\phi_c}{\phi_m}\left(1 - \frac{\lambda_m}{\lambda_u}\right) + \frac{\phi_u}{\phi_m} - \frac{\lambda_m}{\lambda_u}\right] W_k^*$$

$$\left. + \frac{V_u}{V_t}\frac{r_u}{\lambda_m}\left[1 - \frac{\lambda_m}{\lambda_u}\right]\left[\frac{\phi_u}{\phi_m} - \frac{\lambda_m}{\lambda_u}\right] T_k^*\right\} \quad (9)$$

and

$$\frac{D_{f_k}}{\frac{1}{3}\lambda_m} = 1 + \frac{V_c}{V_t} + \frac{V_u}{V_t}\left(1 - \frac{\lambda_m}{\lambda_u}\right) + \frac{V_c}{V_t}\frac{r_c}{\lambda_m}\left(Q_k^*\right)$$

$$+ 2\frac{V_c}{V_t}\frac{r_u}{\lambda_m}\left(1 - \frac{\lambda_m}{\lambda_u}\right) W_k^* + \frac{V_u}{V_t}\frac{r_u}{\lambda_m}\left(1 - \frac{\lambda_m}{\lambda_u}\right)^2 T_k^* \quad (10)$$

The index k denotes either the radial or vertical direction; m denotes moderator; u, fuel; c, void; and t, the entire cell. The symbol λ represents either fast or slow transport mean free paths; Φ, average flux in a region; V, volume; and r, the width of a region. The functions Q^*, W^*, and T^* are modified collision probabilities evaluated by Benoist[10,11]. In equations (9) and (10) a conventional unit cell is assumed to consist of a homogenized cylinder of fuel (UO_2, cladding, and coolant) surrounded by a cylinder of moderator, the two being separated by an annular region of void when applicable. Flux values throughout the cell were obtained by a P-3 calculation[2]. From these diffusion coefficients, radial and vertical values of the diffusion area L^2 and the Fermi age τ were calculated using the expressions,

$$L_k^2 = \frac{D_{s_k}}{\Sigma_{a_{cell}}} \tag{11}$$

and

$$\tau_k = \tau_m \frac{D_{f_{k_{cell}}}}{D_{f_m}} \frac{(\xi\Sigma_s)_m}{(\xi\Sigma_s)_{cell}} \tag{12}$$

where

$$\tau_m = 129.4 \text{ cm}^2$$

These calculated parameters are given in Table II and the results of using them in equations (7) and (8) in Table III.

Data enabling one to plot the Persson equation for each assembly are given in Table IV. A typical plot is shown in Figure 4.

TABLE II

Parameters Used in Perturbation Theory Analysis

Assembly See Fig. 1	Coolant	D_{f_r} cm(a)	D_{f_z} cm(a)	D_{s_r} cm(b)	D_{s_z} cm(b)	τ_r cm²(c)	τ_z cm²(c)	L_r^2 cm²(d)	L_z^2 cm²(d)
1	Organic	1.200	1.203	0.7611	0.7708	118.7	119.0	178.4	180.6
1	H_2O	1.189	1.193	0.7530	0.7649	106.0	106.3	177.9	180.8
1	Air	1.276	1.281	0.8473	0.8513	147.5	147.9	177.1	177.9
2	Organic	1.212	1.213	0.7788	0.7839	125.8	126.0	176.5	177.6
2	Air	1.249	1.249	0.8293	0.8287	141.7	141.7	179.3	179.2
3	Organic	1.176	1.179	0.7388	0.7522	111.0	111.4	117.9	120.0
3	H_2O	1.157	1.162	0.7245	0.7419	92.6	93.0	115.0	117.8
3	Air	1.310	1.321	0.8776	0.8861	162.8	164.1	120.5	121.6
4	Organic	1.163	1.167	0.7285	0.7437	108.3	108.6	100.0	102.1
4	H_2O	1.143	1.149	0.7179	0.7360	88.1	88.6	97.5	100.0
4	Air	1.321	1.334	0.8942	0.9042	171.2	172.6	105.2	106.4
5	Organic	1.139	1.145	0.7002	0.7200	95.2	95.7	99.9	102.7
6	Organic	1.139	1.145	0.7038	0.7223	95.6	96.1	88.3	90.6
6	D_2O	1.216	1.216	0.8202	0.8186	144.7	144.7	95.5	95.4
6	Air	1.386	1.413	0.9407	0.9645	186.6	190.1	102.8	105.4
7	Organic	1.150	1.154	0.7033	0.7198	98.8	99.1	84.5	86.5
8	Polyethylene	1.402	1.446	0.9230	0.9574	183.9	189.4	98.6	102.3
Reference	D_2O	1.222	1.222	0.8197	0.8186	142.9	142.9	111.8	111.7

(a) Calculated by equation (10)
(b) Calculated by equation (9)
(c) Calculated by equation (12)
(d) Calculated by equation (11)

TABLE III

Derived Parameters Used in Perturbation Theory Analysis

Assembly See Fig. 1	Coolant	$\delta D_{z_3}/D_{z_1}$ (a)	$\delta D_{r_3}/D_{z_1}$ (b)
1	Organic	-0.041	-0.050
1	H_2O	-0.050	-0.061
1	Air	0.044	0.038
2	Organic	-0.027	-0.032
2	Air	0.0166	0.0162
3	Organic	-0.059	-0.069
3	H_2O	-0.074	-0.088
3	Air	0.082	0.071
4	Organic	-0.067	-0.079
4	H_2O	-0.082	-0.096
4	Air	0.097	0.085
5	Organic	-0.092	-0.108
6	Organic	-0.089	-0.103
6	D_2O	-0.003	-0.003
6	Air	0.164	0.149
7	Organic	-0.086	-0.098
8	Polyethylene	0.179	0.140

For the intermediate region $\frac{\delta D_{z_2}}{D_{z_1}} = \frac{1}{2}\frac{\delta D_{z_3}}{D_{z_1}}$, $\frac{\delta D_{r_2}}{D_{z_1}} = \frac{1}{2}\frac{\delta D_{r_3}}{D_{z_1}}$

(a) Calculated by equation (8).
(b) Calculated by equation (7).

TABLE IV

Data for Perturbation Theory Analysis

Assembly See Fig. 1	Coolant	No. of Test Assemblies	Change in Vertical Buckling from Ref. Lattice[a] $\alpha - \alpha_1^2$, m^{-2}	Persson Eqn. Data[b] Left-Hand Side, m^{-2}	$\bar{W}_{r_2}/\bar{W}_r$
1	Organic	1	+0.0002	+0.114	2.00
		3	-0.0196	-0.235	1.32
		7	-0.0714	-0.441	0.828
1	H_2O	1	-0.0179[c]	-0.749	2.00
		3	-0.0780[c]	-1.241	1.33
		7	-0.2237[c]	-1.645	0.842
1	Air	1	+0.0358	+1.587	2.00
		3	-0.0911	+1.369	1.30
		7	-0.1990	+1.264	0.794
2	Organic	1	+0.0183	+0.961	2.00
		3	+0.0362	+0.679	1.32
		7	+0.0593	+0.499	0.818
2	Air	1	+0.0396	+1.833	2.00
		3	+0.0974	+1.534	1.31
		7	+0.2171	+1.445	0.800
3	Organic	1	-0.0250	-1.122	2.00
		3	-0.0762	-1.220	1.33
		7	-0.1828	-1.318	0.842
3	H_2O	1	-0.0485	-2.313	2.00
		3	-0.1472	-2.557	1.34
		7	-0.3493	-2.795	0.857
3	Air	1	+0.0174	+0.661	2.00
		3	+0.0470	+0.603	1.29
		7	+0.1087	+0.594	0.790
4	Organic	1	-0.0333	-1.533	2.00
		3	-0.1019	-1.689	1.33
		7	-0.2343	-1.751	0.849
4	H_2O	1	-0.0591[c]	-2.836	2.00
		3	-0.1766[c]	-3.083	1.34
		7	-0.4084[c]	-3.318	0.864
4	Air	1	+0.0079	+0.189	2.00
		3	+0.0165	+0.099	1.29
		7	+0.0360	+0.0838	0.791
5	Organic	1	-0.0666	-3.286	2.00
		3	-0.1943	-3.499	1.35
		7	-0.4421	-3.711	0.870
6	Organic	1	-0.0585	-2.842	2.00
		3	-0.1685	-2.971	1.34
		7	-0.3806	-3.096	0.864
7	Organic	1	-0.0893	-4.551	2.00
		3	-0.2509	-4.695	1.35
		7	-0.5529	-4.915	0.878
6	D_2O	1	-0.0038	-0.176	2.00
		3	-0.0180	-0.299	1.31
		7	-0.0411	-0.298	0.818
6	Air	1	-0.0014	-0.355	2.00
		3	-0.0136	-0.485	1.28
		7	-0.0408	-0.542	0.782
0	Polyethylene	1	-0.0160	-0.978	2.00
		3	-0.0561	-1.123	1.28
		7	-0.1295	-1.129	0.785

(a) Except where noted the following values applied to the one-region reference lattice:

$B_m^2 = 4.3552\ m^{-2}$ (corrected for guide tube worth of $0.100\ m^{-2}$)

$R = 165.77$ cm

$\alpha_1^2 = 2.1507\ m^{-2}$

(b) Persson Eqn.: $$\frac{(\alpha^2-\alpha_1^2)(1+\xi)}{\bar{W}_r} - \beta^2 \frac{\sum_1 (W_{r_1} - U_{r_1})\, \delta D_{r_1}/D_{z_1}}{\bar{W}_r} = (\alpha_3^2-\alpha_1^2) + \delta\alpha_2^2 \frac{\bar{W}_{r_2}}{\bar{W}_r} \qquad (1)$$

(c) Reference lattice properties differ from those in (a) because of small random changes caused by the removal and reloading of the lattice in the PDP.

$B_m^2 = 4.3674\ m^{-2}$ (corrected for guide tube worth of $0.059\ m^{-2}$)

$R = 164.50$ cm

$\alpha_1^2 = 2.1713\ m^{-2}$

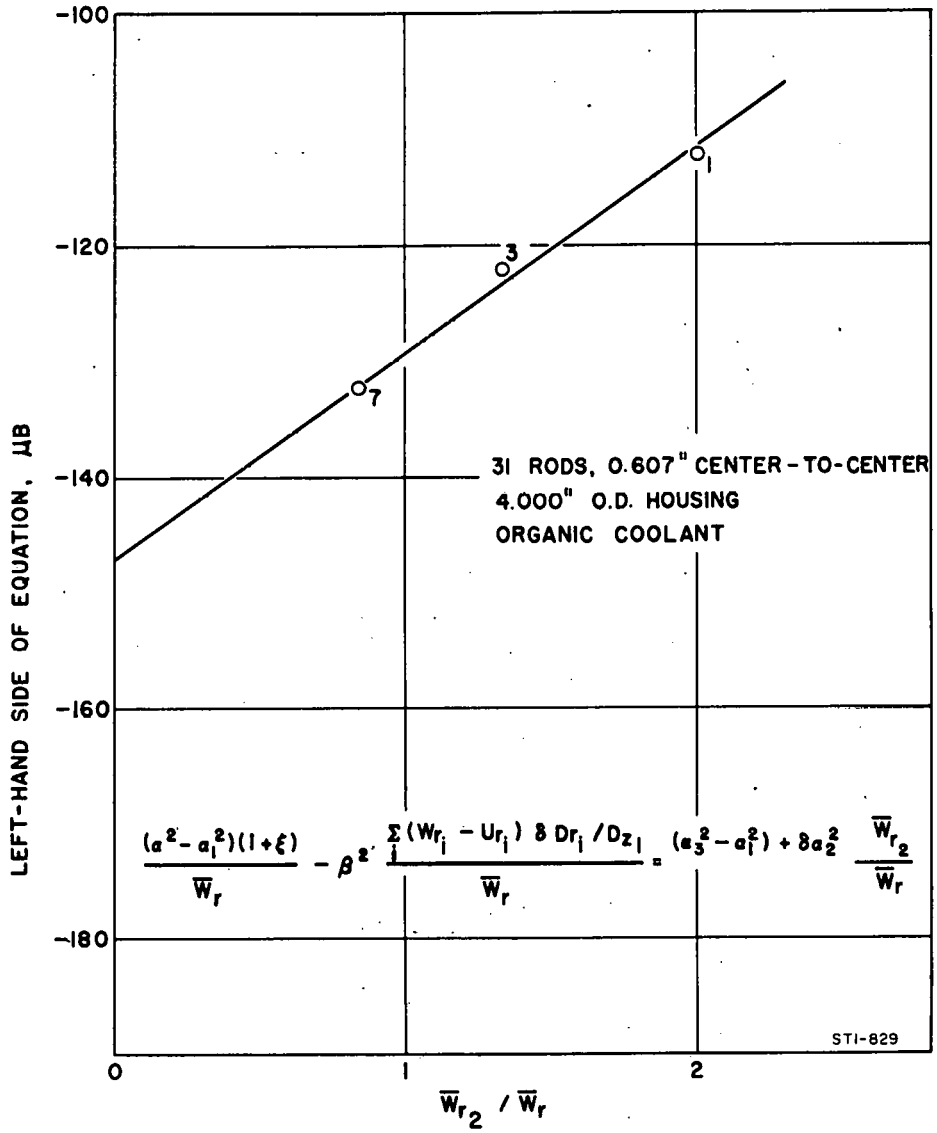

FIG. 4 ONE-GROUP PERTURBATION ANALYSIS PLOT

TWO-GROUP FLUX MATCHING METHOD

The two-group flux matching method[6] required data, in addition to that for the one-region reference pile, only for the case in which the test region consisted of seven assemblies. The test region, shown in Figure 5, was defined in terms of conventional unit cells, in which

$$\text{Cell Area } A = \frac{\sqrt{3}}{2} (\text{pitch})^2$$

rather than the cells used in the perturbation method. The boundary between the test region (a) and the reference region (b) was cylindricized, so that

$$r_1 = \sqrt{7A/\pi}$$

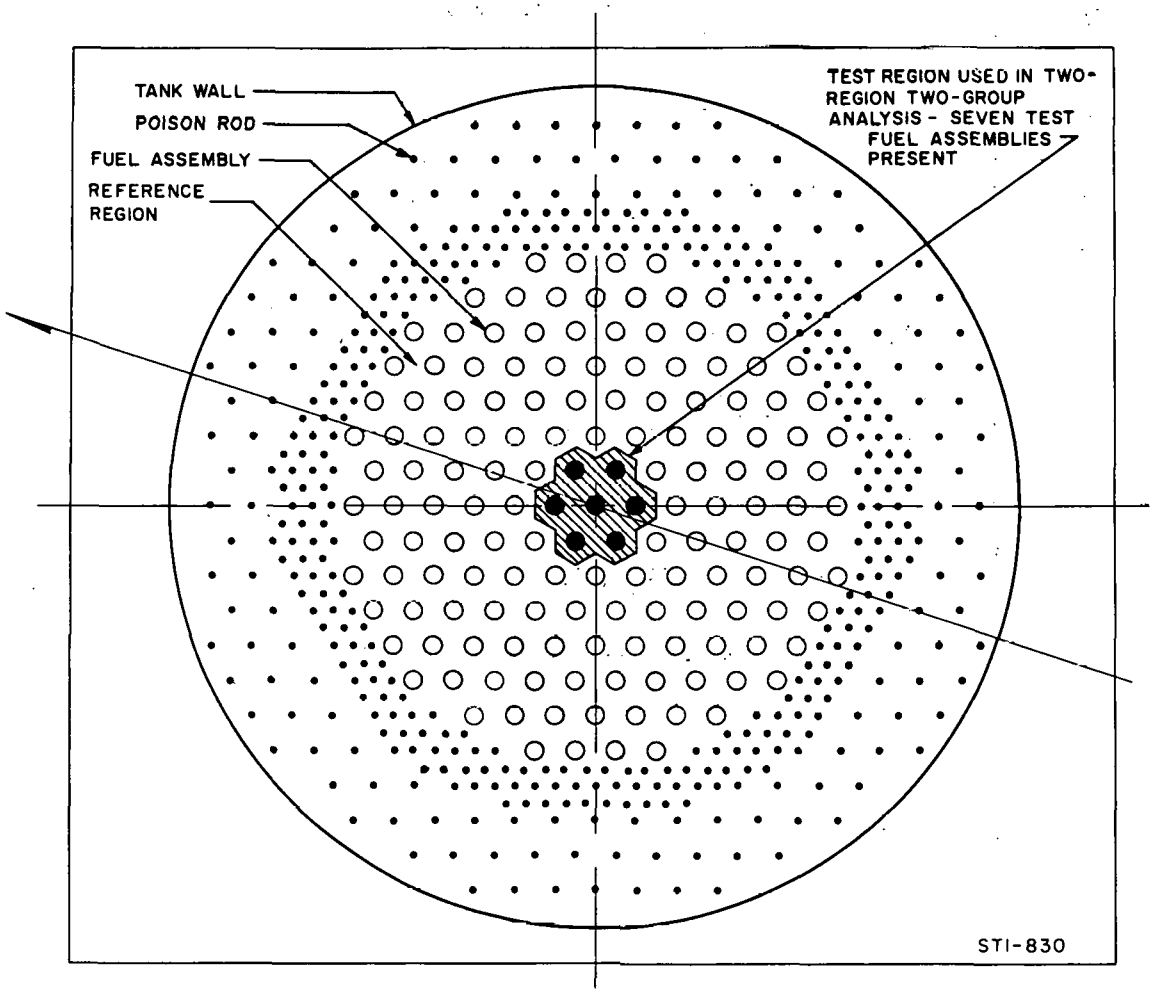

FIG. 5 PILE LOADING DIAGRAM – TWO-GROUP ANALYSIS

The two-group parameters, computed for both the test and reference regions, were the slow diffusion coefficient D_s, the fast diffusion coefficient D_f, the Fermi age τ, the diffusion area L^2, and the resonance escape probability p. With the specification of the material buckling of the reference region B_b^2 (determined in a one-region pile measurement), the critical pile height H, the outer radius of the test region r_1, and the outer radius of the reference region r_2, the problem is completely determined according to two-group theory, and the test region material buckling can be computed.

The two-group calculation proceeds as follows: Radial distributions of the fast flux ϕ_f and the slow flux ϕ_s in region (a) are given as:

$$\phi_{fa} = C_{1a}F_a + C_{2a}G_a$$

$$\phi_{sa} = S_{fa}C_{1a}F_a + S_{sa}C_{2a}G_a$$

where

$$F_a = J_o(\mu_a r)$$

$$G_a = I_o(\nu_a r)$$

$$\mu_a^2 = B_a^2 - \frac{\pi^2}{H^2}$$

$$-\nu_a^2 = -B_a^2 - \frac{\pi^2}{H^2} - \frac{1}{\tau_a} - \frac{1}{L_a^2}$$

$$S_{fa} = \frac{D_{fa}}{\tau_a D_{sa}} \frac{p_a}{\frac{1}{L_a^2} + B_a^2}$$

$$S_{sa} = \frac{D_{fa}}{\tau_a D_{sa}} \frac{p_a}{\frac{1}{\tau_a} + B_a^2}$$

and C_{1a} and C_{2a} are constants, one of which may be arbitrarily chosen.

In region (b) the fluxes take the form:

$$\phi_{fb} = C_{1b}F_b + C_{2b}G_b$$

$$\phi_{sb} = S_{fb}C_{1b}F_b + S_{sb}C_{2b}G_b$$

where

$$F_b = J_o(\mu_b r) - TY_o(\mu_b r)$$

$$G_b = I_o(\nu_b r) - UK_o(\nu_b r)$$

$$T = \frac{J_o(\mu_b r_2)}{Y_o(\mu_b r_2)}$$

$$U = \frac{I_o(\nu_b r_2)}{K_o(\nu_b r_2)}$$

and μ_b, ν_b, S_{fb}, and S_{sb} take the same form as for region (a).

At this stage the neutron current equations are obtained from those of the flux, and these flux and current equations in each region are expressed as matrix products. The appropriate boundary conditions are applied, and B_a^2 is varied until the determinant of the matrix product is zero.

The calculation was coded to enable one to furnish as input the $\eta\epsilon f$ product of the test region and an estimate of B_a^2, rather than p_a. Thus, since both p_a and B_a^2 are unknown, the solution of B_a^2 which makes the determinant vanish is not unique. To provide a unique solution, the code requires the simultaneous solution of the two-group critical equation.

$$p_a(\eta\epsilon f)_a = (1 + L_a^2 B_a^2)(1 + \tau_a B_a^2)$$

The resulting values of p_a and B_a^2 then satisfy both the criticality equation and the flux and current continuity conditions at the boundary.

The values of D_s, D_f, L^2 and τ were the corresponding radial components calculated for the one-group perturbation analysis. The quantity η was assumed to be 1.315 for all lattices, f was obtained from P-3 calculations, and ϵ was calculated as in the SRL Buckling Code[2]. These calculated parameters are listed in Table V.

TABLE V

Parameters Used in Two-Group Flux Matching Analysis

Assembly See Fig. 1	Coolant	D_f (a)	D_s (b)	τ (c)	L^2 (d)	ϵ (e)	f (f)
1	Organic	1.200	0.7611	118.7	178.4	1.024	0.8972
1	H_2O	1.189	0.7530	106.0	177.9	1.022	0.8583
1	Air	1.276	0.8473	147.5	177.1	1.026	0.9590
2	Organic	1.212	0.7788	125.8	176.5	1.025	0.9281
2	Air	1.249	0.8293	141.7	179.3	1.026	0.9584
3	Organic	1.176	0.7388	111.0	117.9	1.037	0.8908
3	H_2O	1.157	0.7245	92.6	115.0	1.014	0.8456
3	Air	1.310	0.8776	162.8	120.5	1.047	0.9635
4	Organic	1.163	0.7285	108.3	100.0	1.031	0.8924
4	H_2O	1.143	0.7179	88.1	97.5	1.028	0.8469
4	Air	1.321	0.8942	171.2	105.2	1.034	0.9662
5	Organic	1.139	0.7002	95.2	99.9	1.029	0.8401
6	Organic	1.139	0.7038	95.6	88.3	1.028	0.8658
6	D_2O	1.216	0.8202	144.7	95.5	1.026	0.9635
6	Air	1.386	0.9407	186.6	102.8	1.032	0.9669
7	Organic	1.150	0.7033	98.8	84.5	1.028	0.8385
8	Polyethylene	1.402	0.9230	183.9	98.6	1.032	0.9585
Reference	D_2O	1.222	0.8197	142.9	111.8	1.028 (g)	0.9569 (g)

(a) Radial value calculated by equation (10).
(b) Radial value calculated by equation (9).
(c) Radial value calculated by equation (12).
(d) Radial value calculated by equation (11).
(e) Calculated as in SRL Buckling Code.
(f) Calculated by P-3 method.
(g) The $\eta\epsilon f$ product of reference region was not used as input for calculation. $p = 0.8760$ was used.

CORRECTIONS

Before either method of analysis was used, the measured vertical bucklings of the one-region reference lattice and of the lattices containing test assemblies were corrected to the condition of a moderator purity of 99.58 mol % D_2O at a temperature of 22.80°C. A calculated temperature correction coefficient of +0.008 $m^{-2}/°C$ was used. Then, since any other buckling change outside of the usual random deviations is due to a change in moderator purity, a plot of the one-region reference lattice buckling versus run number gave the moderator purity correction. These corrections were minor; the purity correction never exceeded 0.015 m^{-2} and the temperature correction never exceeded 0.020 m^{-2}.

Another correction, made after the analysis described above, accounted for the presence of aluminum guide tubes for control rods in the reference region of the lattices. The change in vertical buckling after the insertion of additional guide tubes in the one-region reference lattice was measured. This measured change was then multiplied by the ratio of the calculated statistical weight of the guide tubes already in the pile to the calculated statistical weight of the additional tubes introduced, obtaining a guide tube worth of 0.10 m^{-2}.

When an axial diffusion coefficient difference exists between the test and reference regions, the resulting difference in vertical extrapolation lengths between the two regions should be accounted for. In the case of air-filled test assemblies, this correction was determined from previous one-region lattice measurements[12], which gave the extrapolation length as a function of the ratio of moderator volume to nonmoderator volume in a unit cell. The magnitude of this correction to the test lattice bucklings ranged from 0.01 to 0.07 m^{-2}. In the case of organic- and H_2O-filled test assemblies, no such experimental data were available, and furthermore one expects little change in extrapolation lengths when these coolants are used. Therefore no correction was made for these types of assemblies.

When anisotropy is present, the material buckling is no longer independent of pile geometry. In this case the pile geometry chosen for the presentation of results was one in which the lattice of test assemblies has a minimum critical volume, i.e., when the radial buckling is twice the vertical buckling. The correction needed to put the data in this form was small, never exceeding 0.03 m^{-2}.

A more detailed explanation of these corrections is found in Reference 7.

RESULTS

The results of the measurements are given in Table VI with the previously discussed corrections included. Because there is no reason at present to prefer either method of analysis, the best values were taken to be the average of the results of the two methods. The absolute values of these average bucklings should be good to about ±0.10 m^{-2} in the high buckling lattices and ±0.15 m^{-2} in lattices with bucklings less than 2 m^{-2}.

TABLE VI

Bucklings of Clusters of UO_2 Rods — 9.33-in. Lattice Pitch

Results of Substitution Measurements
Moderator at 22.8°C and 99.58 mol % D_2O

Assembly See Fig. 1	No. of Rods in Cluster	Rod Spacing Center-to-Center, in.	Coolant	Housing Tube Dimensions, in. OD	Housing Tube Dimensions, in. Wall Thickness	Buckling, m^{-2}(a) Pert. Theory	Buckling, m^{-2}(a) Two-Group	Buckling, m^{-2}(a) Average
1	19	0.607	Organic	3.080	0.030	3.53	3.53	3.53
1	19	0.607	H_2O	3.080	0.030	2.06	2.07	2.07
1	19	0.607	Air	3.080	0.030	5.42	5.48	5.45
2	19	0.598	Organic	2.679(b)	0.030	4.54	4.60	4.57
2	19	0.598	Air	2.679(b)	0.030	5.60	5.60	5.60
3	31	0.607	Organic	4.000	0.050	2.89	2.96	2.93
3	31	0.607	H_2O	4.000	0.050	1.17	1.08	1.13
3	31	0.607	Air	4.000	0.050	4.91	4.90	4.91
4	37	0.607	Organic	4.350	0.050	2.45	2.58	2.52
4	37	0.607	H_2O	4.350	0.050	0.64	0.75	0.70
4	37	0.607	Air	4.350	0.050	4.36	4.33	4.35
5	37	0.607	Organic	4.746	0.050	0.28	0.33	0.31
6	37	0.650	Organic	4.746	0.050	1.05	1.16	1.11
7	37	0.650	Organic	4.970	0.162	-0.90	-0.70	-0.80
6	37	0.650	D_2O	4.746	0.050	3.99	4.01	4.00
6	37	0.650	Air	4.746	0.050	3.64	3.65	3.65
8	37	0.650	Polyethylene	5.000	0.056	3.16	3.15	3.16

(a) Organic and H_2O-cooled assembly bucklings not corrected for difference in vertical extrapolation length from reference lattice.
(b) Outer dimension across the flats of a hexagonal housing.

REFERENCES

1. St. John, D. S. and J. W. Wade. "Economics of Heavy-Water Reactors". Nucleonics 20, 12 (1962).

2. Driggers, F. E. and J. C. English. Calculations of Heavy Water Lattice Parameters. E. I. du Pont de Nemours & Co., Savannah River Laboratory, Aiken, S. C. USAEC Report DP-661 (1961).

3. Wade, J. W. A Computer Program for Economic Studies of Heavy Water Power Reactors. E. I. du Pont de Nemours & Co., Savannah River Laboratory, Aiken, S. C. USAEC Report DP-707 (1962).

4. Axtmann, R. C., et al. Initial Operation of the Standard Pile. E. I. du Pont de Nemours & Co., Savannah River Laboratory, Aiken, S. C. USAEC Report DP-32 (1953) (Declassified April 1957).

5. Baumann, N. P., et al. "High Temperature Exponential Measurements and Room Temperature Critical Substitution Measurements on Swedish R3/Adam Fuel Assemblies in Heavy Water". Nukleonik 4, 200-8 (1962).

6. Baumann, N. P. Process Development Pile Measurements of Lattice Parameters of Natural Uranium in Heavy Water. E. I. du Pont de Nemours & Co., Savannah River Laboratory, Aiken, S. C. USAEC Report DP-407 (1959).

7. Graves, W. E. Analysis of the Substitution Technique for the Determination of D_2O Lattice Bucklings. E. I. du Pont de Nemours & Co., Savannah River Laboratory, Aiken, S. C. USAEC Report DP-832 (1963).

8. Persson, R. One-Group Perturbation Theory Applied to Substitution Measurements with Void. A. B. Atomenergie RFX-74 (1961).

9. Persson, R. The Evaluation of Buckling and Diffusion Coefficients from Two-Region Experiments. IAEA Symposium on Exponential and Critical Experiments, SM 42/47, Amsterdam (September 1963).

10. Benoist, P. General Formulation of the Diffusion Coefficient in a Heterogeneous Medium Allowing Cavities. Commissariat à l' Énergie Atomique, Paris, France. Report SPM-522 (1958).

11. Benoist, P. A Simple New Expression for the Radial Diffusion Coefficient for Fueled Channels. Commissariat à l' Énergie Atomique, Paris, France. Report SPM-710 (1962).

12. Graves, W. E., H. R. Fike, and G. F. O'Neill. "Experimental Bucklings and Void Effects in Heavy Water Lattices of Natural Uranium Oxide Rod Clusters". Nucl. Sci. Eng. 16, 186-95 (1963).