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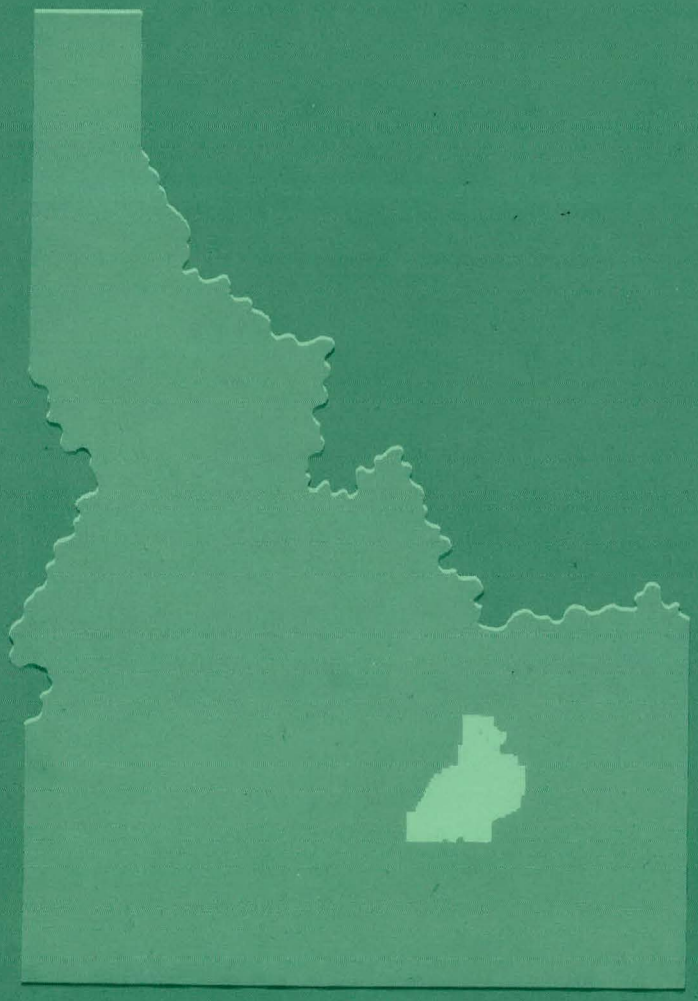
THE DEPENDENCE OF REACTOR BEHAVIOR ON THE SELF-SHUTDOWN MODE

S. G. Forbes

April 25, 1961



PHILLIPS PETROLEUM COMPANY

ATOMIC ENERGY DIVISION

NATIONAL REACTOR TESTING STATION

US ATOMIC ENERGY COMMISSION

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THE DEPENDENCE OF REACTOR BEHAVIOR ON THE SELF-SHUTDOWN MODE

by

S. G. Forbes

Approved:

J. R. Huffman
Assistant Manager, Technical

W. E. Nyer
Manager, Reactor Projects Branch

PHILLIPS PETROLEUM COMPANY
Atomic Energy Division
Idaho Falls, Idaho
Contract AT(10-1)-205

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THE DEPENDENCE OF REACTOR BEHAVIOR ON THE SELF-SHUTDOWN MODE

I. Introduction

One of the primary objectives of the Spert program is to place the evaluation of reactor safety on a consistent, meaningful and quantitative basis. The most idealistic realization of this goal would consist of a universal equation into which each characteristic of a given reactor could be inserted to obtain the figure of merit or safety index for that reactor. In this report a modest beginning has been made in formulating such relationships for the restricted case of reactor self-shutdown for step and ramp reactivity disturbances.

In reactor self-shutdown the reactivity changes resulting from fission energy release during the power excursion frequently involve complicated physical processes which are not sufficiently well understood to permit exact analytical representation. In the absence of such analytical expressions, it is helpful to examine the properties of the reactor kinetics equations for postulated forms of the coupling equation, which relates reactivity to the power behavior, without specifying the underlying physical processes which might give rise to an equation of the assumed form. While this approach begs the question of self-shutdown mechanisms, it can be quite useful in obtaining a general understanding of the way in which the reactor transient behavior is affected by various forms of coupling, and it is with this view in mind that the following discussion is presented.

II. General Comments on Energy Models

One of the simpler general forms for the coupling equation is one in which the reactivity change is assumed to be expressible as a polynomial in the energy release. The polynomial may be conceived as arising in a number of different ways, depending on the reactor type. All of the Spert reactors tested to date have been of the highly-enriched, small-core type in which the principal reactivity changes during a transient are due to a decrease in the moderator density and the consequent increase in fast neutron leakage from the core. These density decreases will be referred to generically as "voids", even though they may arise from such effects

as thermal expansion of the fuel or moderator. In this situation the polynomial expression may be conceived as arising in either of two basic ways:

(1) The void volume is a polynomial in the energy and the void coefficient is a constant. Then,

$$V(T) = g_1 E(T) + \ldots + g_j E^j(T) \quad , \tag{1}$$

and

$$k_c(T) = C_v V(T) \quad . \tag{2}$$

Therefore,

$$k_c(T) = C_v \left[g_1 E(T) + \ldots + g_j E^j(T) \right] \tag{3}$$

where:

$k_c(T)$ = the reactivity change as a function of time T during a power excursion

$C_v = \frac{\partial k_c}{\partial V}$ = the void coefficient of reactivity, a constant here

$V(T)$ = the void volume as a function of time

$E(T)$ = the energy released as a function of time

$g_j = \left(\frac{\partial^j V}{\partial E^j} \right)_o$, a constant.

(2) The void volume is proportional to the energy release and the compensated reactivity is a polynomial in the void volume. Then,

$$V(T) = h_v E(T) \quad , \tag{4}$$

and

$$k_c = g_1 V + \ldots + g_j V^j \tag{5}$$

where:

$$g_j = \left(\frac{\partial^j k}{\partial V^j} \right)_o \text{, a constant,}$$

$$h_v = \frac{\partial V}{\partial E} = \text{void volume per unit energy.}$$

Therefore,

$$k_c(T) = \left[g_1 h_v E(T) + \ldots + g_j h_v^j E^j(T) \right] . \tag{6}$$

The prompt approximation for the reactor kinetics equations following a step insertion of excess reactivity can be written as:

$$\frac{\dot{\phi}(T)}{\phi(T)} \equiv \alpha(T) = \alpha_o - \alpha_c(T) , \tag{7}$$

where

$$\alpha_c \equiv \frac{k_c(T)}{\ell}$$

ℓ = the prompt neutron lifetime

α_o = the initial value of the reciprocal of the asymptotic period corresponding to a step injection

$\phi(T) = \dot{E}(T)$ = power, where the dot denotes differentiation with respect to time.

In either of the cases considered (or a combination of both) the coupling equation can be written in the general form:

$$\alpha_c(T) = \sum_{j=1}^{n} b_j E^j . \tag{8}$$

Equation (7) can be rewritten as

$$\ddot{E} = \alpha_o \dot{E} - \sum_{j=1}^{n} b_j E^j \dot{E} , \tag{9}$$

which can be integrated directly to obtain

$$\dot{E} = \alpha_o E - \sum_{j=1}^{n} \frac{b_j}{j+1} E^{j+1} + C \quad . \tag{10}$$

For transients initiated from low power and zero initial energy, the constant of integration C may be set equal to zero. For simplicity let

$$\alpha_j \equiv \frac{b_j}{j+1} \qquad j = 1, 2 \ldots\ldots, n \tag{11}$$

and absorb the signs of the terms in the general coefficients α_j. Then, rearranging terms in (10) and integrating ,

$$\int dT = \int \frac{dE}{\sum_{j=0}^{n} \alpha_j E^{j+1}} \tag{12}$$

The integrand on the right is a rational function of E and, in principle, can be integrated by separation into partial fractions, provided the roots of the polynomial in the denominator can be found. Thus,

$$T = A_1 \ell n (E-r_1) + \ldots + A_n \ell n (E-r_n) \tag{13}$$

where

r_j = the roots of the polynomial

A_j = the appropriate coefficients, as determined by the α_j.

If E(T) can be found explicitly from (13), analytical expression for $\phi(T)$ and $k_c(T)$ can also be obtained; if not, numerical methods will be required. In many cases certain salient features of the reactor burst behavior will be revealed by examination of the original equations.

III. Considerations of Certain Forms of the Energy Equation

In order to obtain a general understanding of the way in which reactor burst behavior is influenced by the form of the coupling equation (hereafter referred to as the shutdown mode), it is informative to solve the reactor kinetic equations for some of the mathematically simpler shutdown modes and examine the properties of the solutions. The following discussion proceeds from specific cases to more general modes and, for completeness, some shutdown models are included which have been previously treated by the author and by others, as indicated by the references.

A. The Energy Model with No Delay[1]

For step transients initiated from low power, α_o is the initial reciprocal period, which is non-zero in all cases of interest here. If all the other coefficients are zero except that of the highest order term, equation (12) reduces to the form

$$\int dT = \int \frac{dE}{E\,(\alpha_o + \alpha_n E^n)} \quad , \tag{14}$$

which can be integrated directly to obtain an explicit form for the energy as a function of time. That is,

$$E(T) = \left(\frac{\alpha_o}{b_n}\right)^{1/n} (n+1)^{1/n} \frac{1}{\left(1+ne^{-n\alpha_o T}\right)^{1/n}} \tag{15}$$

where

$T \equiv 0$ at the time of peak power

b_n = is the appropriate constant of the type indicated in (8) and will be written hereafter without subscript.

Analytical expressions for the power and reactivity as functions of time have been obtained for this case in Reference 1; however, many of the significant features of the power burst can be obtained directly from the original differential equation or its first integral. For

example, the prompt approximation for the reactor kinetics equation becomes

$$\frac{\ddot{E}}{\dot{E}} = \alpha_o - bE^n \quad . \tag{16}$$

Since $\ddot{E}$ is zero at the time of the power peak, it follows immediately that

$$E_M = \left(\frac{\alpha_o}{b}\right)^{1/n} \tag{17}$$

where the subscript M denotes the value at peak power. Equation (16) integrates to the form

$$\dot{E} - \dot{E}_o = \alpha_o E - \frac{bE^{n+1}}{n+1} \quad , \qquad (E_o \equiv 0) \; . \tag{18}$$

If the final value of the energy (denoted E_f) is defined as that released up to the time the power, $\dot{E}$, returns to its initial value, $\dot{E}_o$, then

$$E_f\left(\alpha_o - \frac{bE_f^n}{n+1}\right) = 0 \quad ,$$

and

$$E_f = \left(\frac{\alpha_o}{b}\right)^{1/n} (n+1)^{1/n} \quad . \tag{19}$$

($E_f = 0$ is a trivial solution)

Thus the energy release at peak and the final energy release are obtainable without recourse to explicit solutions of the kinetics equations; in fact, nothing more need be known about the power behavior other than that it increased from a negligible value, passed through a relatively large maximum, and returned to its initial value.

B. Systems with Initially Positive Coefficients (2)

An interesting example of coupling involving a polynomial in E occurs in systems which have positive coefficients for small energy release, but which eventually display negative coefficients as the energy release increases. The simplest such case leads to an equation of the form

$$\frac{\ddot{E}}{\dot{E}} = \alpha_o + b_1 E - b_2 E^2 \quad . \tag{20}$$

Again, since $\ddot{E}_M = 0$,

$$b_2 E_M^2 - b_1 E_M - \alpha_o = 0 \quad ,$$

and

$$E_M = \frac{b_1 \pm \sqrt{b_1^2 + 4 b_2 \alpha_o}}{2 b_2} \quad . \tag{21}$$

Following the previous example, (20) is integrated to obtain:

$$\dot{E} - \dot{E}_o = \alpha_o E + \frac{b_1 E^2}{2} - \frac{b_2 E^3}{3}$$

and

$$E_f \left(\frac{b_2 E_f^2}{3} - \frac{b_1 E_f}{2} - \alpha_o \right) = 0 \quad ,$$

therefore,

$$E_f = \frac{\frac{b_1}{2} \pm \sqrt{\left(\frac{b_1}{2}\right)^2 + \frac{4 b_2 \alpha_o}{3}}}{\frac{2 b_2}{3}} \quad . \tag{22}$$

These results for E_M and E_f, and other properties of such systems, have been discussed in more detail in Reference 2. Similar treatments can be used on higher order polynomials but with increased analytical difficulty.

C. The Energy Model with Long Delay[1]

The case of a non-linear shutdown with a fixed time delay, τ, long compared to the initial reactor period, has been treated in detail in Reference 1. In making the limiting assumption of long delay, the coupling equation is changed from an implicit function of the energy release, E(T), to an explicit function of time, T, and thus the polynomial form is lost. However, several features of the resulting equation are of interest and will be restated here.

The explicit solution for E(T) has been shown[1] to be of the form

$$E(T) = \frac{\alpha_o^{\frac{n+1}{n}}}{b^{1/n}} e^{\alpha_o \tau} \int_{-\infty}^{T} e^{\alpha_o T} e^{-(1/n)e^{n\alpha_o T}} dT \qquad (23)$$

$$= \left(\frac{\alpha_o}{b}\right)^{1/n} e^{(\alpha_o \tau - 1/n)} q(n,T) , \qquad (24)$$

where

$$q(n,T) \equiv \alpha_o e^{1/n} \int_{-\infty}^{T} e^{\alpha_o T} e^{-(1/n)e^{n\alpha_o T}} dT \qquad (25)$$

The integral in (25) can be expressed in terms of the incomplete gamma function[3] as

$$q(n,T) = (ne)^{1/n} \int_{0}^{n^{-1/n} e^{\alpha T}} e^{-u^n} du \qquad (26)$$

$$= (ne)^{1/n} \frac{\Gamma[1/n, (1/n)e^{n\alpha T}]}{n} \qquad (27)$$

which can be numerically evaluated for some values of n.

The values of the energy at the time of peak, E_M, and the final energy release, E_f, correspond to values at $T = 0$ and $T = \infty$, respectively. These can be expressed as

$$E_M \equiv E(0) = \left(\frac{\alpha_o}{b}\right)^{1/n} e^{(\alpha_o \tau - 1/n)} q(n,o) \qquad (28)$$

and

$$E_f \equiv E(\infty) = \left(\frac{\alpha_o}{b}\right)^{1/n} e^{(\alpha_o \tau - 1/n)} q(n,\infty) \; . \qquad (29)$$

The values of $q(n,o)$ and $q(n,\infty)$ are functions of n only and independent of α_o. The values do not vary greatly over the range $1 \leq n \leq \infty$, as illustrated in Table 1. Therefore, (28) and (29) explicitly express the dependence of energy release on α_o, b, n, and τ, but the dependence on n through $q(n)$ is seen to be weak.

TABLE 1

$q(n,o)$ and $q(n,\infty)$ vs n

n	1	2	∞
$q(n,o)$	1.72 (e-1)	1.42	1
$q(n,\infty)$	2.72 (e)	2.07	1

D. Systems with Multiple Time Constants

In many cases the diffusion of heat into a region exhibiting a shutdown effect can be approximately represented by a lumped parameter system containing a thermal resistance. As an example, consider a system in which heat flows from a fuel plate through a thermal resistance at the metal-water interface into a water moderator. Assume that the reactivity effect is some function of the energy content of the water and that this functional dependence can be expressed in the form of a polynomial. Then the system can be represented by the circuit,

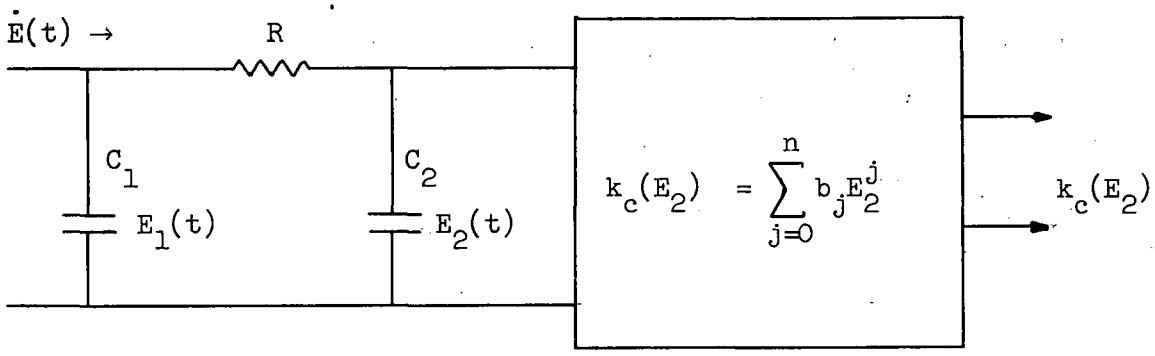

Fig. 1

where:

$\dot{E}(t)$ = the power input to the fuel plate from fission

C_1 = the heat capacity of the fuel plate, assumed constant

C_2 = the heat capacity of the moderator, assumed constant

R = the thermal resistance to heat flow across the interface, assumed constant

E_1 = the energy content of the fuel plate

E_2 = the energy content of the moderator.

Application of conservation of energy and Newton's law of cooling to the network in Fig. 1 leads to the equations

$$E(t) = E_1 + E_2 \tag{30}$$

and

$$R\dot{E}_2 = \frac{E_1}{C_1} - \frac{E_2}{C_2} \quad . \tag{31}$$

Elimination of E_1 from (30) and (31) gives,

$$\dot{E}_2 + E_2 \left(\frac{1}{RC_2} + \frac{1}{RC_1}\right) = \frac{1}{RC_1} E(t) \quad . \tag{32}$$

Integrating (32) and solving for E_2 gives

$$E_2 = \frac{e^{-\lambda t}}{RC_1} \int_0^t e^{\lambda t} E(t)\, dt \quad , \tag{33}$$

where

$$\lambda \equiv \frac{1}{R} \left(\frac{1}{C_1} + \frac{1}{C_2}\right) \quad ,$$

$$E_2(0) = 0 \quad .$$

For illustrative purposes, only the coefficient of the highest order term in the polynomial in E_2 will be taken as non-zero and will be designated as b without subscript. The prompt approximation for the reactor kinetics equation can then be written as,

$$\frac{\ddot{E}}{\dot{E}} = \alpha_o - b \left[\frac{e^{-\lambda t}}{RC_1} \int_0^t e^{\lambda t} E(t)\, dt\right]^n \quad . \tag{34}$$

This can be solved for the two limiting cases of lambda very large and lambda very small. In the case of large lambda, E(t) will be a slowly varying quantity in comparison to $e^{\lambda t}$, and in the limit can be considered as a constant which can be taken through the integral sign to give

$$\frac{\ddot{E}}{\dot{E}} = \alpha_o - b \left[\left(\frac{C_2}{C_1 + C_2} \right) E(t) \right]^n , \tag{35}$$

where the constant of integration has been taken as zero, corresponding to the case of a very low initial power. This equation is the same as (16), previously treated as the energy model with no delay, except for the partition of energy between C_1 and C_2. Lambda very large corresponds to a negligible thermal resistance.

In the case of small lambda, $e^{\lambda t}$ will be slowly varying in comparison to E(t) and (34) can be represented approximately by

$$\frac{\ddot{E}}{\dot{E}} = \alpha_o - b \left[\frac{1}{RC_1} \int_0^t E(t)\, dt \right]^n . \tag{36}$$

This equation can be reduced to dimensionless form by the following substitutions.

Let

$$z(t) = \int_0^t E(t)\, dt \tag{37}$$

$$\psi(x) = z(t) \tag{38}$$

$$x = \alpha_o t \tag{39}$$

$$\zeta(x) = \frac{\psi(x)}{\psi(M)} \tag{40}$$

$$\psi(M) = \text{value of } \psi \text{ when } \ddot{E} = 0, \text{ i.e., at the time of peak power.} \tag{41}$$

Then the following relations exist:

$$\psi(M) = \left(\frac{\alpha_o}{b}\right)^{1/n} RC_1 \tag{42}$$

$$E_M = \dot{\phi}(M) = \alpha_o \psi'(M) = \alpha_o \psi(M) \zeta'(M)$$

$$= \alpha_o \left(\frac{\alpha_o}{b}\right)^{1/n} RC_1 \zeta'(M) \quad , \tag{43}$$

where the primes indicate differentiation with respect to x, and

$$E_f = \alpha_o \left(\frac{\alpha_o}{b}\right)^{1/n} RC_1 \zeta'(\infty) \quad , \tag{44}$$

where ζ is obtained from the equation

$$\frac{\zeta'''(x)}{\zeta''(x)} = 1 - \zeta^n(x) \quad . \tag{45}$$

The solution of (45) to find $\zeta(x)$ can be carried out by the power series method. The initial conditions determine the first few coefficients and the remaining coefficients can be generated from the recursion formula that satisfies (45). Note that $\zeta(x)$ is the dimensionless analog of E_2 and that $\zeta'(x)$ is the dimensionless analog of the reactor energy release E(t).

That $\zeta'(x)$ is bounded for all positive values of x can be shown as follows. From the nature of the physical problem, it can be stated that, since negative values of the reactor power are excluded, z(t) and likewise $\zeta(x)$ are monotone increasing functions. From (45) and the definition of $\zeta(M)$ as the value of ζ at peak power (i.e., $\zeta''' = 0$), it follows that

$$\zeta^n(M) = 1 \tag{46}$$

and, since $\zeta(x)$ is monotone increasing

$$\zeta^n (M-\epsilon) < 1$$

and

$$\zeta^n (M+\epsilon) > 1 \quad . \tag{47}$$

Equation (45) can be integrated to obtain the form

$$\ell n\, \zeta'' = x - \int \zeta^n \, dx$$

or

$$\zeta'' = e^x \, e^{-\int \zeta^n \, dx} \quad . \tag{48}$$

The mean value theorem provides the relationship

$$\int_0^x \zeta^n (x)\, dx = \zeta^n(x_o)(x), \text{ for } 0 < x_o < x \; . \tag{49}$$

Thus (48) can be written as

$$\zeta''(x) = e^{Ax} \tag{50}$$

where

$$A \equiv \left[1 - \zeta^n(x') \right] \quad .$$

Integrating (50) gives

$$\zeta'(x) = \int e^{Ax} \, dx + c \quad . \tag{51}$$

In the bounded region $M > x > 0$, A is everywhere positive from (47), but e^{Ax} is everywhere finite so $\zeta'(M)$ is bounded. In the unbounded region $x > M$, (47) shows that A is everywhere negative and the integral is finite for all $x > M$; therefore, $\zeta'(x)$ is bounded for all $x > 0$ and $\zeta'(\infty)$ is finite.

It is interesting to extend the reactor self-shutdown problem to the case of a linear network in the feedback loop followed by a non-linear function generator of the type considered above. If the number of integrations between the reactor power and the non-linear mechanism is μ, then making the same substitutions as before, the large α_o approximation to the prompt kinetics can be written in the general form

$$\frac{(\mu+1)\zeta}{(\mu)\zeta} = 1 - \zeta^n \quad , \tag{52}$$

where

$$(\mu)_\zeta \equiv \frac{d^\mu \zeta}{dx^\mu} \quad ,$$

that is, the superscript written before the symbol implies successive differentiation of $\zeta(x)$. The general terms for the energy release will be

$$E(M) = \alpha_o^{\mu-1} \left(\frac{\alpha_o}{b}\right)^{1/n} \left[(\mu-1)\zeta(M)\right] F(\lambda_i) \tag{53}$$

and

$$E(f) = \alpha_o^{\mu-1} \left(\frac{\alpha_o}{b}\right)^{1/n} \left[(\mu-1)\zeta(\infty)\right] F(\lambda_i) \tag{54}$$

where

$F(\lambda_i)$ = a function of the system time-constants.

If $\mu = 1$, (52), (53) and (54) correspond to the zero delay model. If $n = 1$ also, then these equations correspond to the simple Fuchs' model.[4]

It will be noted that as α_o increases, the time constants, λ_i, can be treated first as large and finally as small. If the λ_i are well separated from each other, the number, μ, of successive integrations will

increase as α_o increases, and the dependence of E(M) and E(f) on α_o will be increased to successively higher powers of α_o at points corresponding to the poles in the transfer function for the feedback network. It would appear, therefore, that in at least some cases, it should be possible to establish a direct relationship between the transfer function for the feedback loop and the slopes of the energy release vs alpha curves, or for peak-power vs alpha curves for step transients.

General consideration of the nature of the burst behavior shows that ${}^{(\mu-1)}\zeta(M)$ and ${}^{(\mu-1)}\zeta(\infty)$ are not only bounded, but lie between the equivalent functions for the zero delay and long delay cases of the empirical model. In discussing this conclusion, the terms "power" and "energy" will be used to describe the behavior of their dimensionless analogs that satisfy the dimensionless equations which result from the substitutions indicated in (38), (39) and (40). The initial behavior of the power is always an exponential rise of many periods duration for the initial conditions that have been considered in all the previous analyses. The power finally passes through a maximum and subsequently decreases monotonically toward zero. The energy, which is the integral of the power, rises exponentially at the start, has an inflection point at the time of maximum power, and subsequently approaches its final value asymptotically. The first integral of the energy also begins its rise exponentially, but gradually approaches a constant slope as the energy approaches its final value. Successive integrations produce curves that have greater and greater slopes after peak power but which always lie below the extrapolation of their initial exponential rise.

Two items should be noted: first, that the exponential portion of the power rise is assumed to be sufficiently long that all starting transients arising from initial conditions have become negligible; and, second, that in the dimensionless form of the equation all integrals of the initial exponential power rise are identical, that is

$$\int P(x)dx = \int P_o e^x dx = P_o e^x = P(x), \text{ etc.} \qquad (55)$$

Since these integrals represent the reactivity feedback functions, it is clear that the reactivity compensation lies between that described by

the zero-delay model, in which the compensation is proportional to the energy, and that described by the long delay in which the compensation is proportional to the extrapolated exponential rise. The long-delay model is therefore the limiting case for a large number of integrations in reactivity feedback. Since, as soon as the self-shutdown becomes appreciable, the feedback for the long delay model is always higher than that for μ integrations, the power is always lower and the energy is correspondingly lower. Therefore, the long-delay model provides a lower bound for the energy release. Physical considerations make it clear that $^{(\mu-1)}\zeta$ is the dimensionless analog of the energy release. From a practical viewpoint there is no great difficulty in evaluating $^{(\mu-1)}\zeta(M)$ and $^{(\mu-1)}\zeta(\infty)$ by solution of the dimensionless equation (52) on a digital computer.

E. Systems with Power Coefficients

For reactors which have significant heat removal it is necessary to take the energy escape term into account in writing the coupling equation. If the rate of energy escape is proportional to the energy content of the reactor, the kinetics equations for a reactivity step may be written as

$$\frac{\dot{\phi}}{\phi} = \alpha_o - bE^n \qquad (56)$$

and

$$\dot{E} = \phi - \lambda E \quad , \qquad (57)$$

where

ϕ = power,

E = the instantaneous energy content of the core,

λ = the reciprocal of the time constant for escape.

Integration of (57) to find E and substitution into (56) gives the integro-differential equation

$$\frac{\dot{\phi}}{\phi} = \alpha_o - b \left[e^{-\lambda t} \int_{-\infty}^{t} e^{\lambda t} \phi \, dt \right]^n \quad . \tag{58}$$

Solutions are readily obtained for two limiting cases. If lambda is very small, the exponential terms may be treated as constants to give

$$\frac{\dot{\phi}}{\phi} = \alpha_o - b \left[\int \phi \, dt \right]^n \quad , \tag{59}$$

which is the zero-delay model already considered in III. A., above. Small lambda corresponds to negligible energy removal.

If lambda is very large, the reactor power, ϕ, may be treated as a constant to give, after integration,

$$\frac{\dot{\phi}}{\phi} = \alpha_o - b \frac{\phi^n}{\lambda^n} \quad . \tag{60}$$

Large lambda corresponds to the situation in which the heat removal is sufficiently rapid compared to the rate of power change that the energy content of the system is always in quasi-equilibrium. This situation will occur for slow transients or for very rapid coolant flow. Under these conditions the reactor is said to have a power coefficient of reactivity.

Equation (60) can be readily integrated and simplified to obtain the expression for the power as a function of time, which is

$$\phi(t) = \frac{\phi_o}{\left[\left(1 - \frac{b \phi_o^n}{\alpha_o \lambda^n} \right) e^{-n\alpha_o t} + \frac{b \phi_o^n}{\alpha_o \lambda^n} \right]^{1/n}} \quad . \tag{61}$$

At the time of maximum power $\dot{\phi} = 0$, therefore from (60)

$$\phi_M = \lambda \left(\frac{\alpha_o}{b}\right)^{1/n} . \tag{62}$$

Examination of (61) shows that the only non-trivial condition that will satisfy (62) is $t_M \to \infty$; therefore, there is no power overshoot and the power approaches its final value asymptotically.

The energy content of the core at the time of maximum power is simply

$$E_M = \frac{\phi_M}{\lambda} = \left(\frac{\alpha_o}{b}\right)^{1/n} , \tag{63}$$

which is the same result as was obtained for the case of small lambda (i.e., the zero-delay model). In fact, E_M is defined by (56) and is completely independent of the defining equation (57) for E; thus, in the case under consideration

$$E_M = \left(\frac{\alpha_o}{b}\right)^{1/n} \tag{64}$$

for all values of the escape rate, λ. The final energy, E_f, does depend on the definition of E, and in the case of large lambda $E_f = E_M$. Note that, although the energy content of the core is bounded, the total energy produced by the core is unbounded.

IV. Comparison of Ramps and Steps

In many practical cases the concept of a reactivity step is unrealistic and it becomes necessary to consider the reactor transient behavior while the external reactivity addition is still in progress. The simplest such case is that of a ramp in which the reactivity is added at a constant rate called the ramp rate. The analysis of ramp excursions is usually more difficult than that of step transients because of the presence of the external driving function in the kinetics equations. This difficulty can be partially eliminated if a relationship can be found between the ramp rate and an equivalent step which would produce the same

physical result, such as maximum temperature or maximum power. If this equivalence can be established, the simpler, and usually more exact, step analysis can be employed.

A. Ramps with Simple Energy Shutdown

For the simple case in which the reactor self-shutdown is proportional to the energy release, the prompt approximation to the reactor kinetics equation for a combination of a ramp and an initial step can be written as:

$$\frac{\dot{\phi}}{\phi} = \alpha_o + at + \alpha_c(t) \tag{65}$$

and

$$\dot{\alpha}(t) = a - b\,(\phi - \phi_o) \quad , \tag{66}$$

where

$\phi(t)$ = reactor power,

ϕ_o = the initial power,

$\alpha_o = \frac{\Delta k_o}{\ell}$, the initial step divided by the prompt neutron lifetime,

$a = \frac{\dot{k}}{\ell}$, the ramp rate divided by the lifetime,

$b = \frac{C_E}{\ell}$, the energy coefficient of reactivity divided by the lifetime.

Equations (65) and (66) may be combined to give

$$\frac{\dot{\phi}}{\phi} = \alpha_o + at - b\int_0^t [\phi(t) - \phi_o]\,dt \quad . \tag{67}$$

This equation, which involves both the explicit function of time, at, and the implicit function $\phi(t)$ can be reduced[5],[6] to an autonomous form by the following substitutions.

Let

$$P = \frac{\phi(t)}{\phi_o}$$

$$\alpha(t) = \alpha_o + at - bE$$

$$\dot{E}(t) = \phi - \phi_o = \phi_o(P-1)$$

then from (67),

$$\frac{dP}{dt} = \alpha P \tag{68}$$

and

$$\frac{d\alpha}{dt} = a - b\,\phi_o\,(P-1) \quad . \tag{69}$$

Combining (68) and (69) gives

$$\frac{dP}{d\alpha} = \frac{dP}{dt}\,\frac{dt}{d\alpha} = \frac{\alpha P}{a - b\,\phi_o\,(P-1)}$$

or

$$\frac{dP}{d\alpha} = \frac{\alpha P}{a + b\,\phi_o - b\,\phi_o P} \quad , \tag{70}$$

in which the variables are separable, which permits direct integration. That is,

$$\int_{\alpha_o}^{\alpha} \alpha d\alpha = \int_1^P \frac{dP}{P}\,(a+b\phi_o) - \int_1^P b\phi_o dP \quad , \tag{71}$$

which integrates to the form

$$\frac{\alpha^2}{2} - \frac{\alpha_o^2}{2} = (a+b\phi_o)\ \ell n\ P - b\phi_o(P-1) \quad . \tag{72}$$

From (72), the ramp rate and equivalent initial step which will lead to the same value of P_M can be found as follows. At the time of peak power, designated by the subscript M, $\alpha_M = 0$; therefore

$$P_M - 1 - \left(1 + \frac{a}{b\phi_o}\right) \ell n\ P_M = \frac{\alpha_o^2}{2b\phi_o} \quad . \tag{73}$$

First, consider the step case in which the ramp rate is taken as zero, then,

$$P_M - \ell n\ P_M = \frac{\alpha_o^2}{2b\phi_o} + 1 \quad . \tag{74}$$

Similarly, for the ramp case in which the initial step is taken as zero, (73) gives

$$P_M - \left(1 + \frac{a}{b\phi_o}\right) \ell n\ P_M = 1 \quad . \tag{75}$$

If the values of P_M are to be the same in both cases, then subtracting (75) from (74) gives

$$\frac{a}{b\phi_o}\ \ell n\ P_M = \frac{\alpha_o^2}{2b\phi_o} \quad , \tag{76}$$

which can be simplified to

$$\alpha_o = \sqrt{2a\ \ell n \frac{\phi_M}{\phi_o}} \quad . \tag{77}$$

In the above treatment, the self-shutdown effect was taken as the integral of the power increase above the initial equilibrium value, rather than the integral of the absolute power which is used in adiabatic models. This form of the shutdown equation, which is frequently referred to as "the constant power removal model", leads to more realistic analytical results than the adiabatic or "Fuchs' Model" when the initial power level is large compared to the maximum power, since it permits a physically achievable equilibrium initial condition. However, the assumption of constant power removal introduces a term $b\phi_o$ which is analogous to the ramp rate, a, and the resulting equations have all the characteristics of a ramp transient, even for the step case in which $a = 0$.

It will be noted that, although (77) implicitly includes ϕ_M, and therefore is not the desired explicit relationship between α_o and a, the logarithmic term will be very slowly varying for transients initiated from low power, and it is possible to use approximate values of this term to obtain estimates of the equivalent step α_o corresponding to ramp rate a. For example, in the Spert reactors a step transient with $\alpha_o = 10$, initiated from about 1 watt, leads to a maximum power of about 10^7 watts. If this power rise is used as a first approximation for P_M, then (77) gives[7]

$$\alpha_o \cong \sqrt{30\ a} \quad . \tag{78}$$

This form can be used to obtain an approximate value for α_o for any given value of the ramp rate, a. If b is known, (74) and (77) can then be used repeatedly to obtain successive improvements in the estimated value for the equivalent step, α_o. The very slow variation in $\ell n\ P_M$ leads to rapid convergence of the iterative process when P_M is large. Since the derivations above were based on the prompt approximation to the reactor kinetics equation, these relations should not be used unless the resulting value of α_o is greater than that for prompt criticality.

B. Ramps with Nonlinear Shutdown Having Multiple Time Constants

For the more general case in which the reactivity feedback loop involves μ integrations of the reactor power, followed by a nonlinear shutdown mechanism of the power-law type, the prompt approximation to the reactor kinetics equations for a ramp may be written as

$$\frac{\dot{\phi}}{\phi} = at - b\psi^n \tag{79}$$

and

$$\psi \equiv \int \cdots^{\mu} \cdots \int (\phi - \phi_o)\, dt^{\mu} \quad ,$$

where

$\int \cdots^{\mu} \cdots \int$ indicates μ successive integrations of the integrand.

The relationship between the ramp rate, a, and the equivalent step, α_o, cannot be found directly as it was in A., above, since (79) cannot, in general, be reduced to autonomous form. An approximate relationship can be found, however, if the power rise during the ramp is sufficiently great. In this case, it is approximately true that the shutdown effect up to the time of maximum α is negligible in comparison to the reactivity added by the ramp. It is also approximately true that the additional external reactivity addition is negligible during the interval between maximum alpha and maximum power. That is, the burst behavior can be characterized by the behavior which would result from a reactivity step in α equal α maximum. With these approximations in mind, it remains only to show how and under what conditions maximum alpha can be found from (79). In the following discussion the subscript "m" will be used to designate the values of quantities at the time of maximum alpha.

A first order approximation to α_m can be found from (79) by assuming that, at the time of maximum alpha

$$at_m \gg b\psi_m^n \quad , \tag{80}$$

then

$$\frac{\dot{\phi}}{\phi} \cong at$$

and, by integration,

$$\ell n \frac{\phi_m}{\phi_o} \cong \frac{at_m^2}{2} \quad . \tag{81}$$

Since

$$\frac{\dot{\phi}}{\phi} \equiv \alpha(t) \cong at \tag{82}$$

then

$$\frac{at_m^2}{2} \cong \frac{\alpha_m^2}{2a} \quad , \tag{83}$$

and from (81) and (83)

$$\alpha_m \cong \sqrt{2a \ \ell n \frac{\phi_m}{\phi_o}} \quad . \tag{84}$$

It will be noted that the first order result (84) is similar to the expression (77) obtained in the linear shutdown case.

A sufficient condition for the validity of this first order result is that the first order solution must satisfy the assumptions used in its derivation. The conditions under which this is true can be found by the iterative process of using the first order solution for $\phi(t)$ to determine $b\psi^n$ in the original equation (79). If $b\psi^n$ satisfies the condition (80), then the second order correction is small and the first order solution is approximately correct. Condition (80) can be put in a more convenient form by noting that at the time of α_m, $\ddot{\phi} = 0$; therefore,

differentiation of (79) gives for $t = t_m$

$$a = bn\, \psi_m^{n-1}\, \dot{\psi}_m \quad , \tag{85}$$

which can be combined with (80) to obtain

$$(bn\, \psi_m^{n-1}\, \dot{\psi}_m)\, t_m \gg b\psi_m^n \quad ,$$

or

$$\left(\frac{n\dot{\psi}_m}{\psi_m}\right) t_m \gg 1 \quad , \tag{86}$$

as the sufficient condition for the use of (84). If $n = 1$, condition (86) is equivalent to the statement that $\dot{\psi}_m$ is much greater than the mean value of $\dot{\psi}$ in the interval t_o to t_m.

The first order solution for $\phi(t)$ is, from (81),

$$\phi = \phi_o\, e^{\frac{1}{2}at^2} \quad . \tag{87}$$

Therefore, by expansion of the exponential in a power series and from the definition of ψ,

$$\psi = \phi_o \int_0^t \cdots^{\mu} \cdots \int_0^t (e^{\frac{1}{2}at^2} - 1)\, dt^{\mu}$$

$$= \phi_o \int_0^t \cdots^{\mu} \cdots \int_0^t \left[\sum_{j=1}^{\infty} \frac{a^j t^{2j}}{2^j\, j!}\right] dt^{\mu} \quad . \tag{88}$$

Successive integrations of (88) term by term gives

$$\psi = \phi_o \sum_{j=1}^{\infty} \frac{(2j)!}{(2j+\mu)!} \frac{a^j t^{2j+\mu}}{2^j j!} \quad . \tag{89}$$

Differentiating (89), setting $t = t_m$ and multiplying by t_m yields

$$t_m \dot{\psi}_m = \phi_o \sum_{j=1}^{\infty} \frac{(2j)!}{(2j+\mu-1)!} \frac{a^j t_m^{2j+\mu}}{2^j j!} \tag{90}$$

Substituting (89) and (90) in (86) and dividing both numerator and denominator by t_m^{μ} gives

$$(n) \frac{\left[\sum_{j=1}^{\infty} \frac{(2j)!}{(2j+\mu-1)!} \frac{S^j}{j!}\right]}{\left[\sum_{j=1}^{\infty} \frac{(2j)!}{(2j+\mu)!} \frac{S^j}{j!}\right]} \gg 1 \tag{91}$$

where

$$S \equiv \frac{a t_m^2}{2} \cong \ell n \frac{\phi_m}{\phi_o} \quad .$$

Some simplification in interpretation of (91) can be made by performing the first few steps of the division indicated by the ratio of the sums in the numerator and the denominator.

Let

$$X_j \equiv \frac{(2j)!}{(2j+\mu)!} \frac{s^j}{j!} \tag{92}$$

then the division to be made is

$$\frac{\left[\sum_{j=1}^{\infty} (2j+\mu)\, X_j\right]}{\left[\sum_{j=1}^{\infty} X_j\right]} = (2+\mu) + \frac{\sum_{j=1}^{\infty} (2j)\, X_{j+1}}{\sum_{j=1}^{\infty} X_j} \,. \tag{93}$$

Note that from (92) it follows that

$$2j\, X_{j+1} = K_j S X_j \tag{94}$$

where

$$K_j = \frac{(2j+2)\,(2j+1)\,(2j)}{(2j+2+\mu)\,(2j+1+\mu)\,(j+1)} \,. \tag{95}$$

Therefore, the remainder, R_1, in (93) can be expressed as

$$R_1 = \frac{\sum_{j=1}^{\infty} K_j S X_j}{\sum_{j=1}^{\infty} X_j} \,, \tag{96}$$

which can be divided to give

$$R_1 = K_1 S + R_2 \quad , \tag{97}$$

where

$$R_2 = \frac{\sum_{j=1}^{\infty} (K_j - K_1) \; SX_j}{\sum_{j=1}^{\infty} X_j} \quad . \tag{98}$$

In order for R_2 to be positive it is sufficient that $K_j \geq K_1$ since all the other terms in (98) are positive; that is

$$\frac{K_j}{K_1} = \left[\frac{(2j+2)\;(2+2+\mu)}{(2+2)\;(2j+2+\mu)}\right]\left[\frac{(2j+1)\;(2+1+\mu)}{(2+1)\;(2j+1+\mu)}\right]\left[\frac{2j}{(j+1)}\right] \geq 1 \quad , \tag{99}$$

where the grouping of terms has been selected to facilitate reduction. Algebraic reduction of (99) gives

$$\frac{K_j}{K_1} = \left[\frac{(2 + \frac{\mu}{2})}{(2 + \frac{\mu}{2j})}\right]\left[\frac{(1 + \frac{\mu}{2+1})}{(1 + \frac{\mu}{2j+1})}\right]\left[\frac{2j}{(j+1)}\right] \geq 1 \quad , \tag{100}$$

which is clearly satisfied for all values of μ as long as $j \geq 1$ since every term satisfies the inequality; therefore, $R_2 > 0$.

By substitution of (97) and (93) in (91) and from the definitions of S and K_1, a sufficient condition for the use of the (84) is

$$n\left[2 + \mu + \frac{(3)(4)}{(4+\mu)\;(3+\mu)} \; S + R_2\right] \gg 1 \quad . \tag{101}$$

To summarize,

$$\alpha_m \cong \sqrt{2a \ \ell n \frac{\phi_m}{\phi_o}} \quad , \tag{102}$$

if

$$n\left[2 + \mu \frac{\ell n \frac{\phi_m}{\phi_o}}{\left(1 + \frac{\mu}{4}\right)\left(1 + \frac{\mu}{3}\right)}\right] \gg 1 \quad . \tag{103}$$

The demonstration of the conditions under which α_m, as given by (102), can be used as the equivalent step is somewhat unsatisfactory from the standpoint of mathematical rigor. The necessary condition is that

$$t_M = t_m (1 + \epsilon), \quad \epsilon \ll 1 \quad ; \tag{104}$$

that is, that the reactivity addition be negligible during the interval between maximum alpha and maximum power. In this case, reactor power will proceed along essentially the same trajectory as it would for a step of magnitude α_m, and all related quantities such as the energy release to peak will be the same as for the equivalent step.

The argument begins with the observation that in the simple Fuchs' model ($n = \mu = 1$) the peak power is reached within a few periods after the self-shutdown rate first becomes appreciable. It is also noted that for the same power trajectory (monotonically increasing up to the time of peak) successive integrations beyond the first (i.e., $\mu > 1$) lead to functions $\Psi(t)$, which are successively more rapidly increasing functions of time. This can be seen from the appearance of the higher order terms in t when $\phi(t)$ is expanded in a power series in t and integrated μ times. Raising the functions $\Psi(t)$ to the nth power ($n > 1$) introduces even higher order terms in t; therefore, it is apparent that the general model with both $\mu > 1$ and $n > 1$ exhibits a more rapidly increasing shutdown term

than the linear model with $\mu = n = 1$. It follows, therefore, that the general model has a narrower burst width than the linear model, and the time between the first appearance of significant self-shutdown effects and peak power is less than for the linear model.

Let W represent the number of periods required to reach the power peak after shutdown begins. Then, since the self-shutdown rate first becomes appreciable at t_m,

$$t_M = t_m + \frac{W}{\alpha_m} \quad . \tag{105}$$

Setting,

$$\alpha_m = at_m \quad ,$$

then,

$$t_M = t_m \left(1 + \frac{W}{at_m^2}\right) \quad , \tag{106}$$

and since

$$\frac{at_m^2}{2} \cong \ell n \frac{\phi_m}{\phi_o} \quad ,$$

then,

$$t_M = t_m \left(1 + \frac{W/2}{\ell n \frac{\phi_m}{\phi_o}}\right) \quad . \tag{107}$$

Conditions (104) and (105) will thus be satisfied if

$$\frac{W/2}{\ell n \frac{\phi_m}{\phi_o}} \ll 1 \quad . \tag{108}$$

From analysis of step transients it is known that W is not much greater than unity and that introducing a nonlinear shutdown function decreases W. Therefore, the condition to use α_m as the equivalent step is

$$\ell n \frac{\phi_m}{\phi_o} \gg 1 \quad , \tag{109}$$

which is similar to the requirement, (103), for the use of the approximate form (102) maximum alpha. It is suspected, but not proved, that W varies approximately inversely with n and μ, so that (103) is probably a sufficient condition for both (102) and (104), as long as the log power rise is greater than unity.

C. Ramps with Generalized Shutdown Function

A more general formulation of the prompt approximation to the kinetics equation involving a ramp and a self-shutdown mode which can be expressed as a function F of the reactor power is

$$\frac{\dot{\phi}}{\phi} = \alpha_o + at - F(\phi) \quad , \tag{110}$$

or alternatively,

$$\dot{\phi} = \alpha(t)\, \phi \tag{111}$$

and

$$\dot{\alpha}(t) = a - \dot{F}(\phi) \quad . \tag{112}$$

Equations (111) and (112) can be combined to obtain

$$\frac{d\phi}{d\alpha} = \frac{\alpha\, \phi}{a - \dot{F}(\phi)} \quad , \tag{113}$$

which can be rearranged to obtain

$$\frac{d\phi}{\phi}\left[a - \dot{F}(\phi)\right] = \alpha d\alpha \quad . \tag{114}$$

Integration of (104) gives

$$a\,\ell n\left(\frac{\phi}{\phi_o}\right) - G(\phi) = \frac{\alpha^2}{2} - \frac{\alpha_o^2}{2} \quad , \tag{115}$$

where

$$G(\phi) \equiv \int_{\phi_o}^{\phi} \frac{d\phi}{\phi}\,\dot{F}(\phi) \quad . \tag{116}$$

Again, letting the subscript M indicate values of the functions at the time of peak power and remembering that $\alpha_M \equiv 0$, (115) becomes

$$a\,\ell n\left(\frac{\phi_M}{\phi_o}\right) - G_M(\phi) = -\frac{\alpha_o^2}{2} \quad . \tag{117}$$

For a step only (a = 0),

$$G_{MS}(\phi) = \frac{\alpha_o^2}{2} \quad , \tag{118}$$

and for a ramp only ($\alpha_o = 0$),

$$G_{MR}(\phi) = a\,\ell n\,\frac{\phi_M}{\phi_o} \quad . \tag{119}$$

If the equivalence relationship is taken as

$$G_{MS}(\phi) = G_{MR}(\phi) \quad , \tag{120}$$

then

$$\alpha_o = \sqrt{2a \ \ell n \left(\frac{\phi_M}{\phi_o}\right)} \quad . \tag{121}$$

The objection to this development is, of course, that there appears to be no physical justification for using the "equivalence" relation (120), nor does there seem to be any simple interpretation of the physical meaning of $G_M(\phi)$ as defined by (116). In the linear shutdown case it can be shown that

$$G_M(\phi) \equiv \phi_M \quad , \tag{122}$$

but even in this simple case there is no real justification for using equality of peak power as the equivalence relationship between steps and ramps. From a reactor safety standpoint the equivalent quantities should be related to some hazard to the reactor, such as maximum temperature, total energy release, or maximum transient pressure, and these are not necessarily proportional to the peak power.

On the other hand, if the approximate conditions outlined under B., above, are satisfied; that is, if there is no appreciable shutdown up to α_m and no appreciable reactivity addition after α_m, then the power trajectories after α_m are essentially the same for steps and ramps, giving

$$G_{MS}(\phi) \cong G_{MR}(\phi) \quad ; \tag{123}$$

and therefore, (121) is essentially correct.

On these somewhat tenuous grounds, it is concluded that, for systems with shutdown functions which vary at least as fast as the simple Fuchs'

model and for transients initiated from sufficiently low power levels to permit at least several decades of power rise, the equivalence between ramps and steps can be expressed as

$$\alpha_o = \sqrt{a} \quad L = \sqrt{\dot{k}/\ell} \quad L \tag{124}$$

where $L = \sqrt{2\ \ell n\ (\phi_M/\phi_o)}$ is nearly constant.

V. Dependence of the Figure of Merit for a Reactor on the Design Parameters and the Shutdown Mode

For illustrative purposes consider that the figure of merit for reactor safety is to be taken as either the maximum reactivity step, Δk_M, or the maximum reactivity addition rate, $\dot{k}_M$, that a given reactor can tolerate without damage. Assume that the adjustable parameters available to the reactor designer are the void coefficient C_v, the prompt neutron lifetime ℓ, and the maximum energy release E_M, which will just produce core damage; for example, fuel plate melting. Assume further that, from the general nature of a given reactor type, the general form of coupling equation is known. This general form of the coupling equation will be called the "shutdown mode"; e.g., linear energy, nonlinear, multiple time constant, etc. The problem of the reactor designer is to adjust the design parameters to give the highest figure of merit that is compatible with other design considerations. In order to do this, he must have at least a qualitative understanding of the functional dependence of the figure of merit on the design parameters. By way of illustration, these functional dependences will be derived for those cases treated in Section III and Section IV.

The quantities to be determined are Δk_M and $\dot{k}_M$ which will lead to an energy release E_M. These can be obtained from the previous derivations of E_M by use of the following relations:

(a) from the prompt approximation to the kinetics equation,

$$\Delta k_o = \ell\ \alpha_o \ ; \tag{125}$$

(b) from (3) and (8),

$$b_n = \frac{C_v}{\ell} g_n \quad ; \quad \text{and} \tag{126}$$

(c) from (124) and the prompt approximation,

$$\dot{k} = \ell \alpha_o^2 \frac{1}{L^2} = \frac{(\Delta k_o)^2}{\ell L^2} \quad . \tag{127}$$

The shutdown modes to be considered are: positive coefficient, long delay, and multiple time constants. The Fuchs' model and the zero-delay model, which are both special cases of the multiple time constant model, are included for convenience.

The desired functional dependences can be found by combining (21), (25) and (53) with (125), (126) and (127). The results are summarized in Table 2, in which it should be noted that all constants (including L) and other quantities which are not functions of the design parameters C_v, ℓ and E_M, are omitted.

Examination of these results leads to some interesting observations. For example, in the positive coefficient case, if E_M is large, the most effective way to increase the figure of merit is to increase the coefficient C_{v2}, which is the nonlinear term in the expression for the void coefficient, rather than to decrease the magnitude of the initial positive value C_{v1}. Decreasing the prompt neutron lifetime will increase the permissible ramp rate but has no effect on the permissible step.

In the long delay case the only effective controls are through the prompt neutron lifetime ℓ and the delay time τ. Increasing β improves the figure of merit for both steps and ramps, in contrast to the result for the positive coefficient case. For the special case in which the delay time τ is a constant number of reactor periods, $\alpha_o \tau$ becomes a a constant and the functional relationships reduce to the same forms as for the zero-delay model.

TABLE 2

FUNCTIONAL DEPENDENCE OF Δk_M AND $\dot{k}_M$ ON THE PARAMETERS E_M, C_v AND ℓ

FOR VARIOUS SHUTDOWN MODES

Shutdown Mode	E_M	Δk_M	$\dot{k}_M$
Positive Coefficient Model	$\frac{b_1 + \sqrt{b_1^2 + 4b_2 \alpha_o}}{b_2}$	$C_{v2}E_M^2 - C_{v1}E_M$	$\frac{(C_{v2}E_M^2 - C_{v1}E_M)^2}{\ell}$
Long Delay Model	$\left(\frac{\alpha_o}{b}\right)^{1/n} e^{\alpha_o \tau}$	$\ell\left[\frac{\ln\left(E_M\left(\frac{C_v}{\Delta k_M}\right)^{1/n}\right)}{\tau}\right]$	$\ell\left[\frac{\ln\left(E_M\left(\frac{C_v}{\Delta k_M}\right)^{1/n}\right)}{\tau}\right]^2$
Multiple Time Constants Model	$\alpha_o^{\mu-1}\left(\frac{\alpha_o}{b}\right)^{1/n}$	$E_M^{1/\kappa} C_v^{1/n\kappa} \ell^{1-1/n\kappa}$ $(\kappa = \mu - 1 + 1/n)$	$E_M^{2/\kappa} C_v^{2/n\kappa} \ell^{1-2/n\kappa}$
Fuchs' Model $(n = \mu = 1)$	$\frac{\alpha_o}{b}$	$E_M C_v$	$E_M^2 C_v^2 \ell^{-1}$
Zero Delay Model $(\mu = 1)$	$\left(\frac{\alpha_o}{b}\right)^{1/n}$	$E_M^n C_v$	$E_M^{2n} C_v^2 \ell^{-1}$

The general multiple time constant mode is subject to considerable variation through the indices μ and n. Since κ can have any value between $1/n$ and μ, the dependence of the figure of merit on E_M can be either very weak or very strong. The dependence on C_v is always linear or less than linear. The exponent on ℓ is always between zero and plus one for steps, but for ramps may extend from minus one to plus one.

There are, of course, many other considerations to be taken into account, such as the fact that μ may be in itself a function of α, as noted earlier in Section III. If reactors having different shutdown modes are to be intercompared, it will be necessary to evaluate all the constants and, of course, the expressions can only be used for transients which are well into the prompt region.

VI. Conclusion

As was stated at the outset, one of the primary objectives in reactor safety studies is to be able to evaluate the safety of various reactors and reactor types in a consistent, meaningful, and quantitative manner. In the foregoing analysis simplifying approximations and assumptions have been used liberally, and the indices of safety chosen were rudimentary at best. Nevertheless, the results from even this greatly simplified treatment show that the functional dependence of the reactor safety index on the design parameters is strongly dependent on the nature of the reactivity feedback. Therefore, the reactor designer must, at the very least, have a general understanding of the predominant reactivity feedback processes in a given reactor design; otherwise, he will not only be in doubt as to which parameters will produce the most noticeable changes in the reactor dynamic behavior, but will not even be sure whether changing a particular design parameter will increase or decrease the overall safety index of the system. Conversely, even an approximate analysis based on a knowledge of the physical processes involved in the feedback will indicate the directions to be taken in order to produce the most significant gains in reactor safety.

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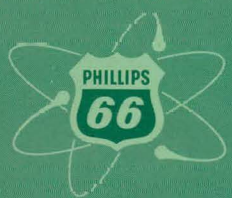

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