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# AN OHMICALLY HEATED HIGH-DENSITY Z PINCH

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## ABSTRACT

The gross properties of a high-density ($n \approx 10^{27}$ m$^{-3}$), small-radius, ($r \approx 10^{-4}$ m) gas-imbedded Z pinch have been examined considering only classical processes. The rate equation using only ohmic heating along with bremsstrahlung and radial heat transport shows that ohmic heating will rapidly take the pinch to thermonuclear temperatures for currents, I, > 1 MA. The radial heat loss for the pinch is very small for I > 1.5 MA. This suggests that the pinch could tolerate being driven to a nearby wall by an m = 1 kink.

The laser technology for initiation of the small-diameter filament and the high-voltage technology for giving a 30-ns rise to a MA or more are available now.

Some reactor considerations have been included.

## I. INTRODUCTION

The density region of $10^{27}$ m$^{-3}$ has been given very little attention in CTR. It appears that recent developments in technology would make this density region worthy of serious consideration for experimental and theoretical study. For these very high densities, the problems to consider are those associated with short times and small radii, which imply very high-voltage, high-current density systems. Modern high-voltage techniques combined with laser initiation could possibly supply the short time scale and small dimensions required to put a dense Z pinch into the reactor regime.

For this report only gross features are considered. Primary consideration is given to a Z pinch with the parameters of particle density n, plasma radius r, pinch current I, and length ℓ:

TABLE I

$n_i + n_e = n = 10^{27}$ m$^{-3}$

$r = 10^{-4}$ m

I = 1 MA with a L/R of 35 ns

$\ell \approx 10^{-1}$ m

pinch time = 100 ns

Assuming only classical processes, an estimate of the temperature reached by the discharge is obtained by considering ohmic heating, bremsstrahlung, and radial heat loss.

## II. REQUIREMENTS FOR MAGNETIC CONFINEMENT

To get an idea of the current required for the Z pinch, first consider only ohmic heating and bremsstrahlung.

$$\frac{dw}{dt} = \frac{I^2\ 5.4 \cdot 10^{-8}}{T^{3/2}\ \pi r^2}\ \mathrm{J\ m^{-1}\ s^{-1}} \qquad (1)$$

is the ohmic heating with ln Λ set equal to 10.[1] The bremsstrahlung radiation loss is given by

$$\frac{dw}{dt} = 4.8 \cdot 10^{-37}\ n^2 T^{1/2} \cdot \pi r^2\ \mathrm{J\ m^{-1}\ s^{-1}}. \qquad (2)$$

When these are equated with the gas particle energy 3 nkT, the temperature rate equation is given by

$$\frac{dT}{dt} = \frac{3.3 \cdot 10^{8}\ I^2}{(\pi r^2)^2\ nT^{3/2}} - 3 \cdot 10^{-21}\ nT^{1/2} \qquad (3)$$

with T in keV and mks units are used otherwise.

For pressure equilibrium with the magnetic field the Bennett relation is used:

$$\pi r^2 n T = 3 \cdot 10^{8} I^2 .$$

If the current is programmed to give pressure balance at all times Eq. (3) becomes

$$\frac{dT}{dt} = \frac{1.1}{\pi r^2 T^{1/2}} - 3 \cdot 10^{-21} n T^{1/2} . \qquad (4)$$

The solution for this equation is

$$Bt = -2\sqrt{T} + \sqrt{\frac{A}{B}} \ln \frac{\sqrt{\frac{A}{B}} + \sqrt{T}}{\sqrt{\frac{A}{B}} - \sqrt{T}}$$

with $A = \frac{1.1}{\pi r^2}$ $\quad B = 3 \cdot 10^{-21} n$ .

A plot of T vs t is shown in Fig. 1.

The equilibrium temperature is found by setting $\frac{dT}{dt} = 0$ giving

$$T_{eq} = \sqrt{\frac{3.7 \cdot 10^{20}}{\pi r^2 n}} .$$

The corresponding current is

$$I = 1.1 \cdot 10^{6} \text{ A}.$$

This current, a constant for all pinches under the above assumptions was first noted by Pease.[2] For larger currents the pinch collapses, while for small currents the pinch blows up.

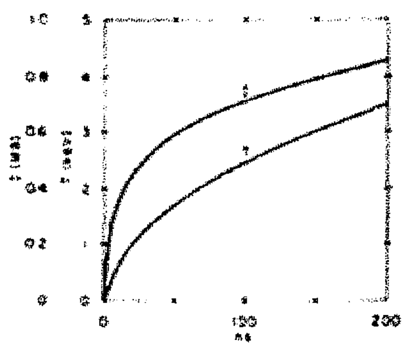


Fig. 1. Current and temperature for equilibrium pinch, $n_e = n_i = 10^{27}$ $m^{-3}$, $r = 10^{-4}$ m.

III. RADIAL HEAT CONDUCTIVITY

The situation is altered for a gas-imbedded pinch because of the radial heat loss. Taking the radial heat conductivity to be given by ion-ion collisions the heat flow as given by Schmidt[3] becomes

$$Q = 3/2 \, k \, \nabla T \, \frac{R_c^2}{\tau_c} \cdot n \quad J\text{-}m^{-2}\text{-}s^{-1}$$

where $R_c$ = larmor radius

$\tau_c$ = ion-ion collision time.

For the temperature drop across the radius of the pinch set equal to the pinch temperature

$$\frac{dW}{dt} = \frac{4 \cdot 10^{-24} \sqrt{T} \, n^2 r^2}{I^2} \quad J \cdot m^{-1} \, s^{-1} . \qquad (5)$$

W is energy per meter, and T is in keV. When this loss is added to Eq. (4), the Bennett equilibrium can no longer be satisfied. With these estimates of rates, the ohmic input is just about equal to the radial heat loss when pressure balance is assumed.

Of course, there is no requirement for exactly satisfying the Bennett relation so if the current rises somewhat faster than that shown in Fig. 1 the radial heat loss is reduced below the ohmic heating rate. With radial heat loss included, Eq. (3) becomes

$$\frac{dT}{dt} = \frac{I^2 \, 3.6 \cdot 10^{7}}{r^4 \, n \, T^{3/2}} - 3 \cdot 10^{-21} n T^{1/2} - \frac{4 \cdot 10^{-9} n T^{1/2}}{I^2} . \qquad (6)$$

From Eq. (5) it is seen that the radial heat loss is less than the bremsstrahlung loss for I > 1.1 MA. The temperature rise given by Eq. (6) is shown in Fig. 2 for the parameters of Table I. Any implosion heating is ignored. Voltage and impedance required to give the current rise used are available in machines now in use at Sandia and in the future at L-Division, LASL.

The D-D neutron yield for this case would be $6 \times 10^{12}$. D-T yield for this case would be about 100 times as large and would give a nuclear yield of $1.6 \cdot 10^{3}$ joules. The losses to radial heat

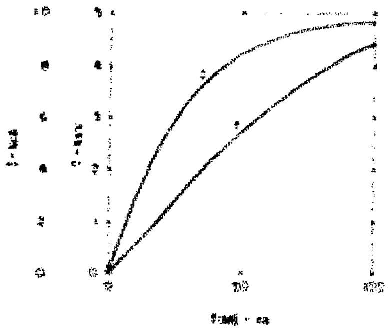

Fig. 2. Temperature and current for nonequilibrium pinch driven by 1-MV, 1-MA line. $n = 10^{27}\ m^{-3}$, $r = 10^{-4}$ m.

conduction and bremsstrahlung are $7 \cdot 10^3$ joules and $2.1 \cdot 10^3$ joules, respectively. The plasma energy is $4 \cdot 10^3$ joules. The energy from the driving line is about 100 kJ.

## IV. INSTABILITIES

The only instabilities to be considered will be MHD. The two modes which are the most dangerous are $m = 0$ and $m = 1$.

In the typical "low"-density pinch experiment, the current forms in an imploding sheath and this configuration is subject to an $m = 0$ sausage formation. If, however, a more gentle distribution of pressure can be formed the $m = 0$ will be stabilized. The prescription for stability is

$$- \frac{d \ln p}{d \ln r} < \frac{4\gamma}{2 + \gamma\beta} \quad \text{with } \beta = 2\,\mu_0 p/B^2 \ . \qquad (7)$$

For $\gamma = 5/3$ the pressure must not fall off more rapidly than $r^{-10/3}$ (Ref. 4).

The current will be distributed throughout the pinch because of the small radius of the pinch and the fact that the current is established before the highest temperatures are reached. The resultant pressure distribution could very well be within the stable region.

The $m = 1$ problem is quite different. For this instability, two favorable effects are of importance. The first is the slowing of the kink growth because of the surrounding dense gas. This problem has been examined by Lampe, Manheimer, and Boris.[5] The kink growth is slowed by shock wave production in the surrounding gas. It is proposed here, however, that the most important effect is that for large I and small r, the heat loss as given by Eq. (5) is smaller than the ohmic heating even when the discharge is resting against a cold surface. Therefore in the framework of classical processes it appears that the kink could be tolerated. The kink will grow slowly and the plasma column will finally rest against a nearby wall and still maintain kilovolt temperatures.

## V. ELECTRON DRIFT SPEED

For the parameters of Table I the conduction electron speed will be of the order of $2 \times 10^5$ m/s. This puts it well below the average thermal electron speed. The electrons drift only 2 cm during the 100-ns time of the discharge.

## VI. DATA FROM HIGH-DENSITY EXPERIMENTS

The two experiments which are of interest are those of Sears,[6] and the NRL group.[7] Hartman is conducting a high-density, gas-imbedded pinch experiment, but using e-beam initiation.[8] The experiment of Sears was a slow experiment in 1 atm of $H_2$ at $10^5$ A. It showed only the $m = 1$ kink instability which was inhibited by shocks driven in the surrounding gas.

The result which is most pertinent for the experiment proposed in this report is that of R. E. Pechacek[9] with a laser-initiated high-density pinch. In this experiment, a pinch driven by a slow bank (~ 30-μs rise) was initiated over a 60-cm length by a 70-joule neodymium glass laser. The experiment was performed with 1-atm $H_2$. The original discharge diameter was probably about 100 μm in diameter. The discharge rapidly grew in diameter as would be expected from the slow rise in pinch current. The small radius in the early phase of the Pechacek experiment is most encouraging and of great importance for high-density research. Discharge initiation in 20 atm with a similar diameter discharge should be possible with similar or slightly smaller laser power.[10]

## VII. EXPERIMENTAL PROPOSAL

The conditions of Table I are tailored for a high-voltage line such as Hydra at Sandia and an initiation laser pulse from a 70-joule neodymium glass laser. The configuration is shown in Fig. 3. The size of the polyethylene cylinder is that derived from the J. C. Martin notes for these experimental conditions, i.e., 1 MV and 100 ns.

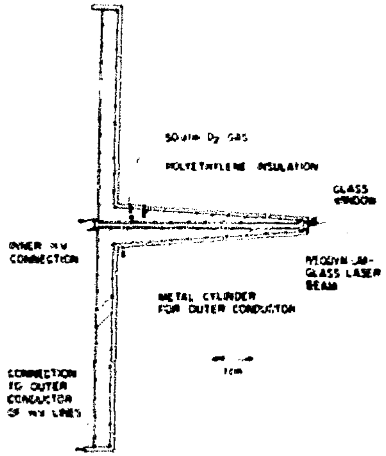


Fig. 3. Possible pinch configuration for 1-MV line.

The simplicity of the experiment is evident; however, many important facts could be determined from this modest effort. Of great importance are:

1. Initiation of the discharge and resultant pinch diameter
2. Stability of the pinch and particularly the effect of the kink on the discharge
3. The heat loss from the pinch and the resultant corona[11] surrounding the discharge
4. The effects of nearby wall on pinch containment
5. The many surprises at this density.

## VIII. REACTOR SCALING

A very rough idea of the current required for break-even can be found from equating ohmic heating with bremsstrahlung loss since the rate calculations [Eq. (6)] show that in small radii the temperature will be approached within the assumed pinch time. The radial heat loss is neglected above 1.5 MA. This gives the relation between I and n with a required temperature of 10 keV.

$$n^2 = \frac{I^2}{r^4}\ 1.2 \cdot 10^{26}\ .$$

The nuclear yield for D-T is

$$Y = 1/2\ n^2\ \langle\sigma V\rangle.\ (\text{energy/reaction})\ \text{vol} \cdot t$$

$$= 1/2\ n^2\ (11.10^{-17} \cdot 10^{-6})$$

$$\cdot\ 19 \cdot 10^6 \cdot 1.6 \cdot 10^{-19}\ \text{vol} \cdot t$$

$$= \frac{7.2 \cdot 10^{-8}}{r^2}\ \ell \cdot t\ I^2.$$

The input energy goes primarily into the inductance of the pinch. This energy is given by

$$W_{In} = 1/2\ L_o\ \ell \cdot I^2.$$

This makes a $Y/W_{In}$ ratio

$$\frac{Y}{W_{In}} = \frac{1.4 \cdot 10^{-7}\ t}{r^2\ L_o}\ .$$

For $r = 10^{-4}$ m, $L_o = 0.7\ \mu H - m^{-1}$

$$\frac{Y}{W_{In}} = 2.10^7\ t\ .$$

The yield to input ratio being proportional to time is the result if the pinch is crowbarred at design current. For energy in the field equal to yield,

$t = 50$ ns.

Using the scaling as a guide, the following sample reactor parameters were calculated.

$n = 2.5 \cdot 10^{27}\ m^{-3}$

$r = 10^{-4}$ m

$V = 5$ MV

$I_{max} = 5$ MA

Length = 50 cm

The results of the actual calculation for the temperature, yield, etc., with actual circuit conditions are

T = 13.8 keV

Nuclear yield = 6.5 MJ

Plasma energy = 17 kJ

Energy in field = 2.5 MJ

*Radial heat loss* = 1 kJ

Bremsstrahlung loss = 6.5 kJ

End loss by streaming in the θ field is slower than a field-free streaming and perhaps a few percent end loss is to be expected in the 150-ns containment time in the 0.5-m pinch length.

This report cannot give reactor design considerations other than the most elementary comments. Certain features can be tabulated:

1. The yield to input ratio is very favorable for this small-radius, high-density pinch.
2. The 5-MV, 5-MA high-voltage sources assumed in the calculation do not require a great advance over those now in existence.
3. Radiation damage problems are not severe because renewable parts are reasonable in such a small volume.

4. Reactor feasibility experiments are not greatly different in size from reasonable first experiments.

## IX. CONCLUSIONS

The great advantage in CTR of an ohmically heated, high-density pinch is evident. Of course, many questions on the feasibility of such a system cannot be answered with present data. The experiments needed to investigate the high-density pinch are not expensive and do not require extraordinary advances in technology. The laser initiation phase could be done on a modest scale with neodymium glass lasers now available at LASL. The first phase of a high-voltage discharge experiment could also be done with a slight modification to present e-beam power supplies at Sandia or even at LASL.

It is felt that the main questions (can the small-diameter pinch be established and can it survive the kink by means of magnetic field insulation?) will be answered only by experiment.

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