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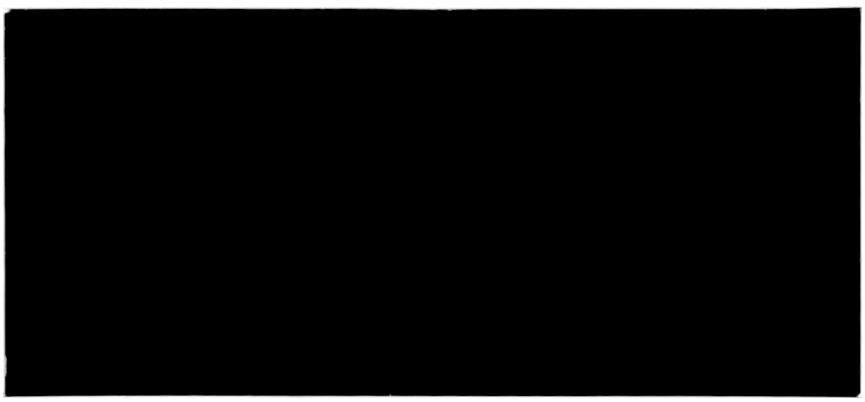

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HETEROGENEOUS REACTOR CALCULATION METHODS

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ABSTRACT

Considerations pertaining to the effect of the finite size of the fuel elements - large source theory - on the kernels used in heterogeneous calculations are explored. The use of kernels going from the spatially distributed sources in one fuel element to spatially distributed sinks within another fuel element permits one to consider complex fuel elements with detailed interior structure. Self-effects of the fuel elements are calculated for some simple geometries. The effect of rod interactions on the kernels is also considered and a simple homogenization recipe is given to take such interactions into account to first order.

A comparison is made between heterogeneous calculations and experiments on critical configurations in a graphite lattice. The calculated reactivities agree with the experimental values to the extent expected.

Heterogeneous calculations of control rod configurations in a graphite lattice are presented. The results indicate that both in reactivity and in power distortion, control rod effects are quite sensitive to the control rod pattern.

Further calculations on spiking effects are given.

Section 1. Introduction

Heterogeneous calculations up to the present have been "small source" calculations. The kernel functions have been obtained from a line source and the resulting inaccuracies in the calculated flux have been partially nullified by calculating the fuel element parameters from experimental quantities by the self-consistent method.[1]

We have explored the possibility of a more accurate treatment of the kernels which takes into account the finite size of the sources and sinks. The use of such kernels we shall call "large source theory". There are at least two classes of problems where large source kernels will be important. First, for fuel elements with a complicated internal structure. For such fuel elements the self-effects, i.e., the kernel values from a fuel element to itself, depend on the detailed structure of the element. Second, for lattices which are not loosely-packed, the finite size of the elements is important.

Large source calculations add an important degree of realism to the heterogeneous reactor physics method and extend its applicability to new problems. The general approach is to solve two separate problems: The internal flux distribution within a fuel element which gives the sources from which the kernels are to be calculated, and the interaction between individual fuel elements, which is treated by the heterogeneous formalism previously developed.[1] It seems reasonable that unless

the lattice is very close packed these two problems can be separated.

The basic ideas of large source calculations are developed in Section 2 of this report. The concept of the self-effect of a fuel element is developed by rewriting the heterogeneous equations in a form where absorptions are labelled by the rod of neutron origin. It is found that at least two alternative ways exist to express the equations. In Section 2.3 the heterogeneous equations are derived from the basic differential equations in a way that explicitly introduces the finite distribution of sources and sinks within each element. The resulting formulas are applied to the calculation of large source kernel self terms in Section 2.4. Significant differences between small and large source kernels are obtained even when the fuel elements are simple rods.

The basic approach has been to calculate the asymptotic flux,[2] i.e., the flux that would be present if the fuel element were replaced by moderator, while its mathematical source and sink effects are retained. The large source kernels describe the rod in a homogeneous moderator. It is shown in Section 2.5 that the neglect of interference effects between rods in these kernel functions is justified when the interference consists of neutron absorption alone. The effect on the kernels of scattering and slowing down by the other rods must, however, be introduced. This can be done, to first approximation, by replacing the real moderator by a fictitious homogenized material that includes nuclear effects of both moderator and fuel element.

In Section 3 a comparison between HERESY 1 results and experimental results is made. The measured reactivity of near critical configurations of the Brookhaven Graphite reactor[3] is compared with HERESY 1 heterogeneous results[4] for a similar configuration. The experimental value and the calculated reactivity values agree to within one-half percent, which is within the difference to be expected between the experimental configuration and the somewhat simplified configuration that was calculated.

An important application of heterogeneous reactor physics is in calculating control rod effects. A study of reactivity and power distribution effects of alternative patterns of control rod placement in a graphite lattice of natural uranium rods is presented in Section 4. Various patterns are considered and it is found that one cannot predict a priori which pattern will lead to the largest reactivity difference or which pattern will lead to the greatest degree of flux peaking. One concludes that the conventional homogeneous calculations of control rod patterns are insufficient for design purposes, and that heterogeneous calculations are necessary.

In Section 5 further work on the effects of spiking on reactivity and average-to-maximum power in graphite lattices is reported. These results indicate that the "something-for-nothing" region,[5] in which one can increase the core lifetime while maintaining a given average-to-maximum power by appropriate spiking, is even greater than previously found.

Section 2. Large Source Calculations

2.1. Basic Considerations

The heterogeneous calculations we have carried out to date may be termed small source calculations. By small source calculations we mean (1) that both source and sink kernels are derived from a line source of zero radial thickness, (2) that the finite size of the fuel elements enters only in the sink kernel $f(R_o)$ at the rod surface and in the method of evaluating[6] the coefficient γ, (3) that no account is taken of the presence of other fuel elements on the kernel for the fuel element in question. It is important in some problems to take the finite size of the fuel elements into explicit account in the calculations. It is also important to examine in closer detail the assumptions inherent in small source calculations in order to understand to what extent the calculations can be generalized towards what we shall call "large source calculations" in which the finite volume of the fuel elements will explicitly affect the kernels.

The finite valume of the fuel element will affect the calculation in several ways:

a. Fuel element "self effects" must be considered. By the self-effect we mean (1) explicit consideration of those absorptions in the element due to neutrons born in the element and thermalized in the surrounding moderator (including moderator within the fuel element itself, as a coolant), (2) explicit consideration of the effect of

the absorption pattern within the rod on the flux depression due to the rod.

b. The effect of the finite size of the source distribution within the rod on the rod-to-rod source kernel[7] F_{nm} must be considered. The kernels do not realistically come from a line source but from a distributed source within the fuel element. There is a similar effect for the sink kernel, since it can be regarded as coming from a distributed absorption pattern.

c. The effects of displacement of moderator by fuel volume and the effects of the particular configuration of fuel elements on the kernel functions for each fuel element should be considered.

One would expect effects a and b to be particularly important when the fuel element contains sizeable amounts of moderator. Consider, for example, a fuel element consisting of a pressure tube containing a number of fuel pins immersed in a coolant. If we wish to treat this complex structure as a single entity it is necessary to take account of the moderating effect of the coolant inside the tube, it is necessary to specify that the fissions take place within the fuel pins, and it is necessary to consider the distribution of absorptions between fissionable material and structural material within the fuel element. If one wishes to treat this complex structure as a single "black box" entity, the

self-effects and kernel effects of the "black box" fuel element must be properly considered.

Furthermore, such a complex fuel element is apt to be of appreciable size relative to the spacing of the elements within the core. The interactions between these fuel elements will be appreciable. This is the effect c.

In Section 2.2 self-effects of the fuel element will be introduced into the formalism of heterogeneous calculations. In Section 2.3 the assumptions of small source theory will be examined. The requirements on the kernels for generalization of the procedure to large sources will be specified. Various methods for obtaining large source kernel values will be considered. In Section 2.4 some calculations of large source kernel functions will be given, in particular the kernel values related to self effects. In Section 2.5 the effects of rod interactions in the kernels are considered. One useful prescription for homogenizing non-absorption interference effects is given, and speculations on other possible treatments are made.

Section 2.2 Self Effects in Heterogeneous Calculations

In order to consider self effects it is convenient to label the absorption rates i_m in the individual rods in a more detailed way. Let i_{mn} be the absorption rate in the m-th rod of fission neutrons which are emitted from the n-th rod. Then i_{nn} will be the self absorption rate in the n-th rod, i.e., the

absorption rate in the n-th rod of fission neutrons emitted from the n-th rod. We shall assume for convenience in this discussion that all fissions and absorptions take place at thermal energy, though this assumption is not essential. Thus i_{mn} gives the absorption rate of fission neutrons emitted from rod n, thermalized in the moderator, (both inside and outside the fuel element considered) and then absorbed at the same rod n. The following relation holds between i_n and i_{nm}:

$$i_n = \sum_m i_{nm} \tag{2.1}$$

One can write the heterogeneous equations in terms of the detailed absorptions i_{nm} as follows:

$$\gamma_n \, i_{nm} = \frac{\eta_m}{k} \, i_m \, F_{nm} - \sum_{s=1}^{N} i_{sm} \, f_{ns} \tag{2.2}$$

$$n = 1, 2, \ldots N$$

$$m = 1, 2, \ldots N$$

We shall first assume that this equation is also valid when n = m.

If one sums each side of (2.2) over all values of m, i.e., over all rods from which neutrons are emitted to reach rod n, one obtains the heterogeneous equations that have been used previously.

$$\gamma_n \, i_n = \sum_m \frac{\eta_m}{k} \, i_m \, F_{nm} - \sum_m i_m \, f_{nm} \tag{2.3}$$

Equation (2.2) can be written for the self-absorptions i_{nn}.

$$\gamma_n \, i_{nn} = \frac{\eta_n}{k} \, i_n \, F_{nn} - \sum_{s=1}^{N} i_{sn} \, f_{ns} \tag{2.4}$$

With this formulation the problem of including self effects becomes one of using appropriate values of the kernels, F_{nn} and f_{nn}.

The problem can also be formulated in another way. We now assume that equation (2.2) is valid when $n \neq m$. When $n = m$ we write the following equation to replace (2.4).

$$\gamma_n \, i_{nn} = \frac{\eta_n}{k} \, i_n \, c_n \, \gamma_n - \sum_s{}' \, i_{sn} \, f_{ns} \tag{2.5}$$

The primed summation is over all rods but the n-th. c_n is the probability that a fission neutron emitted from rod n would return and be absorbed at rod n if no other rods were present. Thus both source and sink effects are contained in c_n. The quantity $\frac{\eta_n}{k} \, i_n \, c_n$ is the number of absorptions due to neutrons emitted from the rod and returning to it. Multiplying this by γ_n gives the flux that must be present when this number of absorptions takes place. The primed summation gives the sink effect due to absorptions in the other rods of neutrons emitted from rod n.

If one sums each side of (2.2) over all values of m except $m = n$, and takes equation (2.5) when $m = n$ one obtains the following equations:

$$\gamma_n i_n = \frac{\eta_n}{k} \, i_n \, c_n \, \gamma_n + \Sigma' \, \frac{\eta_n}{k} \, i_m \, F_{nm} - \Sigma' \, f_{nm} \, i_m$$

$$- f_{nn} \left[i_n - i_{nn} \right]$$

$$n = 1, 2, \ldots N \tag{2.6}$$

The primes denote sums over all rods but the n-th. The quantities i_{nn} cannot be eliminated from these equations. Hence there are two sets of dependent variables, the i_n and the i_{nn}.

This formulation is more difficult to apply than the previous one. There is no a priori reason, however, for preferring one formulation over the other. Our interest is in the self effect terms, which must be obtained for either procedure. These are F_{nn} and f_{nn} for the first procedure, and c_n and f_{nn} for the second. Methods of calculating these quantities will be considered in Section 2.4. The second procedure lumps the self-source and self-sink effects for a single rod in the quantity c_n, which may be easier to calculate than the separate effects.

We thus have two alternative ways to formulate the heterogeneous equations. Although they are not formally equivalent they must be equivalent physically. The consequence of this equivalence is obtained by equating expressions (2.4) and (2.5) for the asymptotic flux. One obtains

$$c_n \gamma_n = F_{nn} - \frac{i_{nn}}{i_n} f_{nn} \cdot \frac{k}{\eta_n} \tag{2.7}$$

This identity may be used to obtain F_{nn} from a calculation using c_n and f_{nn}, or it may be used to obtain any one quantity if the others are known. Note that i_{nn}, i_n and k depend on the configuration, while the other quantities are presumably properties of a given rod in a given moderator. If c_n, γ_n, η_n, F_{nn}, and f_{nn} are independent of the rod configuration, as we have assumed them to be,

the quantity

$$b_n = \frac{i_{nn}}{i_n} \quad k \tag{2.8}$$

must also be independent of the configuration.

Equation (2.6) can be written for an infinite single component lattice (one rod type) as follows:

$$\gamma = \frac{1}{f}\left[c\gamma + \sum_m{}' F_{nm} + \frac{b}{\eta} f_{nn}\right] - \left[f_{nn} + \sum_m{}' f_{nm}\right] \tag{2.9}$$

The primes denote the omission of the diagonal (self) terms from the summations. In this equation resonance absorption is ignored. f is the thermal utilization of the lattice. There are four self-effect parameters in this equation: γ, c, b, f_{nn}. Hence, if we wanted to determine them from measured f values for different one-component infinite lattices (as we have done in the past for γ) four values of f corresponding to four different infinite lattices would be necessary. However it is quite possible that such a simultaneous determination of the self-parameters from a set of measured f values would not be very accurate. That is, small errors in f or in the kernels might lead to large errors in some or all of the parameters.

Equation (2.3), which corresponds to our original formulation of the heterogeneous equations, takes the following form for an infinite one-component lattice, when the self terms F_{nn} and f_{nn} are written separately:

$$\gamma = \frac{1}{f}\left[F_{nn} + \Sigma' F_{nm}\right] - \left[f_{nn} + \Sigma' f_{nm}\right] \tag{2.10}$$

In this formulation there are only three self-effect parameters to be determined.

Section 2.3 Generalization of Small Source Theory to Large Sources

One can begin from the differential equations which can lead to heterogeneous theory for a thermal reactor.[8]

$$\nabla^2 q - \frac{\partial q}{\partial \tau} = - S(\underline{r}, \tau) \tag{2.11}$$

$$- D\nabla^2 \phi + \sigma_a \phi + G(\underline{r}) = q_{th} \tag{2.12}$$

$q(\underline{r}, \tau)$ is the slowing down density past energy E corresponding to age τ, and at the point $\underline{r}$ in space. $\phi(\underline{r})$ is the thermal flux. σ_a is the moderator absorption cross section and D is the moderator diffusion coefficient. q_{th} is the slowing down density at a point $\underline{r}$ and at thermal energy. $S(\underline{r}, \tau)$ is the source density at point $\underline{r}$ and at age τ, per unit age, due to fission neutrons from the fuel elements. $G(\underline{r})$ is the sink density due to fuel elements.

The differential operators

$$L_1 q = - (\nabla^2 - \frac{\partial}{\partial \tau}) q \tag{2.13a}$$

and

$$L_2 \phi = (-D\nabla^2 + \sigma_a)\phi \tag{2.13b}$$

have integral operators as their inverses

$$L_1^{-1} = \int d\tau' \int d\underline{r}' \; g(|\underline{r} - \underline{r}'|, \tau - \tau') \tag{2.14a}$$

$$L_2^{-1} = \int d\underline{r}' \; f(|\underline{r} - \underline{r}'|) \tag{2.14b}$$

L_1^{-1} operates on the source distribution $S(\underline{r}, \tau)$ of (2.11) and L_2^{-1} operates on the source and sink distributions

$$q_{th} - G(\underline{r})$$

of equation (2.12). Thus

$$q(\underline{r}, \tau) = L_1^{-1} S(\underline{r},\tau) = \int d\tau' \int d\underline{r}' \; g(|\underline{r}-\underline{r}'|, \tau-\tau') \; S(\underline{r}',\tau') \tag{2.15a}$$

$$\begin{aligned} \phi(\underline{r}) &= L_2^{-1}\left[q(\underline{r},\tau_{th}) - G(\underline{r})\right] \\ &= \int d\underline{r}' \; f(|\underline{r}-\underline{r}'|) \; \left[q(\underline{r}',\tau_{th}) - G(\underline{r}')\right] \end{aligned} \tag{2.15b}$$

$g(|\underline{r} - \underline{r}'|, \tau - \tau')$ is the Fermi age kernel in an infinite medium. $f(|\underline{r} - \underline{r}'|)$ is the thermal diffusion kernel.

Equations (2.15) are no longer differential equations. Rather they express the slowing down density and thermal flux in terms of kernel functions which are integrated over the source and sink distributions that the rods present. One important assumption implicit in equations (2.15) is that the medium is uniformly filled with moderator, i.e., that in the equations (2.11) and (2.12) the nuclear properties τ, σ_a and D are independent of position in space. In particular one assumes that the volume occupied by fuel elements has the same values for these quantities as the moderator itself. The fuel elements are regarded as mathematical sources and sinks characterized by $S(\underline{r}, \tau)$ and $G(\underline{r})$ and do not otherwise enter into the equations (2.11) and (2.12).

We now make another assumption about the fuel elements. We assume that

$$G(\underline{r}) = \sum_{m=1}^{N} i_m A_m (\underline{r} - \underline{r}_m) \tag{2.16a}$$

$$S(\underline{r},\tau) = \nu(\tau) \sum_{m=1}^{N} \frac{\eta_m}{k} i_m A_m(\underline{r} - \underline{r}_m) \tag{2.16b}$$

The index m characterizes the m-th rod (fuel element) whose center is located at $\underline{r}_m$. i_m is the number of absorptions per second per unit length of the rod. $A_m(\underline{r} - \underline{r}_m)$ is the spatial distribution of absorptions within the rod, normalized to unity over the rod volume.

$$\int_{\text{rod volume}} A_m(\underline{r} - \underline{r}_m)\, d\underline{r} = 1 \tag{2.17}$$

$\nu(\tau)$ is the fission spectrum normalized to unity.

$$\int \nu(\tau)\, d\tau = 1$$

The factor $\frac{1}{k}$ ensures that a steady state holds. k is the static[9] reactivity, which is not in general equal to the (observed) dynamic reactivity.

When equations (2.16) are inserted into (2.15) one obtains

$$q(\underline{r},\ \tau) = \sum_m g_m(\underline{r},\ \tau) \frac{\eta_m}{k} i_m \tag{2.18}$$

where

$$g_m(\underline{r},\ \tau) = \int \nu(\tau')\, d\tau' \int d\underline{r}'\, g(|\underline{r} - r'|, \tau - \tau')\, A_m(\underline{r}' - \underline{r}_m) \tag{2.19}$$

The spatial integral is taken over the rod volume since A_m is zero outside the rod.

$$\phi(\underline{r}) = - \sum_m f_m(\underline{r})\, i_m + \sum_m F_m(\underline{r})\, \frac{\eta_m}{k}\, i_m \tag{2.20}$$

where

$$f_m(\underline{r}) = \int f(|\underline{r} - \underline{r}'|)\, A_m(\underline{r}' - \underline{r}_m)\, d\underline{r}' \tag{2.21}$$

$$F_m(\underline{r}) = \int f(|\underline{r} - \underline{r}'|)\, g_m(\underline{r}', \tau_{th})\, d\underline{r}' \tag{2.22}$$

The integral in (2.21) is taken over the rod volume while the integral in (2.22) is taken over the entire reactor.

We now integrate $\phi(\underline{r})$ of (2.20) over the volume of the n-th rod, multiplying it by a weighting function $B_n(\underline{r} - \underline{r}_n)$ which will be discussed below. We define this function to give

$$\phi_m \equiv \int \phi(\underline{r})\, B_m(\underline{r} - \underline{r}_m)\, d\underline{r} \tag{2.23}$$

Equation (2.20) becomes

$$\phi_n = -\sum_m f_{nm}\, i_m + \sum_m i_m\, \frac{\eta_m}{k}\, F_{nm} \tag{2.24}$$

where

$$f_{nm} \equiv \int f_m(\underline{r})\, B_n(\underline{r} - \underline{r}_n)\, d\underline{r} \tag{2.25a}$$

$$F_{nm} \equiv \int F_m(\underline{r})\, B_n(\underline{r} - \underline{r}_n)\, dr \tag{2.25b}$$

The integrals (2.25) are taken over the volume of the n-th rod. We assume that the weighting function B_n is normalized to unity.

$$\int_{\text{rod volume}} B_n(\underline{r} - \underline{r}_n)\, d\underline{r} = 1 \tag{2.26}$$

If one defines $\gamma_n \equiv \frac{\phi_n}{i_n}$ equations (2.25) take the form of the conventional heterogeneous equation set

$$\gamma_n i_n = - \sum_m f_{nm} i_m + \sum_m i_m \frac{\eta_m}{k} F_{nm} \qquad (2.27)$$

A number of observations can be made about this derivation:

1. The kernel functions should be defined by equations (2.25) and by (2.21) and (2.22)
2. The weighting function $B_n(\underline{r} - \underline{r}_n)$ is arbitrary. It is not clear how sensitive the calculations are to this weighting function.
3. The distribution of absorptions within each rod, $A_n(\underline{r} - \underline{r}_n)$, enters the kernel functions. For the self terms f_{nn} and F_{nn} this is quite important.
4. It has been assumed that the rods have the same nuclear properties as moderator, aside from acting as a mathematical source and sink. This assumption is not necessary for the source rod m or the sink rod n in F_{nm}. The assumption is necessary for the intervening rods, however. In any case, it simplifies the calculation of the kernels.

A more correct treatment of the source and sink rods would take into account the dissimilarity of nuclear properties between rod and moderator. For example, in the absorbing rod the larger scattering cross section in the fuel element (due to a higher density) might be described by providing a somewhat different sink

function $A'_n(\underline{r} - \underline{r}_n)$. Another important characteristic that would alter the sink function is the strongly anisotropic flux angular distribution within the rod. Similarly in the source rod scattering and slowing down effects may perhaps be described by altering the source density to $A_m''(\underline{r} - \underline{r}_m)$. $v(\tau)$ should then correspond to a modified fission spectrum. Following these ideas, equations (2.16) would read as follows:

$$G(\underline{r}) = \sum_{m=1}^{N} i_m A'_m(\underline{r} - \underline{r}_m) \qquad (2.16a')$$

$$S(\underline{r}, \tau) = v(\tau) \sum_{m=1}^{N} \frac{\eta_m}{k} i_m A_m''(\underline{r} - \underline{r}_m) \qquad (2.16b')$$

We now turn to the weighting function $B_n(\underline{r} - r_n)$. While this is arbitrary with regard to the derivation given above, one is interested in using the best weighting function to give a correct neutron balance. One wishes to calculate the absorptions correctly, hence one should weight $\phi(\underline{r})$ with the importance[10] of a unit of flux in causing fissions. Hence we shall take $B_n(\underline{r} - \underline{r}_n)$ to be the probability that a unit of flux at $\underline{r}$ will result in a fission. If $\phi(\underline{r})$ were the actual flux in the rod, instead of the asymptotic flux this weighting function would be proportional to

$$B_n(\underline{r}) \propto \sigma_f \psi(\underline{r}) \qquad (2.28)$$

Where σ_f is the fuel absorption cross section and $\psi(\underline{r})$ is the thermal adjoint flux. Such an importance weighted flux would be expected to minimize the error in reactivity with respect to

errors in the flux calculation. In the absence of an adjoint function one may perhaps use the sink distribution $A'_m(\underline{r} - \underline{r}_m)$ or some approximation to it as the weighting function.

In conventional small source theory the following arbitrary sink and source distributions are used:

To calculate	source rod m	sink rod n
f_{nm}	$A'_m(\underline{r}-\underline{r}_m) \rightarrow$ a delta function line sink at the rod center	$B_n(\underline{r}-\underline{r}_n) \rightarrow$ a cylindrical shell delta function at the rod surface
F_{nm}	$A''_m(\underline{r}-\underline{r}_m) \rightarrow$ a delta function line source at the rod center	$B_n(\underline{r}-\underline{r}_n) \rightarrow$ a delta function line sink at the rod center

In large source calculations more realistic functions must be used. As a first approximation we shall take for each of the four quantities above the distribution of thermal absorptions $A_m(\underline{r}-\underline{r}_m)$ in the rod considered. For loosely packed lattices the importance of the finite volume source and sink corrections will appear principally in the self terms F_{nn} and f_{nn}. These quantities can be quite important, especially for a lattice with several different rod types in close proximity. The values for these quantities that one obtains by considering the finite volume source and sink distributions can be quite different from the small source values. An example to illustrate this will be given in Section 2.4.

Section 2.4 Calculation of Large Source Kernels

In this section we shall indicate how one can calculate the source and sink kernel functions for fuel elements of finite size. We first consider the calculation of f_{nn} for a cylindrical rod. According to equations (2.21) and (2.25a) this is given by

$$f_{nn} = \int d\underline{r}\; B_n(|\underline{r}-\underline{r}_n|)\int f(|\underline{r}-\underline{r}'|)\; A_n(|\underline{r}'-\underline{r}_n|)\,d\underline{r} \qquad (2.29)$$

We make the assumption that the rod volume is replaced by moderator, and that the rod is represented by a source (or sink) distribution $A_n(|\underline{r}-\underline{r}_n|)$. Hence the kernel function $f(|\underline{r}-\underline{r}'|)$ is that appropriate for an infinite medium with sources at points $\underline{r}'$. If the source points $\underline{r}'$ constitute a cylindrical shell the kernel function depends only on r, the radial distance from the rod center and

$$f_{nn} = \int 2\pi r dr\; B_n(r)\int f(r,r')\; A_n(r')\; 2\pi r'\; dr' \qquad (2.30)$$

$$f(r,r') = C\; I_0(\alpha r)\; K_0(\alpha r') \qquad \text{for } r < r' \qquad (2.31a)$$

$$= C\; I_0(\alpha r')K_0(\alpha r) \qquad \text{for } r > r' \qquad (2.31b)$$

The constant $C = \dfrac{1}{2\pi L^2 \sigma_a}$ and $\alpha = \dfrac{1}{L}$ where L is the thermal diffusion length in the moderator.

We shall show that the following assumptions correspond to small source theory.

$$A_n(r) = \frac{\delta(r-R_0)}{2\pi R_0} \qquad (2.32a)$$

$$B_n(r) = \frac{1}{\pi R_0^2} \tag{2.32b}$$

where R_0 is the rod radius. The solution (2.31a) applies .

$$f(r,R_0) = C\, I_0(\alpha r)\, K_0(\alpha R_0) \qquad \text{for} \quad r < R_0$$

Inserting these values in (2.30) one obtains

$$f_{nn} = \frac{2}{\alpha R_0} I_1(\alpha R_0) \cdot C\, K_0(\alpha R_0) \tag{2.33}$$

If the rod radius is small compared to the thermal diffusion length

$$I_1(\alpha R_0) \approx \frac{\alpha R_0}{2} \tag{2.34}$$

Thus one obtains

$$f_{nn} = C\, K_0(\alpha R_0) \tag{2.35}$$

which is the value used in our small source calculations. One should note, however, that if αR_0 is not small the approximation (2.34) cannot be made, and even with the assumptions (2.32) one does not obtain the small source result. This would be pertinent to a strongly absorbing moderator, e.g. water.

When the source is not on the rod surface equation (2.30) gives a result different from that of small source theory. Consider a cylindrical shell source at $r' = a$ where $a < R$. One then has, following (2.32)

$$A_n(r) = \frac{\delta(r-a)}{2\pi a} \tag{2.36a}$$

$$B_n(r) = \frac{1}{\pi R_0^2} \tag{2.36b}$$

One obtains

$$f_{nn} = C\,K_0(\alpha a)\cdot\frac{2a}{\alpha R_0^2} = I_1(\alpha a)$$

$$+ C\,I_0(\alpha a)\cdot\frac{2}{R_0^2}\left[\frac{a}{\alpha}K_1(\alpha a) - \frac{R_0}{\alpha}K_1(\alpha R_0)\right] \qquad (2.37)$$

If αa and αR_o are small one can use the asymptotic expression

$$K_1(x) \approx \frac{1}{x} + (\gamma + \ln\frac{x}{2})\,I_1(x) - \frac{x}{4}$$

This yields the following result:

$$f_{nn} = \frac{a^2}{R_0^2}\,C\,K_0(\alpha a) + C\,\frac{a^2}{R_0^2}\left[\gamma + \ln\frac{\alpha a}{2} - \frac{1}{2}\right] - C\left[\gamma + \ln\frac{\alpha a}{2} - \frac{1}{2}\right] \qquad (2.38)$$

where γ is Eulers constant.

The value (2.38) for f_{nn} may differ appreciably from the conventional value used in small source calculations. The radius a may be considered the effective radius of absorption of thermal neutrons within the rod.

We now consider an arbitrary distribution A(r) of absorptions within the cylindrical rod. Following the recipe of large source theory adopted at the end of Section 2.3 we take

$$A_n(r) = A(r)$$

$$B_n(r) = A(r)$$

One then obtains

$$f_{nn} = C \int_0^{R_0} I_0(\alpha r) A(r) 2\pi r dr \int_r^{R_0} K_0(\alpha r') A(r') 2\pi r' dr'$$

$$+ C \int_0^{R_0} K_0(\alpha r) A(r) 2\pi r dr \int_0^{r} I_0(\alpha r') A(r') 2\pi r' dr' \qquad (2.39)$$

Thus if $A(r) = \frac{\delta(r-a)}{2\pi a}$, where a may be interpreted as the effective radius where absorptions take place, one obtains

$$f_{nn} = C\ I_0(\alpha a)\ K_0(\alpha a)$$

Since $I_0(\alpha a) \approx 1$ when $\alpha a \ll 1$

$$f_{nn} \approx C\ K_0(\alpha a)$$

For low enrichment rods of moderate radius the source distribution is relatively constant through the rod and an effective radius $\sim$ 2/3 R_0 is appropriate. Since K_0 varies rapidly with its argument when αa is small this may make a difference of a factor of two in f_{nn}.

These calculations of f_{nn} for cylindrical solid rods are only illustrative, since it is for composite rods containing both fuel and moderator that f_{nn} will depend strongly on the fuel element absorption pattern.

The calculation of F_{nn} from the source distribution follows similar considerations. It will be much less sensitive than f_{nn} to the form of A(r) since the effective source of thermal neutrons is the slowing down density in the moderator at a distance from the rod.

We now consider the calculation of the parameter C_n for the rod. This is the fraction of fast neutrons emitted by the rod which are reabsorbed in the rod in the absence of other rods. This quantity is of interest in the second formulation of the heterogeneous equations. Unlike F_{nn} it is sensitive to the details of the fuel element and therefore can be expected to contribute to a more precise reactivity and power distribution calculation. In principle C_n can be measured. Analogous quantities have been measured in single fuel element experiments.[11]

It is convenient to associate with C_n a finite source kernel $\overline{F}_n(\underline{r})$ which takes account of the diminution of thermal flux by the rod itself. Thus $\overline{F}_n(\underline{r})$ includes both source and sink effects of the n-th rod. This finite kernel function gives the thermal flux which would be observed at $\underline{r}$ per fission neutron emitted by the rod. The finite source kernel corresponds to a fraction C_n of the emitted neutrons being absorbed by the n-th rod in the absence of other rods. It satisfies the normalization condition

$$\int \sigma_a \, \overline{F}_n(\underline{r}) \, d\underline{r} \;=\; 1 - C_n$$

where the integral is taken over an infinite moderator outside the rod. This finite kernel can be measured in single rod experiments. It would be convenient to write the heterogeneous equations in terms of this directly measurable kernel instead of the $f_n(\underline{r})$ and $F_n(\underline{r})$ kernels, which are not separately measurable. This can

be expressed as follows:

$$\bar{F}_n(\underline{r}) = F_n(\underline{r}) - C_n f_n(\underline{r})$$

We shall now consider the simultaneous calculation of $\bar{F}_n(\underline{r})$ and C_n for a fuel element of simple geometry: an infinite cylinder of circular cross section with radius R_0. Cylindrical symmetry exists around the fuel element axis.

Let r_s be the radial distance to any point in the moderator at which neutrons are thermalized. $q(r_s)$ will denote the slowing down density to thermal at this point. $G(r-r_s)$ will denote the thermal flux at r due to a cylindrical shell source at r_s. The source kernel function $\bar{F}_n(r)$ is then calculated as follows:

$$\bar{F}_n(r) = 2\pi \int_{R_0}^{\infty} G(r-r_s) q(r_s)\, r_s\, dr_s \qquad (2.40)$$

For the slowing down density $q(r_s)$ we take the formula for a line source normalized to unity in the moderator.

$$q(r_s) = e^{R_0^2/4\tau} \cdot \frac{e^{-r_s^2/4\tau}}{4\pi\tau} \qquad (2.41)$$

The function $G(r-r_s)$ is the Green's function of the diffusion equation for a shell source in an infinite medium which satisfies a boundary condition at the rod surface related to the flux distribution in the rod intereior. If we assume that the thermal flux in the fuel rod can be calculated by diffusion theory we obtain

$$G(r-r_s) = B(r)\ K_0(K\ r_s) \qquad \text{for } R_0 < r < r_s$$
$$= D(r)\ K_0(K\ r_s) + D(r)\ .\ I_0(K\ r_s) \qquad \text{for } r > r_s \tag{2.42}$$

where K is the inverse thermal diffusion length in the moderator, and B, C, D can be obtained by satisfying the boundary conditions.

Another way to specify the function G is to give the "blackness factor" β_n of the rod. The thermal absorptions within the rod are given by

$$C_n = 2\pi R_0 \cdot \beta_n \cdot J_-(R_0) \tag{2.43}$$

$J_-(R_0)$ is the inward current at the rod surface. C_n is determined from the condition that

$$\int_{R_0}^{\infty} \sigma_a\ \overline{F}_n(r)\ 2\pi r\ dr = 1 - C_n \tag{2.44}$$

Equations (2.43) and (2.44) enable one to solve uniquely for $\overline{F}_n(r)$.

Both calculations are quite tedious. We have been able to express $\overline{F}_n(r)$ as an integral which apparently must be evaluated numerically. In either case it is clear that once one has found $\overline{F}_n(r)$ one can obtain C_n directly from equation (2.44).

2.5 Interference Effects in the Kernels

It is clear that the kernel functions used up to now in heterogeneous calculations have taken into account only the origin

rod and the terminus rod. (In fact the small source kernel functions depend only on the distance between the rod centers.) The kernels have contained no interference effects due to the other rods in the configuration, but have regarded the origin rod as being immersed in a homogeneous moderator with no other rods present (not even the terminus rod) while the terminus rod is similarly regarded.

It should be pointed out, however, that the heterogeneous formalism does take interference effects due to neutron absorption into account. It is only the multiple rod interference effects due to the dissimilarity in slowing down and scattering properties of rods and moderator, which are not taken into account in the kernel functions. The formalism does not require the kernels to take interference effects due to neutron absorption into account. This can be seen from equation (2.4) or (2.5)

$$\gamma_n \, i_{nn} = \frac{\eta_n}{k} \, i_n C_n \gamma_n - \sum_s{}' \, i_{sn} \, f_{ns} \qquad (2.5)$$

The first term on the right hand side gives the effects of the rod absorption on its own flux. The second term, a summation over the other rods present, gives the flux depression at the n-th rod due to neutrons emitted from the n-th rod and absorbed in the other rods. Hence the <u>absorption</u> effects of the other rods on the neutrons emitted by the n-th rod can be expressed in terms of kernels which do not include other-rod effects. This same characteristic appears in the first formulation of equation (2.4)

$$\gamma_n \, i_{nn} = \frac{\eta_n}{k} \, i_n F_{nn} - i_{nn} f_{nn} - \sum_s{}' i_{sn} f_{ns} \qquad (2.4)$$

Self effects are contained in the first two terms on the right hand side. The primed sum over other rods gives the absorption effects of the other rods on neutrons emitted by the n-th rod and on the flux at the n-th rod. Equation (2.2) shows a similar absorption interference for neutrons emitted by the n-th rod and absorbed by the n-th rod .

$$\gamma_n \, i_{nm} = \frac{\eta_m}{k} \, i_m F_{nm} - \sum_s i_{sm} f_{ns} \qquad (2.2)$$

By summing over all rods from which a neutron may originate one obtains the conventional heterogeneous equations. We therefore conclude that the heterogeneous formalism does take interference effects between rods into account insofar as these interference effects are due to neutron absorptions alone. In other words, the kernels may correctly ignore other-rod interactions as long as neutron absorptions are the only interactions of importance.

In most problems, however, other-rod interactions due to scattering of neutrons and slowing down of neutrons by the rods are of considerable importance, when (as is usually the case) the neutron scattering and slowing down properties of the rod and of the moderator are quite different. The simplest approach towards including these rod properties in the kernel is to calculate the kernels for a fictitious homogenized moderator which includes

the scattering and slowing down properties of the rods. The scattering cross section of this fictitious moderator would be obtained by averaging the actual scattering cross section at all points over the flux

$$\sigma_s = \frac{\int \sigma_s(\underline{r})\ \phi(\underline{r})\, d\underline{r}}{\int \phi(\underline{r})\, d\underline{r}} \qquad (2.45)$$

where the integrals are taken over the reactor volume. The slowing down cross section, diffusion coefficients, etc. should be similarly averaged. Such an average value would be computed at each neutron energy of interest.

This prescription has been followed in the analysis of the Brookhaven critical experiments in Section 3 of this report. At thermal energy the flux in the fuel rods is small. Hence the fuel element contribution to the scattering cross section at thermal is negligible when it is weighted by the flux. The net effect is to treat the fuel element volume as if it were vacuum, and thus the net effect is to use a reduced density moderator. At higher energies the flux in the rods is considerable, and no such simple treatment is possible.

In the case of a tightly packed lattice the rod-interaction effect in the kernel may be appreciable. One possibility may be to use a fictitious homogenized moderator. Another possibility is to describe rod scatterings by a more complicated formalism. For example, a neutron scattered by a rod may be considered as an

absorption of that neutron followed by an emission of a new neutron. Let the kernel f_{nm} include interaction effects. One might write it as follows in terms of the ordinary kernels f_{nm}

$$\tilde{f}_{nm} = f_{nm} + \sum_s f_{sn} B_{nm}^{s} f_{sm} \qquad (2.46)$$

where the parameter B_{nm}^{s} characterizes scattering by the s-th rod. Equation (2.46) is cited only as a suggestion for what might be considered. The validity and usefulness of such a scheme is open to question.

Another possibility in treating tightly packed lattices is the following. One writes the heterogeneous equations in the form

$$\gamma_n i_n = \sum_m R_{nm} \frac{\eta_m}{k} i_m \qquad (2.47)$$

The kernel function R_{nm} includes both source and sink effects. This form (2.47) implies a kernel function R_{nm} that includes all interference effects (including those of neutron absorption by other rods) can be obtained by solving the Boltzmann equation in the moderator with appropriate boundary conditions at moderator-rod boundaries. Such a kernel cannot generally be found. However, for tightly packed lattices it may be possible to consider a fictitious homogenized material combining fuel and moderator nuclear properties. In such a fictitious homogenized material kernels of the R_{nm} type may conceivably be useful. Another possibility is that of the partially homogenized kernel, which

treats nearby rods as if they are in pure moderator, and more distant rods as being in a homogenized region. We mention these possibilities (2.46) and (2.47) and the partially homogenized kernel, simply to indicate the types of heterogeneous formulations which are possible in close-packed systems. There is no reason to think at this time that either one is advantageous.

Section 3. Comparison of Heterogeneous Calculations with Experiment - The Brookhaven Graphite Reactor

The HERESY 1 Code was used to compute the reactivities of certain graphite-natural uranium critical assembly configurations for which experimental reactivities had been obtained by Brookhaven.[12] The calculations were performed using line source diffusion-age theory kernels with rod parameters determined by the self-consistent method. In order to apply HERESY 1 to the Brookhaven experiments, certain idealizations in geometry were made which were of such a nature as to make the calculated reactivities higher than the experimental ones. These include considering the moderator in the radial direction as infinite and completely ignoring a 7 cm gap in the center of the reactor which permitted neutron streaming to occur. Therefore, the comparison of calculated and experimentally determined reactivities cited in the table below and indicating that the analytical reactivities are of the order of 1/2% higher than the experimental ones seems consistent with the approximations made.

Table 1

Experimental and Calculated Reactivities for the Brookhaven Reactor

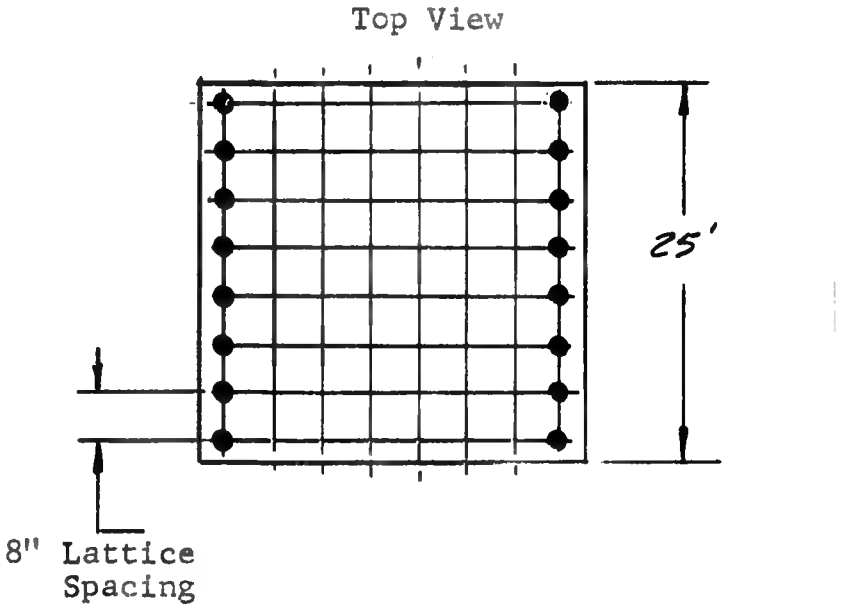
No. Rods in Configuration	Reactivities	
	Critical Exp.	HERESY 1
100	.902	.912
144	.936	.943
196	.961	.967
256	.978	.982

We will now consider in detail the setup of the critical experiments and the techniques and approximations utilized in the HERESY 1 calculations. In Figure 1 we show schematically the geometry of the Brookhaven graphite reactor lattice. In the top view of the reactor there is a total (not shown) of 1369 channels in the graphite. Since the critical experiments were made only on configurations of a few hundred rods in the central channels, most of the channels were left unfilled while the experiments were performed. Therefore, when we performed our calculations of the near-critical configurations kernel functions were used corresponding to a graphite moderator of decreased density. In the front view of the reactor the 7 cm air gap is shown, for which no account was made in the HERESY 1 calculations. In Figure 2 a channel containing a fuel element is shown schematically. The air gap extends from 1.53 cm to 3.39 cm.

The preparation of input for the HERESY 1 calculations of the Brookhaven critical assembly configurations went as follows:

STEP 1 -- To obtain the thermal utilization and resonance escape probability of an infinite system of rods. This can be done either experimentally or analytically. The Brookhaven group determined f (f = .890) experimentally for an infinite square lattice of pitch 8". They also utilized a modified diffusion theory calculation to find the thermal utilization of aluminum, f_{al} = .0081. The resonance escape probability was determined as .8783 by the same group.

There are 37 channels for rods
in any row of the square lattice



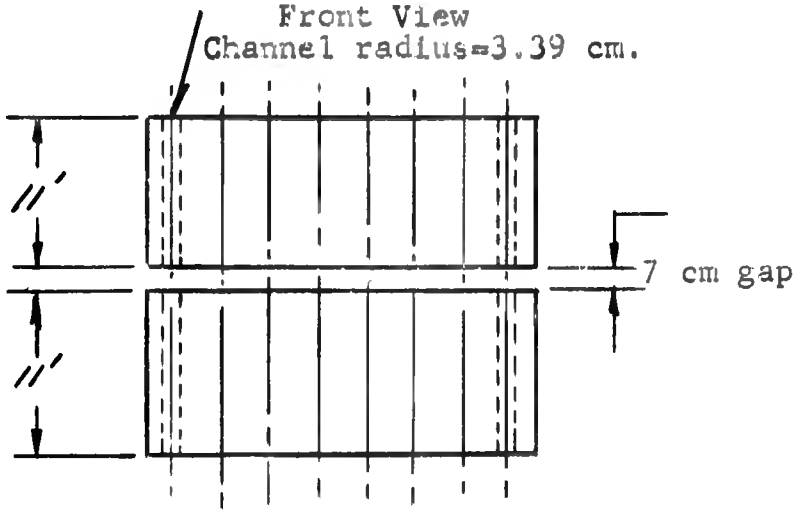


Figure 1: Brookhaven Reactor

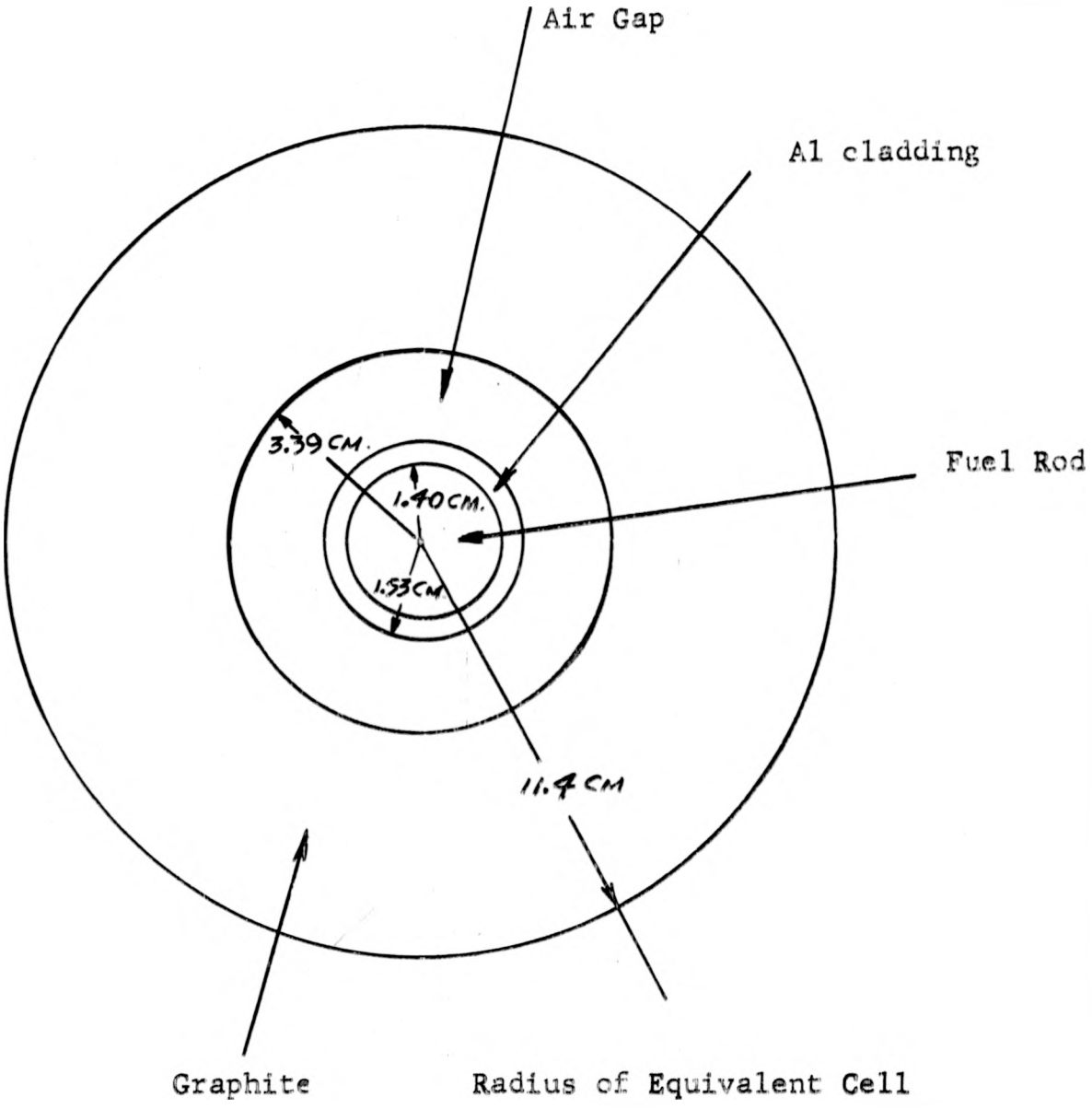


Figure 2. Fuel Element and Channel Schematic

STEP 2 -- Choosing the infinite moderator kernel functions to be used in our calculations of the rod parameters γ and A. Since most of the channels in the assembly were unfilled and even those which were filled contained a large air gap, it seemed reasonable to use kernel functions for a graphite moderator of reduced density. That is, defining

$$\frac{V_{cell}}{V_{cell} - V_{hole}} = R$$

we used

$$L^2_{RD} = L^2_{graphite} \cdot R^2$$

$$\tau_{RD} = \tau_{graphite} \cdot R^2$$

in our kernel functions. We used $L^2_{graphite}(exp) = 2800\ cm^2$; $\tau_{graphite} = 350\ cm^2$; $R = 1.096$ in the kernel functions giving an $L^2_{RD} = 3363\ cm^2$ and $\tau_{RD} = 420.4\ cm^2$.

STEP 3 -- Obtaining the rod parameters γ, η, A. In doing this one must consider the rod as a whole including the aluminum cladding. Therefore, the thermal utilization of the rod was taken as the sum of the uranium and aluminum thermal utilizations which is equal to .8981. Then we used the following equations to obtain the rod parameters.

$$\gamma = \frac{1}{f_{ROD}} \sum_n F_{mn} - \sum_n f_{mn} \qquad f_{ROD} = f + f_{a\ell}$$

$$\eta_{ROD} = \frac{\eta f}{f + f_{a\ell}} \qquad \eta = \eta_U \epsilon$$

$$A = (1-p)a^2$$

In the above calculations we took η to include the fast fission factor (ϵ). This had been determined experimentally as 1.035. η_U(exp) was taken as 1.315. "a" is the lattice spacing. The above calculations gave γ = .5177, A = 50.25 and η_{ROD} = 1.348.

STEP 4 -- Approximations made in the diffusion length and in η to account for leakage out the top and bottom of the reactor. (See Appendix 1). When considering the actual critical assembly configurations we took:

$$\frac{1}{L^2_{eff}} = \alpha_Z^2 + \frac{1}{L^2_{RD}}$$

$$\eta_{eff} = \eta_{ROD} e^{-\alpha_Z^2 \tau_{RD}}$$

where

$$\alpha_Z^2 = \left(\frac{\pi}{H + 2\Delta}\right)^2 \cdot \left(\frac{L_Z^2}{L_R^2}\right)$$

Δ = reflector savings which was determined experimentally as 61 cm and

$\frac{L_Z^2}{L_R^2}$ was determined by standard methods.[13]

$$\frac{L_Z^2}{L_R^2} = \frac{1 + 2\phi + 3/2\ Q\ r\ \phi/\lambda}{1 + 2\phi + 3/4\ Q\ r\ \phi/\lambda}$$

where

ϕ = $\frac{\text{volume of holes}}{\text{volume of moderator}}$

r = radius of hole

Q = 1.333 for circular cylinders

λ = mean free path in the moderator

We found $\frac{L_Z^2}{L_R^2}$ to be 1.0884 leading to an L^2_{eff} = 3123 for the kernel functions to be used in the critical assembly calculations. We also found η_{eff} = 1.335.

STEP 5 -- Use the rod parameters obtained in Step 3 (except for η_{ROD}) and the η_{eff} and kernel functions using an L^2_{eff} as input for the HERESY 1 calculations. The positions of the individual rods in the actual reactor configurations must also be used as input.

Section 4. Study of Control Rods in Interstitial and Substitutional Locations

4.1 Substitutional Location

A study of the effects of control rods in substitutional locations in a natural uranium graphite lattice has been performed with the heterogeneous equations using HERESY 1. In this study of a square array of 121 rods, the numbers of natural uranium (101) and control rods (20) were kept constant while the control rod pattern within the lattice was varied. The standard 11 x 11 configuration with all the rod symmetries indicated is shown in Figure 3. The rod parameters for both natural uranium and boron control rods are listed below. We used infinite line graphite moderator ($L^2 = 2800$, $\Sigma_a = 4 \times 10^{-4}$) kernel functions in our calculations.

Natural Uranium Rod Parameters

$\eta = 1.34$

$\gamma = .255\ cm^{-1}$

$A = 51\ cm^2$

$R_o = 2\ cm$

Boron Control Rod Parameters

$\eta = 0$

$\gamma = .0692\ cm^{-1}$

$A = 0$

$R_o = 2\ cm$

Rods with identical labels have identical symmetry.

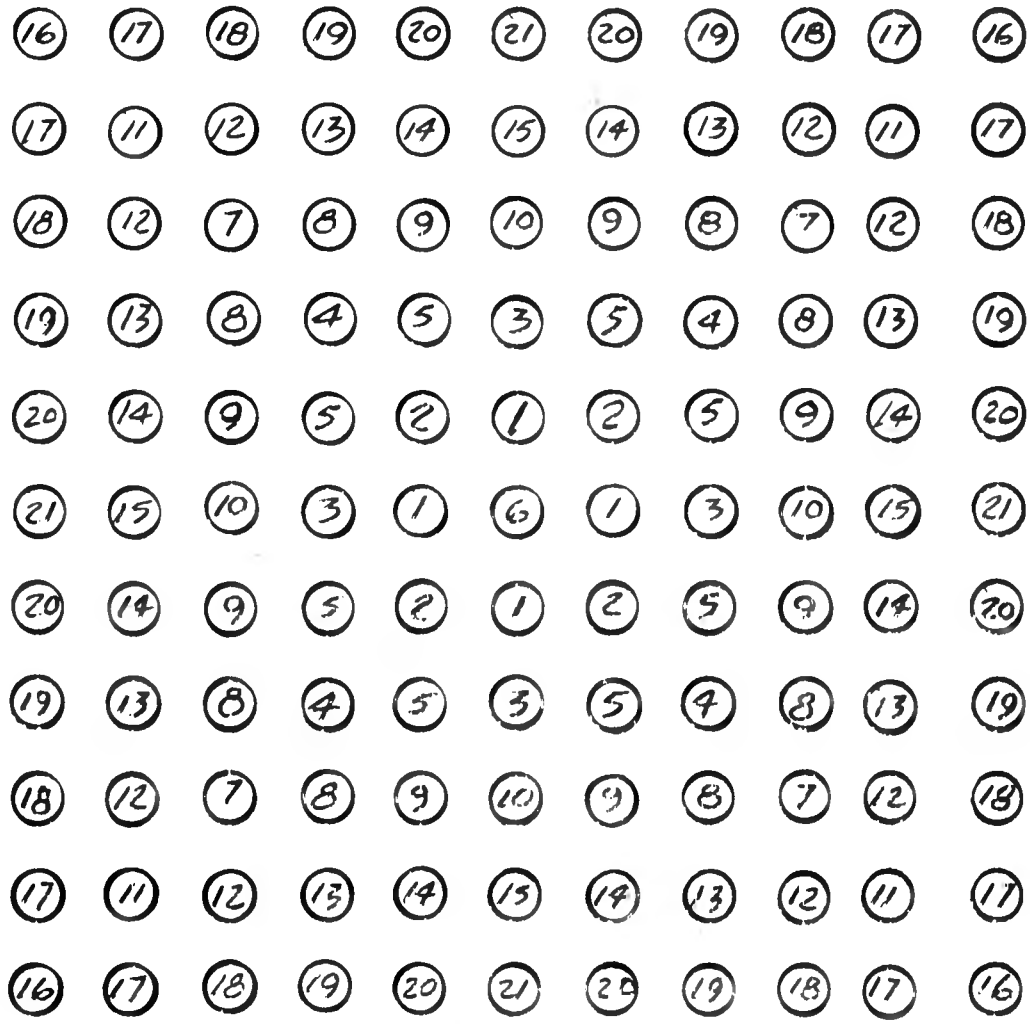


Figure 3. The 11 x 11 Configuration

The results of these calculations cited below indicate that the control rods are least effective when placed on the outside of the array. This is to be expected. However, because of interference effects of the neighboring control rods on one another, the reactivity diminution is not a maximum when all the control rods are placed at the center. For the cases studied, the configuration with control rod symmetries 3, 9, 12 proved to be most effective -- a result which certainly could not be determined apriori. Thus, to determine the most effective placement of the control rods heterogeneous calculations such as the ones performed are essential to the core designer.

Table 2

Control Rod Study

Control Rods	k	f	p
1, 2, 3, 5	.8610	.7106	.9045
2, 4, 7, 11, 16	.8252	.6807	.9050
16, 17, 18	.9554	.8041	.8868
3, 9, 12	.7913	.6481	.9916
1, 5, 10, 15	.8189	.6719	.9093

4.2 Interstitial Location

A study was also made of the effects of control rods on the 11 x 11 lattice when 16 control rods are inserted in various interstitial configurations within the fixed 121 rod natural uranium square lattice. This is the type of problem a core designer faces when deciding where to place control rods within a fixed fuel configuration to achieve a certain reactivity and to avoid flux peaking. The fuel element configuration for the 11 x 11 lattice is identical with that of Figure 3. The control rod configurations within the first 7 x 7 rods are shown in Figures 4 through 8. The heterogeneous calculation results indicate that the configuration with minimum reactivity is difficult to determine a priori since the interference effects of control rods on one another is very important. The results also point up the fact that if the control rods are not carefully placed, one can have severe flux peaking within the reactor. To illustrate this we cite $\frac{i_{max}}{i_{min}}$ for each configuration in addition to k, p, f, in the following Table 3.

Table 3

Interstitial Control Rod Results

Configuration	k	i_{max}/i_{min}	f	p
1	.8054	6.34	.6683	.8986
2	.7509	4.32	.6232	.8992
3	.6868	2.98	.5714	.8971
4	.7661	4.50	.6362	.8980
5	.6882	2.51	.5721	.8977

Since the reactivity of the square lattice without control rods was found to be 1.007, the results for the control rod configurations studied, show the reactivity was reduced by 20% to 32% by their insertion.

A comparison of the relative absorptions in the fuel elements for each configuration of control rods is given in Table 4. This table indicates the extreme variations in the power pattern that one can obtain with different control rod configurations.

Table 4

Fuel Rod Absorptions with Interstitial Control Rods

Rod Type	Config. 1	Config. 2	Config. 3	Config. 4	Config. 5
1	.1906	.3130	.2748	.3505	.2504
2	.1577	.2076	.1624	.4440	.3225
3	.2050	.3150	.3037	.3959	.2715
4	.1837	.2993	.2540	.3595	.2281
5	.1804	.3039	.2243	.3369	.2158
6	.2404	.3558	.3751	.4495	.3331
7	.6195	.5588	.2085	.6797	.2414
8	.4466	.3467	.2325	.4045	.1861
9	.4358	.2801	.1773	.1961	.1920
10	.4943	.4052	.2945	.4941	.2509
11	.8383	.8132	.2457	.8146	.2680
12	.8187	.7528	.2353	.7719	.2391
13	.7774	.5593	.2649	.5712	.2592
14	.7654	.4220	.2137	.4196	.2071
15	.8222	.6223	.3200	.6509	.3111
16	.8740	.8831	.4157	.8647	.4299
17	.8903	.8819	.3995	.8691	.4069
18	.9364	.8902	.4174	.8824	.4159
19	.9513	.8488	.4348	.8416	.4248
20	.9620	.8235	.4466	.8160	.4311
21	1.000	.8975	.4844	.8963	.4676
Control Rods 22	.2771	.4300	.3837	.6022	.3969
Control Rods 23	.6194	.9020	.6819	1.000	.4875
Control Rods 24	.9843	1.000	1.000		.5721
Control Rods 25					1.000

The following 5 figures show the interstitial control rod configurations discussed in Section 4. The control rods are indicated with a ○

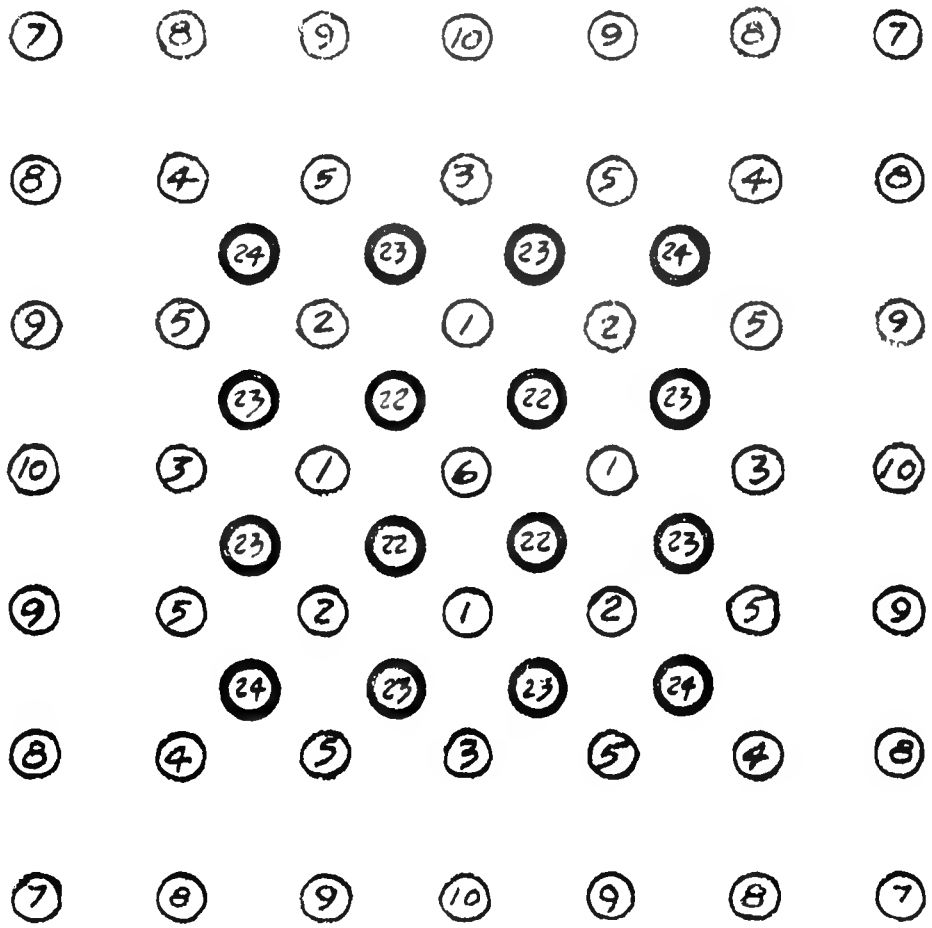


Figure 4 - 1st case

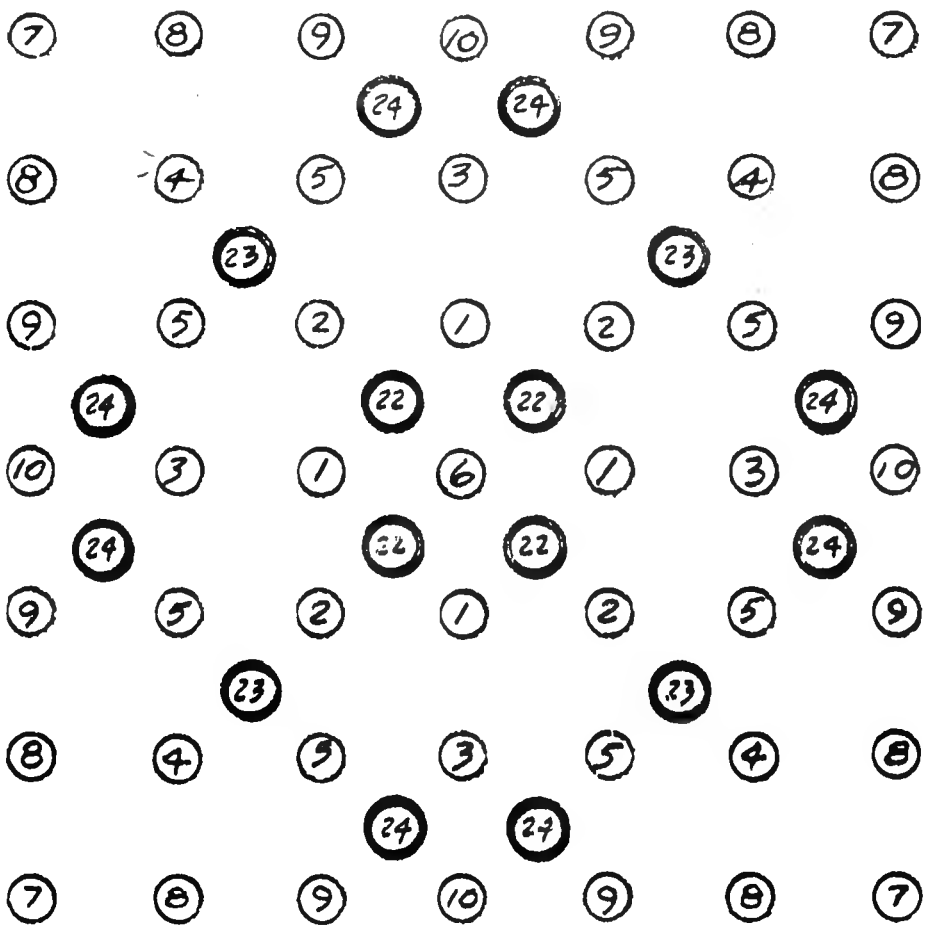


Figure 5 - 2nd case

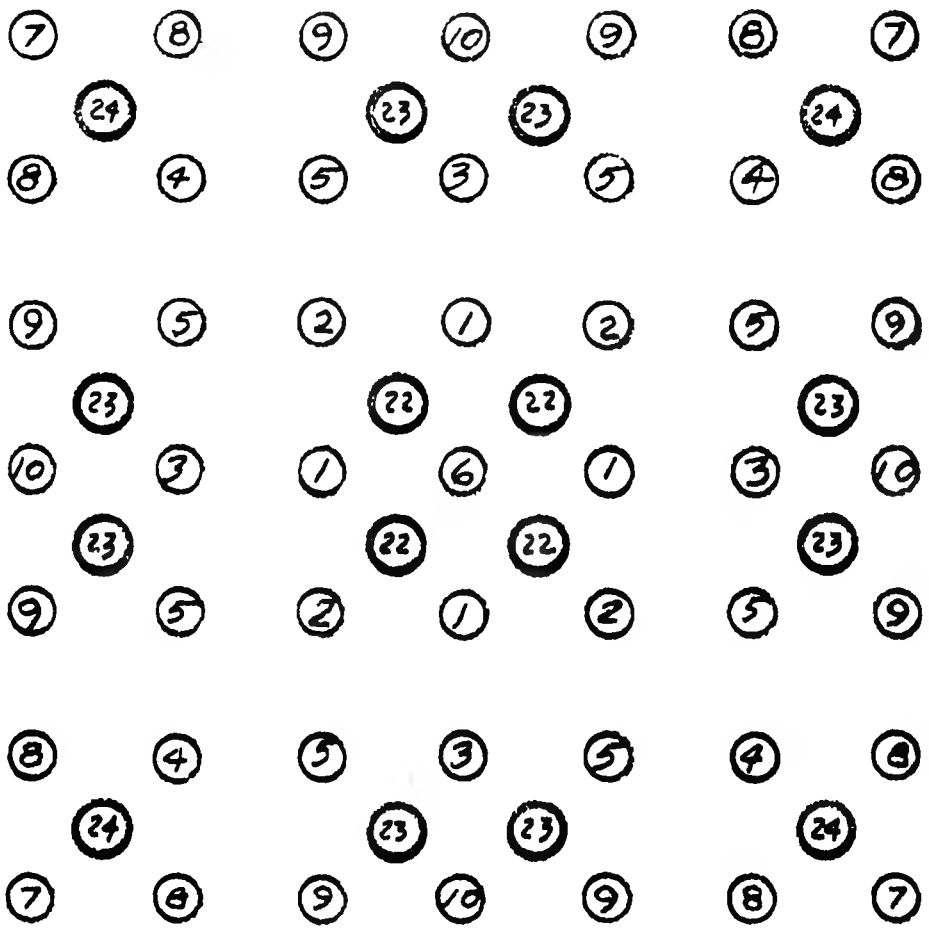


Figure 6 - 3rd case

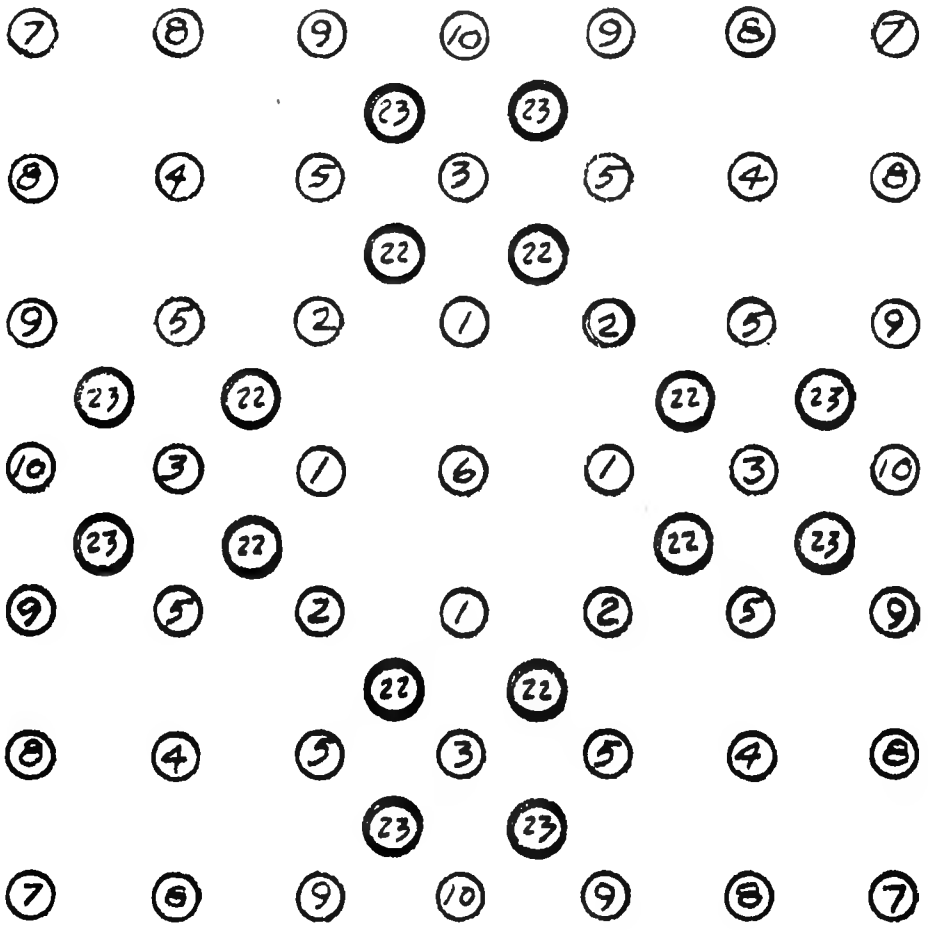


Figure 7 - 4th case

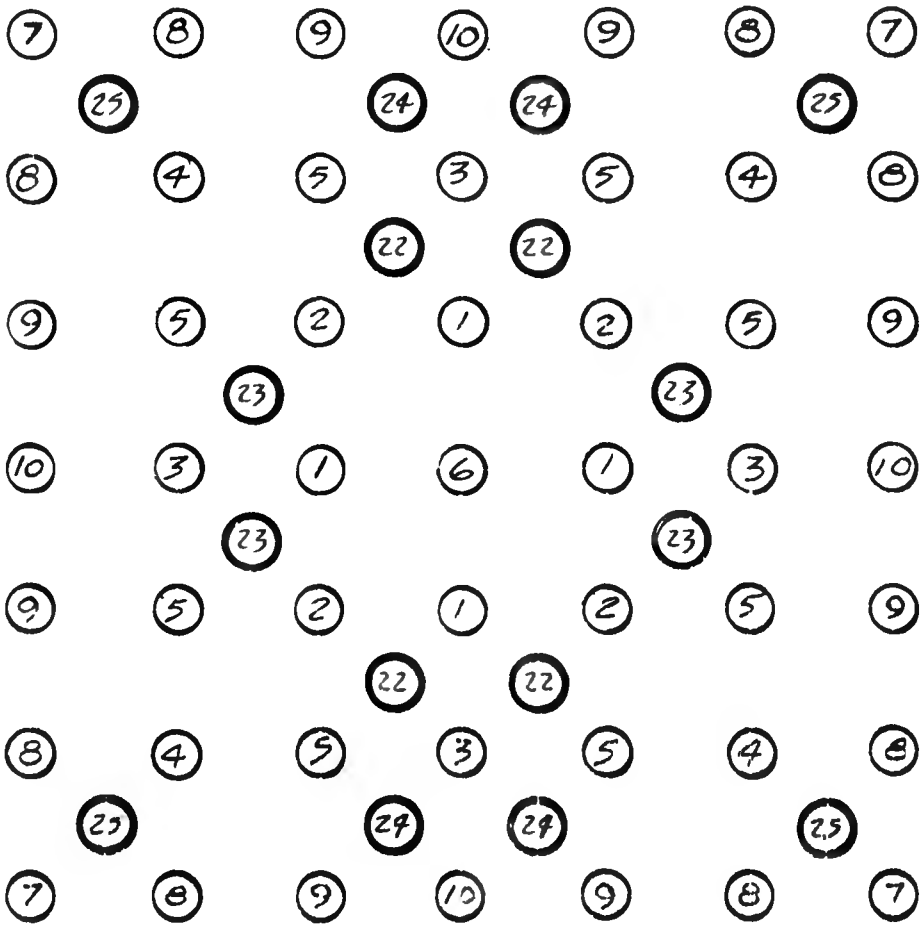


Figure 8 - 5th case

Section 5. Investigation of the Effects of Spiking on Reactivity and Average Power in Graphite - Natural Uranium Lattices

In Quarterly No. 3 calculations were performed on a square configuration of 81 rods where 20 of the rods were replaced by enriched uranium rods. By varying the positions of the enriched rods within the array we found that we could vary the reactivity and average power/max power considerably. We found, however, that in most cases, when reactivity is increased the average power is decreased.

We have subsequently studied various configurations differing only very slightly from those already considered in order to get some quantitative estimate of how much of a gain in reactivity and average power the core designer can obtain with optimum spiking. We cite the results of these calculations for configurations which gave $.76 \leq \frac{\bar{I}}{I_{max}} \leq .771$ in the following table. It should be noted that for approximately the same average to maximum power ratio the first configuration gives a 1.5% greater reactivity than the last configuration. Thus optimum spiking can be of great importance to the core designer.

The configurations referred to are those of the third quarterly report.[14]

Table 5

Spiking Results

Enriched Rod Symmetries	K	i/i_{max}
5, 10, 14	1.042	.769
5, 10, 13	1.041	.769
3, 9, 14	1.038	.762
3, 9, 13	1.037	.771
3, 9, 11, 15	1.034	.763
3, 9, 12	1.032	.768
3, 10, 13, 15	1.026	.768

Appendix 1. Parameter Values to Account for Axial Leakage

For a homogeneous anisotropic cylindrical reactor which is finite in the z direction, the diffusion equation is of the form:

$$D_r \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \phi + D_z \frac{\partial^2 \phi}{\partial z^2} - \Sigma_a \phi = 0 \tag{1}$$

Separating variables setting $\phi = \phi_R \phi_z$, we obtain,

$$D_r \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \phi_R - \Sigma_a = - D_z \frac{\partial^2 \phi}{\partial z^2} = \alpha^2 D_z \tag{2}$$

where the separation constant $\alpha^2 D_z$ is chosen to obtain a cos αz dependence for ϕ_z. This leaves the radial variation of the flux determined by

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \phi_R - \left(\frac{1}{L_R^2} + \frac{D_z}{D_r} \alpha^2 \right) \phi_R = 0 \tag{3}$$

Defining $\alpha_z^2 = \frac{D_z}{D_r} \alpha^2 = \frac{L_z^2}{L_R^2} \alpha^2$ we see from (3) that:

$$\frac{1}{L_{eff}^2} = \alpha_z^2 + \frac{1}{L_R^2} \tag{4}$$

To obtain η_{eff} we once again consider a homogeneous anisotropic reactor. For this case the age equation becomes:

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} q + \frac{D_z}{D_r} \frac{\partial^2}{\partial z^2} q = \frac{\partial q}{\partial \tau} \tag{5}$$

Once again separating variables and setting $q = q_r \cdot q_z$ where

$$q_z = \cos \alpha z$$

we obtain:

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} q_r - \frac{D_z}{D_r} \alpha^2 q_r - \frac{\partial q_r}{\partial \tau} = 0 \tag{6}$$

A solution of this equation is

$$q_r = \bar{q}_r e^{-\frac{D_z}{D_r} \alpha^2 \tau} \tag{7}$$

where $\bar{q}_r$ satisfies

$$\nabla_r^2 \bar{q} - \frac{\partial \bar{q}}{\partial \tau} = 0 \tag{8}$$

From(7) we see that leakage through the top and bottom of the reactor reduces the slowing down density by $e^{-\frac{D_z}{D_r} \alpha^2 \tau}$ at every point. This is equivalent to having an η_{eff} of the form:

$$\eta_{eff} = \eta e^{-\alpha_z^2 \tau} \tag{9}$$

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13. "Reactor Physics Constants" ANL 5800 Page 363

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