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SOME HEAT TRANSFER AND FLUID FLOW CONSIDERATIONS FOR A PACKED-BED FUEL ELEMENT

by

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ABSTRACT

The problem of heat transfer and fluid flow in a heat-generating porous media has been studied analytically. The study is limited to the range of parameters of interest to the packed-bed fuel element concept. The available heat transfer and pressure drop correlations are reviewed, and a system of partial differential equations which govern the velocity and temperature fields in an isotropic porous media is derived. Steady-state temperature distribution in a one-dimensional packed bed is studied, and a numerical method is presented for calculating transient temperature distributions. Pressure drop in a heat-generating packed bed is considered, and flow and temperature stabilities are examined.

1. INTRODUCTION

There is no theoretical upper limit to the rate of energy release by fission. In practice, however, the maximum power level of a reactor is frequently determined by the heat-removal rate. Thus, in a nuclear reactor operating at a sustained power level, the design of the core depends just as much on the thermal aspects as on nuclear considerations. The transfer of heat from fuel to coolant is facilitated by increasing both the contact area and the coolant volume. However, the increase of the coolant volume generally requires additional fissionable material to make the reactor critical.

The heat generated in the fuel is transferred across the solid-fluid interface to the coolant. From the properties of the fluid and the flow characteristics a reasonable prediction or an experimental determination of heat transfer coefficient can be made. Experimentally, a critical heat flux of 0.01728 Mw/cm^2 has recently been reported for subcooled water by Gambill and Greene,(1) but the pressure drop in a test section 1.43 cm long and 0.485 cm in ID was 56.6 atm. It is doubtful, however, that a reactor will be designed in a near future with a design heat flux of this order of magnitude. Thus, to increase appreciably the power density, we can increase the surface area per unit volume of the core. This, of course, is to be done by

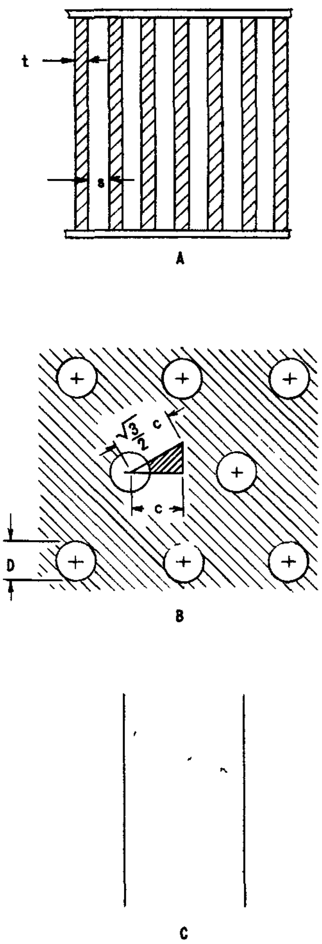

FIG. 1
GEOMETRICAL ARRANGEMENT OF EXISTING AND POSSIBLE FUEL ELEMENTS

keeping the coolant volume constant so as not to decrease the multiplication factor. For purposes of comparison three of many possible and existing fuel elements are given below (see Fig. 1):

(A) The core is composed of a number of fuel-bearing plates with the coolant flowing between them.

(B) The core is made from a block of fuel-bearing material containing a great number of small holes for coolant flow.

(C) The core is composed of a porous fuel or of a bed of small particles, the coolant flowing through the interstices.

The approximate formulas for voids and area densities of these three systems are given as functions of pertinent parameters in Table 1.

The area densities for these systems calculated from the formulas given in Table 1 are presented in Table 2 for three values of voids. The area density of porous material composed of spherical particles is considerably higher. Packed-bed fuel elements would yield an area-density advantage of at least an order of magnitude higher over equivalent, more realistic elements of type A or B.

Table 1

FORMULAS FOR VOIDS AND AREA DENSITIES OF THE THREE FUEL ELEMENTS

	A	B	C
Voids	$\frac{s}{s+t}$	$\frac{\pi D^2}{8\sqrt{3}\,c^2}$	ϵ
Area Density	$\frac{2}{s+t}$	$\frac{\pi D}{2\sqrt{3}\,c^2}$	$\frac{6(1-\epsilon)}{D_p}$

Table 2

AREA DENSITIES FOR $t = D = D_p = 0.02$ cm
AS A FUNCTION OF VOIDS

Voids	A	B	C
0.3	70	60	210
0.4	60	80	180
0.5	50	100	150

The possibility of using a packed-bed fuel element has recently been emphasized by Rodin.[2] The study further shows that the packed-bed fuel element appears to offer some significant advantages: (1) possibility of achieving very high power densities and temperatures; (2) high fuel surface per unit volume reconciles high power densities with small temperature differences between the fuel particles and the coolant; (3) thermal stresses produced in the particles should be relatively unimportant since the temperature gradients are small; and (4) good heat transfer characteristics and more uniform temperatures. Thus, the possible use of a packed bed as a fuel element gives ample incentive for a study of fluid flow and heat transfer characteristics of this system.

The present study has been undertaken with the hope that it will contribute toward better understanding of the potentialities of a packed-bed fuel element. The purpose of the study was twofold: (1) formulation of the general heat transfer equations for flow of fluid in porous media; and (2) solution and clarification of some specific problems.

To this end, a short literature survey of both fluid flow and heat transfer pertinent to heat-generating packed beds was made. The general heat transfer problem was then formulated mathematically. Methods of determining the flow field and temperature distribution were then examined.

To fulfill the second purpose of this study, solution of equations for steady one-dimensional flow were then considered. Temperature distribution and the temperature difference between the fuel and the coolant were obtained. A method of determining the transient temperature distribution in a one-dimensional packed bed was presented. Finally, pressure drop for a one-dimensional bed with temperature-dependent viscosity and constant heat-generation rate was studied.

2. REVIEW OF PRESSURE DROP AND HEAT TRANSFER LITERATURE

2.1 Introduction

The study of flow of fluids and of heat through porous media has become basic for many scientific and technical applications. The subject has been studied by many investigators in such diversified fields as soil mechanics, petroleum and chemical engineering, filtration, powder metallurgy, and many others. All of these branches of science and engineering have contributed vast amounts of information on the subject. The papers on pressure drop and heat transfer through porous media and packed beds have been published in a number of journals. No attempt has been made to give a complete literature of these studies, and only a few pertinent references are cited. In this review, emphasis is placed on more recent experimental contributions. Before we proceed to review the literature, let us define some parameters characteristic of porous media.

A characteristic particle dimension, D_p, is used to represent the size of any particle. This particle dimension is defined as follows:

$$D_p = \text{Effective particle diameter} = \sqrt{A_p/\pi} \quad ,$$

where A_p is the particle surface area. For a packed bed, the surface per unit volume of bed is given by Carman[3] as

$$a = 6(1 - \epsilon)/D_p \, \phi_s \quad , \tag{2.1}$$

where ϵ is defined as the fraction of voids in the bed or the porosity of the bed:

$$\epsilon = \text{Porosity} = \text{Volume of voids in bed/Volume of bed} .$$

The Carman shape factor ϕ_s is a function of sphericity,[3] defined as

$$\psi = \text{Sphericity}$$

$$= \frac{\text{Surface area of a sphere having volume equal to that of the particle}}{\text{Surface area of the particle}}$$

Essentially Eq. (2.1) is a relationship between the effective particle diameter and a shape factor based upon the true particle area. The shape factor ϕ_s is unity for a sphere and less than unity for all other shapes.

For spherical particles the particle-to-particle contacts are point contacts, and the entire particle surface area is effective in the transfer of heat. However, this is not true for particles other than spheres. In addition to having a larger value of a, the nonspherical particles have only a portion of their surface area available for solid-fluid heat transfer due to significant particle-to-particle contacts. The effective heat transfer area per unit volume of bed can be written as

$$A = a\phi = 6(1 - \epsilon)\phi/D_p\ \phi_s \quad . \tag{2.2}$$

The Carman shape factor ϕ_s should not be confused with the particle shape factor ϕ, defined as

$$\phi = \text{Particle shape factor} = \frac{\text{Effective surface area for heat transfer}}{\text{Particle surface area}},$$

which represents the portion of the particle surface which participates in the particle-fluid heat transfer.

The transfer of heat from stationary particles to fluids flowing through porous media can be expressed by the equation

$$Q = hA\Delta T \quad , \tag{2.3}$$

where h is the conventional heat transfer coefficient and ΔT is the temperature difference between the solid and the fluid. The product hA is sometimes called the volumetric heat transfer coefficient. The results of heat transfer studies are generally correlated in terms of a heat transfer factor-Reynolds number relationship, where

$$j_h = \text{Heat Transfer Factor} = \frac{h}{c_p G}\left[\frac{c_p \mu}{k}\right]_f^{2/3} \tag{2.4}$$

and a Reynolds number (often called modified Reynolds number) based on particle diameter

$$Re = D_p G/\mu \quad . \tag{2.5}$$

In order to correlate data for particles of various shape, the Reynolds number may be modified as follows:

$$Re_m = \frac{6G}{A\mu} = \frac{6G}{a\phi\mu} = \frac{D_p G\ \phi_s}{(1 - \epsilon)\mu\phi} \quad . \tag{2.6}$$

2.2 Fluid Flow

There exist several excellent literature surveys on fluid flow through porous media. The most recent one is that of Scheidegger.[4] This book presents a coherent exposition of the physical principles of flow through porous media; however, it deals primarily with the low-velocity regime for which Darcy's law[4]

$$\vec{u} = -\frac{k}{\mu}(\text{grad } p - \rho\vec{g}) \tag{2.7}$$

is valid. The concept of permeability k as introduced in Darcy's equation permits phenomenological description of the flow through porous media in a low-velocity domain. However, an actual understanding of the phenomena can be obtained only if the concept of permeability can be reduced to more fundamental physical quantities. Reference is made to Scheidegger's treatise for a detailed review of the various theories that have been proposed to obtain correlations between the permeability and the dynamic properties of the porous media.

Many attempts have been made to determine the range of Reynolds numbers for which Darcy's law is valid. So far, however, no general picture has been disclosed, such, for example, as was made available for pipes by O. Reynolds. This is probably due to the fact that there is actually no physical basis for the expectation that flows should be analogous if the Reynolds numbers are. The transition from laminar to turbulent flow is sometimes difficult to define. Cornell and Katz[5] used the term "quasi-turbulent" to designate the situation in which the flow is obviously not viscous, as shown by lack of proportionality between pressure drop and flow rate, but where the small sizes of channels are difficult to reconcile with the usual definition of turbulence. As far as the "critical" Reynolds number is concerned, there exists a great discrepancy regarding the "critical" Reynolds number above which the Darcy's law would be valid. The values range from 0.1 to as high as 75.[4] The uncertainty of a factor 750 about the "critical" Reynolds number may reflect in part the indeterminacy of the particle diameter and the fact that porous media is not equivalent to an assemblage of straight tubes, as postulated by the hydraulic radius theory for porous media.

A commonly encountered correlation of pressure drop versus flow rate data for porous media is formulated in terms of a Reynolds number based on the so-called "equivalent capillary diameter" and a friction factor, calculated by invoking the analogy between the laws of Darcy and Poiseuille, which are valid only for low-velocity flow. This analogy would explain the deviation from Darcy's law at higher flow rates because of the emergence of the inertia effects in laminar flow, not necessarily due to onset of turbulence within the interstices, which are visualized as straight, parallel capillaries.

In reviewing the pressure drop correlations it is found that there is a large variety of them. It is certain that they cannot all be universally valid, since many of them contradict each other. To mention just a few, Leva et al.[6] reviewed the literature on pressure drop and presented their own data. The particle sizes used in the experiments were of the same order of magnitude that are of interest in the packed-bed fuel element design. Correlating equations were given for laminar ($Re < 10$), transition ($10 < Re < 100$), and turbulent ($Re > 100$) regions. In the turbulent flow region, modified friction factor correlations were given for smooth particles, rough particles, and rough granules.

Brownell et al.[7] have found that particle roughness is a minor factor in the flow of fluids through random-packed beds. He was able to correlate pressure drop data for such flow to Moody's friction factors for pipe flow. A single curve was obtained when a modified friction factor was plotted versus a modified Reynolds number. The two latter terms were defined as follows:

$$Re' = D_p \, G \, F_{Re} / \mu \tag{2.8}$$

and

$$f' = 2 \, g_c \, D_p \rho \Delta p / L \, G^2 \, F_f \quad , \tag{2.9}$$

where F_{Re} and F_f are factors which are functions of porosity and sphericity only. These factors were obtained from experimental data and are given graphically[7] as functions of porosity with parameters of particle sphericity.

A pressure drop correlation applicable for all types of flow through beds of granular solids of any shape, size, orientation, and fraction of voids was developed by Ergun,[8] using a theoretical as well as an empirical approach:

$$\left(\frac{2 \, g_c \rho \Delta p}{G^2}\right)\left(\frac{D_p}{L}\right)\left(\frac{\epsilon^3}{1 - \epsilon}\right) = 300 \left(\frac{1 - \epsilon}{Re}\right) + 3.5 \quad . \tag{2.10}$$

This expression is a result of the addition of the Blake-Kozeny equation for laminar flow ($Re < 6$) and $\epsilon < 0.5$ to the Burke-Plummer equation valid for $Re > 6{,}000$. The Ergun equation, like many others that relate pressure drop to polynomials of the fluid mass flow rate, differs from them in that the coefficients have definite theoretical significance.

The flow through porous media is determined by a large number of variables, some of which are statistical in nature. A major obstacle to an adequate analysis of flow through porous beds is the difficulty of securing geometrically similar beds over a wide range of porosity or of varying pore diameter while retaining constant particle diameter. A generally applicable correlation for predicting pressure drop during fluid flow through porous media still awaits development. It seems probable that a controlling element in the measurements is the bed configuration, a statistical concept with which none of the formulas published to date can cope. The available correlations are at best valid each for an application to particular system. In this instance, the correlations may be very useful for engineering applications to particular systems, but proper caution should be used if they are applied in any other system than for which they had been originally obtained.

2.3 Heat Transfer

A study of the literature will reveal that exact mathematical solutions of heat transfer problems have been obtained only in the case in which the following three conditions are met: (1) the coolant is flowing at a constant velocity, (2) the physical properties are constant, and (3) the heat generation is a linear function of temperature of the porous solid. The theory of the transfer of heat between a porous body and a fluid has been developed by Anzelius,(9) Schumann,(10) and others. These authors have used a mathematical model with two independent variables - the time and the distance along the axis. The problem was extended by Brinkley(11) to the case in which the solid is generating heat. The heat generation was assumed to be a linear function of the temperature of the solid and the parameters of this function independent of position and time. Amundson,(12,13) by including both axial and radial conduction, considered the problem from a more general point of view.

Experimental investigations of the fluid-particle heat transfer in packed beds have been made utilizing both steady and unsteady methods. Hougen and associates(14,15,16) studied simultaneous heat and mass transfer in beds of spherical and cylindrical particles, Raschig and partition rings, and Berl saddles. By using the analogy between heat and mass transfer, they obtain heat transfer correlations.

Denton(17) carried out steady-state investigations in which test spheres were placed at various positions in a packed bed and heat generated in the test spheres by means of resistance heaters inside each sphere. The heat was transferred to a fluid flowing through the bed. Using air as the coolant, the average heat transfer coefficient was determined over a large range of Reynolds numbers. Denton found that the influence of random packing on the average heat transfer coefficient was negligible and that for values of $N > 17.5$ the wall effect is small; the average heat transfer coefficient can be correlated by the equation

$$j_h = 0.583\ Re^{-0.30} \quad (5{,}000 < Re < 50{,}000) \quad . \tag{2.11}$$

Glaser and Thodos(18) investigated the steady-state heat transfer in the flow of various gases through fixed, random-packed beds consisting of metallic spheres, cubes and cylinders. A uniform generation of heat in the bed was obtained by passing electric current through the metallic particles which were packed between perforated plate electrodes. External heat was supplied to the bed to eliminate radial heat transfer. Direct temperature measurement of both solids and gases in the bed was accomplished by the insertion of thermocouples in the interstices of the bed and by permanent attachment of thermocouples to the surfaces. The resulting expression for the heat transfer factor for either spheres, cubes, or cylinders when the wall effect is negligible was found to be

$$j_h = \frac{0.535}{(Re'')^{0.3} - 1.6} \qquad (100 < Re'' < 9{,}200) \quad , \tag{2.12}$$

where the Reynolds number was modified to

$$Re'' = \frac{\sqrt{A_p}\, G}{\mu(1 - \epsilon)\phi} \tag{2.13}$$

in order to obtain a single correlation for each of the various particle types employed.

Steady-state heat transfer between a random-packed bed of spheres and a stream of air was again a subject of investigation by Baumeister and Bennett.(19) Heat was generated in the steel spheres comprising the bed by high-frequency induction coils which surrounded the test section. Average heat transfer coefficients for the bed were calculated from knowledge of total heat generated in the particles, the surface area of the particles, and the area mean temperature difference between the particles and the air stream. For the case of no wall effect, the following heat transfer correlation was obtained:

$$j_h = 0.918\ Re^{-0.267} \qquad (200 < Re < 10{,}000) \quad . \tag{2.14}$$

Most recently, DeAcetis and Thodos(20) studied simultaneous heat and mass transfer to air from porous spherical particles 1.59 cm in diameter. They correlated their results by the equation

$$j_h = \frac{1.10}{Re^{0.41} - 1.5} \qquad (13 < Re < 2136) \quad . \tag{2.15}$$

The relationship between the j_h-factor and the Reynolds number was found to be independent of the type of packing arrangement employed.

The effect of wall voids on the average heat transfer coefficient for spherical particles observed by various investigators is shown in Fig. 2. It is seen that the heat transfer coefficient depends on the ratio of the container diameter to the particle diameter, N, but the magnitude of the effect changes with the Reynolds number. It is believed that the wall effect on heat transfer and fluid flow will be small for particles and bed sizes of interest to this study.

The heat transfer correlations just reviewed are shown graphically in Fig. 3. They are not in good agreement with each other. Probably, the errors in the measurement of the particle surface and the coolant temperatures contribute the most to the disagreement between the various correlations.

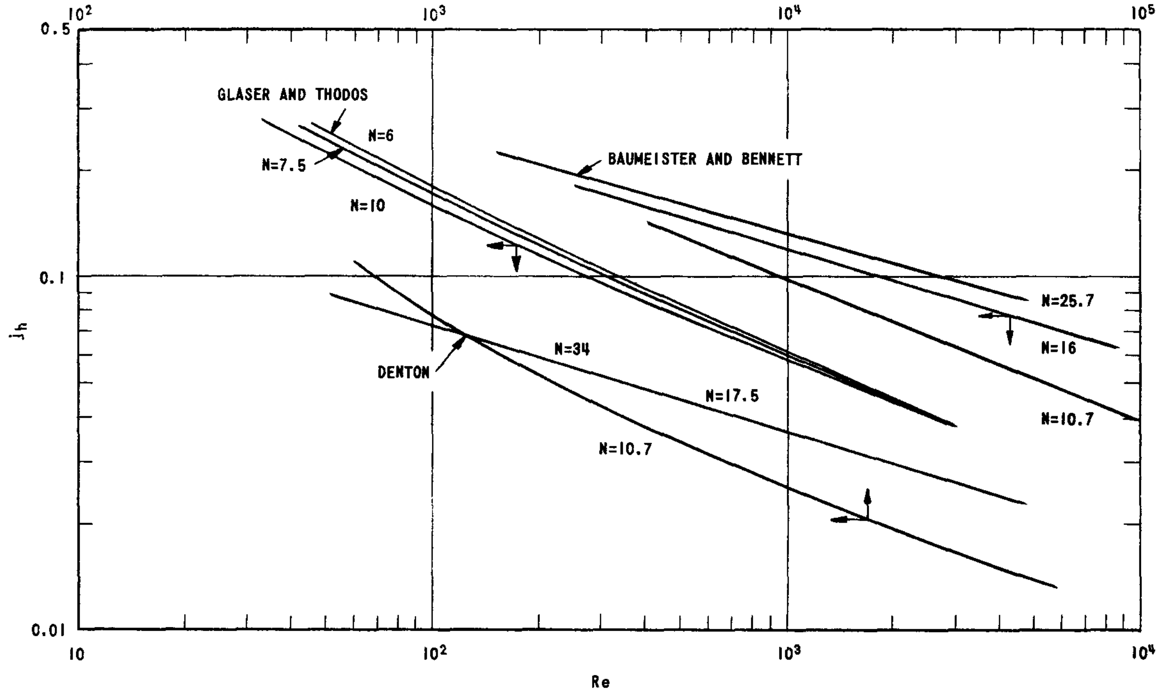

FIG. 2
THE SIGNIFICANCE OF THE WALL EFFECT ON j_h-FACTOR

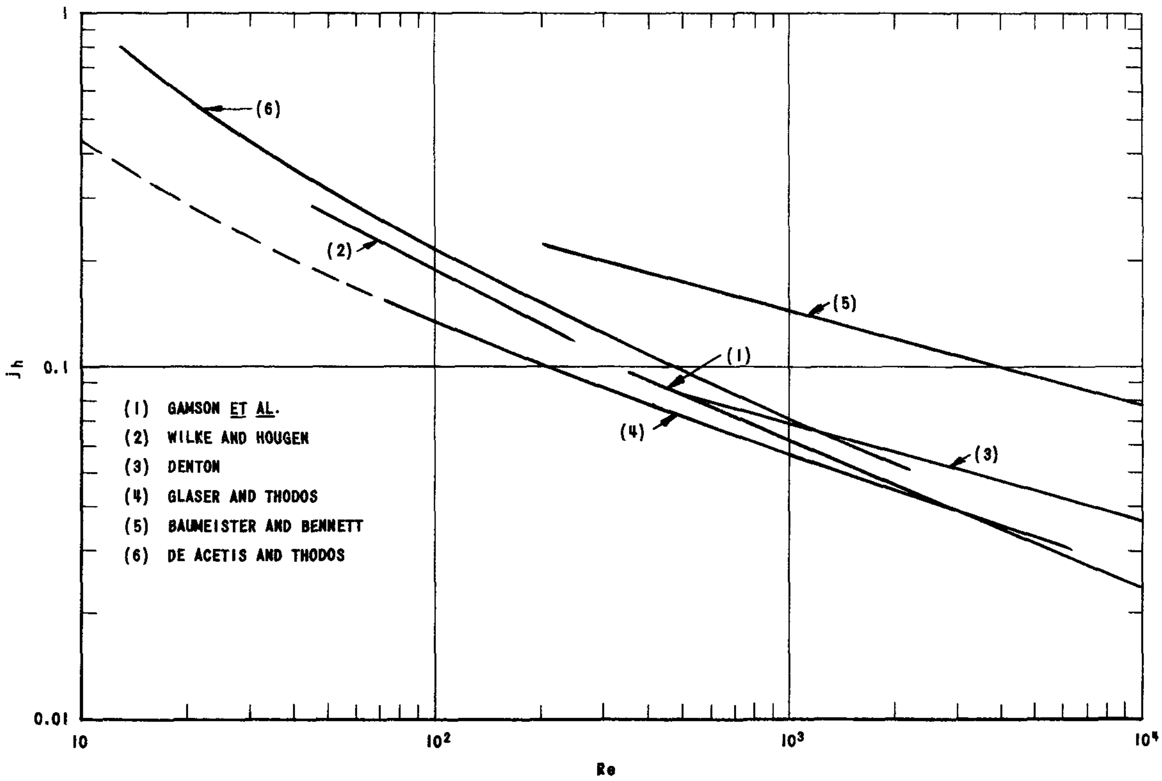

FIG. 3
COMPARISON OF j_h-FACTOR CORRELATIONS

As mentioned earlier, there is no physical basis for the expectation that flows should be analogous if the Reynolds numbers are. For example, it has not been established experimentally that one would obtain the same j_h-factor for a given Reynolds number if in one experiment large particles and low flow rates were used, whereas in another small particles and high flow rates were employed. It is therefore possible that the data of these investigators would be in a better agreement if a different type correlation were found.

Part of the discrepancy between various correlations may in part be due to the fact that the heat transfer coefficients determined in the papers reviewed were based on the total surface area of the particles. The use of the particle shape factor in the heat transfer correlations accounts for the part of the particle surface area which is not available to participate in the particle-fluid heat transfer. In fixed beds, there are two phenomena which contribute to this unavailability of particle surface. The first is the presence of particle-to-particle contacts which, for all shapes other than spherical, cannot be point contacts and will therefore prevent a part of the particle surface area from being available as effective solid-fluid heat transfer area. The second is the inaccessibility of a portion of the particle surface area to the fluid stream. This effect occurs when particles, such as Raschig rings and partition rings, are employed, since these particles present inner surface areas which may be unavailable to the fluid stream.

3. BASIC EQUATIONS

3.1 General

The prediction of the state of flow of fluid and of heat in a packed bed requires the knowledge of the following quantities: the velocity vector, two thermodynamic properties (usually temperature and pressure), and the temperature of the solid. The problem is completely described if these quantities are known at every point in the fluid and the solid, and for all times subsequent to some initial time. The flow may be described by a system of differential equations expressing the conservation of mass, momentum and energy (for the fluid and solid phases), and appropriate boundary conditions. The conservation equations are presented below.

3.2 Continuity Equation

Since the flow space between the particles is irregular, it would be impossible to treat the flow field in detail. We shall therefore take the packed bed as an isotropic porous medium and consider only the mean velocity of coolant flow through a unit area perpendicular to the average mass flow. Consider any volume V in the interior of the bed bounded by a closed surface S, and let $\vec{u}$ be the mean velocity vector. Then for coolant flowing through the bed it follows from the law of the conservation of mass that the net mass of coolant entering any volume V must equal the increase in the mass of coolant in V over the same period of time:

$$-\int_S \rho\vec{u}\cdot d\vec{S} = \int_V \epsilon \frac{\partial \rho}{\partial \tau}\, dV \quad . \tag{3.1}$$

Hence, using Green's lemma, we transform the left-hand side of Eq. (3.1) and obtain

$$\epsilon \frac{\partial \rho}{\partial \tau} + \operatorname{div}(\rho\vec{u}) = 0 \quad . \tag{3.2}$$

Equation (3.2) is the equation of continuity for a coolant flowing through a porous medium.

3.3 Equation of Motion

Darcy's law for flow of fluids through porous media is valid only in the low-velocity domain, outside of which more general flow equations must be used to describe the flow. The limitations of Darcy's law due to turbulence and due to molecular effects are not the only ones. A series of other possible effects can cause Darcy's law to break down.

The physics of fluid flow through porous media, when the temperature of the coolant and the particles are not equal and uniform throughout the space considered, and the flow velocity is quite high, has been studied very little. The interactions between the coolant and the particles are much more difficult to describe analytically; in fact, the equation of motion for these types of conditions is not known. For the problem considered here, Darcy's law must be modified. Thus, assuming that the flow is laminar up to a "critical" Reynolds number, and turbulent above, we can postulate an equation of motion of the form suggested by Engelund:[4]

$$F(\rho|\vec{u}|)\vec{u} = - \text{grad } p + \rho\vec{g} \quad , \tag{3.3}$$

where

$$F(\rho|\vec{u}|) = \begin{cases} \mu/k & (Re < Re_{crit}) \\ a + b\rho|\vec{u}| & (Re > Re_{crit}) \end{cases}$$

Both a and b depend only on viscosity of the coolant and the structure of the porous media. The force terms in the equation of motion that account for the acceleration of the fluid have been neglected. In Section 6 it is shown that the acceleration pressure drop is really negligible compared to the frictional pressure drop, and therefore the simplification of the equation of motion is justifiable.

3.4 Equations of Energy

When heat is generated in a porous media through which fluid is flowing, heat may be transferred from one part of the system to another by four basic mechanisms: (1) conduction of heat through the solid and the fluid phases; (2) convective heat transfer between the solid and the fluid phases; (3) physical movement of the fluid which carries its own heat content; and (4) radiant heat transfer. The equations of energy for the fluid and the solid phases are derived separately, first for the fluid and then for the solid phase.

Consider any element of volume V in the interior of the porous bed bounded by a closed surface S. The equation of energy conservation for the fluid phase at any point in the bed, neglecting energy dissipation, the work of pressure, and gravity forces, can be written as

$$\frac{\partial E}{\partial \tau} = E_1 + E_2 \quad , \tag{3.4}$$

where E is the energy content per unit volume, E_1 is the rate of convection of energy across the boundaries of V, and E_2 is the heat transferred from the solid to the fluid phase. Thermal conduction through the fluid phase was neglected. The validity of this assumption can be checked by direct computation with measured effective conductivities for porous media through which fluids are flowing.[21]

The change in heat content of the fluid in the element of volume V per unit of time is

$$\frac{\partial E}{\partial \tau} = \int_V \epsilon \frac{\partial(\rho e)}{\partial \tau} dV \quad , \tag{3.5}$$

where e is the energy content per unit mass.

The rate of convection of energy, E_1, through the element of area dS is given by

$$E_1 = - \int_S \rho e\vec{u} \cdot d\vec{S} \quad . \tag{3.6}$$

Transforming the right-hand side of Eq. (3.6) by Green's lemma, we obtain

$$E_1 = - \int_V \operatorname{div} (\rho e\vec{u}) \, dV \quad . \tag{3.7}$$

The energy transferred from the solid to the fluid per unit of time, E_2, is given by

$$E_2 = \int_V Ah \, (t - T) \, dV \quad . \tag{3.8}$$

Now substituting Eqs. (3.5), (3.7) and (3.8) into (3.4) and noting that the element of volume is arbitrary, we obtain

$$\epsilon \frac{\partial(\rho e)}{\partial \tau} + \operatorname{div} (\rho e\vec{u}) = Ah \, (t - T) \quad . \tag{3.9}$$

Using the vector identity

$$\operatorname{div} (\rho e\vec{u}) = e \operatorname{div} (\rho\vec{u}) + \rho\vec{u} \cdot \operatorname{grad} e$$

and the continuity Eq. (3.2), we can write Eq. (3.9) as

$$\epsilon\rho \frac{\partial e}{\partial \tau} + \rho\vec{u} \cdot \operatorname{grad} e = Ah \, (t - T) \quad . \tag{3.10}$$

In deriving Eq. (3.10) it was assumed that the fluid does not absorb and emit thermal radiation. The effect of radiation is then only to change the surface temperature of the particles.

Consider a packed bed through which heat is transferred by conduction, heat is generated in the solid, and heat is transferred at the interface of the solid and the coolant by convection. The law of energy conservation for the solid phase in the element of volume V bounded by the surface S can be expressed as

$$\int_V (1-\epsilon)\,\rho_s c_s \frac{\partial t}{\partial \tau}\, dV - \int_S k_{eff}\, \text{grad}\, t \cdot d\vec{S} = \int_V q''' \, dV + \int_V Ah\,(T-t)\, dV \quad . \tag{3.11}$$

Transforming the second term on the left-hand side of Eq. (3.11) by Green's lemma and noting that the element of volume V is arbitrary, we obtain

$$(1-\epsilon)\,\rho_s c_s \frac{\partial t}{\partial \tau} = \text{div}\,(k_{eff}\, \text{grad}\, t) + q''' + Ah(T-t) \quad . \tag{3.12}$$

The effective thermal conductivity k_{eff} depends not only on the temperature, pressure, and chemical composition of the solid and the coolant, but also on the structure of the bed. In a bed of spheres or of particles of other shape where essentially a point contact exists between separate parts of the solid, conduction cannot take place only in the solid but must also occur across narrow gaps existing near each point of contact. Since the energy transfer by thermal radiation cannot be formulated in a rigorous fashion because of the complex geometrical arrangement of the particles, the contribution due to thermal radiation is also included in k_{eff}.

3.5 Discussion of the Basic Equations

In the derivation, the bed was assumed to be nonadiabatic, so that heat transfer by conduction had to be taken into account. The particles were assumed to be so small that the bulk and surface temperatures of the solid phase at any point were thus taken to be the same. This assumption is most valid for small particles of high thermal conductivity. The major resistance to heat transfer in this case is at the particle surface. As the particle size increases, however, radial temperature gradients will arise within the individual particles, and the diffusion of heat in the particle may become the controlling mechanism in the transfer of heat. Equations of energy will have to be then modified to account for the temperature gradients in the solid particles.

Equations (3.2), (3.3), (3.10) and (3.12) together with the equation of state are now completely sufficient for solution of the problem. If the boundary conditions and the heat source q''' are given, the unknown quantities

ρ, p, $\vec{u}$, t, and T can be found. It is evident that an exact solution is impossible in the majority of cases. However, some simplifying assumptions can be made and analytical solutions obtained. For example, as an extremely simplified approach to the problem, Green[22] has suggested the assumption $Ah = \infty$. Setting Ah to infinity is equivalent to assuming that at any point in the system the solid and the fluid temperatures are equal. The assumption is tenable, however, only when the heat generation is small and/or the mass flow rate is very large. A closer approach to physical reality can be made if the main resistance to heat transfer is assumed to be in the solid-fluid interfacial film, and the conduction in the solid phase is negligible. Special cases of equations presented in this section are solved in the following sections.

4. TEMPERATURE DISTRIBUTION IN A HEAT-GENERATING PACKED BED

4.1 Temperature Distribution in One-dimensional Packed Bed

Since no general solution of the problem can be obtained, some simplifications are introduced to make the problem more readily tractable mathematically. The case of one independent space variable is considered in this section. The assumptions involved in the calculation of temperature distribution in a one-dimensional heat-generating packed bed are as follows:

(1) The coolant flow and heat transfer are steady.

(2) The physical properties are independent of temperature.

These assumptions simplify the problem considerably. The equation of motion becomes uncoupled from the energy equation, and the solution of the continuity equation is just a constant. Thus, the basic equations reduce to

$$k_{eff}\frac{d^2t}{dx^2} + Ah(T - t) = -q''' \qquad (4.1)$$

and

$$c_pG\frac{dT}{dx} + Ah(T - t) = 0 \quad . \qquad (4.2)$$

The physical model and the coordinate system are shown in Fig. 4. The bed is assumed to extend indefinitely in the directions normal and parallel to the plane of the figure. If the boundary conditions and the heat source q''' are given, the unknown temperatures T and t can be readily solved. For bed sizes and heat generation rates of interest in this study, the coolant temperature rise upstream of the bed is negligible compared with the temperature rise across the bed, and therefore it is not necessary to consider the differential equation governing the temperature distribution of the coolant in this region.

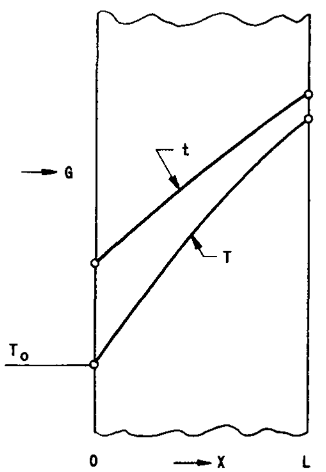

FIG. 4
PHYSICAL MODEL AND COORDINATE SYSTEM

Introducing the heat-generation distribution function, $\Phi = q'''/q'''_{ave}$, a new independent variable defined as $\eta = x/L$, Eqs. (4.1) and (4.2) are written as

$$\frac{d^2t}{d\eta^2} + N_1(T - t) = -N_2\Phi(\eta) \qquad (4.1)$$

and

$$\frac{dT}{d\eta} + N_3(T - t) = 0 \quad , \tag{4.2}$$

where

$$N_1 = \frac{AhL^2}{k_{eff}} \quad , \quad N_2 = \frac{q'''_{ave}\ L^2}{k_{eff}} \quad \text{and} \quad N_3 = \frac{AhL}{c_p G} \quad .$$

The complementary solution of this system of equations is easily found to be

$$T_c = c_1 + c_2\, e^{m_2\eta} + c_3 e^{m_3\eta} \quad , \tag{4.3}$$

where

$$m_2, m_3 = N_3/2\ [-1 \pm \sqrt{1 + 4N_1/N_3^2}] \quad . \tag{4.4}$$

However, before a particular solution can be obtained, the heat-generation rate must be known. For the sake of completeness, uniform and sinusoidal heat-generation distribution functions are considered, but in general the problem can be solved when $\Phi(\eta)$ is any arbitrary function of η. Thus, the particular solution for $\Phi(\eta) = 1$ is

$$T_p = N_2 N_3 \eta / N_1 \quad , \tag{4.5}$$

and for $\Phi(\eta) = \sin\pi\eta$ is

$$T_p = \frac{N_2 N_3^2}{(\pi^2 + N_1)^2 + (\pi N_3)^2} \sin\pi\eta - \frac{N_2 N_3(\pi^2 + N_1)}{\pi[(\pi^2 + N_1)^2 + (\pi N_3)^2]} \cos\pi\eta \quad . \tag{4.6}$$

The general solution of the coolant temperature distribution is

$$T = T_c + T_p = c_1 + c_2 e^{m_2\eta} + c_3 e^{m_3\eta} + N_2 N_3 \eta / N_1 \tag{4.7}$$

and

$$T = c_1 + c_2 e^{m_2\eta} + c_3 e^{m_3\eta} + \frac{N_2 N_3^2}{(\pi^2 + N_1)^2 + (\pi N_3)^2} \sin\pi\eta - \frac{N_2 N_3(\pi^2 + N_1)}{\pi[(\pi^2 + N_1)^2 + (\pi N_3)^2]} \cos\pi\eta \tag{4.8}$$

for $\Phi(\eta) = 1$ and $\Phi(\eta) = \sin\pi\eta$, respectively.

The general solution for temperature distribution in the solid phase is given by

$$t = \left(c_1 + \frac{N_2}{N_1}\right) + \left(1 + \frac{m_2}{N_3}\right) c_2 e^{m_2 \eta} + \left(1 + \frac{m_3}{N_3}\right) c_3 e^{m_3 \eta} + N_2 N_3 \eta / N_1 \quad (4.9)$$

and

$$t = c_1 + \left(1 + \frac{m_2}{N_3}\right) c_2 e^{m_2 \eta} + \left(1 + \frac{m_3}{N_3}\right) c_3 e^{m_3 \eta}$$

$$+ \frac{N_2(N_1 + \pi^2 + N_3^2)}{(\pi^2 + N_1)^2 + (\pi N_3)^2} \sin \pi\eta - \frac{N_1 N_2 N_3}{\pi[(\pi^2 + N_1)^2 + (\pi N_3)^2]} \cos \pi\eta \quad (4.10)$$

for $\Phi(\eta) = 1$ and $\Phi(\eta) = \sin \pi\eta$, respectively.

For the evaluation of the constants of integration in Eqs. (4.7) through (4.10), three boundary and initial conditions are required. In general, the boundary conditions on t are of the form t known, $dt/d\eta$ known, or $f(t, T, dt/d\eta)$ known, and T is known at $\eta = 0$. It is possible to perform an experiment in which the temperature of the incoming coolant and the temperature at both ends of the bed are measured. As only a link in the total heat transfer problem is examined, it is impossible to write down the temperatures t and T at $\eta = 0$ and $\eta = 1$. However, some important conclusions can be reached by considering the magnitude of the parameters m_2 and m_3.

Consider a packed bed composed of spherical UO_2 particles, 200 microns in diameter and of porosity $\epsilon = 0.35$. The bed thickness is 1.0 cm and it is cooled by He at a pressure of 40 atm. Using the correlation of Glaser and Thodos[18] for the heat transfer coefficient, the values of parameters m_2 and m_3 were calculated and are tabulated for various mass flow rates in Table 3. It is evident from the values of m_2 and m_3 and Eqs. (4.7) and (4.8) that heat transfer by conduction is negligible. The effect of conduction is confined to the immediate vicinity at the entrance and exit from the bed, where the temperature distribution departs slightly from a straight line.

Table 3

VALUES OF PARAMETERS m_2 AND m_3

G (kg/sec)	0.5	1.0	2
m_2	79	106	138
m_3	-160	-162	-178

It follows from Eq. (4.2) that the local temperature difference (t - T) is given by

$$(t - T) = m_2 c_2 e^{m_2 \eta} + m_3 c_3 e^{m_3 \eta} + N_2 N_3 / N_1 \qquad (4.11)$$

for the case $\Phi(\eta) = 1$. Since m_2 and m_3 are large (t - T) is constant across the bed, except at values of $\eta = 0$ and $\eta = 1$, for which there is a small departure from a constant value. In view of this fact, the heat transfer by conduction is negligible in problems of interest to packed-bed fuel element design. The equations of energy therefore can be simplified.

4.2 Temperature Rise in the Coolant

In this section the energy Eqs. (4.1) and (4.2) are considered. The energy transfer by molecular conduction is neglected, and only the one-dimensional case is studied. Assuming that the flow is one dimensional and steady, Eqs. (4.1) and (4.2) reduce to

$$\frac{dT}{d\eta} - \frac{q'''_{ave} L}{c_p G} \Phi(\eta) = 0 \quad . \qquad (4.12)$$

If the temperature of the coolant at the inlet to the bed is T_0, the solution of Eq. (4.12) is

$$T = \int_0^{\eta} \frac{q'''_{ave} L}{c_p G} \Phi(\eta) \, d\eta + T_0 \quad . \qquad (4.13)$$

Assuming that the specific heat of the coolant is constant, Eq. (4.13) can be written in dimensionless form as

$$\frac{T - T_0}{q'''_{ave} L / c_p G} = \int_0^{\eta} \Phi(\eta) \, d\eta \quad . \qquad (4.14)$$

The coolant temperature rise calculated from Eq. (4.14) is given in Fig. 5 for uniform and sinusoidal heat generation.

Materials limitations frequently dictate that the maximum surface temperature of the fuel be the critical factor in determining maximum reactor power density. The axial surface and the coolant temperature variation, being a function of the distributions of the heat-generation rate, may be influenced by changing the fuel distribution; however, in packed-bed fuel elements the maximum surface temperature occurs at the exit of the bed, because the temperature drop across the film is very small compared to the coolant temperature. Thus, from the heat transfer standpoint, it is necessary to use a coolant with the highest heat-removal capacity between

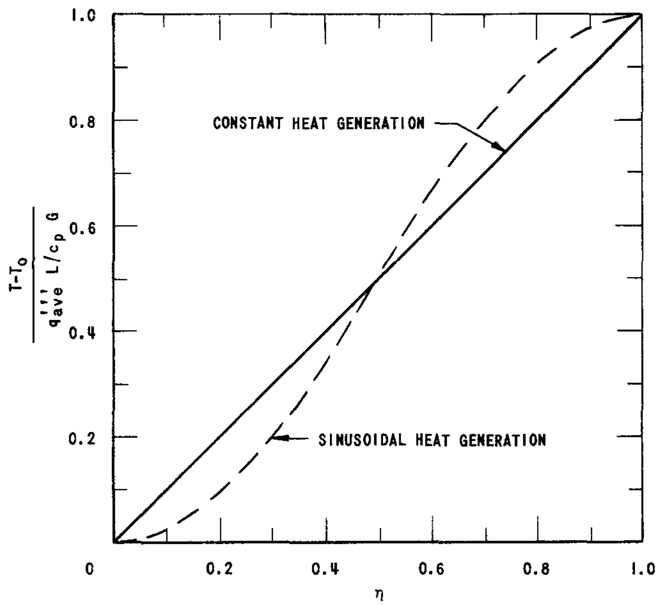

FIG. 5
COOLANT TEMPERATURE VARIATION WITH THE DIMENSIONLESS DISTANCE

the inlet and the outlet temperatures in order to obtain maximum power density. For this purpose four fluids: hydrogen, helium, water, and air, were investigated. The enthalpy and specific heat data for these coolants were taken from Refs. 23, 24, and 25. The results are shown in Figs. 6 and 7. The heat-removal capacity of a liquid metal, such as sodium, would lie between those of water and air. On the basis of mass flow rate, molecular hydrogen has the highest heat removal capacity; then follow helium, water (with vaporization taking place in the bed), and air. On the mole-flow basis, water has the highest heat removal capacity and helium the smallest.

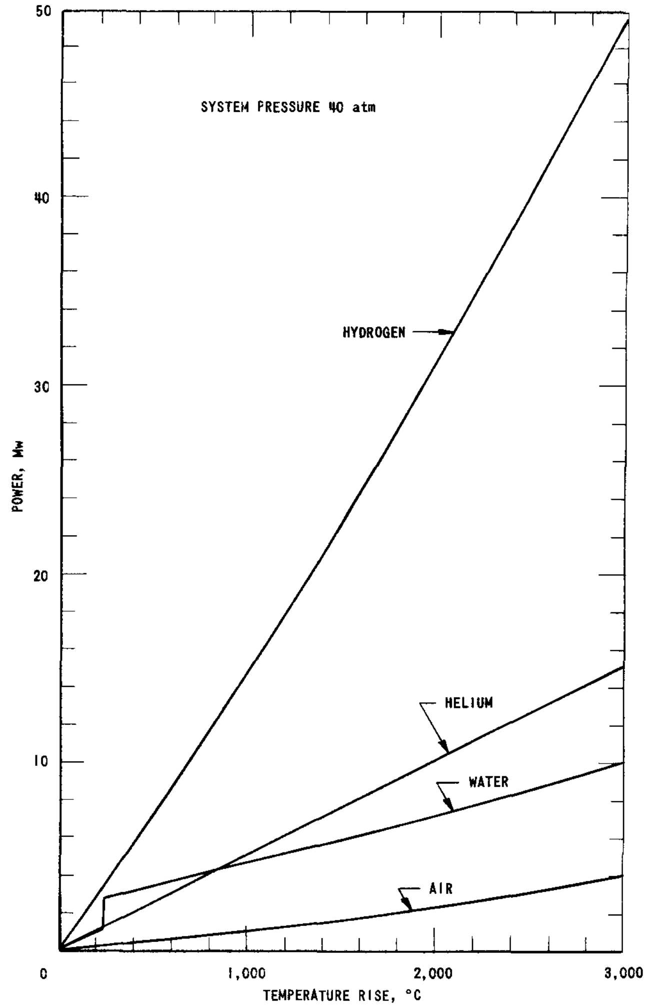

FIG. 6
HEAT REMOVAL CAPACITY OF A COOLANT, MASS FLOW RATE
1 kg/sec, INITIAL TEMPERATURE 0°C

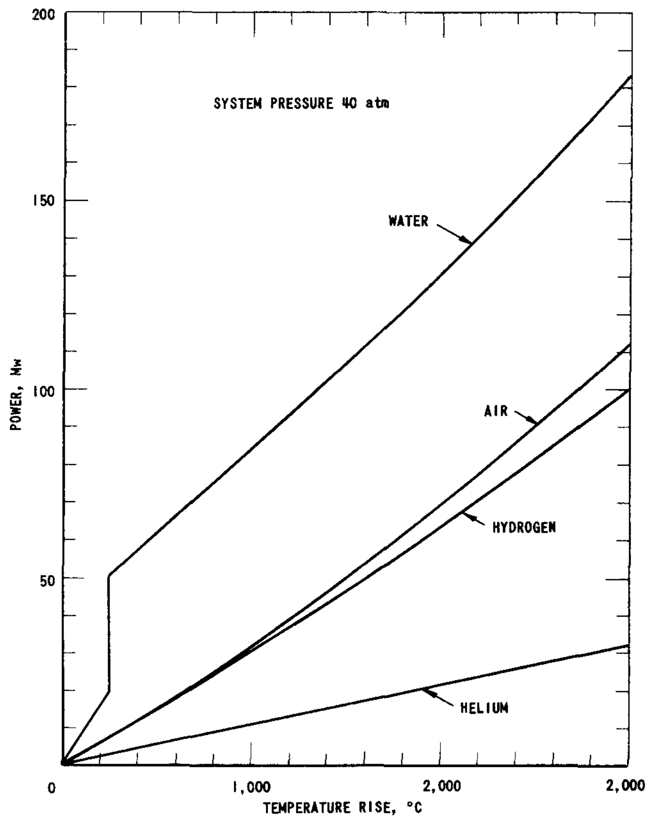

FIG. 7
HEAT REMOVAL CAPACITY OF A COOLANT, MASS FLOW RATE
1 mole/sec, INITIAL TEMPERATURE 0°C

5. TRANSIENT TEMPERATURE DISTRIBUTION IN A HEAT-GENERATING PACKED BED

5.1 Introduction

Most investigators working on heat transfer problems in packed and porous beds have used a mathematical model with two independent variables - the time, and the distance along the axis of the packed bed. This model introduces a number of simplifications in that it assumes plug flow through the bed and neglects the variation of temperature with the radial position. In addition, the heat conduction along the length of the bed is overlooked; even so, this one-dimensional model has proved very useful in studying, both experimentally and theoretically, the performance of packed beds. However, the equations which describe even this simplified model are as a rule too complicated to be solved analytically.

The purpose of this section is to present a numerical procedure, known as the method of characteristics, for solving the equations of the one-dimensional model. This method has been used with success in the field of compressible flow and, because of many attractive features, is being employed for the solution of a large variety of problems. The method of characteristics is an approximate one, but it is capable of producing numerical results of good accuracy.

The transfer of heat from the solid to the fluid for the one-dimensional model is governed by the equations

$$\rho c_p \left(\epsilon \frac{\partial T}{\partial \tau} + u \frac{\partial T}{\partial x}\right) = Ah\,(t - T) \tag{5.1}$$

and

$$\rho_s c_s (1 - \epsilon) \frac{\partial t}{\partial \tau} = q''' + Ah\,(T - t) \tag{5.2}$$

if, for simplicity, the mass flow rate is considered to be constant:

$$G = \rho_0 u_0 = \text{constant} \quad .$$

Therefore the initial conditions are

$$\begin{aligned} T &= T_0 \text{ at } x = 0 \text{ for all } \tau > 0 \\ t &= t_0 \text{ at } \tau = 0 \text{ for all } x \geq 0 \quad . \end{aligned} \tag{5.3}$$

In the analysis, the pressure drop across the bed is assumed to be small in comparison with the absolute pressure of the coolant.

The problem at hand is then the solution of Eqs. (5.1) and (5.2) subject to the two conditions given by Eq. (5.3). The system has been solved to date analytically only under the assumptions $u = u_0$ for all τ and x and q''' is a linear function of t.

Equations (5.1) and (5.2) can, for convenience, be put into the following forms:

$$\frac{\partial T}{\partial \tau} + u^* \frac{\partial T}{\partial \xi} = \frac{Ah}{\epsilon \rho c_p} (t - T) \tag{5.4}$$

and

$$\frac{\partial t}{\partial \tau} = \frac{1}{(1-\epsilon)\rho_s c_s} [q''' + Ah(T - t)] \quad , \tag{5.5}$$

where

$$\xi = \frac{\epsilon x}{u_0}; \quad u^* = \frac{u}{u_0}; \quad u_0 = \frac{G}{\rho_0} = \frac{\text{constant}}{\rho_0} \quad .$$

Therefore, the initial conditions become

$$\left. \begin{aligned} T &= T_0 \text{ at } \xi = 0 \text{ for all } \tau > 0 \\ t &= t_0 \text{ at } \tau = 0 \text{ for all } \xi \geqslant 0 \end{aligned} \right\} \tag{5.6}$$

5.2 The Method of Characteristics

Consider in the τ-ξ plane two families of lines, I and II, such that

$$d\tau/d\xi = 1/u^* \text{ for lines I} \tag{5.7}$$

and

$$\xi = \text{constant for lines II} \quad . \tag{5.8}$$

Such curves are plotted in Fig. 8, and are known as characteristics of the differential Eqs. (5.4) and (5.5). Since, now,

$$\frac{dT}{d\xi} = \frac{\partial T}{\partial \xi} + \frac{\partial T}{\partial \tau}\frac{d\tau}{d\xi} \tag{5.9}$$

along an arbitrary direction in the τ-ξ plane, then, along any characteristic I, from Eqs. (5.4) and (5.7), we have

$$\left(\frac{dT}{d\xi}\right)_I = \frac{Ah}{\epsilon \rho c_p u^*} (t - T) = f_1(t, T, \xi) \quad . \tag{5.10}$$

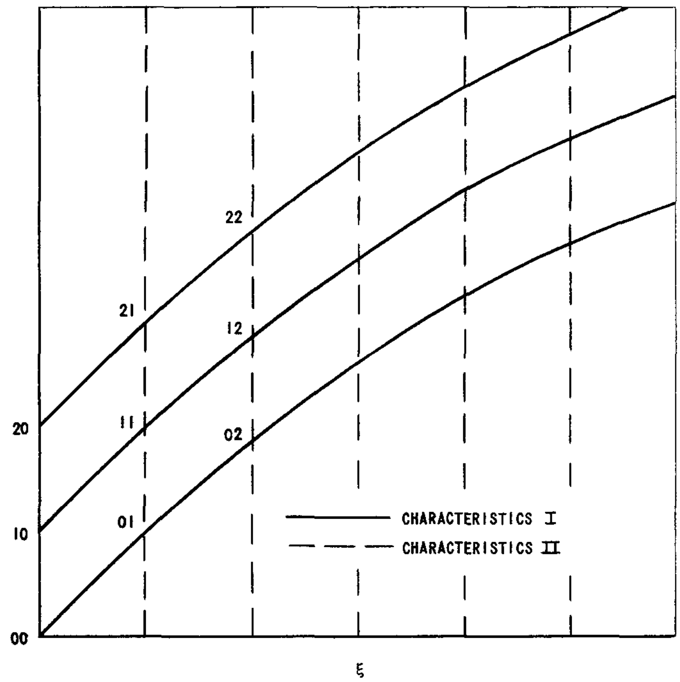

FIG. 8
NETWORK OF TWO FAMILIES OF CHARACTERISTICS FOR A PROBLEM OF TEMPERATURE DISTRIBUTION IN A HEAT-GENERATING PACKED BED

The notation $(dT/d\xi)_I$ denotes the derivative of T along the characteristic

$$\tau = \xi + \text{constant.} \tag{5.11}$$

Along any characteristic II, from Eqs. (5.5) and (5.8), we have

$$\left(\frac{dt}{d\tau}\right)_{II} = \frac{1}{(1-\epsilon)\rho_s c_s}\left[q''' + Ah(T-t)\right] = f_2(t_1, T, \xi, \tau) \quad . \tag{5.12}$$

Because of the initial conditions,

$$T = T_0 \tag{5.13}$$

along the characteristic $\xi = 0$ at all times, whereas

$$t = t_0 \tag{5.14}$$

along the characteristic $d\tau/d\xi = 1/u^*$, with $\tau = 0$ when $\xi = 0$.

Equations (5.10) and (5.12) are now two independent ordinary differential equations which have to be solved numerically. The procedures for such a numerical computation can be found in Ref. 26. It follows, therefore, that the initial conditions and the solution of Eqs. (5.10) and (5.12) provide a knowledge of the two dependent variables, T and t, along the characteristic lines ξ = const and $d\tau/d\xi = 1/u^*$.

The characteristics II are parallel to the τ axis. On the other hand, because u^* is not constant, the characteristics I cannot be drawn a priori. A construction of the mesh network is shown in Fig. 8. First, an arbitrary $\Delta\xi$ is chosen, so that characteristics II are

$$\xi = n\Delta\xi \quad , \tag{5.15}$$

where n is an integer. Then, as $u^* = 1$ at $\xi = 0$, a line is drawn with the slope

$$\frac{d\tau}{d\xi} = 1 \quad .$$

The intersection of this line with the characteristic $\xi = \Delta\xi$ is tentatively point (0,1), denoted by $(0,1)_1$. Now, using the simplest method known, the Euler method, for the solution of a differential equation, the approximate value of $T(0,1)_1$ follows from equation (5.10):

$$T(0,1)_1 = T_0 + f_1(0,0)\Delta\xi \quad . \tag{5.16}$$

Since the temperature of the coolant at the point $(0,1)_1$ is known, the velocity at that point can be readily computed from

$$u^*(0,1)_1 = \rho_0/\rho(0,1)_1 \quad ,$$

where $\rho(0,1)_1$ is the density of the fluid at the point $(0,1)_1$. It must be remembered that because u^*, T, and t are known along the τ axis, $f_1(0,0)$ can be calculated a priori. Of course, because of Eq. (5.6), $t(0,1) = t_0$.

The accuracy of $T(0,1)_1$ and $u^*(0,1)_1$ can be improved by any one of the methods suggested in Ref. 26. This iteration converges quite rapidly and can be continued until both point (0,1) and, T(0,1) have been determined with the desired accuracy.

The point (1,1) is tentatively located at the intersection of the characteristic $\xi = \Delta\xi$ and the straight line drawn from point (1,0) with the slope

$$d\tau/d\xi = 1 \quad .$$

Then,

$$T(1,1)_1 = T_0 + f_1(1,0)\,\Delta\xi \tag{5.17}$$

and

$$t(1,1)_1 = t_0 + f_2(0,1)\,\Delta\tau_{01}'' \quad , \tag{5.18}$$

where $\Delta\tau_{01}'' = \tau(1,1)_1 - \tau(0,1)$. The results of this step can be improved by iteration, and, in identical manner, the values of T, t, and u^* along the characteristic $\xi = \Delta\xi$ can be determined. The same procedure is applied along the characteristics $\xi = n\Delta\xi$, with $n = 2,3,4,\ldots$ until the complete mesh network is constructed.

The theory of the method of characteristics is treated in detail in many standard references(27,28) and therefore further mathematical details are omitted here. The method of characteristics replaces two hyperbolic partial differential Eqs. (5.4) and (5.5) by two ordinary differential equations (5.10) and (5.12), which are solved by a stepwise process. The method is simple and systematic; its accuracy at each step can be checked and improved. By constructing a finer mesh of characteristics, the method can be made as accurate as needed, and, finally, it is well suited for programing on a digital computer.

The analysis here has been limited to those problems where the mass rate of flow and the temperature of the entering coolant were constant. These, however, are not limitations on the method of characteristics, which can be used also, essentially with no modifications, when the rate of flow and the temperature of the entering coolant are known functions of time.

6. PRESSURE DROP

6.1 Derivation of Equation for Pressure Drop

In order to investigate the flow and temperature stability of a heat-generating packed bed, the pressure drop for steady flow of coolant through porous media must be studied. Exact solution of the pressure drop problem would require the solution of the system of conservation equations, including the equation of state for the coolant. Because of the nonlinearity of the conservation equations, an exact solution is not possible to obtain by analytical means, and an alternate approach is taken to obtain the pressure drop in a heat-generating packed bed.

Consider steady one-dimensional flow of incompressible fluid through a porous medium. Since the mass flow rate, $G = \rho_0 u_0 = \rho u$, is constant throughout the medium, ρu may serve as an independent variable. The mechanical work is given by equation

$$vdp + dF + \frac{udu}{g_c} = 0 \quad . \tag{6.1}$$

The frictional force dF is expressed as

$$dF = \frac{G^2}{2g_c\rho^2 D_p}\left[\frac{\alpha}{Re} + \beta\right] dx \quad , \tag{6.2}$$

where the parameters α and β are

$$\alpha = \frac{c_1(1-\epsilon)^2}{\epsilon^3} \quad , \quad \beta = \frac{c_2(1-\epsilon)}{\epsilon^3} \quad .$$

The mechanical work due to the change in elevation has been neglected; however, since the temperature of the fluid increases with the distance x, the effects due to changes in the momentum of the fluid [last term in Eq. (6.1)] cannot be ruled out a priori. Dividing Eq. (6.1) by v and introducing the variable ρu gives

$$\rho dp + \rho^2 dF + \rho(\rho u)\frac{du}{g_c} = 0 \quad . \tag{6.3}$$

Since

$$dG = d(\rho u) = 0 = \rho du + ud\rho \quad ,$$

Eq. (6.3) can be rewritten as

$$\rho dp + \rho^2 dF - \frac{G^2 d\rho}{g_c \rho} = 0 \quad . \tag{6.4}$$

For a constant rate of heat generation (any heat generation can be readily treated), the temperature of the coolant at any position in the medium is obtained from Eq. (4.13) as

$$T = T_0 + \left(\frac{q'''L}{c_pG}\right)\eta \quad . \tag{6.5}$$

Since the viscosity of the coolant appearing in the Reynolds number varies with temperature, we express the Reynolds number as

$$\frac{Re}{Re_0} = \frac{\left(\dfrac{G\,D_p}{\mu}\right)}{\left(\dfrac{G\,D_p}{\mu_0}\right)} = \frac{\left(\dfrac{1}{\mu}\right)}{\left(\dfrac{1}{\mu_0}\right)} = \frac{1}{(T/T_0)^m} = \left(\frac{T_0}{T}\right)^m \quad , \tag{6.6}$$

where Re_0 is the Reynolds number at the entrance to the porous medium and m is a constant determined from the viscosity data. Assuming that the coolant is a gas; the density is given by

$$\rho = \frac{Mp}{ZRT} = \frac{p}{\sigma T} \quad , \tag{6.7}$$

where Z is the compressibility, R is the gas constant, and M is the molecular weight of the coolant. Substituting Eqs. (6.7) and (6.6) into (6.4) and rearranging, we obtain

$$\frac{1}{2}\,d(p^2) + \frac{\sigma T G^2}{2\,g_c\left(\dfrac{D_p}{L}\right)}\left[\frac{\alpha}{Re_0}\left(\frac{T}{T_0}\right)^m + \beta\right] d\eta - \frac{\sigma T G^2}{g_c}\,\frac{d\left(\dfrac{p}{\sigma T}\right)}{\left(\dfrac{p}{\sigma T}\right)} = 0 \quad . \tag{6.8}$$

The last term on the right-hand side of this equation cannot be readily integrated. For problems of interest here, the variation of coolant temperature is at least an order of magnitude greater than the pressure variation. Therefore as an approximation it is assumed that the pressure is constant. Substituting Eq. (6.5) into Eq. (6.8) and integrating, we get

$$\frac{p^2}{2} + \frac{\sigma G^2\left(T_0 + \dfrac{q'''L\eta}{c_pG}\right)^2}{2g_c\left(\dfrac{D_p}{L}\right)(q'''L/c_pG)}\left[\frac{\alpha\left(T_0 + \dfrac{q'''L\eta}{c_pG}\right)^m}{(2+m)\,Re_0\,T_0{}^m} + \frac{\beta}{2}\right] + \frac{\sigma G^2}{2}\left[T_0 + \frac{q'''L\eta}{c_pG}\right] = 0 \quad . \tag{6.9}$$

Multiplying by 2 and putting in the limits of integration, $p = p_1$ at $\eta = 0$ and $p = p$ at $\eta = \eta$, we finally get

$$\frac{p_1^2 - p^2}{\sigma T_0 G^2/g_c} = \frac{(L/D_p)}{\chi}\left\{(1 + \chi\eta)^2\left[\frac{\alpha(1 + \chi\eta)^m}{(2 + m)\,Re_0} + \frac{\beta}{2}\right] - \left[\frac{\alpha}{(2 + m)\,Re_0} + \frac{\beta}{2}\right]\right\} + 2\chi\eta \quad , \tag{6.10}$$

where

$$\chi = \frac{q'''L}{c_p T_0 G} \quad .$$

If there is no heat generation in the porous media, $\chi = 0$, and the temperature of the coolant remains constant. Without making the assumption that the change in pressure is small in comparison with the absolute pressure, Eq. (6.8) can be integrated exactly, and we get

$$\frac{p_1^2 - p^2}{\sigma T_0 G^2/g_c} = \left(\frac{L}{D_p}\right)\eta\left[\frac{\alpha}{Re_0} + \frac{\beta}{2}\right] + 2\ln\frac{p_1}{p} \quad . \tag{6.11}$$

6.2 Discussion of Results

The results given here were based on the Ergun correlation for the pressure drop [see Eqs. (6.2) and (2.10)], except that the constants c_1 and c_2 were taken to be 350 and 4, respectively. The first term on right-hand side of Eq. (6.2), representing viscous energy loss, is a most important factor at low flow rates and of little significance at high flow rates, whereas the second term on the right-hand side of Eq. (6.2), representing the kinetic energy loss, is the most significant at high flow rates, and of little importance at very low flow rates.

The exponent m in Eq. (6.10) depends on the coolant. Helium, for which $m = 0.57$, was chosen as a coolant for the calculations reported here. In general, the viscosity of gases increases with increase in temperature, and the exponent m will be greater than zero; therefore the results reported here should agree in trend with those for other gases as well.

In Section 3 it was assumed that the acceleration pressure drop is negligible compared with the frictional pressure drop. It can be seen from Eq. (6.10) that, even in cases where the gas accelerates through a region of strong positive temperature gradient, the acceleration pressure drop, that is, the last term in Eq. (6.10), is indeed negligible. The dependence of pressure drop on the temperature rise parameter is shown in Fig. 9. The additional pressure drop due to microscopic increase in the momentum of the expanding coolant has been neglected.

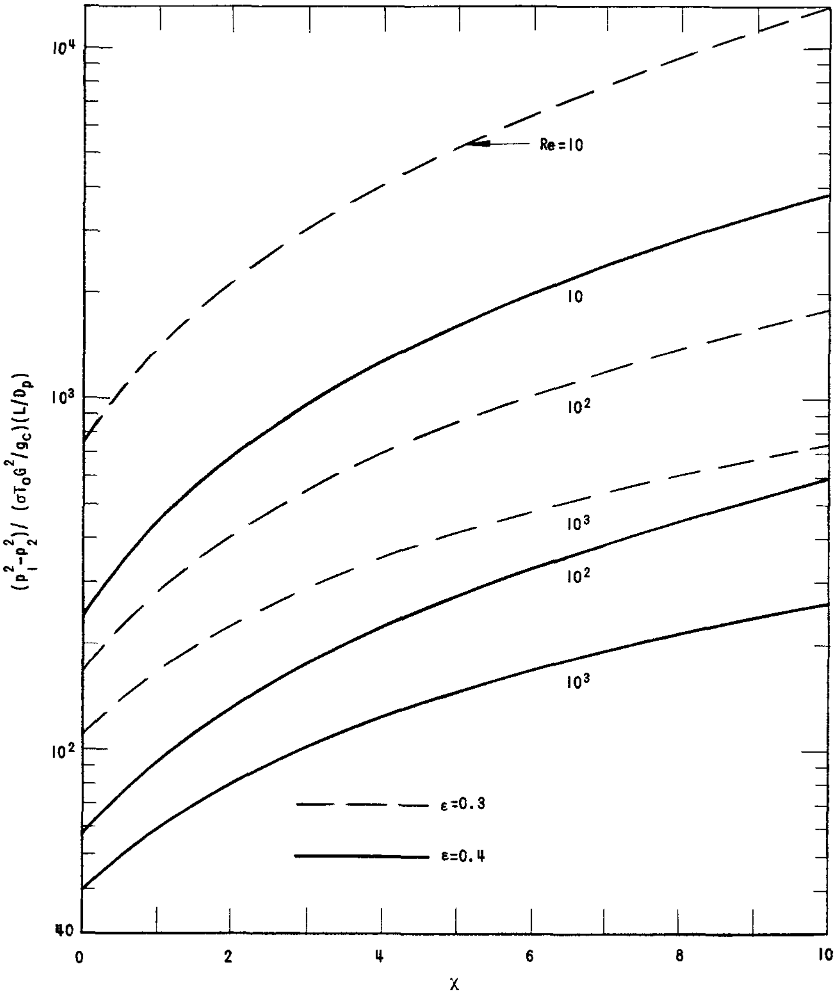

FIG. 9
VARIATION OF THE PRESSURE DROP WITH THE TEMPERATURE RISE PARAMETER

The curves in Figs. 10 and 11 suggest operation of the packed-bed fuel element at the point of minimum pressure drop. However, this may not be always advantageous. A choice of two alternative positions, high temperature, low flow rate or low temperature, high flow rate, is available for each value of the pressure difference established across the bed.

Note, however, the flow and temperature instability may arise because of the temperature dependence of the coolant viscosity. Should a localized heating occur in the bed because of some inhomogeneity of the porous material, the increase in viscosity would tend to decrease the flow in that region, thus increasing the temperature and aggravating the situation. The temperature could increase in this fashion until melting occurred. In view of this, it would appear that stability considerations may require operation on the high-flow-rate side of the optimum point, where the pressure drop is dominated by the quadratic nonviscous term of Eq. (6.2). In addition, it is expected that stability would be promoted by high heat capacity of the coolant.

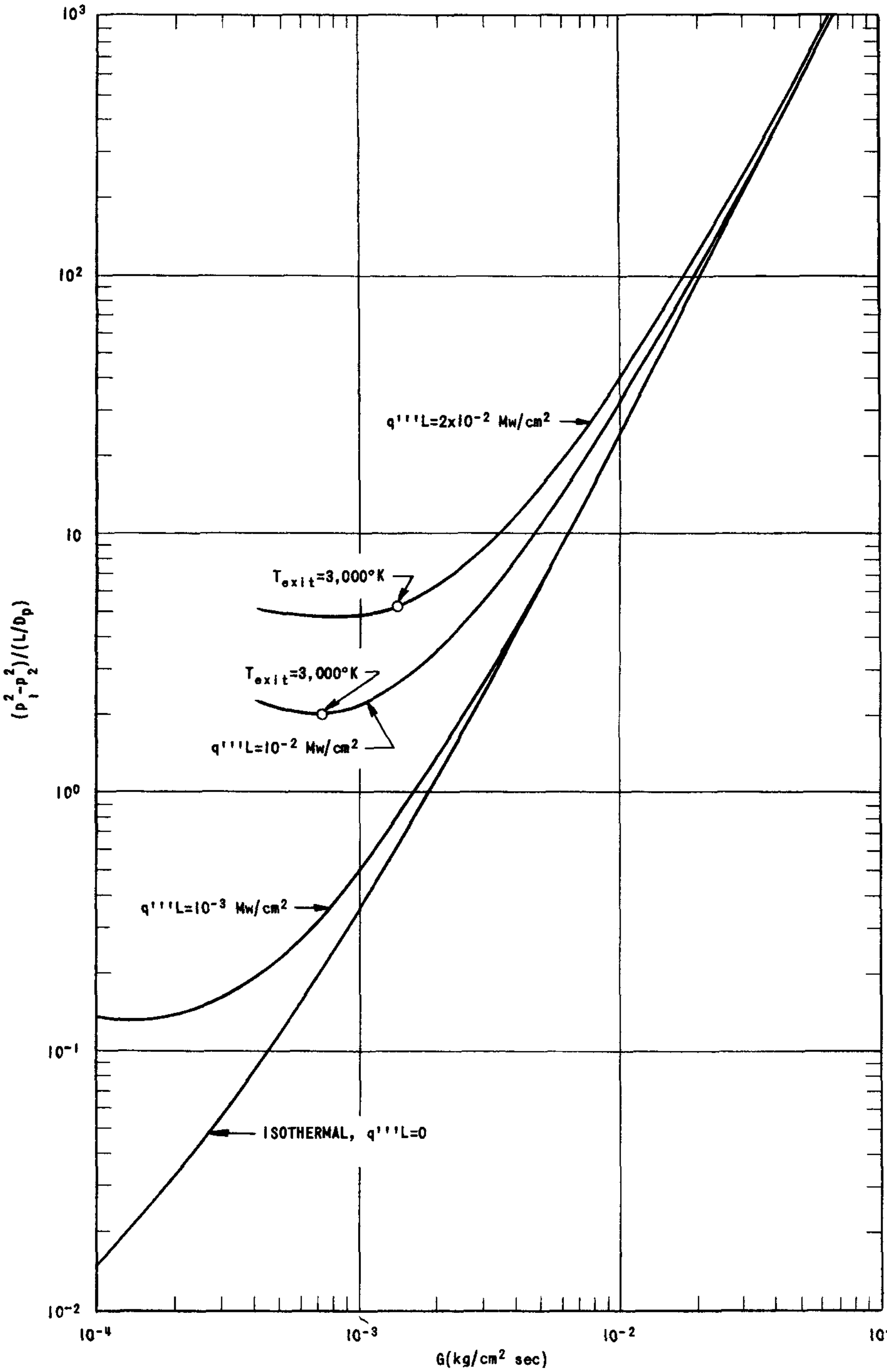

FIG. 10
VARIATION OF PRESSURE DROP WITH MASS FLOW RATE FOR T_0=300°K AND ε=0.4

Figure 12 shows the relationship between the pressure drop and the fractional thickness of the bed for various mass flow rates and bed thicknesses. It should be noted that the pressure gradient increases with the decrease in the coolant pressure, thus suggesting operation of the system at a high pressure.

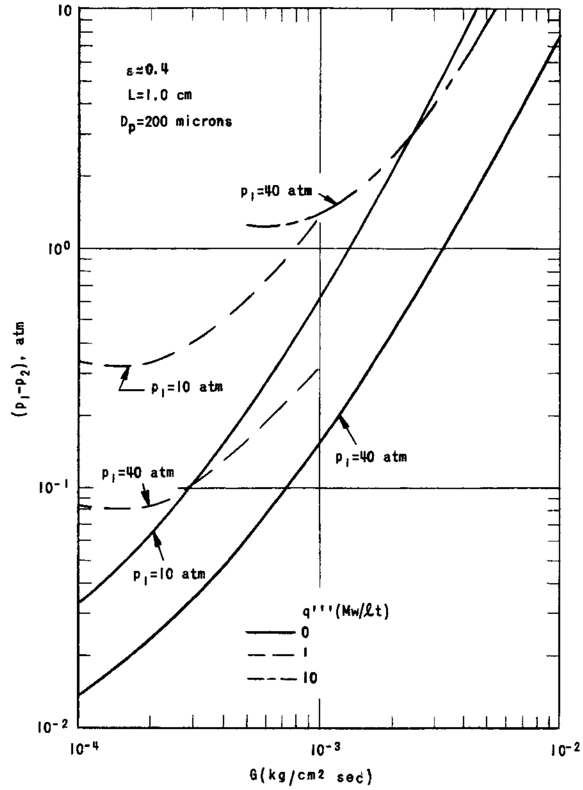

FIG. 11
PRESSURE DROP ACROSS THE BED VS. MASS FLOW RATE FOR DIFFERENT INLET PRESSURES AND HEAT GENERATION RATES

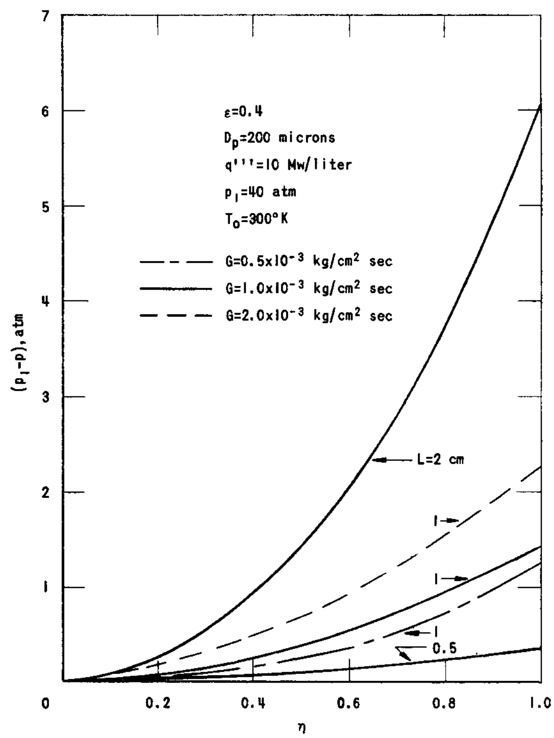

FIG. 12
PRESSURE DROP VS. FRACTIONAL BED DEPTH

7. CONCLUSIONS

Consideration is given in this report to some heat transfer and fluid flow problems of a packed-bed fuel element. The conclusions listed below are based on packed beds made of particles ranging in size from 50 to 300 microns in diameter and with Reynolds numbers ranging from 1 to 500.

(1) Little experimental work in the range of particle size and fluid flow rates of interest to the packed-bed fuel element design has been done. In particular, there has been no experimental work reported on fluid-particle heat transfer (without simultaneous mass transfer) at Reynolds numbers below 60. This is evident from the correlations given in Fig. 3. The investigations reviewed in this report involve particles for which the effective diameters range from 0.23 to 8.4 cm. Because the heat transfer correlations are based on data for large particles and low flow rates it is not certain whether the correlations would hold for small particles and high flow rates, even though the Reynolds numbers might be the same.

(2) The steady-state temperature distributions calculated for a one-dimensional, heat-generating packed bed show that, for conditions of interest to the packed-bed fuel element design, the interparticle conduction is negligible compared to the heat generation. In view of this fact, the conduction term can be neglected from the energy equation, thus simplifying the solution of the problem.

(3) In general, the temperature drop across the film is small, and therefore the highest temperature in the bed will occur at the exit. To increase the power density it is suggested that a coolant with the highest heat-removal capacity be used. From the study of a few coolants it was found that molecular hydrogen has this capability.

(4) A study of pressure drop indicates that instability of the flow rate and the bed temperature may arise from the temperature dependence of the coolant viscosity. The stability considerations require operation on the low-temperature, high-flow-rate side of the minimum pressure drop point. The stability of flow through the bed is improved by high heat capacity of the coolant and high flow rates, which tend to decrease the temperature rise for a given power density.

(5) Calculations based on the heat transfer correlations available at the present time indicate that the packed-bed fuel element is capable of operating at power densities of 50 Mw(t)/ℓ (of core), and possibly higher. The power density of this type of fuel element will likely be limited by considerations other than heat transfer, i.e., the sintering temperature and pressure of the particles, radiation damage to the fuel, length of operation, and so forth.

(6) In order to obtain an analytical solution of a heat transfer and fluid flow problem for a packed bed, it is necessary to make many simplifications and assumptions. Thus, in some cases an experimental approach would appear to be more fruitful.

8. RECOMMENDATIONS

In view of the potential of a packed-bed fuel element, some general heat transfer and fluid flow experiments for porous media are suggested. Even if the feasibility of the packed-bed fuel element concept is not demonstrated experimentally, information obtained in the experiments would be useful for other applications.

(1) Undertake fluid flow studies with coolant and particle sizes of interest to the packed-bed fuel element concept, and approach realistic conditions of temperature, pressure, and mass flow rate.

(2) Investigate heat transfer to porous media in the region where no experimental work has been reported. The author recognizes the difficulties involved in an experimental study of this type. Two major problems are: (a) the measurement of particle surface temperature; and (b) the generation of about Mw(t)/ℓ of heat in a bed - in out-of-pile or in-pile experiments. The power density of 10 Mw(t)/ℓ (of bed) is about the minimum necessary to really prove the feasibility of the concept.

(3) Because of the difficulty of analyzing the flow and temperature stability in a heat-generating porous media, an experimental approach is suggested.

NOMENCLATURE

Symbol	Definition
A	Effective area density for heat transfer defined by Eq. (2.2)
A_p	Surface area of the particle
a	Area density of the bed defined by Eq. (2.1)
c_1, c_2, c_3	Integration constants in Eq. (4.3)
c_p	Specific heat of the coolant at constant pressure
c_s	Specific heat of the solid
D_p	Effective particle diameter defined as $\sqrt{A_p/\pi}$.
E	Energy content of the coolant per unit volume
E_1	Rate of convection of energy across the boundaries of a unit volume
E_2	Rate of energy transfer to the coolant at the solid-fluid interface
e	Energy content of the coolant per unit mass
F	Frictional force defined in Eq. (6.2)
$F(\rho\lvert\vec{u}\rvert)$	Function defined in Eq. (3.3)
f'	Modified friction factor defined in Eq. (2.9)
G	Mass flow rate
g_c	Gravitational constant
$\vec{g}$	Acceleration vector due to gravity
h	Heat transfer coefficient
j_h	Parameter defined by Eq. (2.4)
k	Permeability
k_{eff}	Effective thermal conductivity

L	Bed Thickness
M	Molecular weight of the coolant
m	Constant defined in Eq. (6.6)
m_2, m_3	Parameters defined by Eq. (4.4)
N	Ratio of the container diameter to particle diameter
N_1, N_2, N_3	Parameters defined in Eqs. (4.1) and (4.2)
p	Pressure
p_1	Pressure at the inlet of the bed
p_2	Pressure at the exit from the bed.
Q	Volumetric heat transfer rate defined by Eq. (2.3)
q'''	Heat generation rate per unit volume
R	Gas constant
Re	Reynolds number defined by Eq. (2.5)
Re_m	Modified Reynolds number defined by Eq. (2.6)
Re'	Reynolds number defined by Eq. (2.8)
Re''	Reynolds number defined by Eq. (2.13)
T	Temperature of the coolant
t	Temperature of the solid
u	Superficial coolant velocity in the x direction
$\vec{u}$	Superficial coolant velocity vector
u^*	Dimensionless velocity defined by $u^* = u/u_0$
v	Specific volume of the coolant
Z	Compressibility factor of the coolant

Greek Symbols

α	Parameter defined in Eq. (6.2)
β	Parameter defined in Eq. (6.2)
ϵ	Porosity
η	Dimensionless thickness of a bed, y/L
μ	Dynamic viscosity
ξ	Independent variable defined as $\epsilon x / u_0$
ρ	Density of the coolant
ρ_s	Density of the solid
σ	Parameter defined as ZR/M
τ	Time
Φ	Heat generation distribution function defined as q'''/q'''_{ave}
ϕ	Particle shape factor for heat transfer
ϕ_s	Carman shape factor
ψ	Sphericity
χ	Parameter defined as $q''' L/c_p T_0 G$.

Subscripts

f	Refers to the film
0	Refers to inlet conditions

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