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Various Types of Fluctuations in the $C^{12}(C^{12},\alpha)Ne^{20}$ Reaction

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The reaction to be discussed is $C^{12}(C^{12},\alpha)Ne^{20}$, where the alpha particles are to resolved, low-lying levels of Ne^{20}. The measurements were made at the tandem Van de Graaff at Risø in collaboration with Jørn Borggreen and Bent Elbek. The interpretation of the data was largely by Jakob Bondorf at Copenhagen.

Slide 1 presents a small sample of the data to show the general nature of the observations and the difficulties in understanding them on the basis of earlier concepts. Here is shown the excitation function of alpha particles emitted to the 0+ ground state of Ne^{20} and at the observation angle of 90° (center-of-mass). As in the other data to be shown, the solid carbon target was roughly 50-keV thick, the reaction alpha particles were observed by semiconductor counters with angular resolution of about 3°, and the spectra of alpha particles were recorded in multichannel analyzers. In all the data, a pump oil build-up on the target contributed an appreciable number of unresolved alpha particles to the spectrum and thus caused some uncertainty in the final analysis of the data.

The resonance-like structure of the excitation function in this slide is striking. Note particularly the strong, 150-keV wide "resonance" at 11.4 MeV. It will be discussed later in greater detail, but we note here that the approximately 50-keV experimental resolution is considerably less than the "resonance" widths observed. If, for the moment, angular momentum considerations are left out of our discussion, the "apparent resonances" at separations of several hundred keV can be shown to have

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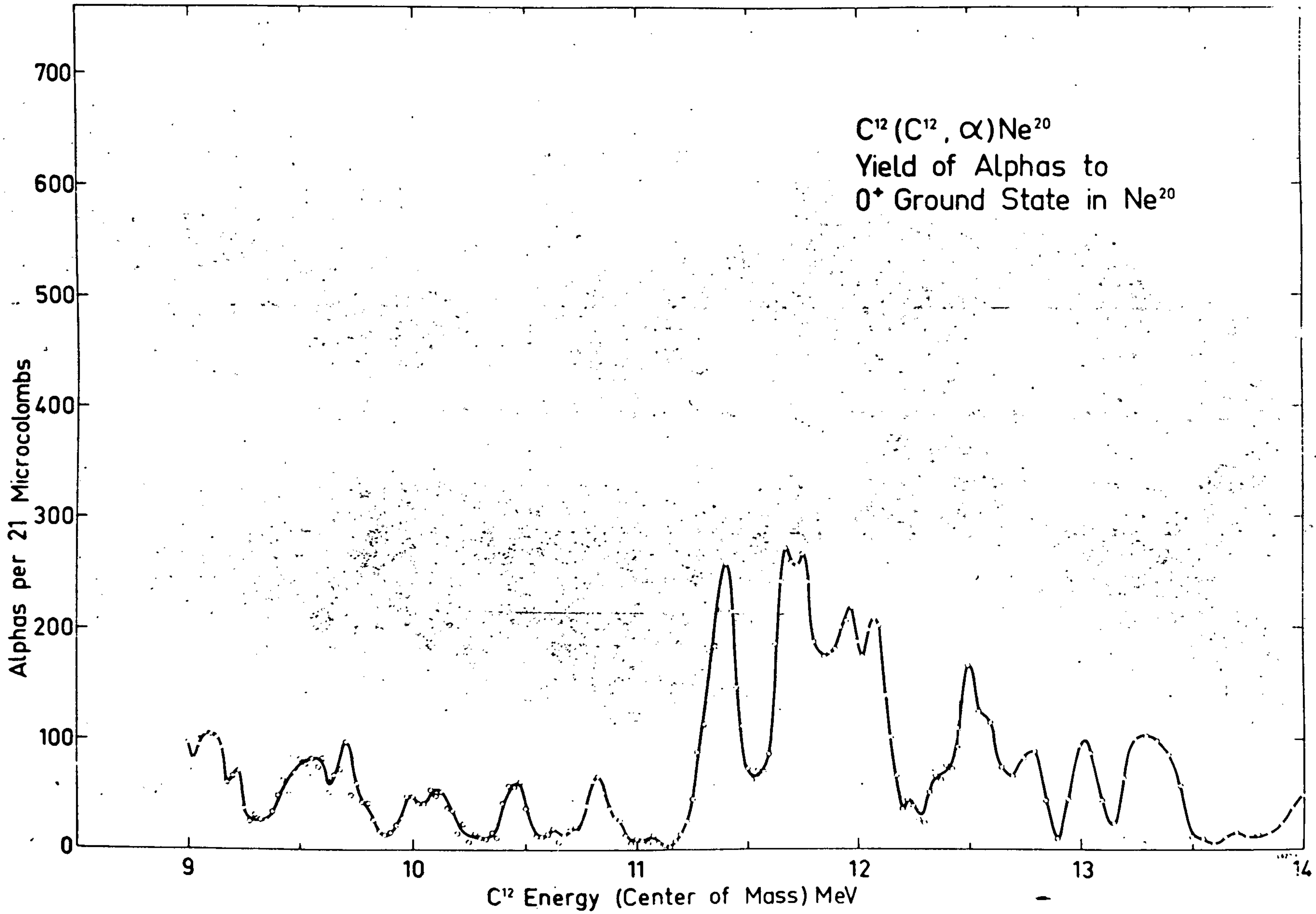
$C^{12}(C^{12}, \alpha)Ne^{20}$
Yield of Alphas to
0^{+} Ground State in Ne^{20}
Alphas per 21 Microcolombs
0
100
200
300
400
500
600
700
9
10
11
12
13
14
C^{12} Energy (Center of Mass) MeV

Slide 1

-2-

a separation many hundreds of times more than the keV separations expected from the statistical distributions of levels. This small separation is expected on the basis of the large excitation energy of the compound nucleus as a result of both the bombarding energy and the large 13.9-MeV separation energy of a C^{12} from the compound Mg^{24} nucleus.

Actually, there are other reasons for doubting a "resonance" interpretation of these data. As we shall see later, a second reason is that the alpha particle yields to the excited states of Ne^{20} do not show enhanced yields at the same bombarding energies of C^{12}; that is, there are no correlations. This is true both for our differential cross sections and for the total cross sections[1] reported by Chalk River. Thus if these cross section peaks result from isolated levels in the Mg^{24} compound nucleus, highly unusual selection rules must operate to eliminate correlations.

A third reason for doubting a "resonance" interpretation of the data is seen in Slide 2. Here is shown the angular distribution of alpha particles from the 11.4-MeV cross section peak noted in the previous slide. Again the results are spectacular; here because of the deep valleys in the distribution. In fact, the positions of the peaks and valleys are rather well fitted by the square of the eighth order Legendre polynomial, $[P_8(\cos\theta)]^2$, and the angular distribution itself is moderately well fitted by this function. However, just this lack of a good fit gives us the third reason for doubting a "resonance" interpretation: an isolated level of the compound nucleus would have a definite J value, and for a decay by the spinless alpha particle to the 0+ ground state of Ne^{20} the angular distribution should be exactly the J order of the squared Legendre polynomial. Incidentally, we note that the fact that $C^{12} + C^{12}$ involves identical spinless bosons requires the J values involved always to be even, and this J restriction contributes to the simplicity of the angular distributions.

Having now this introduction to the type of data we wish to interpret, we turn to Slide 3 to see the important exit channels for a compound nucleus reaction. We primarily consider compound nucleus interactions because the

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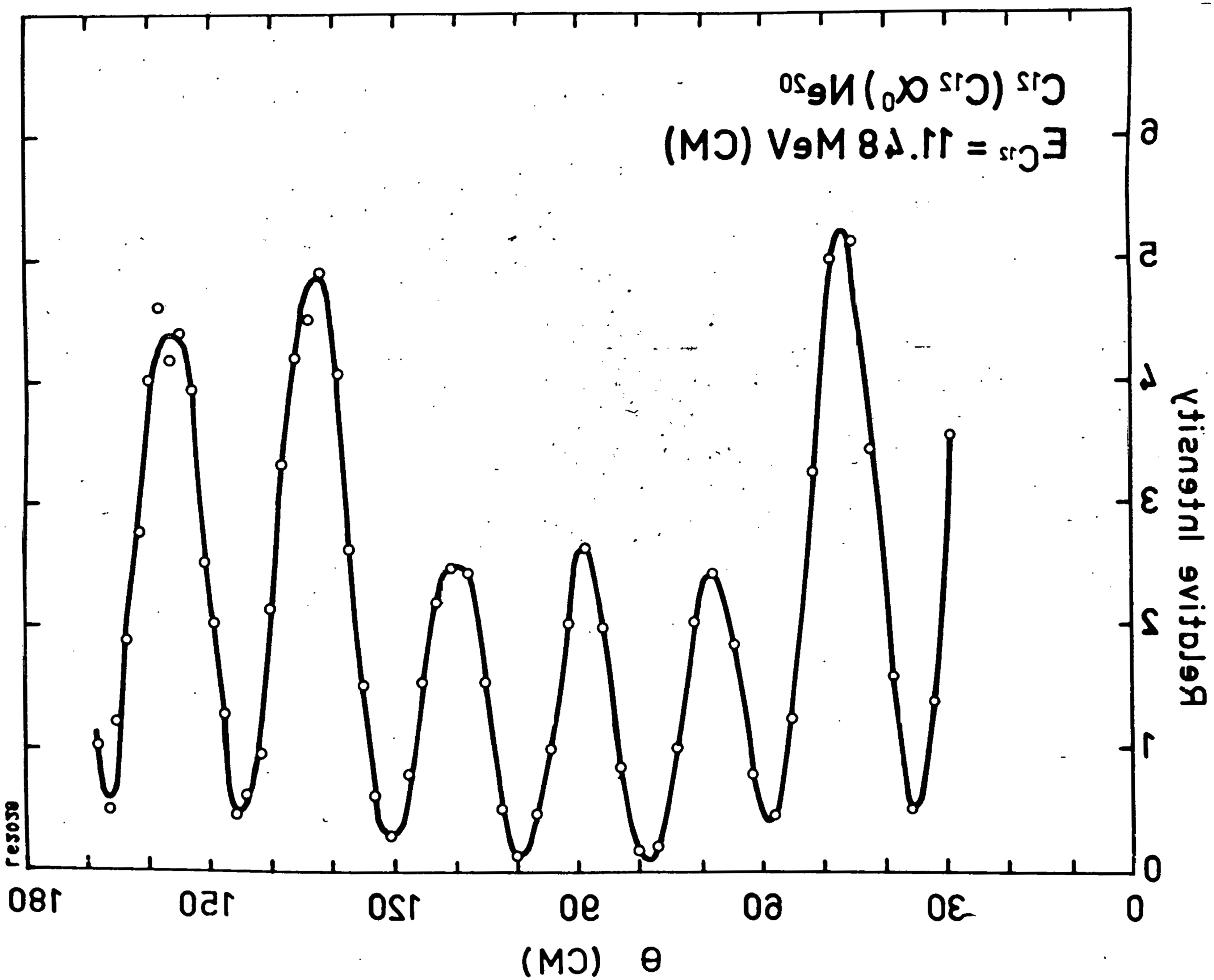
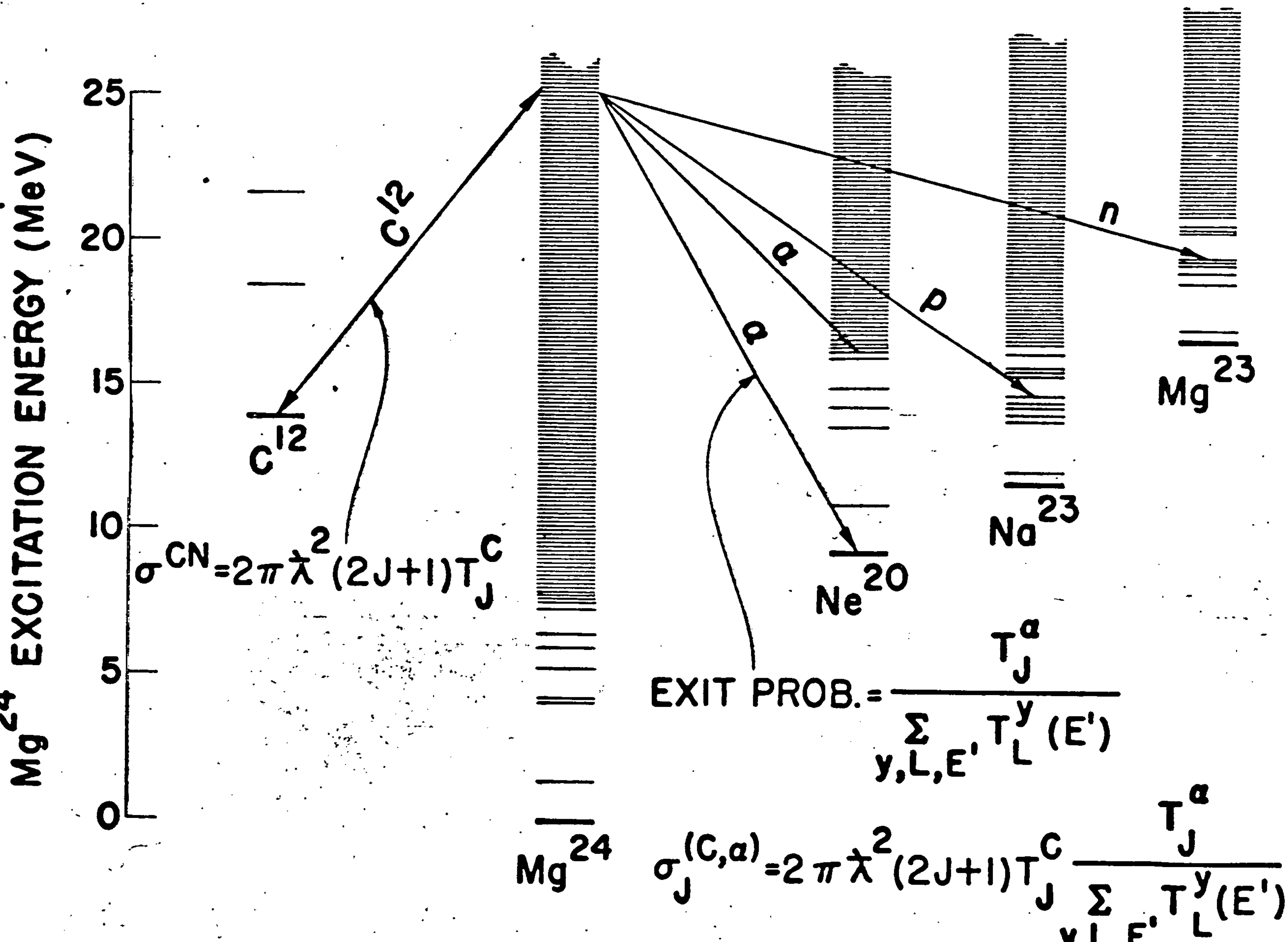
[1] J. A. Kuehner, J. D. Prentice, and E. Almqvist, Phys. Letters 4, 332(1963).


C^{12} (C^{12} α_0) Ne^{20}
$E_{C^{12}}$ = 11.48 MeV (CM)
Relative Intensity
θ (CM)
0 1 2 3 4 5 6
180 150 120 90 60 30 0
re5038

Slide 2

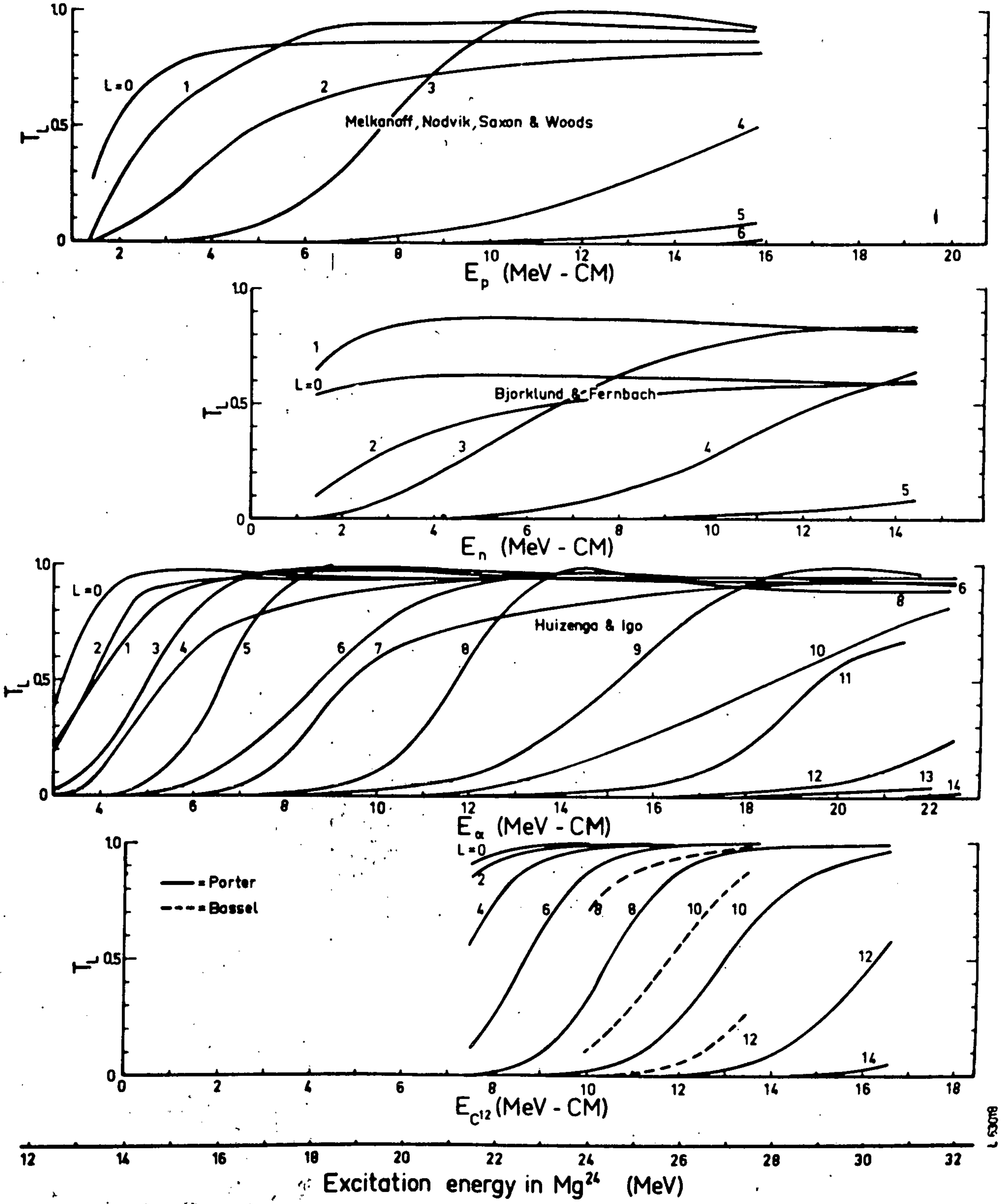

Mg^{24} EXCITATION ENERGY (MeV)
0
5
10
15
20
25
C^{12}
C^{12}
Mg^{24}
Ne^{20}
Na^{23}
Mg^{23}
α
α
p
n
$\sigma^{CN} = 2\pi\lambda\!\!\!^{-2}(2J+1)T_J^C$
EXIT PROB. $= \dfrac{T_J^{\alpha}}{\sum_{y,L,E'} T_L^{y}(E')}$
$\sigma_J^{(C,\alpha)} = 2\pi\lambda\!\!\!^{-2}(2J+1)T_J^C \dfrac{T_J^{\alpha}}{\sum_{y,L,E'} T_L^{y}(E')}$

Slide 3

relatively slow, heavy particles involved in heavy ion interactions indicate compound nucleus interactions and because the other extreme, direct interactions, would not contribute to the cross section fluctuations observed. In this slide, the cross section for the formation of the compound nucleus with spin J involves the wavelength λ and the statistical weight factor $(2J+1)$. The various exit probabilities are proportional to their transmission coefficients $T_L(E')$ to the residual energies E', and normalization is by the summation over all these exit channels for all particles y. The resulting cross section, which it should be emphasized is for a cross section $\langle\sigma_J\rangle$ averaged over many compound levels of spin J, is of the Hauser-Feshbach form.

In the next slide, Slide 4, are shown the T_L values calculated for these exit channels of the previous slide. These T_L values were calculated by Leona Stewart at Los Alamos from parameters given by the indicated persons. Abscissae are displaced for the various exit channels to result in a common excitation energy of the compound Mg^{24} for each. It is seen from this slide that the transmission coefficients of both alphas and C^{12} for the 11.4-MeV case we examined earlier suppress the $J = 10$ ~~for channels~~. When we remember that the $(2J+1)$ statistical factor populates the low J states relatively little, we can expect that the average cross section $\langle\sigma\rangle$ might have a dominant J value as was observed. This is expecially so for our case where odd J values are excluded.

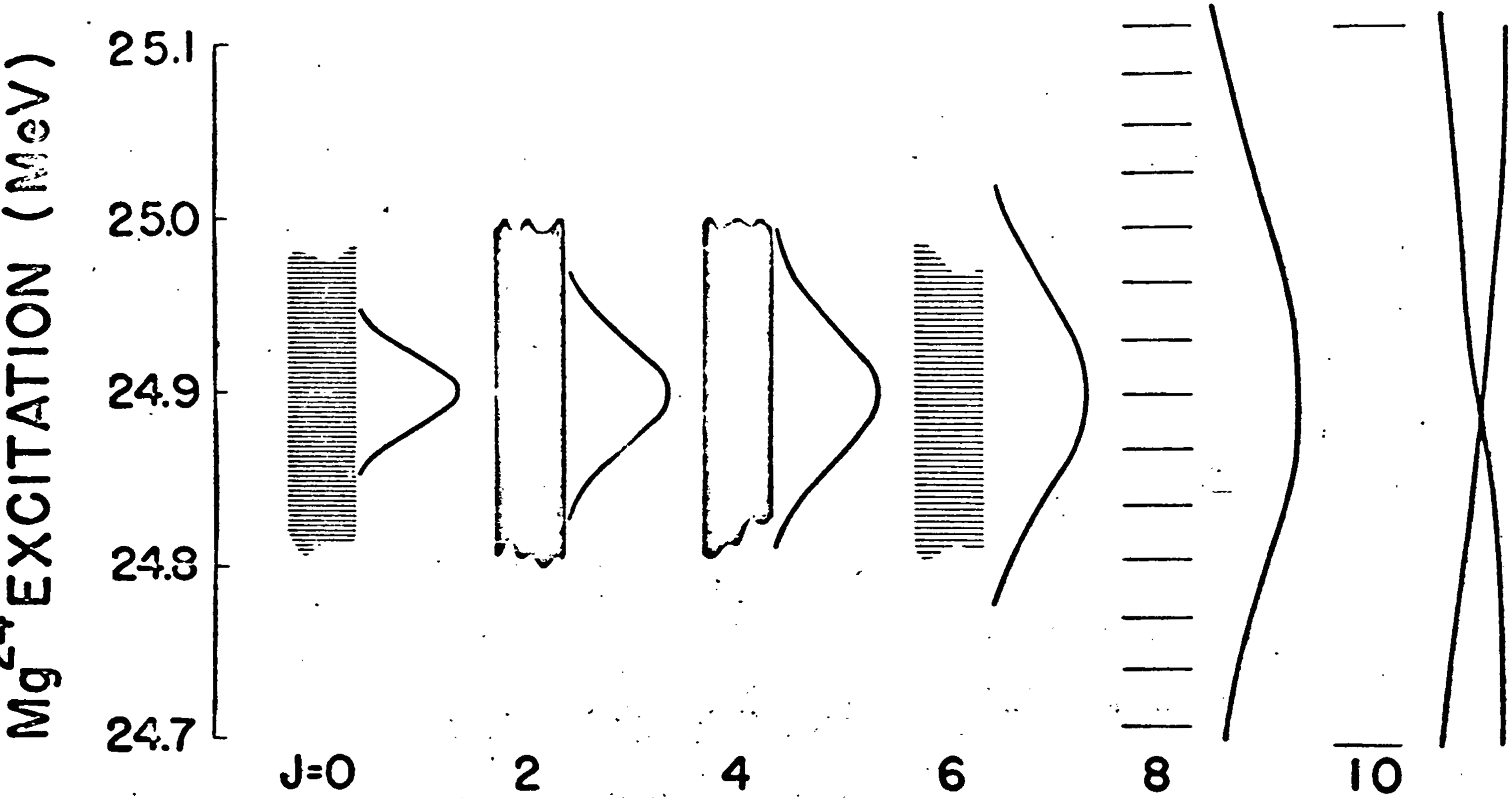
It is now apparent that we must concentrate our attention on single J values of the compound system, and this is done in Slide 5. Compound-nucleus levels in a narrow range of energies near 11-MeV C^{12} energy are considered. Here we start from the observed width of 150 keV for the predominantly $J = 8$ state at 11.4 MeV, which we found to be a minimum observed width, and have used the familiar relation $\Sigma T_J = 2\Pi\langle\Gamma_J\rangle/\langle D_J\rangle$ to determine the spacing. The spacings D_J for the other spins J are obtained from the so-called "spin cutoff" expression of the statistical distribution of levels shown on the slide, and the widths are then obtained through the sum of the transmission coefficients. Of course, we expect the levels to have a nonuniform spacing



1.0
0.5
0
T_L
L=0
1
2
3
4
5
6
Melkanoff, Nodvik, Saxon & Woods
2 4 6 8 10 12 14 16 18 20
E_p (MeV - CM)
L=0
1
2
3
4
5
Bjorklund & Fernbach
0 2 4 6 8 10 12 14
E_n (MeV - CM)
L=0
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Huizenga & Igo
4 6 8 10 12 14 16 18 20 22
E_α (MeV - CM)
— = Porter
--- = Bassel
L=0
2
4
6
8
10
12
14
0 2 4 6 8 10 12 14 16 18
$E_{C^{12}}$ (MeV - CM)
L 53019
12 14 16 18 20 22 24 26 28 30 32
Excitation energy in Mg^{24} (MeV)

Slide 4

Slide 5



$$\frac{\langle \Gamma_J \rangle}{\langle D_J \rangle} = \frac{\Sigma T_L}{2\pi} = 17.4 \quad 40.7 \quad 32.4 \quad 15.1 \quad 4.8 \quad 0.7$$

$\Gamma_8(\text{obs.}) = 0.15$ MeV

$$\frac{D_J}{D_0} = 0.07\,(2J+1)\exp\frac{-J(J+1)}{2(2.6)^2}$$

$$\left(\text{STRICTLY, } T_L = \frac{\tau}{(1+\frac{\tau}{4})^2}, \quad \text{WHERE } \tau = 2\pi\frac{\langle \Gamma_J \rangle}{\langle D_J \rangle}\right)$$

instead of the average spacing shown; however the arguments of Porter-Thomas[2] distributions indicate little fluctuation in the widths for our case of a large number of exit channels.

Of particular note here is that the spin of interest in the vicinity of 11-MeV C^{12} energy, namely $J = 8$, has an average level width about five times the average spacing. Thus even when we consider only the specific spin involved, the overlap still argues against isolated levels in the compound nucleus, as we discussed earlier. This overlap is roughly a factor of two greater when we instead use the correct relation between widths and spacing given by Moldauer[3] and shown at the bottom of the slide. A historical note is that the Chalk River group[1] at first concluded that isolated resonances were possible, although improbable, with the width a few times the average spacing; however, Vogt[4] later pointed out that the greater overlap resulting from the Moldauer correction significantly decreases the probability of isolated levels being seen.

The explanation for fluctuations in the cross section even with these large overlaps of levels in the compound nuclei was provided independently by Ericson,[5] by Brink and Stephen,[6] and by Mottelson.[7] Slide 6 shows the starting point of their argument. All spinless particles for both the entrance and exit channel are first considered, just as was encountered for the data of the first two slides. Even though the levels are strongly overlapping, we know from R-matrix theory that the differential cross section as given in first equation contains a denominator of the Breit-Wigner form, although the exact form of the complex $a_n^{(L)}$ term in the numerator is uncertain in its dependence on the energies E_n of the compound nucleus

[2] R. G. Thomas and C. E. Porter, Phys. Rev. 104, 483 (1956).

[3] P. A. Moldauer, Phys. Rev. 129, 754 (1963).

[4] Paper B.6 of the Compound Nuclear States Conference, Gatlinburg, October 1963, to be published, Bull. Am. Phys. Soc.

[5] T. Ericson, Phys. Letters 4, 258 (1963).

[6] D. M. Brink and R. O. Stephen, Phys. Letters 5, 77 (1963).

[7] B. R. Mottelson, private communication (1963).

All particles spinless:

$$\sigma(\theta) = 2\pi\lambda\!\!\!^{-}{}^2 \left| \sum_L \sum_n \frac{a_n^{(L)} \; Y_L^0(\cos\theta)}{E - E_n^{(L)} + i\frac{\Gamma_n^{(L)}}{2}} \right|^2$$

$$= 2\pi\lambda\!\!\!^{-}{}^2 \left| \sum_L (\xi_L + i\eta_L) \; Y_L^0(\cos\theta) \right|^2$$

$$= 2\pi\lambda\!\!\!^{-}{}^2 \{[\xi(\theta)]^2 + [\eta(\theta)]^2\}$$

$$\sigma = 2\pi\lambda\!\!\!^{-}{}^2 \sum_L \left[\left(\xi_L\right)^2 + \left(\eta_L\right)^2\right]$$

One final product having spin:

$$\sigma(\theta) = 2\pi\lambda\!\!\!^{-}{}^2 \sum_M \left\{\left[\xi^M(\theta)\right]^2 + \left[\eta^M(\theta)\right]^2\right\}$$

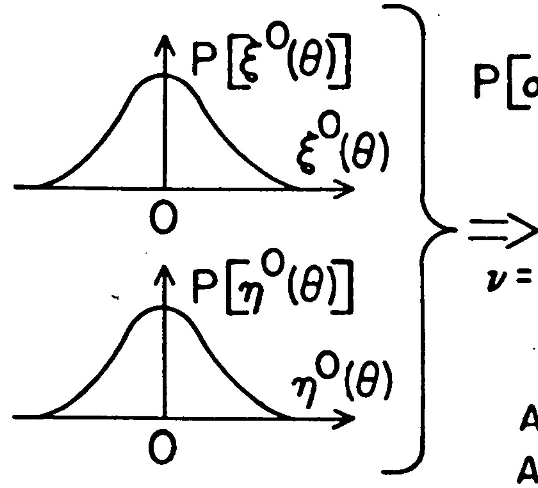
states n and orbital angular momentum L. The spherical harmonic $Y_L^0(\cos\theta)$ gives the angular dependence and includes the familiar $(2L+1)^{1/2}$ factor. Of importance here is the coherent combination of the partial waves indicated by the fact that the summations over both the states and the orbital angular momentum are within the brackets.

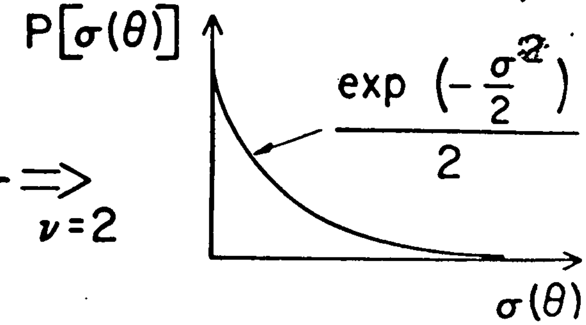
Since we are ignorant of the exact form of $a_n^{(L)}$, we combine it with the denominator, after summation over states n, into a real part ξ_L and an imaginary part η_L shown in the second equation. Summation over the orbital angular momentum L and squaring gives the third equation, which emphasizes the point that the differential cross section is dependent on two important variables, $\xi(\theta)$ and $\eta(\theta)$, which by containing the spherical harmonic are angle dependent. The fourth equation, on the other hand, shows the total cross section σ, obtained by integrating the second equation over all angles, is a function of a number of variables that is twice the number of angular momentum partial waves L. This results from the orthonormal properties of the spherical harmonics.

We continue in the last equation to the more general case where one of the final products has spin. In our example this is Ne^{20} in one of its excited states. Now the cross section is of the third equation form, but the incoherence of the M partial waves results in a number of variables that is twice the number of M substates. This incoherence in M can be understood by the fact that M is, in principle, measurable by polarization, and this measurement destroys the coherence.

It remains now to associate these numbers of variables determining the cross section with the fluctuations in the cross section. This association is shown in Slide 7. In the upper left is again the more general expression of the cross section in terms of variables. Now we notice the similarity of $\sigma(\theta)$ with the χ^2 term of statistics defined by the second equation. If each of the variables is Gaussian, has equal and unit variance, and ~~are~~ is independent, then the probability distribution in χ^2, which is $P(\chi^2)$, is given by the bottom equation. Thus, to use fluctuation theory we assume these Gaussian properties for each of the $\xi^M(\theta)$ and $\eta^M(\theta)$ variables, and we can then expect

$$\sigma(\theta) = 2\pi ƛ^2 \sum_M \left\{ [\xi^M(\theta)]^2 + [\eta^M(\theta)]^2 \right\}$$



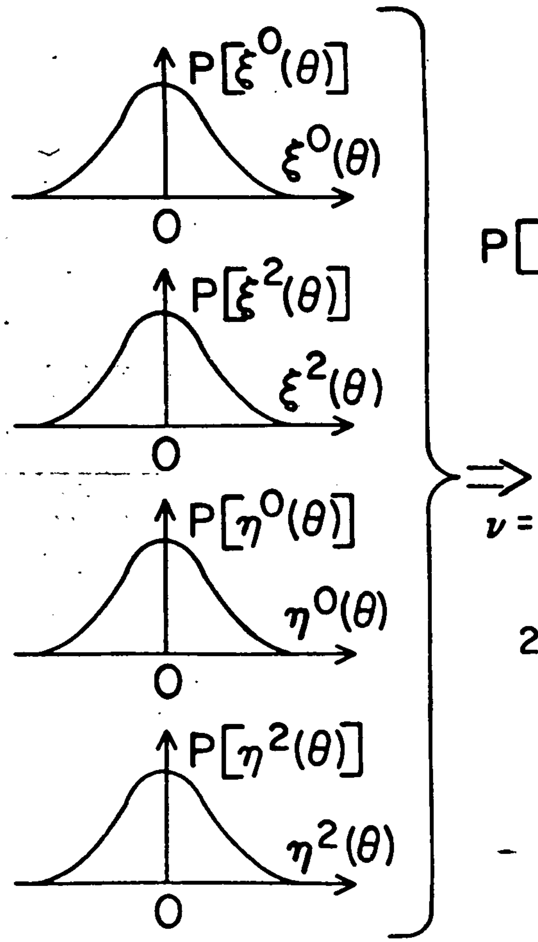


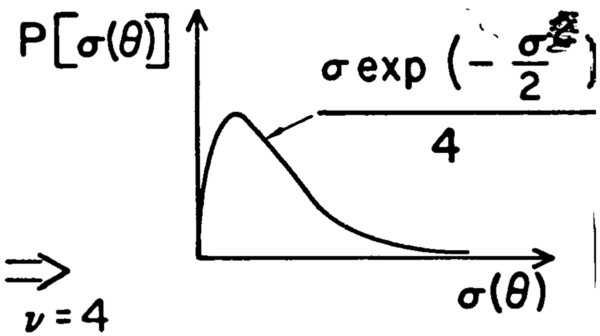
ALL FINAL STATES AT 0°
ALSO ALL ANGLES FOR
0+ STATE

$$\chi^2 = \sum_{\mu=1}^{\nu} z_\mu^2$$

IF $P(z_\nu)$ IS GAUSSIAN, THEN

$$P(\chi^2) = \frac{(\chi^2)^{\frac{\nu}{2}-1} \exp(-\frac{\chi^2}{2})}{2^{\frac{\nu}{2}}\, \Gamma(\frac{\nu}{2})}$$





2+ FINAL STATE AT 90°
(|M − L| = EVEN)

Slide 7

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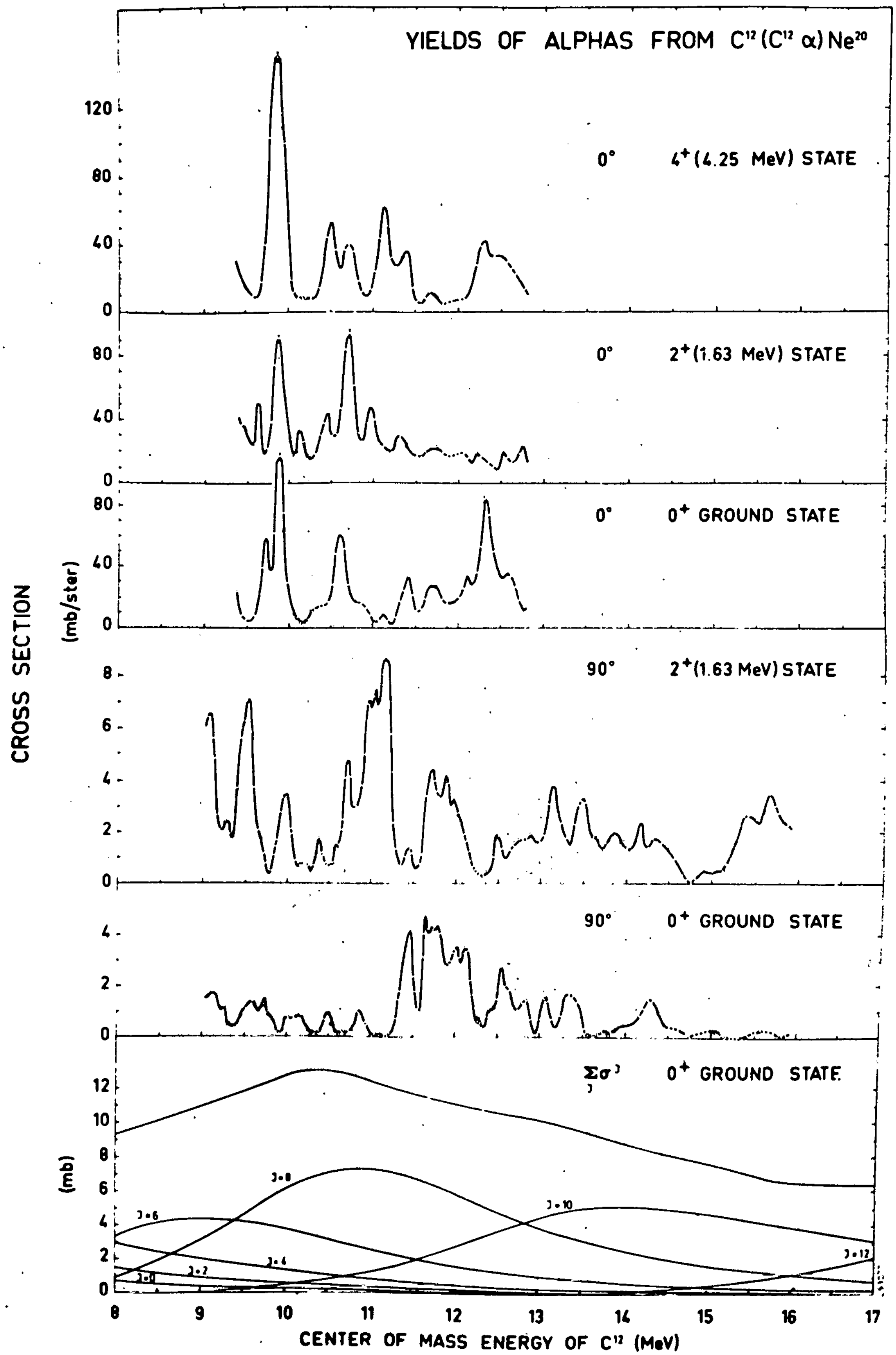
a probability distribution in $\sigma(\theta)$ that is just the form of probability distribution for χ^2. Here the number of degrees of freedom ν is the number of these variables.

The figure illustrates the two simplest examples. At the top we consider the $\nu = 2$ case of zero spin throughout, which allows only M = 0. Also the condition of M = 0 applies for $\theta = 0^o$ because of angular momentum properties. For these M = 0 conditions we see the combination of two Gaussian variables to be an exponential of $(-\ \sigma^2/2)$ argument. Under this is the next more complicated case of M = 0,2, which is the condition of the 2+ final state of Ne^{20} at 90^o, where $|M - L|$ must be even. Here we have four variables, and so $\nu = 4$. The four Gaussian variables combine as shown, which is a cross section probability distribution distinctly different than that resulting from $\nu = 2$, and so the difference is open to experimental verification. We note in passing that the zero probability of zero cross section for $\nu = 4$ results from changing the first power variables ξ and η in the constituent Gaussian probabilities to second power variables $(\xi)^2$ and $(\eta)^2$ in the cross section.

Having this background, we now turn to the experimental data of Slide 8. This is a composite of excitation functions for resolved alpha particles to the indicated excited states of Ne^{20}. The upper group is for 0^o observations, and the lower group is for 90^o observations. We remember that 0^o results in only M = 0 and so the most probable cross section should be zero for this group. The same is true for the spinless case of 0+ at 90^o observation. However, the 2+ and 4+ final states at 90^o observation are expected to result in progressively larger most probable cross sections. The lack of energy correlations between the cross section peaks is to be noted. A single exception is the possible correlation at 9.9 MeV for 0^o.

At the bottom of this slide is the result of Hauser-Feshbach calculations of $\langle \sigma_J \rangle$, the total cross section for all angles, going through compound nucleus states of various spin J. Also shown is $\langle \sigma \rangle$ for all J values. Due to the statistical nature of the Hauser-Feshbach formulism, these cross sections are smooth with energy and so are useful in normalizing data for


YIELDS OF ALPHAS FROM $C^{12}(C^{12}\alpha)Ne^{20}$
0° 4+ (4.25 MeV) STATE
0° 2+ (1.63 MeV) STATE
0° 0+ GROUND STATE
90° 2+ (1.63 MeV) STATE
90° 0+ GROUND STATE
$\sum_J \sigma^J$ 0+ GROUND STATE
CROSS SECTION
(mb/ster)
(mb)
J=0
J=2
J=4
J=6
J=8
J=10
J=12
CENTER OF MASS ENERGY OF C^{12} (MeV)

Slide 8

fluctuation analyses. Also, these calculations predict the dominant J values for the various ranges of C^{12} energy. We emphasize that the cross sections are expected to fluctuate above and below these averages from the Hauser-Feshbach calculations; thus we should expect occasionally to observe for the 0+ case an angular distribution characterized by an L value other than the calculated dominant J for that reaction.

Slide 9 confirms these expectations. Here the angular distributions for the 0+ ground state reaction are generally $L = 8$ in the 9.5-MeV to 12.8-MeV region and generally $L = 10$ above 12.8 MeV as calculated. The expected exceptions are an $L = 6$ angular distribution at 11.2 MeV and an $L = 10$ angular distribution at 15.1 MeV; these exceptions occur with the expected frequency of about one in six.

To compare the measured cross section probability (of data normalized in an approximate manner with the calculated cross sections of Slide 8) we look at Slide 10. Data are given by the histograms, while the smooth curves are the cross sections expected from the χ^2 form of fluctuation theory. Agreement between theory and experiment is moderately good. The numbers in parentheses are the final states of Mg^{24} populated by alpha particles from the reaction $O^{16}(C^{12},\alpha)Mg^{24}$ that are unresolved from the alpha group in the $C^{12}(C^{12},\alpha)Ne^{20}$ reaction. Oxygen is expected as a contaminant from pump oil deposits. The presence of such incoherent alpha particles would have the effect of increasing the apparent number of freedom ν.

Finally, Slide 11 is a compilation of fits, all with $\nu = 2$ expected. It is strking that the data for the 0+ state, which results in $\nu = 2$ for all angles, provide generally good fits with theory while, on the other hand, the fits for the 0° cases have few, or no, observations of very low cross section, and thus are poorer fits. In the case of the 4+ state at 0° there is not even the excuse of unresolved alpha groups from $O^{16}(C^{12},\alpha)Mg^{24}$. An explanation has been provided by William Gibbs.[8] He points out that $\nu = 2$ for these 2+, 4+, and 3- states is true only exactly at 0°. For the finite angles away from 0° included in our angular resolution,

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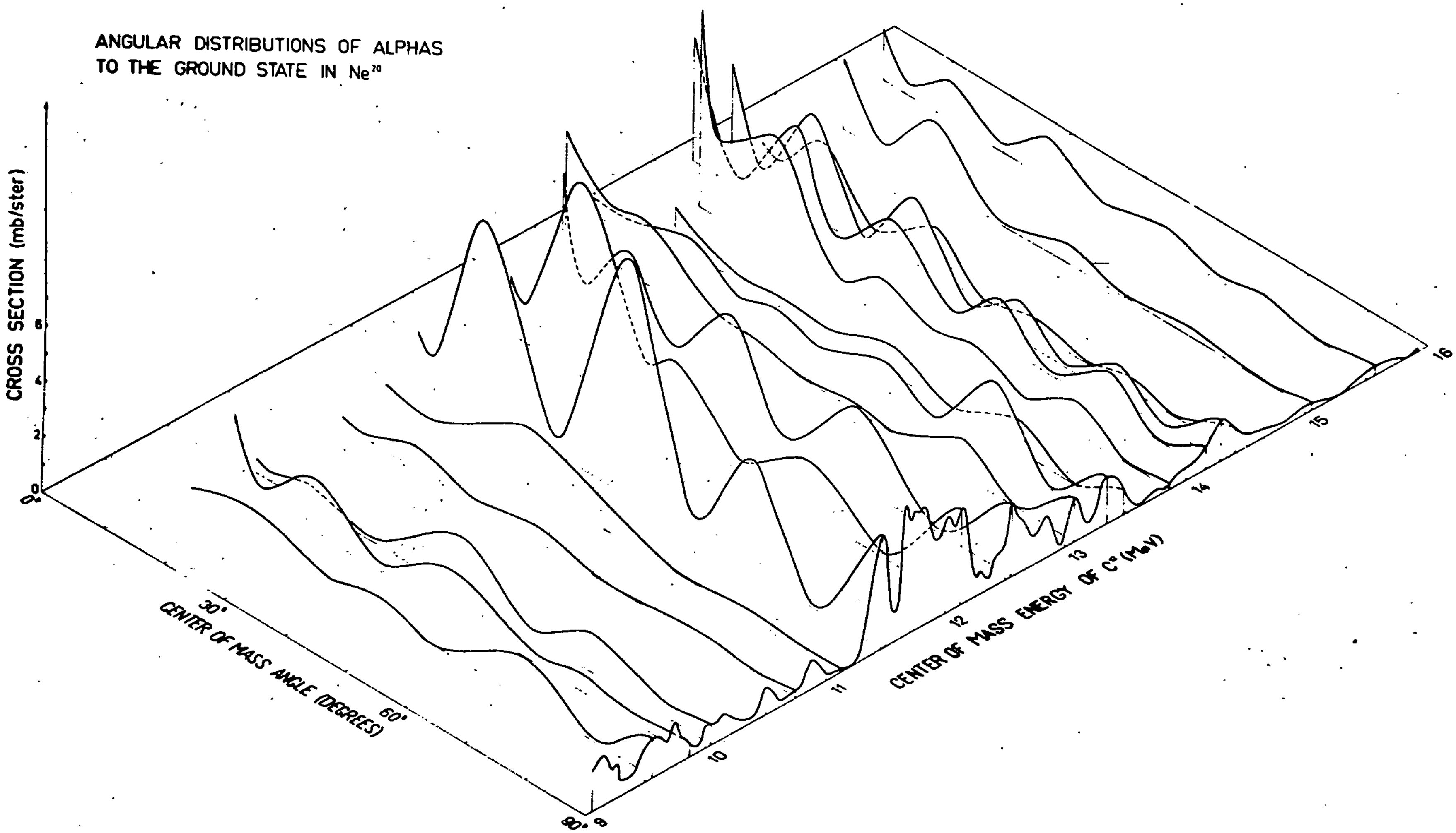
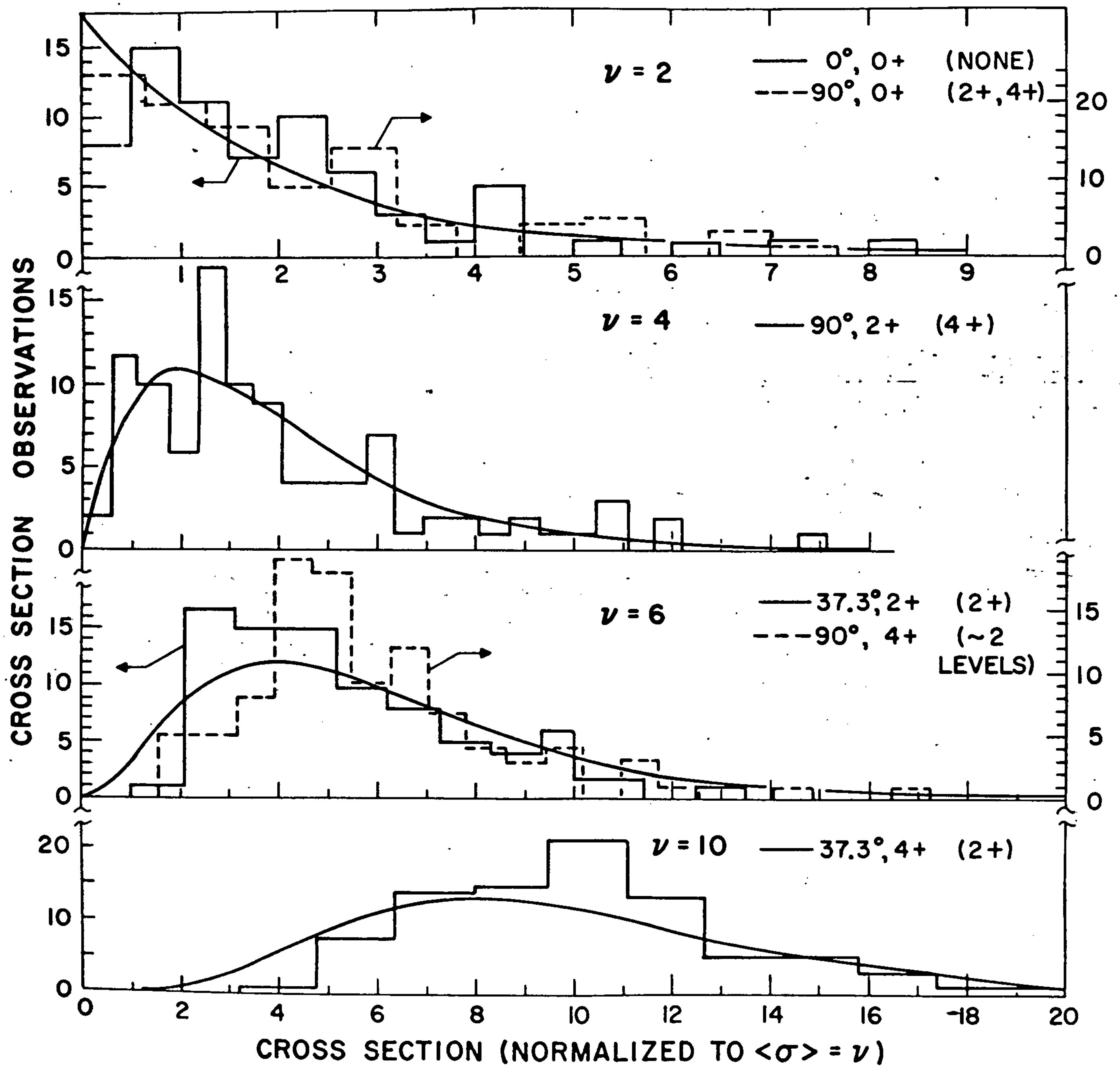
[8] William Gibbs, private communication (1964).


ANGULAR DISTRIBUTIONS OF ALPHAS
TO THE GROUND STATE IN Ne^{20}
CROSS SECTION (mb/ster)
0
2
4
6
0°
30°
60°
90°
CENTER OF MASS ANGLE (DEGREES)
9
10
11
12
13
14
15
16
CENTER OF MASS ENERGY OF C^{12} (MeV)

Slide 9

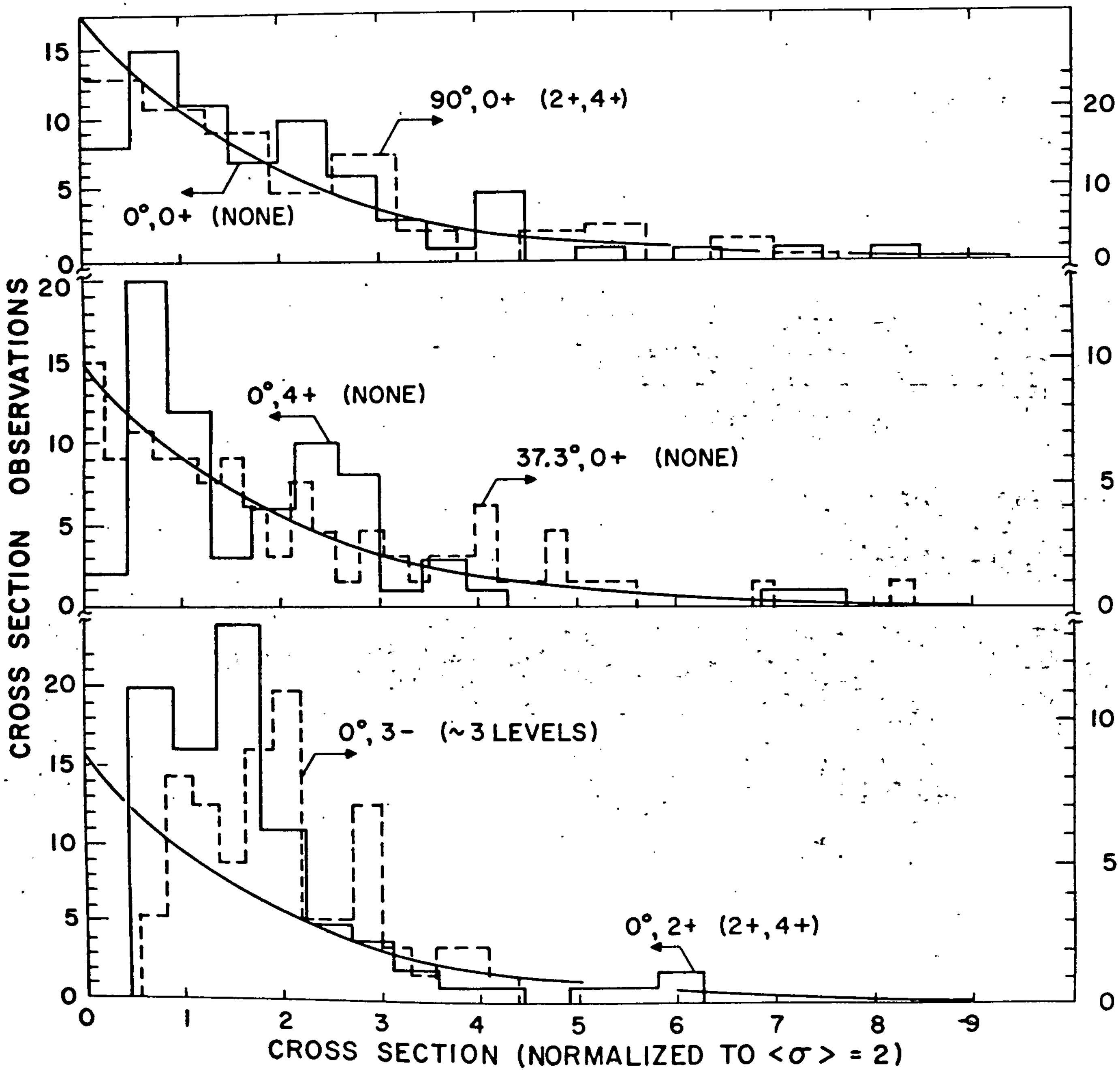
-16-



$\nu = 2$
0°, 0+ (NONE)
90°, 0+ (2+, 4+)
$\nu = 4$
90°, 2+ (4+)
$\nu = 6$
37.3°, 2+ (2+)
90°, 4+ (~2 LEVELS)
$\nu = 10$
37.3°, 4+ (2+)
CROSS SECTION OBSERVATIONS
CROSS SECTION (NORMALIZED TO $\langle\sigma\rangle = \nu$)

-17-

Slide 10



90°, 0+ (2+, 4+)
0°, 0+ (NONE)
0°, 4+ (NONE)
37.3°, 0+ (NONE)
0°, 3- (~3 LEVELS)
0°, 2+ (2+, 4+)
CROSS SECTION OBSERVATIONS
CROSS SECTION (NORMALIZED TO <σ> = 2)

Slide 11

-18-

admixtures of $\xi_L^1(\theta)$ and $\eta_L^1(\theta)$ will result in a theoretically expected zero yield of zero cross section, just as observed.

In summary, we conclude our data are in accord with the compound nucleus theory and no significant contribution of direct interactions or isolated levels is required to explain the data. The cross section data confirm in detail the various fluctuations expected from statistical theory.