

TRANSIENT DYNAMICS SIMULATIONS:  
PARALLEL ALGORITHMS FOR CONTACT DETECTION  
AND SMOOTHED PARTICLE HYDRODYNAMICS

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**Abstract.** Transient dynamics simulations are commonly used to model phenomena such as car crashes, underwater explosions, and the response of shipping containers to high-speed impacts. Physical objects in such a simulation are typically represented by Lagrangian meshes because the meshes can move and deform with the objects as they undergo stress. Fluids (gasoline, water) or fluid-like materials (earth) in the simulation can be modeled using the techniques of smoothed particle hydrodynamics. Implementing a hybrid mesh/particle model on a massively parallel computer poses several difficult challenges. One challenge is to simultaneously parallelize and load-balance both the mesh and particle portions of the computation. A second challenge is to efficiently detect the contacts that occur within the deforming mesh and between mesh elements and particles as the simulation proceeds. These contacts impart forces to the mesh elements and particles which must be computed at each timestep to accurately capture the physics of interest. In this paper we describe new parallel algorithms for smoothed particle hydrodynamics and contact detection which turn out to have several key features in common. Additionally, we describe how to join the new algorithms with traditional parallel finite element techniques to create an integrated particle/mesh transient dynamics simulation. Our approach to this problem differs from previous work in that we use three different parallel decompositions, a static one for the finite element analysis and dynamic ones for particles and for contact detection. We have implemented our ideas in a parallel version of the transient dynamics code PRONTO-3D and present results for the code running on a large Intel Paragon.

**1. Introduction.** Large-scale simulations are increasingly being used to complement or even replace experimentation in a wide variety of settings within science and industry. Computer models can provide a flexibility which is difficult to match with experiments and can be a particularly cost-effective alternative when experiments are expensive or hazardous to perform. As an example, consider the modeling of crashes and explosions which can involve the interaction of fluids with complex structural deformations. The simulation of gas-tank rupture in an automobile crash is a prototypical example.

Transient dynamics codes are able to model the material deformations occurring in such phenomena, and smoothed particle hydrodynamics (SPH) provides an attractive method for including fluids in the model. There are several commercial codes which simulate structural dynamics including LS-DYNA3D, ABACUS, EPIC, and Pam-Crash.

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Of these, EPIC is the only one which includes an SPH capability [10]. PRONTO-3D is a DOE code of similar scope that was developed at Sandia National Laboratories [18, 2, 17] which includes both structural dynamics and SPH models.

A complicated process such as a collision or explosion involving numerous complex objects requires a mesh with fine spatial resolution if it is to provide accurate modeling. The underlying physics of the stress-strain relations for a variety of interacting materials must also be included in the model. Coupling of mechanical and hydrodynamic effects further increases the complexity. Running such a simulation for thousands or millions of timesteps can be extremely computationally intensive, and so is a natural candidate for the power of parallel computers.

Unfortunately, these kinds of simulations have resisted large-scale parallelization for several reasons. First, multi-physics simulations which couple different types of phenomenological models (e.g. structures and fluids), are often difficult to parallelize. A strategy which enables efficient parallel execution of one portion of the model may lead to poor performance of other portions. Second, the problem of contact detection, which is described in the next section, is central to structural dynamics simulations, but has resisted efficient parallelization. We believe our parallel algorithm for this problem is the first to exhibit good scalability.

In this paper we describe the algorithms we have developed to enable an efficient parallel implementation of PRONTO-3D, coupling structural dynamics with SPH. Although there have been previous attempts to parallelize contact detection and SPH individually, we believe this work is the first effort to parallelize them together in the same code. A key idea in our approach is the use of different parallel decompositions for each of the three dominant computations that occur in a timestep: finite element analysis, SPH calculation, and contact detection. Although we must communicate information between the different decompositions, the cost of doing so turns out to be small, and thus is more than compensated for by the load-balance and high parallel performance we achieve in each individual stage.

In the next section we present a brief overview of transient dynamics simulations including SPH. We also introduce our parallelization strategy and contrast it with previous approaches. We omit a detailed discussion of the numerics of either deformation modeling or SPH, since this can be found in the references and is beyond the scope of this paper. In §3 we introduce the kernel operations which we use in our parallel algorithms. These kernels are applied to the parallelization of SPH in §4 and the contact detection task in §5. This is followed in §6 by some performance results using PRONTO-3D.

We note that our algorithms have targeted large distributed-memory parallel supercomputers such as the Intel Paragon and Cray T3D. As such, the algorithms that we describe here have been implemented in PRONTO-3D using C and F77 with standard MPI message-passing calls.

**2. Overview.** Transient structural dynamics models are usually formulated as finite element (FE) simulations on Lagrangian meshes. Unlike Eulerian meshes which remain fixed in space as the simulation proceeds, Lagrangian meshes can be easily fit-

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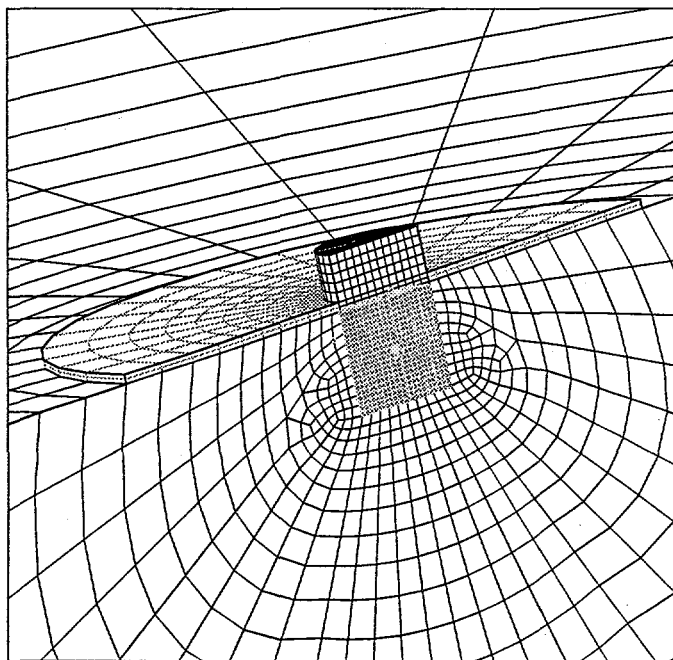
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ted to complex objects and can deform with the objects as they change shape during a simulation. The deformation of an individual mesh element is computed by calculating the material-dependent stresses and strains imparted by neighboring elements within the topology of the FE mesh. We term this calculation the FE portion of the transient dynamics simulation.

Smoothed particle hydrodynamics (SPH) models are gridless Lagrangian techniques used to model regions of extreme deformation where traditional FE meshes become too distorted and break down [14, 5, 16]. Essentially the material is modeled as a dense collection of particles. In this setting, the term "hydrodynamics" is somewhat of a misnomer, since the particles can carry along stress and strain tensors to model the strength of solid materials. A group of nearby SPH particles is treated as a collection of interpolation points. Spatial gradients of various quantities are computed as a sum of pairwise interactions between a particle and its neighbors and this information is used in material equations-of-state to compute the forces on individual particles. We term this calculation the SPH portion of a hybrid particle/mesh transient dynamics simulation.

For illustration purposes, Fig. 1 shows a schematic of the initial state of a simulation containing both FE meshes and SPH particles. This is a model of an underwater explosion where an explosive charge is detonated beneath a steel plate with a protruding plug in the center [17]. In this cutaway view, the figure shows a central square region of SPH particles (the explosive = cyan), with a flat circular mesh (the plate = blue) on top, surrounded by a global mesh (the water = black). A second circular mesh (the plug = red) sits on top of the plate.



**Fig. 1.** *Initial state of a combination finite element and smoothed particle hydrodynamics simulation.*

In most transient dynamics simulations, there is a third major computation which must be performed every timestep. This is the detection of *contacts* between pairs of unconnected mesh elements and between SPH particles and mesh elements. For example, in Fig. 2, initial and 5 millisecond snapshots are shown of a simulation of a steel rod colliding with a brick wall. Contacts occur any time a surface element on one brick interpenetrates a surface element on another brick. These contacts impart forces to the impacting objects which must be included in the equations-of-motion for the interpenetrating elements. If SPH particles are included in the model, then contacts also occur whenever an SPH particle penetrates a surface element of a meshed object. Finding the set of all such mesh/mesh and particle/mesh contact pairs is termed the contact detection portion of the simulation.

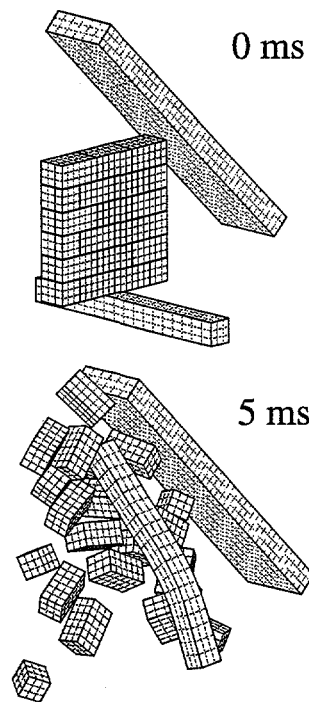


Fig. 2. *Simulation of a steel rod hitting a brick wall.*

To achieve high performance on a massively parallel machine with a hybrid particle/mesh transient dynamics code such as PRONTO-3D all three of the computational tasks outlined above must be effectively parallelized. We now discuss some of the issues that must be considered in such an effort. Consider a single timestep of the simulation as outlined in Fig. 3.

In step (1), the FE portion of the timestep is performed. Within a single timestep in an explicit timestepping scheme, each mesh element interacts with only the neighboring elements it is connected to within the topology of the FE mesh. These kinds of FE calculations can be parallelized straightforwardly. This is also true for implicit schemes if iterative methods are used to solve the resulting matrix equation. In either case, the key is to assign each processor a small cluster of elements so that the only interprocessor communication will be the exchange of information on the cluster



- (1) Compute FE forces on each mesh element
- (2) Compute SPH forces on each particle
- (3) Move mesh elements and particles
- (4) Detect mesh/mesh and particle/mesh contacts
- (5) Generate push-back forces
- (6) Final update of mesh and particle positions

Fig. 3. One timestep of a PRONTO-3D calculation.

boundary with a handful of neighboring processors. It is important to note that because the mesh connectivity does not change during the simulation (with a few minor exceptions), a *static* decomposition of the elements to processors is sufficient to insure good performance. A variety of algorithms and tools have been developed that optimize this assignment task. For PRONTO-3D we use a software package called Chaco [7] as a pre-processor to partition the FE mesh so that each processor has an equal number of elements and interprocessor communication is minimized. Similar FE parallelization strategies have been used in other transient dynamics codes [9, 12, 13, 15]. In practice, the resulting FE computations are well load-balanced and scale efficiently (over 90%) when large meshes are mapped to thousands of processors in PRONTO-3D. The chief reason for the scalability is that the communication required by the FE computation is *local* in nature.

In step (2), the SPH portion of the timestep is performed. In PRONTO-3D, individual SPH particles are spheres with a variable smoothing length or radius that can grow or shrink as the simulation progresses. The details of how SPH particles interact with each other and how physical quantities such as stresses and forces are derived from these interactions is beyond the scope of this paper, but the interested reader is referred to [2] for a description of the SPH formulation within serial PRONTO-3D. For parallelism considerations, the key concept is that two SPH particles interact if their spheres overlap. A single SPH particle may interact with several nearby neighbors and with distant particles whose radius is large. Thus an efficient parallel implementation of SPH has two requirements: (1) the number of SPH particles per processor must be balanced, and (2) the spatial region owned by a processor must be geometrically compact to enable neighbors to be found quickly and with a minimum of communication. A static decomposition of particles to processors cannot meet both these demands, since the spatial density of SPH particles can change quite dramatically over time.

There have been a few implementations of parallel SPH calculations described in the literature [19, 20], although SPH is more commonly used for astrophysical simulations than in transient dynamics. The paper by Warren and Salmon gives an excellent overview of issues involved in SPH parallelization and discusses their implementation using an oct-tree approach. This method results in processors owning geometrically compact, although somewhat irregularly-shaped sub-domains. We have opted for a different approach in PRONTO-3D, which is to use recursive coordinate bisectioning

(RCB) as a dynamic decomposition technique for the SPH particles. The technique is detailed in the next section, but in essence it assigns a simple rectangular sub-domain to each processor which contains an equal number of particles. We use this technique for two reasons in addition to the fact that it is simpler to implement. First it allows us to take advantage of the fast sorting and searching routines that are already part of serial PRONTO-3D that work within a rectangular sub-domain to optimize the neighbor finding on a single processor. Second, as we shall see in the next sections, we use the RCB technique in both the SPH and contact detection tasks and thus can reuse the same routine for both purposes.

In step (3) of Fig. 3, the forces previously computed on mesh elements and SPH particles are used to advance their positions and velocities in time. This step is perfectly parallel within the context of the separate decompositions for the FE and SPH steps. In step (4), the element and particle interpenetrations resulting from step (3) must be detected. As illustrated in Fig. 4, several kinds of contacts can occur. In (a), mesh elements from distant regions of one object may come in contact as when a car fender crumples. In (b) elements on two separate objects can also contact, as in the bricks simulation above. Formally, a "contact" is defined as a geometric penetration of a *contact surface* (face of a mesh element) by a *contact node* (corner point of an element). Thus in the 2-D representation of (c), node B has contacted surface EF and likewise node F has contacted surface BC. In this context, the centers of SPH particles in PRONTO-3D are treated as additional contact nodes. Thus, in (d), particle B has contacted the top surface of the lower object, but particle A has not.

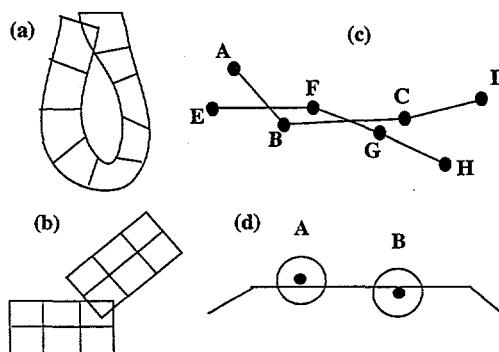


Fig. 4. Various types of mesh/mesh and particle/mesh contacts.

Although in any one timestep only a few contacts typically occur, checking for all such contacts requires a global search of the simulation domain and in practice can require 30-50% of the overall CPU time when PRONTO-3D runs on a vector machine like the Cray Y-MP. This is because, in principle, any contact node and contact surface can interpenetrate at some time during the simulation. Efficient schemes for spatially sorting and searching lists of contact nodes and surfaces have been devised to speed this computation in the serial version of PRONTO-3D [6].

Unfortunately, parallelizing contact detection is problematic. First, in contrast to the FE portion of the computation, some form of *global* analysis and communication is

now required. This is because a contact node and surface pair can be owned by any two processors in the static mesh decomposition described above. Second, load-balance is a serious problem. The contact nodes associated with SPH particles are balanced by the RCB decomposition described above. However, the contact surfaces and nodes come from mesh elements that lie on the surface of meshed object volumes and thus comprise only a subset of the overall FE mesh. Since the static mesh decomposition load-balances the entire FE mesh, it will not (in general) assign an equal number of contact surfaces and nodes to each processor. Finally, finding the one (or more) surfaces that a contact node penetrates requires that the processor who owns the node acquire information about all surfaces that are geometrically nearby. Since the surfaces and nodes move as the simulation progresses, we again require a dynamic decomposition technique which results in a compact sub-domain assigned to each processor if we want to efficiently search for nearby surfaces.

Given these difficulties, how can we efficiently parallelize the task of contact detection? The most commonly used approach [12, 13, 15] has been to use a single, static decomposition of the mesh to perform both FE computation and contact detection. At each timestep, the FE region owned by a processor is bounded with a box. Global communication is performed to exchange the bounding box's extent with all processors. Then each processor sends contact surface and node information to all processors with overlapping bounding boxes so that contact detection can be performed locally on each processor. Though simple in concept, this approach is problematic for several reasons. For general problems it will not load-balance the contact detection because of the surface-to-volume issue discussed above. This is not as severe a problem in [15] because only meshes composed of "shell" elements are considered. Since every element is on a surface a single decomposition can balance both parts of the computation. However, consider what happens in Fig. 2 if one processor owns surface elements on two or more bricks. As those bricks fly apart, the bounding box surrounding the processor's elements becomes arbitrarily large and will overlap with many other processor's boxes. This will require large amounts of communication and force the processor to search a large fraction of the global domain for its contacts.

An alternative approach, more similar in spirit to our work and which was developed concurrently, is described in [9]. This approach uses a different decomposition for contact detection than for the FE analysis. In their method, they decompose the contact surfaces and nodes by overlaying a fine 1-D grid on the entire simulation domain and mapping the contact surfaces and nodes into the 1-d "slices". Then a variable number of slices are assigned to each processor so as to load-balance the number of contact elements per processor. Each processor is responsible for finding contacts within its collection of slices which it can accomplish by communicating with processors owning adjacent slices. While this approach is likely to perform better than a static decomposition, its 1-D nature limits its utility on large number of processors. The implementation described in [9] suffered from some load imbalance on as few as 32 processors of a Cray T3D.

Our approach to parallel contact detection in FE simulations as implemented in

PRONTO-3D has been described in [8, 1]. In this paper we generalize the discussion to include SPH particles. The key idea is that we use the same RCB approach described above to dynamically balance the entire collection of contact nodes and SPH particles at each timestep as part of step (4) in Fig. 3. Note however that this is a different decomposition than used in step (2) for the SPH particles alone. In other words, the RCB routine is called twice each timestep, with a different set of input data. The details of how contacts are detected within the context of this new decomposition are presented in §5.

Once contacts have been detected, restoring or “push-back” forces are computed in step (5) that move the interpenetrating mesh elements and particles back to non-penetrating positions. This adjustment is performed in step (6). Both steps (5) and (6) only work on the relatively small set of pairs of contact nodes (including SPH particles) and contact surfaces detected in step (4). Thus they consume only a small fraction of the timestep which is dominated by the computations in steps (1), (2), and (4).

In summary, we have outlined a parallelization strategy for transient dynamics simulations that uses 3 different decompositions within a single timestep: a static FE-decomposition of mesh elements, a dynamic SPH-decomposition of SPH particles, and a dynamic contact-decomposition of contact nodes and SPH particles. In the early stages of this project we considered other parallelization strategies that were less complex, such as sharing particles and mesh elements evenly across processors. However our final decision to use 3 separate decompositions was motivated by two considerations. First, the only “overhead” in such a scheme is the cost of moving mesh and particle data between the decompositions. Once that cost is paid (and it turns out to be minimal in practice), we have a highly load-balanced decomposition in which to perform each of the three major computational stages within a timestep. We have not found any other viable path to achieving scalable performance on hundreds or thousands of processors. Second, in each of the three decompositions we end up reducing the global computational problem to a single-processor computation that is conceptually identical to the global problem, e.g. find all contacts or compute all SPH forces within a rectangular sub-domain. This allows the parallel code to re-use the bulk of the intricate serial PRONTO-3D code that performs those tasks in the global setting. Thus the parallel parts of the new parallel version of PRONTO-3D simply create and maintain the new decompositions; the transient dynamics computations are still performed by the original serial code with only minor modifications.

In the following sections we provide more detail as to how these new decompositions are used to parallelize the SPH and contact-detection computations. But first, in the next section, we provide some details about two kernel ideas that are central to all of the higher level algorithms.

**3. Kernels.** Our parallel algorithms for SPH and contact detection involve a number of unstructured communication steps. In these operations, each processor has some information it wants to share with a handful of other processors. Although a given processor knows how much information it will send and to whom, it doesn’t know how much it will receive and from whom. Before the communication can be performed effi-

ciently, each processor needs to know about the messages it will receive. We accomplish this with the approach sketched in Fig. 5.

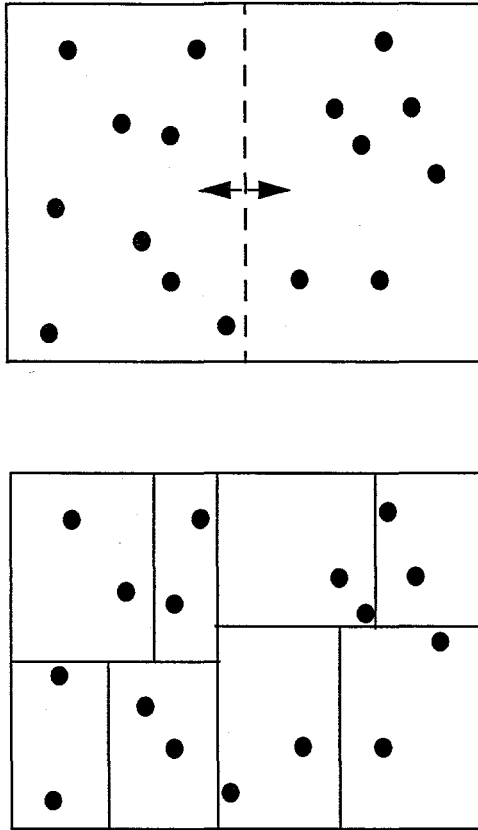
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|---|
| <ol style="list-style-type: none"> <li>(1) Form vector of 0/1 denoting who I send to</li> <li>(2) Fold vector over all <math>P</math> processors</li> <li>(3) <math>nrecvs = \text{vector}(q)</math></li> <li>(4) For each processor I have data for,<br/>          send message containing size of the data</li> <li>(5) Receive <math>nrecvs</math> messages with sizes coming to me</li> <li>(6) Allocate space &amp; post asynchronous receives</li> <li>(7) Synchronize</li> <li>(8) Send all my data</li> <li>(9) Wait until I receive all my data</li> </ol> |
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Fig. 5. Parallel algorithm for unstructured communication for processor  $q$ .

In steps (1)–(3) each processor learns how many other processors want to send it data. In step (1) each of the  $P$  processors initializes a local copy of a  $P$ -length vector with zeroes and stores a 1 in each location corresponding to a processor it needs to send data to. The fold operation [4] in step (2) communicates this vector in an optimal way; processor  $q$  ends up with the sum across all processors of only location  $q$ , which is the total number of messages it will receive. In step (4) each processor sends a short message to the processors it has data for, indicating how much data they should expect. These short messages are received in step (5). With this information, a processor can now allocate the appropriate amount of space for all the incoming data, and post receive calls which tell the operating system where to put the data once it arrives. After a synchronization in step (7), each processor can now send its data. The processor can proceed once it has received all its data.

Another kernel operation used to create both our SPH- and contact-decompositions is recursive coordinate bisectioning (RCB). The RCB algorithm was first proposed as a static technique for partitioning unstructured meshes [3]. Although for static partitioning it has been eclipsed by better approaches, RCB has a number of attractive properties as a dynamic partitioning scheme [11]. The sub-domains produced by RCB are geometrically compact and well-shaped. The algorithm can also be parallelized in a fairly inexpensive manner. And it has the attractive property that small changes in the positions of the points being balanced induce only small changes in the partitions. Most partitioning algorithms do not exhibit this behavior.

The operation of RCB is illustrated in Fig. 6 for a 2-D example. Initially each processor owns some subset of “points” which may be scattered anywhere in the domain. The first step is to choose one of the coordinate directions,  $x$ ,  $y$ , or  $z$ . We choose the direction for which the box bounding the points is longest, so that when we cut orthogonal to that direction, the resulting sub-domains will be as cubic as possible. The next task is to position the cut, shown as the dotted line in the figure, at a location which puts half the points on one side of the cut, and half on the other. This is equivalent to



**Fig. 6.** *Top: First cut of RCB operation. Bottom: Final partitioning for 8 processors.*

finding the median of a distributed set of values in parallel. We do this in an iterative fashion. First we try the midpoint of the box. Each processor counts the number of points it owns that are on one side of the cut. Summing this result across processors determines which direction the cut should be moved to improve the median guess. In practice, within a few iterations we find a suitable cut that partitions the points exactly. Then we divide the processors into two groups, one group on each side of the cut. Each processor sends its points that fall on the far side of the cut to a partner processor in the other group, and likewise receives a set of points that lie on its side of the cut. These steps are outlined in Fig. 7. Note that if we start with a nearly-balanced assignment of points to processors (e.g. the assignment from a previous timestep), very little data will actually be exchanged in steps (3) and (4).

After the first pass through steps (1)–(4), we have reduced the partitioning problem to two smaller problems, each of which is to partition  $N/2$  points on  $P/2$  processors within a new bounding box. Thus we can recurse on these steps until we have assigned  $N/P$  points to each processor, as shown in Fig. 7 for an 8-processor example. The final geometric sub-domain owned by each processor is a regular parallelepiped. Note that it is simple to generalize the RCB procedure for any  $N$  and non-power-of-two  $P$  by adjusting our desired “median” criterion at each stage to insure the correct number of

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| <ol style="list-style-type: none"> <li>(1) Choose a coordinate axis (<math>xyz</math>)</li> <li>(2) Position cut so as to partition points equally</li> <li>(3) Send points that lie on far side of cut</li> <li>(4) Receive points that lie on my side of cut</li> <li>(5) Recurse</li> </ol> |
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Fig. 7. Parallel algorithm for recursive coordinate bisection.

points end up on each side of the cut.

**4. Parallel Smoothed Particle Hydrodynamics.** Our parallel algorithm for the SPH portion of the global timestep (step (2) of Fig. 3) is listed in Fig. 8. Since the SPH particles have moved during the previous timestep, we first rebalance the assignment of SPH particles to processors by performing an RCB operation in step (1). This results in a new SPH-decomposition where each processor owns an equal number of SPH particles contained in a geometrically compact sub-domain. This will load-balance the SPH calculations which follow and enable the neighbors of each particle to be efficiently identified.

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| <ol style="list-style-type: none"> <li>(1) Perform RCB on SPH particles to rebalance</li> <li>(2) Share RCB cut info with all processors</li> <li>(3) For all my particles             <ul style="list-style-type: none"> <li>If particle extends beyond my sub-domain</li> <li>Determine which processors it overlaps</li> </ul> </li> <li>(4) Send overlapping particles to nearby processors</li> <li>(5) Compute pairwise interactions with COI in my sub-domain</li> <li>(6) Communicate results back to processors owning particles</li> <li>(7) Update particle equation-of-state, forces, etc.</li> </ol> |
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Fig. 8. A parallel algorithm for smoothed particle hydrodynamics.

The geometry of the SPH-decomposition can be represented as a set of  $P - 1$  cuts, where  $P$  is the number of processors. One of these cuts is stored by each processor as the RCB operations are carried out. In step (2) we communicate this cut information so that every processor has a copy of the entire set of cuts. This is done optimally via an *expand* operation [4].

Before the SPH calculations can be performed, each processor must know about nearby particles that overlap its sub-domain and are thus potential neighbors of its own particles. This information is acquired in steps (3) and (4). First in step (3), each processor checks which of its SPH particles extend beyond its sub-domain. For those that do, a list of processors whose sub-domain is overlapped is created. This is done using the RCB vector of cuts created in step (2). The information in this vector enables a processor to know the bounds of the sub-domains owned by every other processor. In step (4) the data for overlapping SPH particles is communicated to the appropriate processors.

Now we are ready to begin computing SPH interactions. As indicated earlier, this consists of two kinds of computations. First, in step (5), pairwise interactions are computed between all pairs of particles which overlap. Each processor determines the interacting neighbors for each of its owned particles. Then the interactions are computed for each pair. This is done in exactly the same way as in the serial PRONTO-3D code (including the sorts and searches for neighbors) with one important exception. It is possible that a pair of interacting particles will be owned by 2 or more processors after extra copies of particles are communicated in steps (3) and (4). Yet we want to insure that exactly one processor computes the interaction between that pair of particles. We enforce this by defining a unique point, which we call the center-of-interaction (COI) between two spheres A and B. Let  $x$  be the point within A which is closest to the center of B, and let  $y$  be the point within B which is closest to the center of A. The COI of A and B is  $(x+y)/2$ . A processor computes a pairwise interaction if and only if the COI for that pair is within its sub-domain.

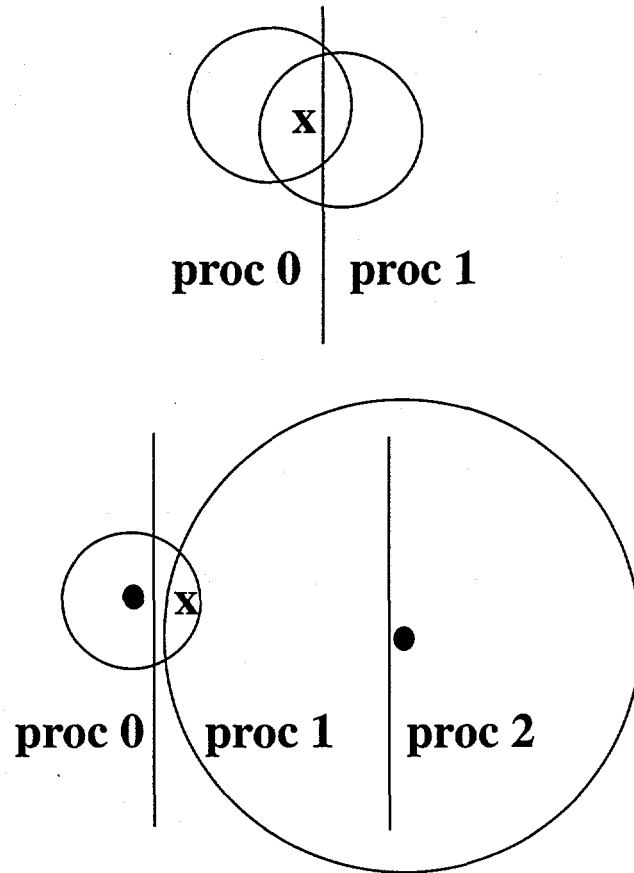
This is illustrated in Fig. 9 for two kinds of cases. In the upper example two particles are owned by processors responsible for neighboring sub-domains. After steps (3) and (4), both processors will own copies of both particles. However, only processor 0 will compute the pairwise interaction of the particles since the COI for the pair is within processor 0's sub-domain. The lower example illustrates a case where processors 0 and 2 own the particles but the COI is on processor 1. Thus only processor 1 will compute the pairwise interaction for those particles. Note that this methodology adds only a tiny check (for the COI criterion) to the serial SPH routines and allows us to take full advantage of Newton's 3rd law so that the minimal number of pairwise interactions are computed.

When step (5) is complete, as a result of computing the pairwise interactions based on the COI criterion, the processor owning a particular particle may not have accumulated the entire set of pairwise contributions to that particle, since some of the contributions may have been computed by other processors. In step (6) these contributions are communicated to the processors who actually own the particles within the SPH-decomposition. Then in step (7) each processor can loop over its particles to compute their equation-of-state and the total force acting on the particle. In actuality, steps (5)–(7) are iterated on four times in PRONTO-3D [2], to compute several necessary SPH quantities in the proper sequence. From a parallel standpoint however, the calculations are as outlined in Fig. 8: pairwise computations followed by communication followed by particle updates.

In summary, step (4) requires unstructured communication of the kind outlined in Fig. 5. Steps (1), (2) and (6) are also primarily devoted to communication. Steps (5) and (7) are solely on-processor computation. They invoke code which is only slightly modified from the serial version of PRONTO-3D. This is because the SPH-decomposition produced in step (1) reformulated the global SPH problem in a self-similar way: compute interactions between a group of particles contained in a box.

**5. Parallel Contact Detection Algorithm.** Our parallel algorithm for the contact detection portion of the global timestep (step (4) of Fig. 3) is listed in Fig. 10. It





**Fig. 9.** *Upper: A pair of SPH particles owned by two processors whose center-of-interaction (COI) shown by an "x" is owned by the left processor. Lower: A pair of SPH particles owned by two processors whose COI is owned by neither.*

uses the same kernels as our SPH algorithm, but in different ways. Recall that for load-balancing purposes we desire a decomposition of all objects that will potentially come in contact (contact nodes, contact surfaces, and SPH particles) that is different than the FE- and SPH-decompositions. In step (1), the current position of each contact node is communicated by the processor who owns and updates it in the FE-decomposition to the processor who owned that node in the contact-decomposition of the previous timestep. (On the first timestep this step is simply skipped.) Likewise current SPH particle positions are communicated from the current SPH-decomposition to the processors who owned the SPH particles in the contact-decomposition of the previous timestep. Both of these operations involve unstructured communication as detailed in §3. The purpose of step (1) is to give the contact-decomposition a starting point that is close to the correctly balanced answer, since the finite elements and SPH particles do not move far in any one timestep. In step (2) we perform an RCB operation as described in §3 on the combined set of contact nodes and SPH particles, based on their current positions.

- |   |
|---|
| <ol style="list-style-type: none"> <li>(1) Send minimal contact node and SPH data to old RCB decomposition</li> <li>(2) Perform RCB to rebalance</li> <li>(3) Send full contact and SPH data to new RCB decomposition</li> <li>(4) Share RCB cut info with all processors</li> <li>(5) For all my surfaces             <ul style="list-style-type: none"> <li>If surface extends beyond my sub-domain                 <ul style="list-style-type: none"> <li>Determine which processors it overlaps</li> </ul> </li> </ul> </li> <li>(6) Send overlapping surfaces to nearby processors</li> <li>(7) Find contacts within my sub-domain</li> <li>(8) Send contact results to FE and SPH owners</li> </ol> |
|---|

**Fig. 10.** A parallel algorithm for contact detection.

Once the RCB operation is complete, in step (3) we send full information about the contact nodes and SPH particles to their new contact-decomposition owners. This is much larger set of information than was sent in step (1) and we delay its sending until after the RCB operation for two reasons. First, it does not have to be carried along as “points” migrate during the RCB stages. More importantly, in step (3) we also send contact surface information to processors in the new contact-decomposition by assigning each contact surface to the contact-decomposition processor who owns one (or more) of the 4 contact nodes that form the surface. By delaying the sending of contact surfaces until step (3), the surfaces do not have to be included in the RCB operation.

Once step (3) is finished, a new contact-decomposition has been created. Each processor owns a compact geometric sub-domain containing equal numbers of contact nodes and SPH particles. This is the key to load-balancing the contact detection that will follow. Additionally each processor in the new decomposition owns some number of contact surfaces.

Steps (4)–(6) are similar to steps (2)–(4) in the SPH algorithm. The only difference is that now contact surfaces are the objects that overlap into neighboring sub-domains and must be communicated to nearby processors since they may come in contact with the contact nodes and SPH particles owned by those processors. The extent of this overlap is computed by constructing a bounding box around each contact surface that encompasses the surface’s zone of influence.

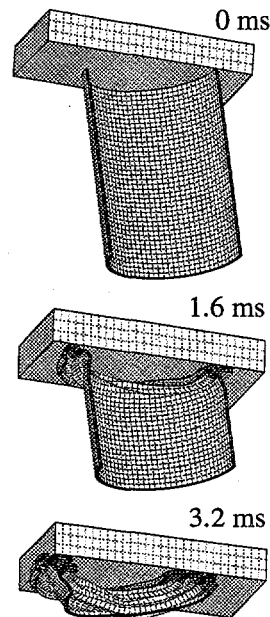
In step (7) each processor can now find all the node/surface and particle/surface contacts that occur within its sub-domain. Again, a nice feature of our algorithm is that this detection problem is conceptually identical to the global detection problem we originally formulated, namely to find all the contacts between a group of surfaces, nodes, and particles bounded by a box. In fact, in our contact algorithm each processor calls the original unmodified serial PRONTO-3D contact detection routine to accomplish step (7). This enables the parallel code to take advantage of the special sorting and searching features the serial routine uses to efficiently find contact pairs [6]. It also means we did not have to recode the complex geometry equations that compute intersections

between moving 3-D surfaces and points! Finally, in step (8), information about found contacts is communicated back to the processors who own the contact surfaces and contact nodes in the FE-decomposition and who own the SPH particles in the SPH-decomposition. Those processors can then perform the appropriate force calculations and element/particle push-back in steps (5) and (6) of Fig. 3.

In summary, steps (1), (3), (6) and (8) all involve unstructured communication of the form outlined in Fig. 5. Steps (2) and (4) also consist primarily of communication. Steps (5) and (7) are solely on-processor computation.

**6. Results.** [Unfortunately, we are in the process of implementing our parallel SPH ideas in PRONTO-3D. Thus we do not yet have parallel SPH timing results; we plan to include them in the final version of our SuperComputing paper. For now, we just present parallel FE timing results for PRONTO-3D, which include parallel contact detection of mesh/mesh contacts.]

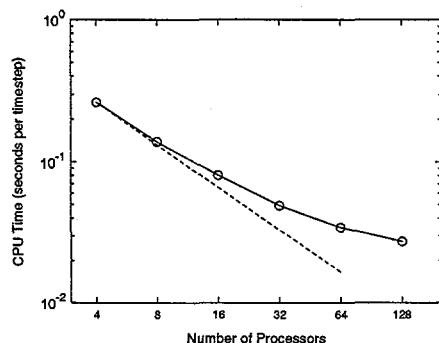
Fig. 11 shows the results of a parallel PRONTO-3D simulation of a steel shipping container being crushed due to an impact with a flat inclined wall. The front of the figure is a symmetry plane; actually only one half of the container is simulated. As the container crumples, numerous contacts occur between layers of elements on the folding surface. We have used this problem to test and benchmark the implementations of our parallel FE analysis and contact detection algorithms in PRONTO-3D.



**Fig. 11.** Simulation of a crushed shipping container from initial impact to final state after 3.2 milliseconds.

The first set of timing results we present is for a fixed-size problem containing 7152 finite elements. Both the container and wall were meshed 3 elements thick, so roughly

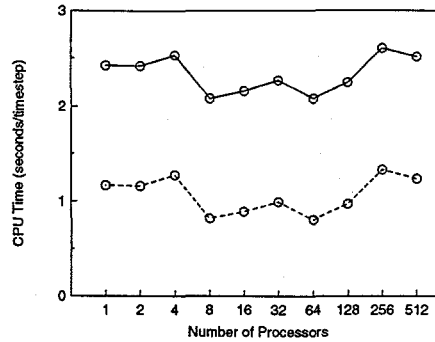
2/3 of the elements are on a surface. Since each surface element contributes both a surface and node, there were about 9500 contact surfaces and nodes in the problem. The average CPU time per timestep for simulating this problem on various numbers of Intel Paragon processors from 4 to 128 is shown in Fig. 12. Whether in serial or parallel, PRONTO-3D spends virtually all of its time for this calculation in two portions of the timestep – FE computation and contact detection. For this problem, both portions of the code speed-up adequately on small numbers of processors, but begin to fall off when there are only a few dozen elements per processor.



**Fig. 12.** Average CPU time per timestep to crush a container with 7152 finite elements on the Intel Paragon. The dotted line denotes perfect speed-up.

Fig. 13 shows performance on a scaled version of the crush simulation where the container and surface are meshed more finely as more processors are used. On one processor a 1875-element model was run. Each time the processor count was doubled, the number of finite elements was also doubled by halving the mesh spacing in a particular dimension. Thus all the data points are for simulations with 1875 elements per processor; the largest problem is nearly a million elements on 512 processors.

In contrast to the previous graph, we now see excellent scalability. The upper curve is the total CPU time per timestep; the lower curve is the time for parallel contact detection. The difference between the two curves is essentially all FE computation which scales nearly perfectly. Together these curves show that the parallel contact detection algorithm requires about the same fraction of time on 1 processor (the serial code) as it does on large numbers of processors, which was our original goal with this work. In fact, since linear speed-up would be a horizontal line on this plot, we see apparent super-linear speed-up for some of the data points! This is due to the fact that we are really not exactly doubling the computational work involved in contact detection each time we double the number of finite elements. First, the mesh refinement scheme we used does not keep the surface-to-volume ratio of the meshed objects constant, so that the contact algorithm may have less (or more) work to do relative to the FE computation for one mesh size versus another. Second, the timestep size is reduced as the mesh is refined. This slightly reduces the work done in any one timestep by the serial contact search portion of the contact detection algorithm (step (7) in Fig. 10), since



**Fig. 13.** Average CPU time per timestep on the Intel Paragon to crush a container meshed at varying resolutions. The mesh size is 1875 finite elements per processor at every data point. The solid line is the total time; the dotted line is the time for contact detection.

contact surfaces and nodes are not moving as far in a single timestep. More generally, the number of actual contacts that occur in any given timestep will not exactly double just because the number of finite elements is doubled.

**7. Conclusions.** We have described the parallelization of the multi-physics code PRONTO-3D, which combines transient structural dynamics with smoothed particle hydrodynamics to simulate complex impacts and explosions in coupled structure/fluid models. Our parallel strategy uses different decompositions for each of the three computationally dominant stages, allowing each to be parallelized as efficiently as possible. This strategy has proven effective in enabling PRONTO-3D to be run scalably on hundreds of processors of the Intel Paragon.

We use a static decomposition for finite element analysis and a dynamic, geometric decomposition known as recursive coordinate bisectioning for SPH calculations and contact detection. RCB has the attractive property of responding incrementally to small changes in the problem geometry, which limits the amount of data transferred in updating the decomposition. Our results indicate that the load balance we achieve in this way more than compensates for the cost of the communication required to transfer data between decompositions.

One potential concern about using multiple decompositions is that mesh and particle data may be duplicated, consuming large amounts of memory. We reduce this problem by using a dynamic decomposition of the SPH particles, so that each particle is only stored once (although the processor which owns it may change over time). However, we do suffer from data duplication in the contact detection portion of the code both for contact surfaces and nodes and for SPH particles. This has not been a major bottleneck for us because the duplication is only for surface elements of the volumetric meshes and because we are typically computationally bound, not memory bound, in the problems that we run with PRONTO-3D.

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