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Singular Perturbation Applications in Neutron Transport

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A singular perturbation technique, described earlier[1], was developed for neutron transport analysis by postulating expansion in terms of a small ordering parameter ε. Our perturbation analysis is carried, without approximation, through $O(\varepsilon^2)$ to derived a material interface correction for diffusion theory. Here we present results from an analytical application of the perturbation technique to a fixed source problem and then describe an implementation of the technique in a computational scheme.

Our fixed source test calculation uses a symmetric reflected slab geometry shown in Figure 1. The core region has a uniform source with $c = 0.8$, where c is the number of secondary neutrons per collision. The reflector is source-free and more absorptive, $c = 0.6$. Each region is 2 mfp's thick, so $\varepsilon = \frac{1}{2}$ which fulfills, yet strains, our assumption that ε is small. Table I in Ref. 1 gives the interior equations and boundary conditions that we solve analytically using the transport-corrected diffusion equations where $c_4 = -\frac{4}{5} c_2^2$. Equation (5) of Ref. 1 is added to the interior solutions forming the composite solution.

The figure compares the normalized perturbation results with similar diffusion theory results and with a transport solution calculated by the ANISN code[2] in S^{16} quadrature. For this rather severe case, the perturbation technique provides a clear improvement over diffusion theory. The figure also shows the $O(\varepsilon)$ component of the perturbation solution that adds a correction at the vacuum boundary in a way nearly equivalent to using the extrapolated endpoint in conventional diffusion theory. The $O(\varepsilon^2)$ component provides a correction at the material interface similar to recent P_2 derivations[3], except that our boundary layer analysis provides flux continuity.

To formulate the perturbation equations for a computational finite-difference implementation, the interior equations of Ref. 1 are transformed to dimensional form using $x/\varepsilon = \Sigma_t z$, obtaining:

$$-\frac{d}{dz} C_g(z) D_g(z) \frac{d}{dz} \Psi_g^{j0}(z) + \Sigma_{rg} \Psi_g^{j0}(z) = S_g^j(z) \tag{1}$$

where g is a group index. The removal cross section Σ_{rg} includes absorption, down-scatter and transverse leakage. Here we defer the question of an asymptotically consistent multi-group formulation by using the conventional slowing-down treatment.

Equation (1) has a form identical to the conventional diffusion equation except for the factor $C(z) = \{1-c(\varepsilon)\}/c_2\varepsilon^2$ that multiplies the leakage term and depends on the asymptotic expansion selected for $c(\varepsilon)$. If we select $c(\varepsilon) = 1 + c_2\varepsilon^2$ then $C(z) = 1$ and the common diffusion equation results.

The O(1) source, $S^0_g(z)$, is the same as in conventional diffusion analysis and models the source due to fission, inscatter and external sources. Our analysis assumes the $O(\varepsilon)$ source includes only inscatter without $O(\varepsilon)$ perturbations from fission or external sources. The $O(\varepsilon^2)$ source derived from Table I of Ref. 1 is:

$$S_g^2(z) = \sum_{g'=1}^{g-1} \Sigma_{sg' \to g} \Psi_{g'}^{20}(z) + \frac{4}{5} \left\{ \Theta \frac{\Sigma_{rg}}{\Sigma_{tg}} [\Sigma_{rg} \Psi_g^{00}(z) - S_g^0(z)] - \frac{d}{dz} D_g(z) \frac{d}{dz} \frac{S_g^0(z)}{\Sigma_{tg}} \right\} \tag{2}$$

This source has additional terms beyond those in the lower-order analysis and depends on the expansion of $c(\varepsilon)$. For the common diffusion equations, $\Theta = 1$; while for the transport-corrected diffusion equations, $\Theta = 0$.

To implement these perturbation equations in a diffusion code, the outer iteration scheme remains essentially intact. The O(1) flux is iterated until the fission source converges; then it is fixed. The O(1) flux is then used to calculate the $O(\varepsilon)$ vacuum boundary conditions and the $O(\varepsilon^2)$ interface conditions. A single outer iteration calculates the $O(\varepsilon)$ flux and adds a vacuum boundary correction. The calculation continues to $O(\varepsilon^2)$ that uses the source in Eq. (2) and has pseudo-source terms arising from the interface condition. With this $O(\varepsilon^2)$ source, solving for the $O(\varepsilon^2)$ flux again requires just one outer iteration.

Several test calculations using both analytic and computational implementations of our proposed perturbation technique showed that the error in flux distributions is typically reduced to about half that achieved with conventional diffusion theory. Although limited to one-dimensional slab geometry, our test calculations included a research reactor case with relatively severe in-core heterogeneity caused by poisons and water holes.

Our comparisons of the common and transport-corrected forms for the diffusion equations showed that the common form generally produces better results. We found that, as previously reported[4], an $O(\varepsilon)$ perturbation analysis is nearly equivalent to conventional diffusion theory. However, the $O(\varepsilon^2)$ perturbation analysis derives material interface corrections that can improve the accuracy of diffusion theory calculations. The $O(\varepsilon^3)$ corrections are generally less important, only contributing to the vacuum boundary corrections.

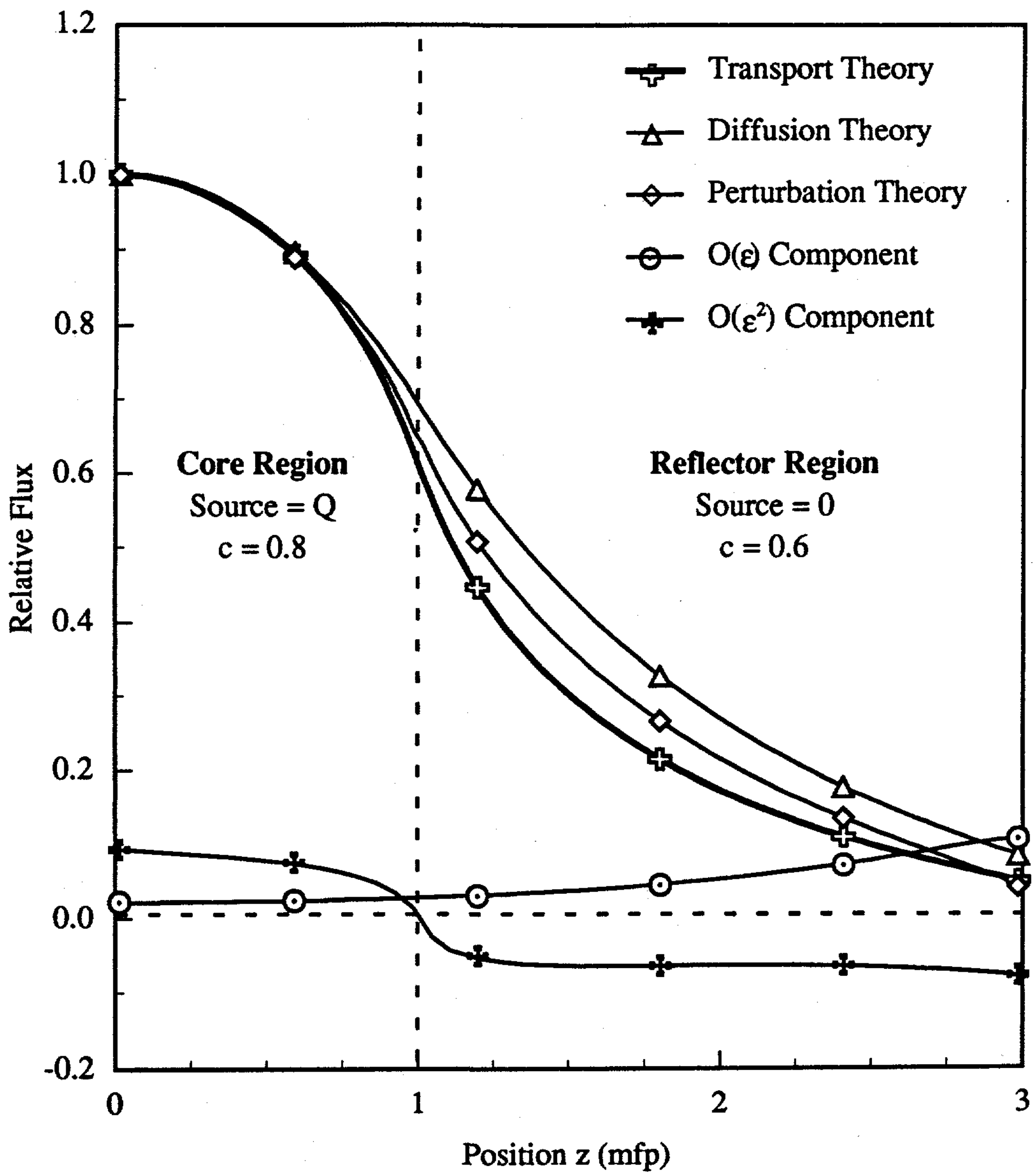

Figure 1. Scalar flux for a reflected slab

1 D. C. LOSEY and J. C. LEE, "Singular Perturbation Analysis of the Neutron Transport Equation", submitted for presentation at the present conference.

2 W. W. ENGLE, Jr., "A User's Manual for ANISN, A One-Dimensional Discrete Ordinates Transport Code with Anisotropic Scattering," K-1693, Oak Ridge Gaseous Diffusion Plant (1967).

3 G. C. POMRANING, R. RULKO and B. SU, "Diffusion Theory with Discontinuities," *Nucl. Sci. Eng.*, **118**, 1 (1994).

4 E. W. LARSEN, "Diffusion Theory as an Asymptotic Limit of Transport Theory for Nearly Critical Systems with Small Mean Free Paths," *Annals of Nuclear Energy*, **7** 249, (1980).