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Title: NON-HERMITIAN QUANTUM MECHANICS AND LOCALIZATION IN PHYSICAL SYSTEMS

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Non-Hermitian quantum mechanics and localization in physical systems

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Recent studies on a delocalization phenomenon of a non-Hermitian random system is reviewed. The complex spectrum of the system indicates delocalization transition of its eigenfunctions. It is emphasized that the delocalization is related to various physical phenomena such as flux-line pinning in superconductors and population biology of bacteria colony.

1. INTRODUCTION

In the last few years, there has been a novel development in the study of localization phenomena, namely non-Hermitian localization and delocalization [1–30]. A new type of delocalization transition was found in a simple quantum-mechanical model with a non-Hermite Hamiltonian. I here describe interesting properties of the non-Hermitian system and a few physical motivations of studying it.

The model is defined by the Hamiltonian [1,2]

$$\mathcal{H} \equiv \frac{1}{2m}(\vec{p}+i\vec{g})^2 + V(\vec{x}), \tag{1}$$

where a constant $\vec{g}$ makes the Hamiltonian non-Hermite. The operator $\vec{p}$ is the momentum and $V(\vec{x})$ is a random potential. The system is reduced to a fundamental model of the Anderson localization in the Hermitian case $\vec{g} = \vec{0}$. In this Hermitian limit, it is well accepted that the eigenfunctions are localized in one and two dimensions and that there are energy regions of localized eigenfunctions in three dimensions and higher. It was pointed out [1,2] that, in any dimensions, as the non-Hermitian field $\vec{g}$ is introduced and increased,

(i) Each of the originally localized states is delocalized at its own critical point $\vec{g}_c$;

(ii) The delocalization transition coincides with the instance where the corresponding eigenvalue becomes complex (Up to this point the eigenvalue is fixed to the real value for $\vec{g} = \vec{0}$.);

(iii) The inverse localization length κ of the original eigenfunction (for $\vec{g} = \vec{0}$) is equal to $|\vec{g}_c|/\hbar$.

It is a remarkable fact to have delocalization even in one dimension. It is also a very characteristic and new feature of this system that one can detect the delocalization transition by calculating the spectrum of the system.

2. PHYSICAL MOTIVATIONS

2.1. Flux-line depinning in high-T_c superconductors

Flux line (or vortex line) in high-T_c materials has spawned a new branch of physics [31]. This one-dimensional object exhibits novel "solid," "liquid" and "glass" phases. One of the interesting topics is flux-line pinning due to impurities and defects in superconductors, particularly due to columnar and planar defects. When the magnetic field applied to the superconductor is parallel to these extended defects, the flux lines generated by the field are easily pinned by the defects, which stabilizes the superconductivity. A depinning transition has been observed when the field is tilted from the axis of the defects [32–34].

The depinning transition was successfully described by a phenomenological model of flux lines [35], where a flux line is regarded as an elastic string. Using the inverse of path-integral mapping, we can further transform the partition function of this classical model to the Green's function of a quantum mechanical system, where the elastic string is regarded as the world line of a quantum particle.

The flux-line depinning due to the tilt of the external magnetic field is then equivalent to the delocalization of the non-Hermitian system described in the previous section [1,2]. The depinning point is hence estimated by calculating the spectrum of the Hamiltonian (1). The criticality of the depinning transition can be also discussed by applying the argument of the Mott variable-range hopping to the non-Hermitian system.

2.2. Localization length of the Hermitian Anderson model

The relation (3) between the inverse localization length of the Hermitian Anderson model and the delocalization point of the non-Hermitian Anderson model (1) enables us to estimate the localization length of the Hermitian Anderson model by calculating the spectrum of the system (1) [2,19,20,29]. (The conventional method of estimating the localization length is to calculate the Lyapunov exponent of the random transfer matrix of a long stripe of the Anderson model.)

2.3. Fokker-Planck system

The imaginary-time Schrödinger equation for the Hamiltonian (1) may be regarded as a Fokker-Planck equation of the form

$$\frac{\partial}{\partial\tau}c(\vec{x},\tau) = D\vec{\nabla}^2 c(\vec{x},\tau) - \vec{v}\cdot\vec{\nabla}c(\vec{x},\tau) + V(\vec{x})c(\vec{x},\tau) + \text{const.} \tag{2}$$

The first and the second terms of the right-hand side represents diffusion in a flow, while the third term yields growth or decay due to a first-order reaction with random reaction rate. Thus the above equation can describe bacteria population in a constant flow with random distribution of nutrients [15,30] and chemical reaction system with random catalyst distribution.

An important difference between the quantum-mechanical problem and the Fokker-Planck problem is that, in the former problem, the particle density distribution is given by the product of the left and right eigenfunctions, while in the latter problem the right eigenfunction itself is the density distribution. In fact, recent studies in one dimension [24–26] showed that, even when the product of the two eigenfunctions is delocalized, each of the eigenfunction shows a behavior intermediate between localized and delocalized. It is also suggested [3,15] that the eigenfunction in two dimensions is a fractal object in the delocalized regime.

2.4. Other motivations

Mudry *et al.* [18] pointed out a relation between the non-Hermitian Anderson model (1) and the Dirac fermion in a weak random gauge field. Using the method of Hermitization [36,8], the spectrum of the non-Hermitian model (1) can be studied by calculating the Green's function of the Hermite matrix

$$H \equiv \begin{pmatrix} 0 & \mathcal{H} - z \\ \mathcal{H}^\dagger - z^* & 0 \end{pmatrix}. \tag{3}$$

In the weak-disorder limit, the low-energy low-momentum expansion of the matrix (3) yields the Hamiltonian of a Dirac Fermion in a random non-Abelian gauge field. The random Dirac Fermion, particularly in two dimensions, has been an interesting topic because the system has randomness-driven phase transitions [37] and the wave function at the critical point was found to be multifractal [38,39].

As a related problem, non-Hermitian random matrix theory has been brought into focus lately. There are various physical motivations for this theory, from a model of chiral symmetry breaking in QCD [40,41,17] to neural networks [42]. A recent paper [43] studied a non-Hermitian $\mathcal{PT}$-symmetric oscillator from the point of view that the Hermiticity of Hamiltonian is not necessarily a physical requirement but the $\mathcal{PT}$ symmetry is.

3. SUMMARY

Many physicists have thought that non-Hermite Hamiltonians have no physical meaning. In fact, the last few years saw a growing number of physical non-Hermitian models. More intensive studies of these models are awaited.

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