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EXTREME NONLINEARITY IN ROCKS: AN INVESTIGATION USING ELASTIC PULSE WAVE PROPAGATION

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INTRODUCTION and SUMMARY

Because of the presence of structural defects such as microcracks and grain boundaries, the effective moduli in a highly disordered material change dramatically as a function of stress. Earth materials (rocks) are an important example of this type of disordered media and are of practical interest in geophysics and seismology. At the laboratory scale, static stress-strain theory and elastic resonance experiments on rocks suggest that the ratio of third order elastic constants to second-order elastic constants in such materials is several orders of magnitude higher than in the case of ordinary uncracked materials. This high degree of nonlinearity means that frequency components mix and energy is transferred from fundamental frequencies to sum and difference frequencies along the wave propagation path well away from an acoustic or elastic wave source. Accurate measurement of nonlinear contributions along the propagation path can be used as a sensitive measure of consolidation and saturation in earth materials as well as symptoms of fatigue or damage.

In this paper we report a model for that describes the nonlinear interaction of frequency components in arbitrary pulsed elastic waves during one-dimensional propagation in an infinite medium. The model is based on the use of one dimensional Green's Function theory in combination with a perturbation method, as has been developed for a general source function by McCall.[1] A polynomial expansion is used for the stress-strain relation in which we account for four orders of nonlinearity. The perturbation expression corresponds to a higher order equivalent of the Burgers' equation solution for velocity fields in solids. It has conceptual clarity and is easy to implement numerically, even with the inclusion of an arbitrary attenuation function. A comparison with experimental data on Berea sandstone[2] is given to illustrate the model when used in an iterative procedure, and good agreement is obtained limiting model parameters up to cubic anharmonicity. The resulting values for the nonlinear parameters are several orders of magnitude larger than those for uncracked materials. Finally we discuss the values obtained for the dynamic nonlinearity parameters in comparison with static and resonance results.

THEORY

As shown in Ref.[1], the one dimensional nonlinear equation of motion for the displacement field can be obtained by representing the stress (σ) as a polynomial expansion in the strain (ε). Because it is believed that nonlinear coefficients of higher order than the cubic term can play a role in the wave behavior characteristics for microcracked materials, we take into account four orders of nonlinearity in this expansion. Representing those coefficients by Greek letters, the resulting nonlinear equation takes the form:

$$\frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left[\frac{\partial u}{\partial x}\left(1+\beta\frac{\partial u}{\partial x}+\delta\left(\frac{\partial u}{\partial x}\right)^2+\eta\left(\frac{\partial u}{\partial x}\right)^3+\xi\left(\frac{\partial u}{\partial x}\right)^4\ldots\right)\right] + S(x,t) \qquad (1)$$

in which u is the particle displacement, c_0 is the linear velocity, $S(x,t)$ is the expression for the source function and β, δ, η and ξ are the nonlinear parameters. In particular we are interested in the response of the displacement field to an arbitrary train of pulsed elastic waves (at carrier frequency

ω_0) at one end of the sample under consideration. The Fourier transform of the source function in this case can be written as follows:.

$$S(x,\omega) = -2i\frac{\omega}{c_0}\delta(x)\,2\pi\sum_{n=-\infty}^{+\infty}U_n\delta(\omega-n\omega_0) \tag{2}$$

in which $U_n=[U_{-n}]^*$ is a complex number describing the amplitude R_n and phase ϕ_n of the n-th harmonic displacement component ($U_n=-\,i/2\,R_n\exp[i\phi_n]$), and $\delta(x)$ is the delta distribution function (as opposed to the nonlinear parameter δ).

Applying an analog procedure combining Green's Function theory and perturbation approach as done by McCall, we found a general expression describing the harmonic distortion of a pulsed signal propagating in a nonlinear medium. In terms of the particle velocity components $V_n=-in\omega_0U_n=-\,n\omega_0/2\,R_n\exp[i\phi_n]$ the perturbation solution is given by:

$$\begin{aligned}
V_n(x+dx) &= V_n(x)\,\frac{dx}{|dx|}\exp[-\frac{n\omega_0}{2Qc_0^2}dx]\\
&+\frac{i\omega_0 n}{2c_0^2}\sum_{m=-\infty}^{+\infty}V_{n-m}(x)\,V_m(x)\,A(n-m,m)\\
&+\frac{-i\omega_0 n}{2c_0^3}\sum_{m,l=-\infty}^{+\infty}V_{n-m-l}(x)\,V_m(x)\,V_l(x)\,B(n-m-l,m,l)\\
&+\frac{i\omega_0 n}{2c_0^4}\sum_{m,l,k=-\infty}^{+\infty}V_{n-m-l-k}(x)\,V_m(x)\,V_l(x)\,V_k(x)\,C(n-m-l-k,m,l,k)\\
&+\frac{i\omega_0 n}{2c_0^5}\sum_{m,l,k,j=-\infty}^{+\infty}V_{n-m-l-k-j}(x)\,V_m(x)\,V_l(x)\,V_k(x)\,V_j(x)\,D(n-m-l-k-j,m,l,k,j)
\end{aligned} \tag{3}$$

where the functions A, B for a symmetrical "breathing" input source ($u(-x,t)=-u(x,t)$) are:

$$A(n,m)=\beta\,dx$$

$$B(n,m,l)=\delta\,dx-\beta^2dx(\frac{2n+3m+3l}{2(n+m+l)})-i\frac{(m+l)\omega_0}{c_0}\beta^2\frac{dx|dx|}{2}$$

for n,m,l all being integer numbers.

For conciseness and also because the theoretical simulation of the experiment described below has been performed taking into account only two-fold and three-fold frequency component interactions, the interaction weight functions C and D have been omitted. Their leading terms start with $\eta\,dx$ and $\xi\,dx$ respectively. In the case of an antisymmetrical "wiggling" source these functions are slightly different.

Eq.(3) corresponds to a higher order equivalent of the Burgers' equation solution for particle velocity fields in solids. Generally only interactions between two frequency components are accounted for, whereas in this case energy mixing can occur between up to 5 harmonics . Rather than calculating the distortion directly at propagation distance L, it is implemented as part of an iterative procedure in which the harmonic distorted signal at distance x is taken as an input for the calculation of the distortion at x+dx. Dividing the distance L in a number of iteration steps has an additional advantage in that 1) harmonics of order higher than 5 (n) can be accounted for using an expansion of stress-strain up to quintic (n-th) anharmonicity due to repeated "source" distortion and that 2) any attenuation law can be substituted. Here we used the common exponential decaying function in which Q is a frequency independent measure of the linear attenuation.

For small distances (only one iteration) and a single frequency input, the analytic form of the solution suggests easily verifiable relations for experimental observations as a function of amplitude, frequency and distance.[1,2]

EXPERIMENT, COMPARISON and DISCUSSION

The theoretical model, in a reduced form limiting ourselves to the functions *A* and *B* in Eq.(3), has been tested with experimental data from a bar of Berea sandstone, which were reported in an previous publication together with an extensive description of the experimental apparatus and method.[2] A single frequency signal is generated at the source and its distorted waveform is detected at a receiver 58 cm away from the source. Figure 1 shows the Fourier amplitude spectra at the source and receiver (continuous lines) for various intensity levels of the input signal. As the input intensity increases, one clearly observes the increased nonlinear interaction that occurs along the propagation path. Harmonics up to (and even greater than) the seventh order are generated. Given a measured value for the linear attenuation in a sandstone bar (Q=68), we reproduced ,with amazingly good agreement, the observed data using the theoretical 1D wave propagation model described above with 20 iterations (shown by solid triangular-topped bars in Figure 1). Except for the input intensity, all model parameters (input frequency, sound velocity, distance AND both nonlinear coefficients) are kept constant in all subfigures. The resulting values for the nonlinear parameters β and δ used in the simulation are -250 and $-3.2 \cdot 10^8$ respectively. These values are several order of magnitude larger than those found in uncracked materials, typically between 3 and 15 for β and where in general δ is of the order β^2. This anomaly illustrates that rocks and earth materials are particularly nonlinear. Furthermore, the observation that odd harmonics dominate the even harmonics is also a striking feature of rock nonlinearity. However, the values of β and δ from the theoretical simulation of this experiment are in disagreement with static stress-strain and resonance results. The first nonlinearity parameter is roughly of the same order of magnitude as predicted by static measurements, but δ is at least 2 orders of magnitude higher than in static modulus-strain data. Also the recently developed resonance model of Guyer *et al.*[3], which includes hysteresis in stress-strain space, predicts considerably lower values for δ. Because the stress and strain excursions in static and resonance experiments are much larger than in pulse mode experiments, we believe that a distinction between dynamic and static nonlinearity parameters may be appropriate. The fact that the dynamic values are larger than the static ones expresses that the mean hysteresis loop is much steeper for the tiny pulse mode type strain variations than what is expected from the static data hysteresis curves. This work is continuing.

CONCLUSION

From Green's function theory and perturbation methods a quite general expression is derived for describing the nonlinear interaction of frequency components in arbitrary pulsed elastic waves along their propagation path in an infinite medium. Nonlinearity is inserted into the wave equation by assuming a polynomial expansion of the stress-strain relation. The 1D propagation model is capable of reproducing observed experimental data[2] for a rock (Berea sandstone) and infer dynamic values for the characterization of material nonlinearity. The relation for these values to other (static) estimations is still a topic of discussion. The nonlinear parameters for Berea sandstone derived by the model are several orders of magnitude higher than found in uncracked materials indicating that this material (and more generally all rocks) exhibit highly nonlinear behavior.

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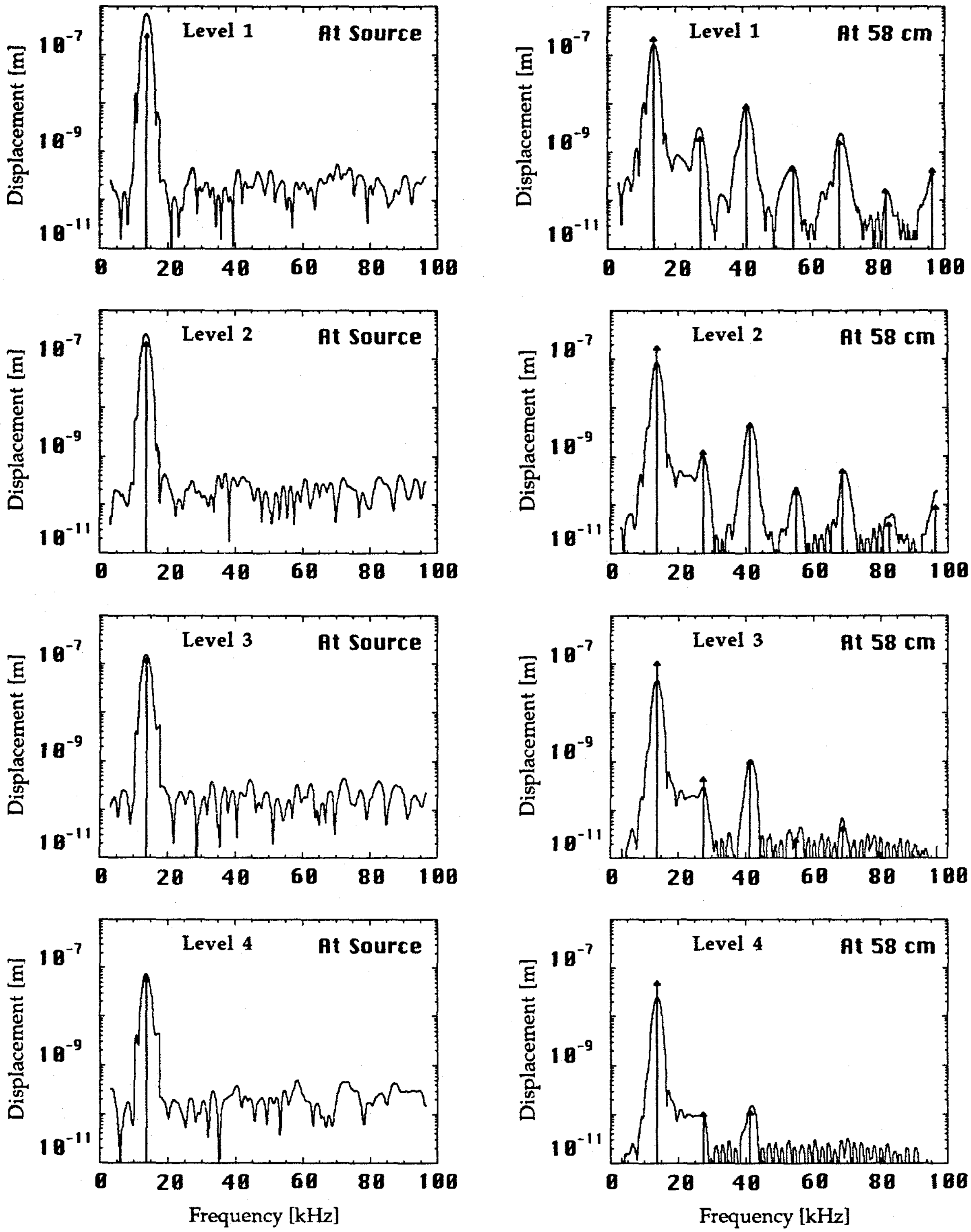

Figure 1: Experimental source and receiver (at 58 cm) spectra for a 13.75-kHz drive at 4 intensity levels (continuous lines) in a Berea sandstone bar and corresponding theoretical simulations using a 1D perturbation method (solid triangular-topped bars).