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ELECTRON-DEUTERON SCATTERING IN A RELATIVISTIC THEORY OF HADRONS

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Electron-deuteron scattering in a relativistic theory of hadrons

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Abstract. We review a three-dimensional formalism that provides a systematic way to include relativistic effects including relativistic kinematics, the effects of negative-energy states, and the boosts of the two-body system in calculations of two-body bound-states. We then explain how to construct a conserved current within this relativistic three-dimensional approach. This general theoretical framework is specifically applied to electron-deuteron scattering both in impulse approximation and when the $\rho\pi\gamma$ meson-exchange current is included. The experimentally-measured quantities A , B , and T_{20} are calculated over the kinematic range that is probed in Jefferson Lab experiments. The role of both negative-energy states and meson retardation appears to be small in the region of interest.

1 Introduction

A number of the experiments being performed at the Thomas Jefferson National Accelerator Facility (TJNAF) involve the elastic and inelastic scattering of electrons off the deuteron at space-like momentum transfers of the order of the nucleon mass. In building theoretical models of these processes, relativistic kinematics and dynamics would seem to be called for. Much theoretical effort has been spent constructing relativistic formalisms for the two-nucleon bound state that are based on an effective quantum field theory lagrangian. If the usual hadronic degrees of freedom appear in the lagrangian then this strategy is essentially a logical extension of the standard nonrelativistic treatment of the two-nucleon system.

Furthermore, regardless of the momentum transfer involved, it is crucial that a description of the deuteron be used which incorporates the consequences of electromagnetic gauge invariance. Minimally this means that the electromagnetic current constructed for the deuteron must be conserved.

Of course, the two-nucleon bound state can be calculated and a corresponding conserved deuteron current constructed using non-relativistic NN potentials which are fit to the NN scattering data. This approach has met with considerable success. (For some examples of this program see Refs. [1, 2].) Our goal here is to imitate such calculations—and, we hope, their success!—in a relativistic framework. To do this we construct an NN interaction, place it in a relativistic scattering equation, and then fit the parameters of our interaction to the NN scattering data. We then calculate the electromagnetic

form factors of the deuteron predicted by this NN model. By proceeding in this way we hope to gain understanding of the deuteron electromagnetic form factors in a model in which relativistic effects, such as relativistic kinematics, negative-energy states, boost effects, and relativistic pieces of the electromagnetic current, are explicitly included at all stages of the calculation.

This program could be pursued using a four-dimensional formalism based on the Bethe-Salpeter equation. Indeed, pioneering calculations of electron-deuteron scattering using Bethe-Salpeter amplitudes were performed by Zuilhof and Tjon almost twenty years ago [3, 4]. However, despite increases in computer power since this early work the four-dimensional problem is still a difficult one to solve. Since the NN interaction is somewhat phenomenological ultimately it is not clear that one gains greatly in either dynamics or understanding by treating the problem four-dimensionally. Therefore, instead we will employ a three-dimensional formalism that incorporates what we believe are the important dynamical effects due to relativity at the momentum transfers of interest.

We will use a three-dimensional (3D) formalism that, in principle, is equivalent to the four-dimensional Bethe-Salpeter formalism. This approach has been developed and applied in Refs. [5, 6, 7]. In this paper we will focus on the calculation of elastic electron-deuteron scattering. Here we review the formalism for relativistic bound states and show how to construct the corresponding electromagnetic current. Calculations of elastic electron-deuteron scattering are performed both in the impulse approximation and with some meson-exchange currents included. The results for the observables A , B and T_{20} are presented.

Many other 3D relativistic treatments of the deuteron dynamics which are similar in spirit to that pursued here exist (see for instance Refs. [8, 9]). Of these, our work is closest to that of Hummel and Tjon [10, 11, 12]. However, in that work approximations were employed for ingredients of the analysis, such as the use of wave functions based on the 3D quasipotential propagator of Blankenbecler-Sugar [13] and Logunov-Tavkhelidze [14], approximate boost operators, and an electromagnetic current which only approximately satisfies current conservation. Calculations of elastic electron-deuteron scattering also were performed by Devine and Wallace using a similar approach to that pursued here [15]. Here we extend these previous analyses by use of our systematic 3D formalism. In this way we can incorporate retardations into the interaction and also use a deuteron electromagnetic current that is specifically constructed to maintain the Ward-Takahashi identities.

The paper is organized as follows. In Section 2 we explain our reduction from four to three dimensions. In Section 3 we present a four-dimensional equation which is a modified version of the ladder Bethe-Salpeter equation. This modified equation has the virtue that it, unlike the ladder BSE, incorporates the correct one-body limit. By applying our three-dimensional reduction technique to this four-dimensional equation we produce an equa-

tion which has the correct one-body limit and contains the correct physics of negative-energy states. In Section 4 we explain the various potentials that are used in calculations of deuteron wave functions. These can be divided into two classes: instant potentials, and potentials that include meson retardation. Within either of these classes versions of the potentials are constructed that do and do not include the effects of negative-energy states, in order to display the role played by such components of the deuteron wave function. Section 5 discusses our 3D reduction of the electromagnetic current that maintains current conservation. This completes the laying out of a consistent formalism that includes the effects of relativity systematically, has the correct one-body limits, and maintains current conservation. In Section 6 we apply this machinery to the calculation of electron-deuteron scattering both in the impulse approximation and when corrections due to some meson-exchange currents are included. Finally, discussion and conclusions are presented in Section 7.

2 The reduction to three dimensions

The Bethe-Salpeter equation,

$$T = K + KG_0T, \quad (1)$$

for the four-dimensional NN amplitude T provides a theoretical description of the deuteron which incorporates relativity. Here K is the Bethe-Salpeter kernel, and G_0 is the free two-nucleon propagator. In a strict quantum-field-theory treatment, the kernel K includes the infinite set of two-particle irreducible $NN \rightarrow NN$ Feynman graphs.

For the two-nucleon system an application of the full effective quantum field theory of nucleons and mesons is impractical and perhaps, since hadronic degrees of freedom are not fundamental, inappropriate. In other words, the Bethe-Salpeter formalism may serve as a theoretical framework within which some relativistic effective interaction may be developed. But, if the NN interaction is only an effective one, then it would seem to be equally appropriate to develop the relativistic effective interaction within an equivalent three-dimensional formalism which is obtained from the four-dimensional Bethe-Salpeter formalism via some systematic reduction technique.

One straightforward way to reduce the Bethe-Salpeter equation to three dimensions is to approximate the kernel K by an instantaneous interaction K_{inst} . For example, if $q = (q_0, \mathbf{q})$ is the relative four-momentum of the two nucleons then

$$K(q) = \frac{1}{q^2 - \mu^2} \quad \rightarrow \quad K(\mathbf{q}) = -\frac{1}{\mathbf{q}^2 + \mu^2}. \quad (2)$$

This, admittedly uncontrolled, approximation, yields from the Bethe-Salpeter equation the Salpeter equation:

$$T_{\text{inst}} = K_{\text{inst}} + K_{\text{inst}} \langle G_0 \rangle T_{\text{inst}}, \quad (3)$$

where the three-dimensional Salpeter propagator $\langle G_0 \rangle$ is obtained by integrating over the time-component of relative momentum,

$$\langle G_0 \rangle = \int \frac{dp_0}{2\pi} G_0(p; P). \quad (4)$$

Throughout this paper we denote the integration over the zeroth component of relative momenta, which is equivalent to consideration of an equal-time Green's function, by angled brackets. We shall consider only spin-half particles, and so

$$\langle G_0 \rangle = \frac{A_1^+ A_2^+}{E - \epsilon_1 - \epsilon_2} - \frac{A_1^- A_2^-}{E + \epsilon_1 + \epsilon_2}; \quad (5)$$

where A^\pm are related to projection operators onto positive and negative-energy states of the Dirac equation, E is the total energy, and $\epsilon_i = (\mathbf{p}_i^2 + m_i^2)^{1/2}$. Note that for spin-half particles, this propagator $\langle G_0 \rangle$ is not invertible.

In order to systematize this kind of 3D reduction one must split the 4D kernel K into two parts. One of these, K_1 , is to be understood as a three-dimensional interaction in the sense that it does not depend on the zeroth component of relative four momentum¹. We then seek to choose this K_1 such that the 3D amplitude T_1 defined by

$$T_1 = K_1 + K_1 \langle G_0 \rangle T_1, \quad (6)$$

has the property that

$$\langle G_0 \rangle T_1 \langle G_0 \rangle = \langle G_0 T G_0 \rangle. \quad (7)$$

It is straightforward to demonstrate that such a K_1 is defined by the coupled equations:

$$K_1 = \langle G_0 \rangle^{-1} \langle G_0 K \mathcal{G} \rangle \langle G_0 \rangle^{-1}, \quad (8)$$

which is three-dimensional, and

$$\mathcal{G} = G_0 + G_0 (K - K_1) \mathcal{G}, \quad (9)$$

which is four dimensional. The K_1 of Eq. (8) does this by ensuring that

$$\langle \mathcal{G} \rangle = \langle G_0 \rangle. \quad (10)$$

The formalism is systematic in the sense that, given a perturbative expansion for the 4D kernel, K , a perturbative expansion for the 3D kernel, K_1 , can be developed. At second order in the coupling this gives:

¹ Of course, this is not a covariant reduction, but covariance can be maintained by a suitable generalization of this idea [6].

$$K_1^{(2)} = \langle G_0 \rangle^{-1} \langle G_0 K^{(2)} G_0 \rangle \langle G_0 \rangle^{-1}. \quad (11)$$

In $++ \rightarrow ++$ states this is just the usual energy-dependent one-particle-exchange interaction of time-ordered perturbation theory, but with relativistic kinematics, i.e. ignoring spin and isospin:

$$K_1^{(2)} = \frac{g^2}{2\omega} \left[\frac{1}{E^+ - \epsilon_1 - \epsilon_2' - \omega} + (1 \leftrightarrow 2) \right], \quad (12)$$

where ω is the on-shell energy of the exchanged particle. Note that $\langle G_0 \rangle$ must be invertible in order for the 3D reduction to be consistent. (Similar connections between three and four-dimensional approaches are discussed in Refs. [14, 16, 17, 18, 19].)

Equation (6) leads to an equation for the bound-state vertex function:

$$\Gamma_1 = K_1 \langle G_0 \rangle \Gamma_1, \quad (13)$$

where Γ_1 is the vertex function in the three-dimensional theory. The 4D vertex function, Γ , and the corresponding 3D one, Γ_1 , are related via

$$G_0 \Gamma = \mathcal{G} \Gamma_1. \quad (14)$$

3 The one-body limit

As mentioned above, and discussed many years ago by Klein [16], the propagator $\langle G_0 \rangle$ is not invertible and therefore the above reduction is not consistent. We shall show in this section that this difficulty is connected to the behavior of the three-dimensional equation in the one-body limit. In this limit we allow one particle's mass to tend to infinity. We expect that the amplitude T_1 then reduces to that given by the Dirac equation for a light particle moving in the static field of the heavy particle. In fact, this does not happen unless we include an infinite number of graphs in the kernel of the integral equation Eq. (3).

In fact, if a scattering equation with a kernel which contains only a finite number of graphs is to possess the correct one-body limit, two distinct criteria must be satisfied. First the 3D propagator should limit to the one-body propagator for one particle (the Dirac propagator in this case) as the other particle's mass tends to infinity. Second, as either particle's mass tends to infinity, the equation should become equivalent to one in which the interaction, K_1 , is static.

Equation (3)'s lack of either of these properties stems from Eq. (1) not having the correct one-body limit if any kernel which does not include the infinite set of crossed-ladder graphs is chosen [20]. Solution of Eq. (1) with such a kernel is impractical in the NN system. Nevertheless, the contributions of crossed-ladder graphs to the kernel may be included in an integral equation for T by using a 4D integral equation for K , the kernel of Eq. (1)

$$K = U + UG_C K. \quad (15)$$

Once G_C is defined this equation defines a reduced kernel U in terms of the original kernel K . The propagator G_C is chosen so as to separate the parts of the kernel K that are necessary to obtain the one-body limit from the parts that are not. U may then be truncated at any desired order without losing the one-body limits. The following 4D equation for the t-matrix is thus equivalent to Eqs. (1) and (15),

$$T = U + U(G_0 + G_C)T. \quad (16)$$

We can now remedy the defects of our previous 3D reduction. Applying the same 3D reduction used above to Eq. (16) gives:

$$T_1 = U_1 + U_1(G_0 + G_C)T_1, \quad (17)$$

where the 3D propagator is

$$\begin{aligned} \langle G_0 + G_C \rangle = & \frac{A_1^+ A_2^+}{P^{0+} - \epsilon_1 - \epsilon_2} - \frac{A_1^+ A_2^-}{2\kappa_2^0 - P^{0+} + \epsilon_1 + \epsilon_2} \\ & - \frac{A_1^- A_2^+}{P^{0-} - 2\kappa_2^0 + \epsilon_1 + \epsilon_2} - \frac{A_1^- A_2^-}{P^{0-} + \epsilon_1 + \epsilon_2}, \end{aligned} \quad (18)$$

and κ_2^0 is a parameter that enters through the construction of G_C . This three-dimensional propagator was derived by Mandelzweig and Wallace with the choice $\kappa_2^0 = P^0/2 - (m_1^2 - m_2^2)/(2P^0)$ [21, 22]. With κ_2^0 chosen in this way $\langle G_0 + G_C \rangle$ has the correct one-body limits as either particle's mass tends to infinity and has an invertible form. The kernel U_1 is defined by Eqs. (8) and (10) with the replacements $G_0 \rightarrow G_0 + G_C$, $K \rightarrow U$, and $K_1 \rightarrow U_1$.

Here we are interested in the scattering of particles of equal mass and so we make a different choice for κ_2^0 . Specifically,

$$\kappa_2^0 = \frac{P^0 - \epsilon_1 + \epsilon_2}{2}. \quad (19)$$

This form avoids the appearance of unphysical singularities when electron-deuteron scattering is calculated [7]. It yields a two-body propagator:

$$\langle G_0 + G_C \rangle = \frac{A_1^+ A_2^+}{P^{0+} - \epsilon_1 - \epsilon_2} - \frac{A_1^+ A_2^-}{2\epsilon_2} - \frac{A_1^- A_2^+}{2\epsilon_1} - \frac{A_1^- A_2^-}{P^{0-} + \epsilon_1 + \epsilon_2}, \quad (20)$$

which is consistent with that required by low-energy theorems for Dirac particles in scalar and vector fields [23]. Another way of saying this is to realize that if we compare the the $++ \rightarrow ++$ piece of the amplitude

$$V_1 \langle G_0 + G_C \rangle V_1 \quad (21)$$

to the amplitude obtained at fourth order in the full 4D field theory then the contribution of negative-energy states agrees at leading order in $1/M$ [7].

For bound states the argument of the previous section leads to the 3D equation:

$$\Gamma_1 = U_1(G_0 + G_C)\Gamma_1. \quad (22)$$

Equation (22) is a bound-state equation which incorporates relativistic effects and the physics of negative-energy states. For instance, fig. 1 is one example of a graph which is included if Eq. (22), even if only the lowest-order kernel $U_1^{(2)}$ is used, because of our careful treatment of the one-body limit.

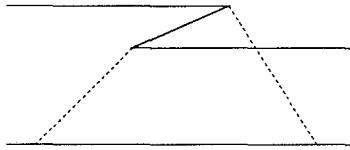


Fig. 1. One example of a Z-graph which is included in our 3D equation (22).

4 Results for the deuteron

To calculate observables in the deuteron we now consider two types of kernels U_1 , both of which are calculated within the framework of a one-boson exchange model for the NN interaction:

1. $U_1 = U_{\text{inst}}$, the instantaneous interaction.
2. A kernel $U_1^{(2)}$ which is a retarded interaction. This is obtained from Eq. (11) by the substitutions $K_1^{(2)} \rightarrow U_1^{(2)}$ and $G_0 \rightarrow G_0 + G_C$.

These interactions are used in a two-body equation with the full ET Green's function given by Eq. (22), and also in an equation in which only the $++$ sector is retained. For the instant interaction, we follow the practice of Devine and Wallace [15] and switch off couplings between the $++$ and $--$ sectors, and between the $+-$ and $-+$ sectors. A partial justification of this rule follows from an analysis of the static limit of our 3D retarded interaction.

The mesons in our one-boson exchange model are the $\pi(138)$, the $\sigma(550)$, the $\eta(549)$, the $\rho(769)$, the $\omega(782)$, and the $\delta(983)$. All the parameters of the model, except for the σ coupling, are taken directly from the Bonn-B fit to the NN phase shifts [24]—which is a fit performed using a relativistic wave equation and relativistic propagators for the mesons. The σ coupling is varied so as to achieve the correct deuteron binding energy for each interaction considered. Of course, we should refit the parameters of our NN interaction using our different scattering equations. However, for a first estimate of the importance of negative-energy states and retardation we adopt this simpler approach to constructing the interaction. Work on improving the NN interaction model is in progress [25].

Once a particular interaction is chosen, the integral equation (22) is solved for the bound-state energy. In each calculation, the σ coupling is adjusted to get the correct deuteron binding energy, producing the results (accurate to three significant figures) given in Table 1. The value given for the instant calculation with positive-energy states alone is that found in the original Bonn-B fit. In all other cases the σ coupling must be adjusted to compensate for the inclusion of retardation, the effects of negative-energy states, etc. We believe that this adjustment of the scalar coupling strength is sufficient to get a reasonable deuteron wave function. The static properties of this deuteron are very similar to those of a deuteron calculated with the usual Bonn-B interaction.

With the bound-state wave function in the center-of-mass frame has been determined in this fashion, it is a simple matter to solve the integral equation (22) in any other frame. We choose to calculate electron-deuteron scattering in the Breit frame. The interaction is recalculated in the Breit frame for a given Q^2 , and then the integral equation is solved with this new interaction. Because the formalism we use for reducing the four-dimensional integral equation to three dimensions is *not* Lorentz invariant there is a violation of Lorentz invariance in this calculation. Estimations of the degree to which Lorentz invariance is violated are displayed in Ref. [7].

Table 1. Sigma coupling required to produce the correct deuteron binding energy in the four different models under consideration here.

Interaction	States included	$g_\sigma^2/4\pi$
Instant	++	8.08
Retarded	++	8.39
Instant	All	8.55
Retarded	All	8.44

5 Current conservation

5.1 Currents in the three-dimensional formalism

As discussed in the Introduction, we now want to compare the predictions of this formalism with experimental data gained in electron scattering experiments. In calculating the interaction of the electron with the hadronic bound state it is crucial to derive a 3D reduction of the electromagnetic current which is consistent with the reduction of the scattering equation we have chosen to use here.

The current in the full four-dimensional formalism is obtained by coupling photons everywhere on the right-hand side of Eq. (1). This produces the following gauge-invariant result for the photon's interaction with the bound state:

$$\begin{aligned} \mathcal{A}_\mu = & \bar{\Gamma}(P')G_0(P')J_\mu G_0(P)\Gamma(P) \\ & + \bar{\Gamma}(P')G_0(P')K_\mu^\gamma G_0(P)\Gamma(P), \end{aligned} \quad (23)$$

where P and P' are the initial and final total four-momenta of the deuteron bound state. Here J_μ contains the usual one-body currents and K_μ^γ represents two-body contributions which are necessary for maintaining the Ward-Takahashi identities. All integrals implicitly are four-dimensional. The connection to the three-dimensional amplitude, Γ_1 , obtained from Eq. (22) is made by inserting Eq. (14) into Eq. (23), giving

$$\mathcal{A}_\mu = \bar{\Gamma}_1(P')\langle\mathcal{G}(P') [J_\mu + K_\mu^\gamma] \mathcal{G}(P)\rangle\Gamma_1(P). \quad (24)$$

Once the effective operator $\langle\mathcal{G}(P') [J_\mu + K_\mu^\gamma] \mathcal{G}(P)\rangle$ is calculated the expression (24) involves only three-dimensional integrals.

Since \mathcal{G} is an infinite series in $K - K_1$ this result would not be much help on its own. But, given a result for Γ_1 obtained by systematic expansion of K_1 , the amplitude \mathcal{A}_μ can be analogously expanded in a way that maintains current conservation. K_1 as defined by Eq. (8) is an infinite series and the condition (10) is imposed order-by-order in the expansion in $K - K_1$ defines K_1 to some finite order. The question is: Does a corresponding 3D approximation for the current matrix element (24) exist that maintains the Ward-Takahashi identities of the theory? *It turns out that the current matrix element (24) is conserved if $\mathcal{G}(J_\mu + K_\mu^\gamma)\mathcal{G}$ on the right-hand side of Eq. (24) is expanded to a given order in the coupling constant and the kernel K_1 used to define Γ_1 is obtained from Eq. (10) by truncation at the same order in the coupling constant.*

This is done by splitting the right-hand side of Eq. (24) into two pieces, one due to the one-body current J_μ , and one due to the two-body current K_μ^γ . If K_1 has been truncated at lowest order—i.e., $K_1 = K_1^{(2)}$ —then, in the J_μ piece, we expand the \mathcal{G} s and retain terms up to the same order in $K^{(2)} - K_1^{(2)}$. A piece from the two-body current, in which we write $\mathcal{G} = G_0$, is added to this. That is, we define our second-order approximation to \mathcal{A}_μ , $\mathcal{A}_\mu^{(2)}$, by

$$\begin{aligned} \mathcal{A}_\mu^{(2)} = & \bar{\Gamma}_1(P')\langle G_{0\mu}^\gamma \rangle\Gamma_1(P) \\ & + \bar{\Gamma}_1(P')\langle G_0(P')(K^{(2)}(P') - K_1^{(2)}(P'))G_{0\mu}^\gamma \rangle\Gamma_1(P) \\ & + \bar{\Gamma}_1(P')\langle G_{0\mu}^\gamma (K^{(2)}(P) - K_1^{(2)}(P))G_0(P) \rangle\Gamma_1(P) \\ & + \bar{\Gamma}_1(P')\langle G_0(P')K_\mu^\gamma G_0(P) \rangle\Gamma_1(P), \end{aligned} \quad (25)$$

where $G_{0\mu}^\gamma = G_0(P')J_\mu G_0(P)$. It can now be shown that if Eq. (10) expanded to second order defines $K_1^{(2)}$, the corresponding amplitude for electromagnetic interactions of the bound state, as defined by Eq. (25), exactly obeys

$$Q^\mu A_\mu^{(2)} = 0. \quad (26)$$

It is straightforward to check that the same result holds if Eq. (10) for K_1 is truncated at fourth order, while the one-body and two-body current pieces are expanded to fourth order.

The amplitude $A_\mu^{(2)}$ includes contributions from diagrams where the photon couples to particles one and two while exchanged quanta are “in-flight”. These contributions are of two kinds. Firstly, if the four-dimensional kernel K is dependent on the total momentum, or if it involves the exchange of charged particles, then the WTIs in the 4D theory require that K_μ^γ contain terms involving the coupling of the photon to internal lines in K . Secondly, even if such terms are not present, terms arise in the three-dimensional formalism where the photon couples to particles one and two while an exchanged meson is “in-flight”. These must be included if our 3D approach is to lead to a conserved current. (See Fig. 2 for one such mechanism.)

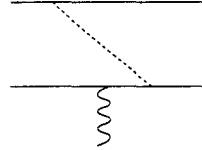


Fig. 2. One example of a two-body current that is required in our formalism in order to maintain current conservation.

A special case of the above results occurs when retardation effects are omitted, i.e., the kernel $K_1 = K_{\text{inst}}$, is chosen, and the bound-state equation (13) is solved to get the vertex function $\Gamma_1 = \Gamma_{\text{inst}}$. Then a simple conserved current is found:

$$A_{\text{inst},\mu} = \bar{\Gamma}_{\text{inst}}(P') \langle G_{0\mu}^\gamma \rangle \Gamma_{\text{inst}}(P) + \bar{\Gamma}_{\text{inst}}(P') \langle G_0(P') \rangle K_{\text{inst},\mu}^\gamma \langle G_0(P) \rangle \Gamma_{\text{inst}}(P), \quad (27)$$

where we have also replaced the meson-exchange current kernel K_μ^γ by the instant approximation to it.

5.2 Current conservation in the 4D formalism with G_C

In Ref. [7] we showed how to construct a conserved current consistent with the 4D equation

$$\Gamma = U(G_0 + G_C)\Gamma. \quad (28)$$

This turns out to be a moderately complicated exercise, because the propagator G_C depends on the three-momenta of particles one and two, not only in the usual way, but also through the choice (19) made for κ_2^0 above. However, a 4D current $\mathcal{G}_{0,\mu}^\gamma = G_{0,\mu}^\gamma + G_{C,\mu}^\gamma$ corresponding to the free Green's function $G_0 + G_C$ can be constructed. Its form is displayed in Ref. [7] and is not really germane to our purposes here, for, as we shall see hereafter, only certain pieces of the current $\mathcal{G}_{0,\mu}^\gamma$ are actually used in our calculations.

5.3 Reduction to 3D and the ET current

Having constructed a 4D current for the formalism involving G_C that obeys the required Ward-Takahashi identity, we can apply the reduction formalism of Section 5.1 to obtain the currents corresponding to the 3D reduction of this 4D theory. The result is:

$$\begin{aligned} \mathcal{A}_\mu^{(2)} = & \bar{\Gamma}_{1,ET}(P') \langle \mathcal{G}_{0,\mu}^\gamma \rangle \Gamma_{1,ET}(P) \\ & + \bar{\Gamma}_{1,ET}(P') \langle (G_0 + G_C)(P') (K^{(2)}(P') - U_1^{(2)}(P')) \mathcal{G}_{0,\mu}^\gamma \rangle \Gamma_{1,ET}(P) \\ & + \bar{\Gamma}_{1,ET}(P') \langle \mathcal{G}_{0,\mu}^\gamma (K^{(2)}(P) - U_1^{(2)}(P)) (G_0 + G_C)(P) \rangle \Gamma_{1,ET}(P) \\ & + \bar{\Gamma}_{1,ET}(P') \langle (G_0 + G_C)(P') K_\mu^{(2)}(G_0 + G_C)(P) \rangle \Gamma_{1,ET}(P), \end{aligned} \quad (29)$$

where $\Gamma_{1,ET}$ is the solution of Eq. (22) with $U_1 = U_1^{(2)}$. This current obeys the appropriate Ward-Takahashi identity. In fact in one-boson exchange models the only contributions to $K_\mu^{(2)}$ give rise to isovector structures, and so their contribution to electromagnetic scattering off the deuteron is zero.

5.4 Impulse-approximation current based on the instant approximation to ET formalism

Just as in the case of the Bethe-Salpeter equation, if the instant approximation is used to obtain a bound-state equation with an instant interaction from Eq. (28) then a corresponding simple conserved impulse current can be constructed:

$$\mathcal{A}_{\text{inst},\mu} = \bar{\Gamma}_{\text{inst}} \langle \mathcal{G}_{0,\mu}^\gamma \rangle \Gamma_{\text{inst}}. \quad (30)$$

Now we note that the full result for $\mathcal{G}_{0,\mu}^\gamma$ was constructed in order to obey Ward-Takahashi identities in the full four-dimensional theory. It is not necessary to use this result if we are only concerned with maintaining WTIs at the three-dimensional level in the instant approximation. Therefore we may construct the corresponding current

$$\begin{aligned} \mathcal{G}_{\text{inst},\mu}^\gamma(\mathbf{p}_1, \mathbf{p}_2; P, Q) = & i \langle d_1(p_1) d_2(p_2 + Q) j_\mu^{(2)} d_2(p_2) \\ & + d_1(p_1) d_2^c(p_2 + Q) j_{c,\mu}^{(2)} d_2^c(p_2) \rangle + (1 \leftrightarrow 2). \end{aligned} \quad (31)$$

Here d_i is the Dirac propagator for particle i , and $j_\mu = q\gamma_\mu$ is the usual one-body current, with q is the charge of the particle in question. Meanwhile d_i^c is

a one-body Dirac propagator used in $G_C(P)$ to construct the approximation to the crossed-ladder graphs. Correspondingly, d_i^c appears in $G_C(P + Q)$, which does *not* equal d_i^c , even if particle i is not the nucleon struck by the photon. Finally,

$$j_{c,\mu}^{(2)} = q_2 \gamma_\mu - \tilde{j}_\mu^{(2)}, \quad (32)$$

where

$$\tilde{j}_\mu^{(2)} = q_2 \frac{\hat{p}'_{2\mu} + \hat{p}_{2\mu}}{\epsilon'_2 + \epsilon_2} \gamma_{20}, \quad (33)$$

with $\hat{p}_2 = (\epsilon(\mathbf{p}_2), \mathbf{p}_2)$. (For further explanation of these quantities and the necessity of their appearance here the reader is referred to Ref. [7].)

If a vertex function Γ_{inst} is constructed to be a solution to Eq. (22) with an instant interaction then the three-dimensional hadronic current:

$$\mathcal{A}_{\text{inst},\mu} = \bar{\Gamma}_{\text{inst}} \mathcal{G}_{\text{inst},\mu}^\gamma \Gamma_{\text{inst}} \quad (34)$$

is conserved. This current is simpler than the full ET current and omits only effects stemming from retardation in the current. Our present calculations are designed to provide an assessment of the role of negative-energy states and retardation effects in the vertex functions. Therefore we use the simple current (34) in *all* of our calculations here—even the ones where Γ_1 is calculated using a retarded two-body interaction. The effects stemming from retardation in the current are expected to be minor, and so we expect this to be a good approximation to the full current in the three-dimensional theory. Future calculations should be performed to check the role of meson retardation in that current.

6 Results for electron-deuteron scattering

6.1 Impulse approximation

We are now ready to calculate the experimentally observed deuteron electromagnetic form factors A and B , and the tensor polarization T_{20} . These are straightforwardly related to the charge, quadrupole, and magnetic form factors of the deuteron, F_C , F_Q , and F_M . These form factors in turn are related to the Breit frame matrix elements of the current \mathcal{A}_μ discussed in the previous section,

$$F_C = \frac{1}{3\sqrt{1+\eta}e} (\langle 0|\mathcal{A}^0|0\rangle + 2\langle +1|\mathcal{A}^0|+1\rangle), \quad (35)$$

$$F_Q = \frac{1}{2\eta\sqrt{1+\eta}e} (\langle 0|\mathcal{A}^0|0\rangle - \langle +1|\mathcal{A}^0|+1\rangle), \quad (36)$$

$$F_M = \frac{-1}{\sqrt{2\eta(1+\eta)}e} \langle +1|\mathcal{A}_+|0\rangle, \quad (37)$$

where $|+1\rangle$, $|0\rangle$ and $| -1\rangle$ are the three different spin states of the deuteron.

We take the wave functions constructed for the four different interactions of Section 4 and insert them into the expression (34). In using any of the interactions obtained with only positive-energy state propagation we drop all pieces of the operator $\mathcal{G}_{\text{inst},\mu}^\gamma$ in negative-energy sectors.

The single-nucleon current used in these calculations is the usual one for extended nucleons. We choose to parametrize the single-nucleon form factors F_1 and F_2 via the 1976 Hohler fits [26]. Choosing different single-nucleon form factors does not affect our qualitative conclusions, although it has some impact on our quantitative results for A , B , and T_{20} .

Using this one-body current we then calculate the current matrix elements via Eq. (24). This is a conserved current if the vertex function Γ_1 is calculated from an instant potential. However, if a potential including meson retardation is used it violates the Ward-Takahashi identities by omission of pieces that are required because of the inclusion of retardation effects in the calculation. Work is in progress to estimate the size of these effects.

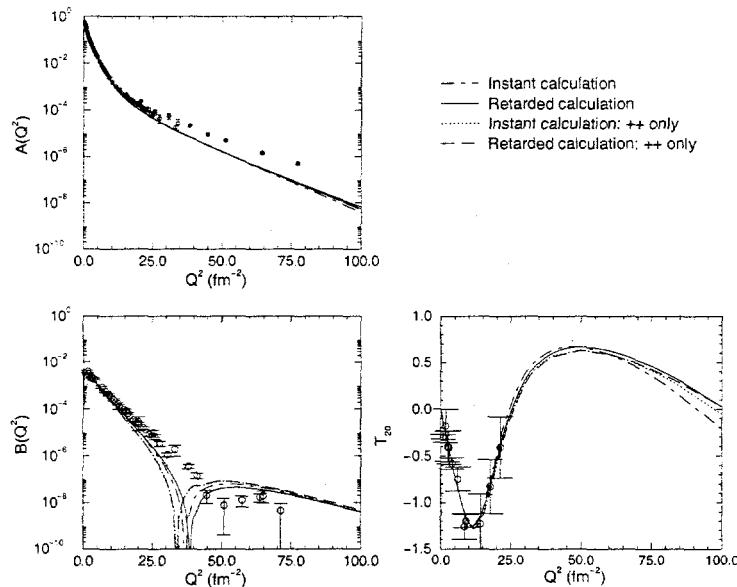


Fig. 3. The form factors $A(Q^2)$ and $B(Q^2)$ and the tensor polarization T_{20} for the deuteron calculated in impulse approximation. The dash-dotted line represents a calculation using a vertex function generated using the instant interaction. Meanwhile the solid line is the result obtained with the retarded vertex function. The dotted and long dashed lines are obtained by performing a calculations with instant and retarded interactions in which no negative-energy states are included.

The results for the impulse approximation calculation of the experimental observables A , B , and T_{20} are displayed in Fig. 3. We also show experimental data from Refs. [27, 28, 29, 30, 31] for A , from Refs. [29, 30, 32, 33] for B and from Ref. [34] for T_{20} . A number of two-body effects must be added to our calculations before they can be reliably compared to experimental data. However, even here we see the close similarity of the results for these observables in all four calculations. The only really noticeable difference occurs at the minimum in B . There, including the negative-energy states in the calculation shifts the minimum to somewhat larger Q^2 . A similar effect was observed by van Orden *et al.* [8] in calculations of electron-deuteron scattering using the spectator formalism. However, note that here, in contradistinction to the results of Ref. [8], the inclusion of negative-energy states does *not* bring the impulse approximation calculation into agreement with the data.

The fact that negative-energy states seem to have a smaller effect on observables in the ET analysis than in the spectator analysis of van Orden *et al.* [8] is somewhat surprising since our “ET” propagator has twice the negative-energy state propagation amplitude of the spectator propagator. Thus, other differences between the ET and spectator models, not just differences in the role of negative-energy states in the two approaches, appear to be responsible for Ref. [8]’s success in reproducing the minimum in B .

For the tensor polarization T_{20} the different models produce results which are very similar. This suggests that this observable is fairly insensitive to dynamical details of the deuteron model, at least up to $Q^2 = 4 \text{ GeV}^2$.

6.2 Meson-exchange currents

As Q^2 increases the cross-section due to the impulse approximation diagrams drops precipitously. Thus we expect that in some regime other interactions may become competitive with the impulse mechanism. One such possibility is that the photon will couple to a meson while that meson is in flight. Because of the deuteron’s isoscalar nature and the conservation of G-parity, the lowest mass state which can contribute in such meson-exchange current (MEC) diagrams is one where the photon induces a transition from a π to a ρ .

This $\rho\pi\gamma$ MEC is a conserved current whose structure can be found in Refs. [10, 35]. The couplings and form factors for the meson-nucleon-nucleon vertices are all taken to be consistent with those used in our one-boson-exchange interaction. Meanwhile, the $\rho\pi\gamma$ coupling is set to the value $g_{\rho\pi\gamma} = 0.56$, and a vector meson dominance form factor is employed at the $\rho\pi\gamma$ vertex: $F_{\rho\pi\gamma}(q) = 1/(q^2 - m_\omega^2)$. The value of this MEC is added to the impulse contribution calculated above and A , B , and T_{20} are calculated. This is done with the vertex function obtained from an instant interaction, and consequently the electromagnetic current is exactly conserved. The results of this calculation are displayed in Fig. 4. We see that at Q^2 of order 2 GeV^2 the $\rho\pi\gamma$ MEC makes a significant contribution to all three observables. However, far from improving the agreement of the position of the minimum in the B form

factor with the experimental data, this particular MEC moves the theoretical result *away* from the data—as noted by Hummel and Tjon [10], and seen within a simplified version of the formalism presented here by Devine [35]. Thus, it would seem that some physics beyond the impulse approximation other than the $\rho\pi\gamma$ MEC plays a significant role in determining the position of the minimum in $B(Q^2)$.

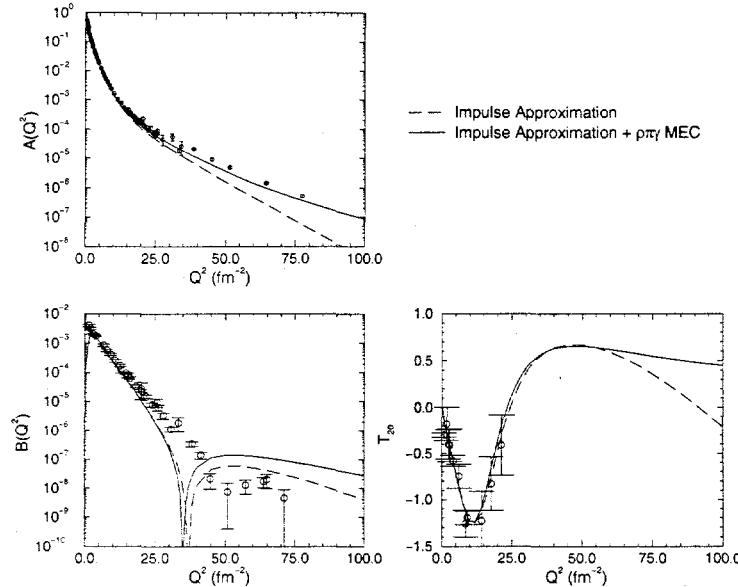


Fig. 4. The form factors $A(Q^2)$ and $B(Q^2)$ together with the tensor polarization for the deuteron. The long dashed line is an impulse approximation calculation with an instant interaction. The solid line includes the effect of the $\rho\pi\gamma$ MEC.

7 Conclusion

A systematic theory of the electromagnetic interactions of relativistic bound states is available in three dimensions. In this formalism integrations are performed over the zeroth component of the relative momentum of the two particles, leading to the construction of “equal-time” (ET) Green’s functions. If the formalism is to incorporate the Z-graphs that are expected in a quantum field theory, then the propagator must include terms coming from crossed Feynman graphs. Here we have displayed a three-dimensional propagator that includes these effects correctly to leading order in $1/M$.

Given a suitable choice for the ET propagator, the electromagnetic and interaction currents which should be used with it can be calculated. If these are truncated in a fashion consistent with the truncation of the NN interaction in the hadronic field theory then the Ward-Takahashi identities are maintained in the three-dimensional theory. A full accounting of the dynamical role played by negative-energy states and of retardations in electromagnetic interactions of the deuteron is thereby obtained.

Calculations have been performed for both the impulse approximation and when the $\rho\pi\gamma$ MEC is included. In our MEC calculations we use an instant approximation for the electromagnetic current. This current satisfies current conservation when used with deuteron vertex functions that are calculated with instant interactions. We also have used this simpler current with vertex functions which are calculated with the retarded interactions obtained within the ET formalism.

Comparing impulse approximation calculations with and without negative-energy states indicates that the role played by negative-energy state components of the deuteron vertex function is small. This corroborates the results of Hummel and Tjon and is in contrast to those obtained in Ref. [8]. Because the ET formalism incorporates the relevant Z-graphs in a preferable way, we are confident that these Z-graphs really do play only a minor role in calculations that are based upon standard boson-exchange models of the NN interaction.

The results for impulse approximation calculations of the electromagnetic observables are relatively insensitive to the distinction between a vertex calculated with retardations included and one calculated in the instantaneous approximation. The results of both calculations fall systematically below experimental data for the form factors A and B for Q of order 1 GeV. This deficiency at higher Q suggests that mechanisms other than the impulse approximation graph should be significant. Indeed, when the $\rho\pi\gamma$ MEC graph is included in our calculation it somewhat remedies the result for $A(Q^2)$. However, it fails to narrow the gap between our result for $B(Q^2)$ and the existing experimental data. The significant gap that remains between our theoretical result for $B(Q^2)$ and the data indicates that it is an interesting observable in which to look for physics of the deuteron other than the simple impulse mechanism or the standard $\rho\pi\gamma$ MEC. Finally, the existing tensor polarization data are reasonably well described. This is consistent with previous analyses which have shown T_{20} to be less sensitive to non-impulse mechanisms.

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