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# $E \times B$ Shearing Rate in Quasi-symmetric Plasmas

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## Abstract

The suppression of turbulence by the  $E \times B$  shear is studied in systems with quasi-symmetry using the nonlinear analysis of eddy decorrelation previously utilized in finite aspect ratio tokamak plasmas [Phys. Plasmas **2**, 1648 (1995)]. The analytically derived  $E \times B$  shearing rate which contains the relevant geometric dependence can be used for quantitative assessment of the fluctuation suppression in stellarators with quasi-symmetry.

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## I. Introduction

Understanding and reducing anomalous transport is one of the major goals of magnetic confinement physics. Since the high(H)-mode has first been discovered in the Axisymmetric Divertor Experiment (ASDEX),<sup>1</sup> there has been considerable experimental and theoretical progress in physics of enhanced confinement regimes. There is accumulating evidence that transport reduction in various forms of the enhanced confinement regimes is due to fluctuation suppression caused by the shear in the radial electric field.<sup>2-9</sup>

Most significant results are the enhanced reversed shear (ERS) plasmas<sup>7</sup> in the Tokamak Fusion Test Reactor (TFTR)<sup>10</sup> and the negative central shear (NCS) plasmas<sup>8</sup> in DIII-D.<sup>11</sup> Their transport barriers in the plasma core are characterized by a very sharp radial gradient of  $E_r$ . In both machines, plasma density fluctuations after the transition to either ERS or NCS phase are suppressed to a level well below that of typical Super Shot or L-mode.<sup>4,6</sup>

Using the  $\mathbf{E} \times \mathbf{B}$  shearing rate in general toroidal geometry<sup>12</sup> is essential in quantitative assessment of the  $\mathbf{E} \times \mathbf{B}$  shear suppression of turbulence at core including very high (VH)-mode,<sup>3</sup> ERS,<sup>4,5</sup> NCS, and weak negative shear (WNS) plasmas,<sup>6,13</sup> since the radial variation of  $B_\theta$  is often as important as that of  $E_r$  in determining the  $\mathbf{E} \times \mathbf{B}$  shearing parameter  $\partial(E_r/RB_\theta)/\partial\psi_p$ .<sup>14,15</sup> Here  $R$ ,  $B_\theta$  and  $\psi_p$  are the major radius, poloidal magnetic field, and the poloidal flux respectively. The importance of  $E_r$  in various plasma confinement devices has been speculated<sup>16</sup> for sometime, and the previous theory in cylindrical geometry<sup>17,18</sup> has been useful in the progress of H-mode physics.

The H-mode has been also obtained in stellarators<sup>19-22</sup> and the  $\mathbf{E} \times \mathbf{B}$  shear suppression of turbulence has been considered as the leading candidate for the L-H transition in stellarator too.<sup>19,20</sup> The central role of the  $\mathbf{E} \times \mathbf{B}$  shear assigned in the nonlinear theories<sup>12,17,18,23</sup> are further supported by the Heliotron-E high ion temperature mode results which indicate the spatial correlation of the transport reduction at core and the  $\mathbf{E} \times \mathbf{B}$  shear, not the  $U_\theta$  (poloidal flow) shear.<sup>24</sup> However, more quantitative assessment of the  $\mathbf{E} \times \mathbf{B}$  shear effect on fluctuation and transport is desirable. In particular, potentially important geometric effect on the  $\mathbf{E} \times \mathbf{B}$  shearing rate<sup>12</sup> which has been exhibited through tokamak experiments<sup>9</sup> has not been utilized for stellarator confinement research.

In this work, we derive the  $\mathbf{E} \times \mathbf{B}$  shearing rate in arbitrary shape quasi-symmetric finite aspect ratio plasma in which the magnetic field strength approximately depends on only one angular coordinate, instead of two, within the constant magnetic surfaces. Specifically,  $B \equiv |\mathbf{B}| \simeq B(\psi, \alpha)$ , with  $\alpha \equiv \theta - N\phi$  a helical coordinate. Here,  $N = 0$  corresponds to quasi-toroidal symmetry<sup>25</sup> and a positive integer  $N$  corresponds to quasi-helical symmetry.<sup>26</sup> We use the flux coordinates  $(\psi, \theta, \phi)$  in which the magnetic field lines are straight.<sup>27</sup> Here,  $\psi$  is the toroidal magnetic flux,  $\theta$  and  $\phi$  are the generalized poloidal angle and the generalized toroidal angle respectively. While deriving the corresponding  $\mathbf{E} \times \mathbf{B}$  shearing rate in general three dimensional system without apparent quasi-symmetry is beyond the scope of this paper, this work provides a useful quantitative guidance.

Principal results of this paper include the following. Our results indicate that fluctuation suppression occurs when the  $\mathbf{E} \times \mathbf{B}$  shearing rate  $\omega_E$  given below, exceeds the decorrelation rate of the ambient turbulence  $\Delta\omega_T$ :

$$\omega_E = \frac{k_\perp}{k_\psi} \frac{|\nabla\psi||\mathbf{B} \times \nabla\psi|}{B^2} \left| (\iota - N) \frac{\partial}{\partial\psi} \left\{ \frac{1}{\iota - N} \frac{\partial}{\partial\psi} \Phi_0(\psi) \right\} \right|.$$

Therefore, not only the radial variation of  $E_r$  but also that of  $\iota - N$  determine the  $\mathbf{E} \times \mathbf{B}$  shearing rate, especially for high magnetic shear operation of a stellarator with quasi-toroidal symmetry ( $N = 0$ ) or with small  $N$ . Furthermore, with an assumption of weak variation of  $\frac{k_\perp}{k_\psi}$  within the constant flux surface, the  $\mathbf{E} \times \mathbf{B}$  shearing rate has a strong helical-angle-dependence through a geometric factor  $\frac{|\nabla\psi||\mathbf{B} \times \nabla\psi|}{B^2}$ . This fact may offer a new insight into search for a stellarator configuration which is favorable for fluctuation suppression. Finally, we show that the previous result for axisymmetric tokamak<sup>12</sup> can be understood in the context of the extended work in this paper via the isomorphism which relates the neoclassical transport properties common to a variety of quasi-symmetric systems.<sup>28</sup> This work, therefore, suggests a strong possibility of utilizing the knowledge accumulated through tokamak research in turbulence suppression for stellarator design.

The remainder of this paper is organized as follows. In Sec. II, the two-point correlation function evolution equation in quasi-symmetric system is derived and analyzed. In Sec. III, the general criterion for flow shear suppression of fluctuation is presented in a form useful

for comparison to experimental data.

## II. $\mathbf{E} \times \mathbf{B}$ Shear Induced Decorrelation of Turbulence

In the flux coordinate system  $\{\psi, \theta, \phi\}$  in which the field lines are straight, magnetic field is given by<sup>27</sup>

$$\mathbf{B} = \nabla\psi \times \nabla\theta + \iota(\psi)\nabla\phi \times \nabla\psi = \nabla\psi \times \nabla\alpha + (\iota(\psi) - N)\nabla\phi \times \nabla\psi. \quad (1)$$

where  $\iota(\psi)$  is the rotational transform. Following the previous work,<sup>12,17,18,23</sup> we start from a one-field fluid model in which the fluctuating field  $\delta H$  is convected by the equilibrium  $\mathbf{E} \times \mathbf{B}$  flow  $\mathbf{u}_E$ , and the fluctuating  $\mathbf{E} \times \mathbf{B}$  flow  $\tilde{\mathbf{u}}_E$ ,

$$(\partial/\partial t + \mathbf{u}_E \cdot \nabla + \tilde{\mathbf{u}}_E \cdot \nabla)\delta H = S, \quad (2)$$

where  $\mathbf{u}_E = \mathbf{B} \times \nabla\Phi_0/B^2$ ,  $\tilde{\mathbf{u}}_E = \mathbf{B} \times \nabla\delta\Phi/B^2$ , and  $S$  is the driving source of the turbulence. Linear dissipation and subdominant nonlinearities other than  $\mathbf{E} \times \mathbf{B}$  nonlinearity are ignored for simplicity.

For a rigorous derivation of the  $\mathbf{E} \times \mathbf{B}$  shearing rate in realistic geometry, it is crucial to use a representation in which the spatial variation of  $\mathbf{k}$  as well as that of  $\mathbf{u}_E$  is clearly captured.<sup>12,23</sup> This is accomplished by using a vector identity,

$$B^2 \nabla B \times \nabla\psi = \mathbf{B}\mathbf{B} \cdot (\nabla B \times \nabla\psi) + (\mathbf{B} \cdot \nabla)B \mathbf{B} \times \nabla\psi$$

which projects the direction of constant  $B$  (along  $\nabla B \times \nabla\psi$ ) into the parallel direction and the perpendicular direction with respect to the magnetic field. Since the  $k_{\parallel}$  correction to the shearing rate<sup>12</sup> is of the order  $(k_{\parallel}/k_{\perp})^2$ , and therefore negligible for most cases, we can write the second term of Eq. (2) in the following form.

$$\mathbf{u}_E \cdot \nabla \delta H = \frac{\partial\Phi_0(\psi)}{\partial\psi} \frac{B}{B^2} \times \nabla\psi \cdot \nabla \delta H \simeq \frac{\partial\Phi_0(\psi)}{\partial\psi} \frac{\nabla B \times \nabla\psi}{(\mathbf{B} \cdot \nabla)B} \cdot \nabla \delta H. \quad (3)$$

For  $B = B(\psi, \alpha)$ , we have

$$\frac{\nabla B \times \nabla\psi \cdot \nabla\beta}{(\mathbf{B} \cdot \nabla)B} = \frac{\nabla\alpha \times \nabla\psi \cdot \nabla\beta}{(\mathbf{B} \cdot \nabla)\alpha} = \frac{1}{N - \iota}$$

, where  $\alpha$  and  $\beta$  are the helical angle and the angle in the direction of constant  $B$  (quasi-symmetry) respectively. Therefore,

$$\mathbf{u}_E \cdot \nabla \delta H = \left( \frac{\partial \Phi_0(\psi)}{\partial \psi} \frac{\nabla B \times \nabla \psi}{(\mathbf{B} \cdot \nabla) B} \right) \cdot \nabla \delta H = \left( \frac{1}{N - \iota} \frac{\partial \Phi_0(\psi)}{\partial \psi} \right) \frac{\partial}{\partial \beta} \delta H. \quad (4)$$

In this work,  $\Phi_0$  is assumed to be a flux function for simplicity, although this constraint can be relaxed.<sup>30</sup> The expression for the  $\mathbf{E} \times \mathbf{B}$  nonlinear term in flux coordinate is given by Frieman and Chen.<sup>31</sup> The two-point correlation evolution equation is then derived following the standard procedure<sup>32</sup> of symmetrization with respect to  $(\psi_1, \alpha_1, \beta_1)$  and  $(\psi_2, \alpha_2, \beta_2)$  followed by ensemble average,

$$\left( \frac{\partial}{\partial t} + \psi_- \Omega_E \frac{\partial}{\partial \beta_-} - D_-^{\text{eff}} \frac{\partial^2}{\partial \beta_-^2} \right) \langle \delta H(1) \delta H(2) \rangle = S_2. \quad (5)$$

Here, the radial shear of the angular rotation frequency associated with  $\mathbf{E} \times \mathbf{B}$  flow is given by

$$\Omega_E \equiv \frac{\partial}{\partial \psi} \left( \frac{1}{N - \iota} \frac{\partial \Phi_0(\psi)}{\partial \psi} \right). \quad (6)$$

In Eq. (5),  $S_2$  is the source term for the two-point correlation function and the  $\mathbf{E} \times \mathbf{B}$  nonlinearity is approximated as a turbulent diffusion along the direction of constant  $B$  following nonlinear theories in axisymmetric tokamak.<sup>33,23,12</sup> At small separation, the relative diffusion  $D_-^{\text{eff}}$  has the following asymptotic form,

$$D_-^{\text{eff}} = 2D^{\text{eff}} \left\{ \left( \frac{\psi_-}{\Delta \psi_0} \right)^2 + \left( \frac{\beta_-}{\Delta \beta} \right)^2 + \left( \frac{\alpha_-}{\Delta \alpha} \right)^2 \right\}, \quad (7)$$

where  $D^{\text{eff}} = \Delta \omega_T \Delta \beta^2 / 4$  is proportional to the diffusion coefficient at large separation. The decorrelation dynamics due to the coupling of the flow shear and turbulent diffusion can be studied by taking various moments of the left hand side (lhs) of Eq. (5).

$$\partial_t \langle \psi_-^2 \rangle = 0, \quad (8)$$

$$\partial_t \langle \alpha_-^2 \rangle = 0, \quad (9)$$

$$\partial_t \langle \beta_-^2 \rangle = 4D_{\text{eff}} \left\{ \frac{\langle \alpha_-^2 \rangle}{\Delta \alpha^2} + \frac{\langle \beta_-^2 \rangle}{\Delta \beta^2} + \frac{\langle \psi_-^2 \rangle}{\Delta \psi_0^2} \right\} + 2\Omega_E \langle \psi_- \beta_- \rangle, \quad (10)$$

and

$$\partial_t \langle \psi_- \beta_- \rangle = \Omega_E \langle \psi_-^2 \rangle. \quad (11)$$

Here,

$$\langle A(\alpha_-, \beta_-, \psi_-) \rangle \equiv \int d\alpha'_- d\beta'_- d\psi'_- G(\alpha_-, \beta_-, \psi_- | \alpha'_-, \beta'_-, \psi'_-) A(\alpha'_-, \beta'_-, \psi'_-),$$

and  $G$  is the two point Green's function for the lhs of Eq. (5). Integration of Eqs. (8) through (11) yields a solution which has the following asymptotic form for  $\Delta\omega_T t > 1$

$$\frac{\langle \beta_-^2 \rangle(t)}{\Delta\beta^2} = \left[ \frac{\psi_-^2}{\Delta\psi_0^2} \left\{ 1 + \left( \frac{\Omega_E}{\Delta\omega_T} \frac{\Delta\psi_0}{\Delta\beta} \right)^2 \right\} + \frac{1}{\Delta\beta^2} \left\{ \beta_- + \frac{\Omega_E}{\Delta\omega_T} \psi_- \right\}^2 + \frac{\alpha_-^2}{\Delta\alpha^2} \right] e^{\Delta\omega_T t}. \quad (12)$$

Equation (12) yields the eddy lifetime and is a function of the initial separation between two nearby points,

$$\tau_{\text{eddy}} \simeq \Delta\omega_T^{-1} \ln([\dots]^{-1}), \quad (13)$$

where  $[\dots]$  is the expression multiplying  $e^{\Delta\omega_T t}$  on the right hand side (rhs) of Eq. (12). We recall that Eq.(7) implies  $[\dots] < 1$ . The radial correlation length in flux unit  $\Delta\psi$ , is reduced by the flow shear relative to its value  $\Delta\psi_0$ , determined by the ambient turbulence alone:

$$\left( \frac{\Delta\psi_0}{\Delta\psi} \right)^2 = 1 + \left( \frac{\Omega_E}{\Delta\omega_T} \frac{\Delta\psi_0}{\Delta\beta} \right)^2. \quad (14)$$

The reduction of radial correlation length due to  $\mathbf{E} \times \mathbf{B}$  shear has been recently confirmed by the measurements at DIII-D edge.<sup>34</sup> Therefore, we expect that fluctuation suppression occurs when the decorrelation rate of the ambient turbulence  $\Delta\omega_T$  is exceeded by the  $\mathbf{E} \times \mathbf{B}$  shearing rate,  $\omega_E$ :

$$\omega_E \equiv \left| \Omega_E \frac{\Delta\psi_0}{\Delta\beta} \right|. \quad (15)$$

### III. Geometric Dependence of the $\mathbf{E} \times \mathbf{B}$ Shearing Rate

The experimental results from TFTR and DIII-D show that,<sup>14,9</sup> not only the radial variation of  $B_\theta$ , but also Shafranov shift and appropriate shaping can enhance the  $\mathbf{E} \times \mathbf{B}$  shearing rate in general toroidal geometry.<sup>12</sup> In this section, we discuss the explicit geometric dependence of the  $\mathbf{E} \times \mathbf{B}$  shearing rate derived in the previous section,

$$\omega_E = \left| \frac{\Delta\psi_0}{\Delta\beta} \frac{\partial}{\partial\psi} \left\{ \frac{1}{\iota - N} \frac{\partial}{\partial\psi} \Phi_0(\psi) \right\} \right|. \quad (16)$$

Since the spatial characteristics of turbulence are often discussed in terms of  $\mathbf{k}$ -spectra, we express  $\Delta\psi_0$  and  $\Delta\beta$  in terms of  $k_\psi$  and  $k_\perp$ ; the components of the  $\mathbf{k}$  vector of fluctuation in the radial ( $\mathbf{e}_\psi$ ) and nonradial perpendicular ( $\mathbf{b} \times \mathbf{e}_\psi$ ) directions. From Eqs. (3)-(4), we have

$$k_\perp = \frac{B^2}{|(\iota - N)\mathbf{B} \times \nabla\psi|} \frac{1}{\Delta\beta}. \quad (17)$$

With Eq. (17) and  $k_\psi \equiv |\nabla\psi|/\Delta\psi$ , we can write Eq. (16) in a form similar to the one widely used for tokamak applications,<sup>35,9</sup>

$$\omega_E = \frac{k_\perp}{k_\psi} \frac{|\nabla\psi| |\mathbf{B} \times \nabla\psi|}{B^2} \left| (\iota - N) \frac{\partial}{\partial\psi} \left\{ \frac{1}{\iota - N} \frac{\partial}{\partial\psi} \Phi_0(\psi) \right\} \right| \quad (18)$$

Here,  $(\iota - N) \frac{\partial}{\partial\psi} \left\{ \frac{1}{\iota - N} \frac{\partial}{\partial\psi} \Phi_0(\psi) \right\}$  is a function of the toroidal flux ( $\psi$ ) only,  $\frac{|\nabla\psi| |\mathbf{B} \times \nabla\psi|}{B^2}$  is the helical-angle ( $\alpha$ ) dependent form factor, and the first factor describes dependence on the eddy shape. While  $\mathbf{k}$  spectrum measurements on stellarators are scarce, density fluctuation measurements on TFTR tokamak<sup>36</sup> indicate that  $\frac{k_\perp}{k_\psi} \simeq 1$ . Then,  $\omega_E$  in Eq. (18) can be expressed in terms of the equilibrium quantities only.

This formula suggests a variety of methods by which  $\omega_E$  can be enhanced. The most obvious one is the  $E_r$  profile control by producing and sustaining plasma rotation. One has much greater chance in quasi-symmetric systems such as the Modular Helias-like Heliac 2 (MHH2)<sup>25</sup> and Helically Symmetric eXperiment (HSX)<sup>38,39</sup> since the rotation is undamped only in the direction of constant  $B^{37}$  (quasi-symmetry). Much experience from tokamaks<sup>9,14</sup> can be most easily utilized in devices with quasi-toroidal symmetry such as MHH2.<sup>25</sup> The second way is through the favorable  $B_\theta$  profiles as evidenced by the reversed magnetic shear experiments on tokamaks.<sup>14,9,40-43</sup> While many stellarators operate with low magnetic shear,



it will be useful to examine whether the core confinement improvement observed in Wendelstein VII-AS (WII-AS) with high magnetic shear operation<sup>44</sup> could be related to the  $B_\theta$ -profile dependence of the  $\mathbf{E} \times \mathbf{B}$  shearing rate. Finally, the plasma shaping can produce a strong helical angle dependence of the  $\mathbf{E} \times \mathbf{B}$  shearing rate. It will be helpful to know whether one can find a configuration in which the  $\mathbf{E} \times \mathbf{B}$  shearing rate can be made larger in the region where the micro-turbulence is expected to be strong. For this, the calculation of local magnetic shear<sup>45,46</sup> can be easily accompanied by the calculation of  $\frac{|\nabla\psi||\mathbf{B} \times \nabla\psi|}{B^2}$ . Note that there is some evidence of larger  $\mathbf{E} \times \mathbf{B}$  shearing rate at the bad curvature side of tokamaks.<sup>9,30</sup>

We can also write the  $\mathbf{E} \times \mathbf{B}$  shearing rate in terms of the poloidal flux  $d\psi_p \equiv \iota d\psi$  and the magnetic safety factor  $q \equiv 1/\iota$ ,

$$\omega_E = \left| \frac{\Delta\psi_{p0}}{\Delta\beta} \frac{\partial}{\partial\psi_p} \left\{ \frac{1}{1 - Nq} \frac{\partial}{\partial\psi_p} \Phi_0 \right\} \right| = \frac{k_\perp}{k_\psi} \frac{|\nabla\psi_p||\mathbf{B} \times \nabla\psi_p|}{B^2} \left| (1 - Nq) \frac{\partial}{\partial\psi_p} \left\{ \frac{1}{1 - Nq} \frac{\partial}{\partial\psi_p} \Phi_0 \right\} \right|. \quad (19)$$

For a quasi-toroidally symmetric stellarator such as MHH2<sup>25</sup> with  $N = 0$  and  $\Delta\beta = \Delta\phi$ .

$$\omega_E = \left| \frac{\Delta\psi_{p0}}{\Delta\phi} \frac{\partial^2}{\partial\psi_p^2} \Phi_0 \right| = \frac{k_\perp}{k_\psi} \frac{|\nabla\psi_p||\mathbf{B} \times \nabla\psi_p|}{B^2} \left| \frac{\partial^2}{\partial\psi_p^2} \Phi_0 \right|. \quad (20)$$

Here, it is important to note that  $\phi$  is the generalized toroidal angle which is different from the cylindrical angle  $\phi_c$  in general.<sup>28</sup> Therefore,  $\phi = \text{constant}$  surfaces are not necessarily planar. For an axisymmetric tokamak, we recover the result of Ref. 12 with  $\phi \equiv \phi_c$ .

$$\omega_E = \left| \frac{\Delta\psi_{p0}}{\Delta\phi_c} \frac{\partial^2}{\partial\psi_p^2} \Phi_0 \right| = \frac{k_\perp}{k_\psi} \frac{(RB_\theta)^2}{B} \left| \frac{\partial}{\partial\psi_p} \left( \frac{E_r}{RB_\theta} \right) \right|. \quad (21)$$

Another limiting case is a poloidally symmetric system such as mirrors. By taking  $N \gg \iota$  limit of Eq. (18), we obtain,

$$\omega_E = \frac{k_\perp}{k_\psi} \frac{|\nabla\psi||\mathbf{B} \times \nabla\psi|}{B^2} \left| \frac{\partial^2}{\partial\psi^2} \Phi_0(\psi) \right| = \frac{k_\theta}{k_\psi} \frac{rB_\phi}{B} \left| \frac{\partial}{\partial r} \left( \frac{E_r}{rB_\phi} \right) \right| \quad (22)$$

This agrees with the previous results in cylindrical geometry.<sup>17,18</sup>

Mathematically, the most fundamental (although not readily useful for application) form of the  $\mathbf{E} \times \mathbf{B}$  shearing rate can be gotten by introducing the "helical flux",

$$\psi_H \equiv \psi_p - N\psi.$$

Then, we have

$$\mathbf{B} = \nabla\psi \times \nabla\alpha + \nabla\phi \times \nabla\psi_H$$

and the  $\mathbf{E} \times \mathbf{B}$  shearing rate can be written as,

$$\omega_E = \left| \frac{\Delta\psi_{H0}}{\Delta\beta} \frac{\partial^2}{\partial\psi_H^2} \Phi_0(\psi) \right| \quad (23)$$

Now it is apparent from Eqs. (18)-(23) that the  $\mathbf{E} \times \mathbf{B}$  shearing rate in various systems with different manifestation of the same class of quasi-symmetries can be written by using an appropriate set of an angle in the direction of symmetry and a flux function which conjugates to the other angle which  $B$  depends on.

Deriving the corresponding  $\mathbf{E} \times \mathbf{B}$  shearing rate in general three dimensional system is beyond the scope of this paper. However, the fact that the relation presented in Eq.(4) can be derived without explicitly assuming the quasi-symmetry<sup>47</sup> gives us some hope of rigorously generalizing the method used in this paper to more complicated configurations. For short term applications to stellarators,  $N$  should be considered as the dominant component in Fourier decomposition of  $B$  in Boozer coordinates.<sup>27</sup>

In conclusion, we have derived the  $\mathbf{E} \times \mathbf{B}$  shearing rate in arbitrary shape quasi-symmetric finite aspect ratio plasmas. This formula can be used for quantitative assessment of the  $\mathbf{E} \times \mathbf{B}$  shear suppression of turbulence in stellarators. With accumulating evidence of various enhanced confinement modes in stellarators,<sup>19-22</sup> this work adds to knowledge for confinement optimization of stellarators.

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## References

- <sup>1</sup>F. Wagner, G. Becker, K. Behringer, D. Campbell, A. Eberhagen, W. Engelhardt, G. Fussmann, O. Gehre, J. Gerhardt, G. v. Vierke, G. Haas, M. Huang, F. Karger, M. Keilhacker, O. Kluber, M. Kornherr, K. Lackner, G. Lisitano, G.G. Lister, H.M. Mayer, D. Meisel, E.R. Muller, H. Murmann, H. Niedermeyer, W. Poschenrieder, H. Rapp, H. Rohr, F. Schneider, G. Siller, E. Speth, A. Stbler, K. H. Steuer, G. Venus, O. Vollmer, and Z. Yu, *Phys. Rev. Lett.* **49**, 1408 (1982).
- <sup>2</sup>K.H. Burrell, E.J. Doyle, P. Gohil, R.J. Groebner, J. Kim, R.J. La Haye, L.L. Lao, R.A. Moyer, T.H. Osborne, W.A. Peebles, C.L. Rettig, T.H. Rhodes, and D.M. Thomas, *Phys. Plasmas*, **1**, 1536 (1994).
- <sup>3</sup>K.H. Burrell, M.E. Austin, T.N. Carlstrom, S. Coda, E.J. Doyle, P. Gohil, R.J. Groebner, J. Kim, R.J. La Haye, L.L. Lao, J. Lohr, R.A. Moyer, T.H. Osborne, W.A. Peebles, C.L. Rettig, T.L. Rhodes, and D.M. Thomas, in *Proceedings of the Fifteenth International Conference on Plasma Physics and Controlled Nuclear Fusion Research*, (Seville, Spain, September 1994) (International Atomic Energy Agency, Vienna, Austria 1995), Vol. I, p. 221.
- <sup>4</sup>E. Mazzucato, S.H. Batha, M. Beer, M. Bell, R.E. Bell, R.V. Budny, C. Bush, T.S. Hahm, G.W. Hammett, F.M. Levinton, R. Nazikian, H. Park, G. Rewoldt, G.L. Schmidt, E.J. Synakowski, W.M. Tang, G. Taylor, and M.C. Zarnstorff, *Phys. Rev. Lett.* **77**, 3145 (1996).
- <sup>5</sup>E.J. Synakowski, S. Batha, M. Beer, M.G. Bell, R.E. Bell, R.V. Budny, C.E. Bush, P.C. Efthimion, G. Hammett, T.S. Hahm, B. LeBlanc, F. Levinton, E. Mazzucato, H. Park, A.T. Ramsey, G. Rewoldt, S.D. Scott, G. Schmidt, W.M. Tang, G. Taylor, and M.C. Zarnstorff, *Phys. Rev. Lett.* **78**, 2972 (1997).
- <sup>6</sup>L.L. Lao, K.H. Burrell, T.S. Casper, V.S. Chan, M.S. Chu, J.C. DeBoo, E.J. Doyle, R.D. Durst, C.B. Forest, C.M. Greenfield, R.J. Groebner, F.L. Hinton, Y. Kawano, E.A. Lazarus, Y.R. Lin-Liu, M.E. Manuel, W.H. Meyer, R.L. Miller, G.A. Navratil, T.H.

- Osborne, Q. Peng, C.L. Rettig, G. Rewoldt, T.L. Rhodes, B.W. Rice, D.P. Schissel, B.W. Stallard, E.J. Strait, W.M. Tang, T.S. Taylor, A.D. Turnbull, R.E. Waltz, and the DIII-D Team, *Phys. Plasmas* **3**, 1951 (1996).
- <sup>7</sup>F. M. Levinton, M.C. Zarnstorff, S.H. Batha, M. Bell, R.E. Bell, R.V. Budny, C. Bush, Z. Chang, E. Fredrickson, A. Janos, J. Manickam, A. Ramsey, S.A. Sabbagh, G.L. Shmidt, E.J. Synakowski, and G. Taylor, *Phys. Rev. Lett.* **75**, 4417 (1995).
- <sup>8</sup>E. J. Strait, L.L. Lao, M.E. Manuel, B.W. Rice, T.S. Taylor, K.H. Burrell, M.S. Chu, E.A. Lazarus, T.H. Osborne, S.J. Thomson, and A.D. Turnbull, *Phys. Rev. Lett.* **75**, 4421 (1995).
- <sup>9</sup>K.H. Burrell, *Phys. Plasmas* **4**, 1499 (1997).
- <sup>10</sup>D.J. Grove and D.M. Meade, *Nucl. Fusion* **25**, 1167 (1985).
- <sup>11</sup>J.L. Luxon and L.G. Davis, *Fusion Technol.* **8**, 441 (1985).
- <sup>12</sup>T.S. Hahm and K.H. Burrell, *Phys. Plasmas* **2**, 1648 (1995).
- <sup>13</sup>C.M. Greenfield, D.P. Schissel, B.W. Stallard, E.A. Lazarus, G.A. Navratil, K.H. Burrell, T.S. Casper, J.C. DeBoo, E.J. Doyle, R.J. Fonck, C.B. Forest, P. Gohil, R.J. Groebner, M. Jakubowski, L.L. Lao, M. Murakami, C.C. Petty, C.L. Rettig, T.L. Rhodes, B.W. Rice, H.E. St. John, G.M. Staebler, E.J. Strait, T.S. Taylor, A.D. Turnbull, K.L. Tritz, R.E. Waltz, and the DIII-D Team, *Phys. Plasmas* **4**, 1596 (1997).
- <sup>14</sup>E.J. Synakowski, S. Batha, M. Beer, M.G. Bell, R.E. Bell, R.V. Budny, C.E. Bush, P.C. Efthimion, T.S. Hahm, G. Hammett, B. LeBlanc, F. Levinton, E. Mazzucato, H. Park, A.T. Ramsey, G. Rewoldt, G. Schmidt, S.D. Scott, G. Taylor, and M.C. Zarnstorff, *Phys. Plasmas* **4**, 1736 (1997).
- <sup>15</sup>C.B. Forest, C.C. Petty, M.E. Austin, F.W. Baity, K.H. Burrell, S.C. Chiu, M.S. Chu, J.S. deGrassie, P. Gohil, A.W. Hyatt, H. Ikezi, E.A. Lazarus, M. Murakami, R.I. Pinsky, M. Porkolab, R. Prater, B.W. Rice, G.M. Staebler, E.J. Strait, T.S. Taylor, and D.G. Whyte, *Phys. Rev. Lett.* **77**, 3141 (1996).

- <sup>16</sup>S.-I. Itoh and K. Itoh, *J. Phys. Soc. Jpn.* **59**, 3815 (1990).
- <sup>17</sup>H. Biglari, P.H. Diamond, and P.W. Terry, *Phys. Fluids B* **2**, 1 (1990).
- <sup>18</sup>K.C. Shaing, E.C. Crume, Jr., and W.A. Houlberg, *Phys. Fluids B* **2**, 1492 (1990).
- <sup>19</sup>V. Erkmann, F. Wagner, J. Baldzuhn, R. Brakel, R. Burhenn, U. Gasparino, P. Grigull, H.J. Hartfuss, J.V. Hofmann, R. Jaenicke, H. Niedermeyer, W. Ohlendorf, A. Rudyj, A. Weller, S.D. Bogdanov, B. Bomba, A.A. Borschegovski, G. Cattanei, A. Dodhy, D. Dorst, A. Elsner, M. Endler, T. Geist, L. Giannone, H. Hacker, O. Heinrich, G. Herre, D. Hildebrandt, V.I. Hiznyak, V.I. Il'in, W. Kasperek, F. Karger, M. Kick, S. Kubo, A.N. Kuftin, V.I. Kurbatov, A. Lazaros, S.A. Malygin, V.I. Malygin, K. McCormick, G.A. Muller, V.B. Orlov, P. Pech, H. Ringler, I.N. Roi, F. Sardei, S. Sattler, F. Schneider, U. Schneider, P.G. Schuller, G. Siller, U. Stroth, M. Tutter, E. Unger, H. Wolff, E. Wursching, and S. Zopf, *Phys. Rev. Lett.* **70**, 2086 (1993).
- <sup>20</sup>F. Wagner, J. Baldzuhn, R. Brakel, R. Burhenn, V. Erkmann, T. Estrada, P. Grigull, H.J. Hartfuss, G. Herre, M. Hirsch, J.V. Hofmann, R. Jaenicke, A. Rudyj, U. Stroth, A. Weller and the W7-AS Teams, *Plasma Phys. Contr. Fusion* **36**, A61 (1994).
- <sup>21</sup>K. Toi, R. Akiyama, H. Arimoto, A. Ejiri, K. Ida, H. Idei, H. Iguchi, O. Kaneko, K. Kawahata, A. Komori, S. Kubo, K. Matsuoka, T. Morisaki, S. Morita, K. Nishimura, S. Okamura, A. Sagara, S. Sakakibara, C. Takahashi, Y. Takita, K. Tanaka, K. Tsumori, J. Xu, H. Yamada, and I. Yamada, *Plasma Phys. Contr. Fusion* **36**, A117 (1994).
- <sup>22</sup>M.G. Shats, D.L. Rudakov, B.D. Blackwell, G.G. Borg, R.L. Dewar, S.M. Hamberger, J. Howard, and L.E. Sharp, *Phys. Rev. Lett.* **77**, 4190 (1996).
- <sup>23</sup>T.S. Hahm, *Phys. Plasmas* **1**, 2940 (1994).
- <sup>24</sup>K. Ida, K. Kondo, K. Nagasaki, T. Hamada, S. Hidekuma, F. Sano, H. Zushi, T. Mizuuchi, H. Okada, S. Besshou, H. Funaba, K. Watanabe, and T. Obiki, *Plasma Phys. Controlled Fusion* **38**, 1433 (1996).
- <sup>25</sup>P.R. Garabedian, *Phys. Plasmas* **4**, 1617 (1997).

- <sup>26</sup>J. Nuhrenberg and R. Zille, Phys. Lett. A **129**, 113 (1988).
- <sup>27</sup>A.H. Boozer, Phys. Fluids **24**, 1999 (1981).
- <sup>28</sup>A.H. Boozer, Phys. Fluids **26**, 496 (1983).
- <sup>29</sup>M.Yu. Isaev, M.I. Mikhailov, and V.D. Shafranov, Plasma Phys. Rep. **20**, 319 (1994).
- <sup>30</sup>T.S. Hahm and K.H. Burrell, Plasma Phys. Controlled Fusion **38**, 1427 (1996).
- <sup>31</sup>E.A. Frieman and L. Chen, Phys. Fluids **25**, 502 (1982).
- <sup>32</sup>T.H. Dupree, Phys. Fluids **15**, 334 (1972).
- <sup>33</sup>P.W. Terry and P.H. Diamond, Phys. Fluids **28**, 1419 (1985).
- <sup>34</sup>S. Coda, M. Pokolab, and K.H. Burrell, *Private Communications* (1996).
- <sup>35</sup>T. S. Hahm, M. Artun, M.A. Beer, G.W. Hammett, W.W. Lee, X. Li, Z. Lin, H.E. Mynick, S.E. Parker, G. Rewoldt, and W.M. Tang "Turbulence and Transport in Enhanced Confinement Regimes of Tokamaks: Simulation and Theory" Sixteenth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Montreal, Canada October 7-11, 1996 in *Proceedings of the Sixteenth International Conference on Plasma Physics and Controlled Nuclear Fusion Research*, (Montreal, Canada, October, 1996) (International Atomic Energy Agency, Vienna, Austria 1997), F1-CN-64/D1-2.
- <sup>36</sup>R.J. Fonck, G. Cosby, R.D. Durst, S.F. Paul, N. Bretz, S. Scott, E. Synakowski, and G. Taylor, Phys. Rev. Lett. **70**, 3736 (1993).
- <sup>37</sup>K.C. Shaing and J.D. Callen, Phys. Fluids **26**, 3315 (1983).
- <sup>38</sup>F.S.B. Anderson, A. Almagri, D.T. Anderson, P.G. Matthews, J.N. Talmadge, and J.L. Shohet, Trans. Fusion Tech. **27**, 273 (1995).
- <sup>39</sup>J.N. Talmadge and W.A. Cooper, Phys. Plasmas **3**, 3713 (1996).
- <sup>40</sup>T. Fujita, S. Ide, H. Shirai, O. Naito, Y. Koide, S. Takeji, H. Kubo, S. Ishida, and M. Kikuchi, Phys. Rev. Lett. **78**, 2377 (1997).

<sup>41</sup>Y. Koide and the JT-60 Team, *Phys. Plasmas* **4**, 1623 (1997).

<sup>42</sup>JET Team, in Ref. 3, p. 423.

<sup>43</sup>X. Litaudon, R. Arslanbekov, G.T. Hoang, E. Joffrin, F. Kazarian-Vibert, D. Moreau, Y. Peysson, P. Bibet, P. Froissard, M. Goniche, G. Rey, J. Ferron, and K. Kupfer, *Plasma Phys. Controlled Fusion* **38**, 1603 (1996).

<sup>44</sup>F. Wagner, *Private Communications* (1997).

<sup>45</sup>J.M. Greene and M.S. Chance, *Nucl. Fusion* **21**, 453 (1981).

<sup>46</sup>H.L. Berk, M.N. Rosenbluth, and J.L. Shohet, *Phys. Fluids* **26**, 2616 (1983).

<sup>47</sup>A.H. Boozer, *Private Communications* (1997).