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Abstract: Localized wave (LW) pulses were produced using a standard Navy array in the anechoic tank at NUWC Keyport. The LW pulses used were the MPS pulse first derived by Ziolkowski, and a new type of pulse based on a superposition of Gaussian beam modes. This new type is motivated by a desire to make a comparison of the MPS pulse with another broad band pulse built from solutions to the wave equation. The superposed Gaussian pulse can be described by parameters which are analogous to those describing the MPS pulse. We compare the directivity patterns and the axial energy decay between the pulses. We find the behavior of the pulses to be similar so that the superposed Gaussian could be another candidate in the class of low diffractive pulses known as localized waves.

INTRODUCTION

Several kinds of low diffraction pulses have been formulated in the last several years. Among these are Greenleaf and Lu's X waves [1], Brittingham's Focused Wave Modes (FWM) [2], Ziolkowski's Modified Power Spectrum (MPS) pulse [3,4], and Stepanishen and Sun's transient Bessel beams [5]. All of these are formulated as direct pulse solutions of the scalar wave equation or superpositions of direct solutions. In this paper we compare the directivity patterns and down range energy decay rates of the MPS pulse and a new broad band pulse formulated as a superposition of single frequency Gaussian beams. The measurements were performed in an anechoic tank at NUWC Keyport. The pulses were produced by a standard acoustic array (AdCap) used by the U. S. Navy. Details of the experiment are reported in the accompanying paper by Lewis and Chambers [6].

FORMULATION OF SUPERPOSED GAUSSIAN PULSE

The original intent of the Keyport experiment was to produce an MPS pulse using the AdCap array. However, a second type of broad band pulse was desired for comparison. Since the MPS pulse was originally formulated as a superposition of Focused Wave Modes, we designed this second pulse as a superposition of Gaussian beam modes. We chose Gaussian beam modes because of their widespread use to describe forward propagating beams even though they are not exact solutions of the wave equation. The fundamental mode Gaussian beam is given by [7]

$$u_k(r,z,t) = \frac{k\,w_0^2 \exp\left(i(\omega t - kz)\right)}{k\,w_0^2 - 2i(z-z_0)} \exp\left(-\frac{k\,r^2}{k\,w_0^2 - 2i(z-z_0)}\right) \quad , \tag{1}$$

where $k = \omega/c$ is the wave number, ω the angular frequency, w_0 the beam waist, and z_0 is the focal distance from the aperture. A superposition of Gaussian beams of different frequencies can be written as

$$\Phi(r,z,t) = \int_0^\infty \hat{\Phi}(\omega) \frac{\omega\,w_0^2 \exp\left(i\omega\,(t - z/c)\right)}{\omega\,w_0^2 - 2ic(z-z_0)} \exp\left(-\frac{\omega\,r^2}{\omega\,w_0^2 - 2ic(z-z_0)}\right) d\omega \tag{2}$$

where $\hat{\Phi}(\omega)$ is the weighting for the beam with angular frequency ω. In order to obtain a simple analytical form the beam waist is chosen to be a function of frequency, $w_0(\omega) = r_0\sqrt{\omega_0/\omega}$, and the weighting is chosen to be $\hat{\Phi}(\omega) = \exp(\omega/\omega_0)/\omega_0$. With these choices the integral can be performed giving the final expression for the acoustic potential of the SG pulse:

$$\Phi(r,z,t) = \mathrm{Re}\left[\frac{1}{r^2/r_0^2 + \left(1 - i\omega_0(t - z/c)\right)\left(1 - 2ic(z - z_0)/\omega_0 r_0^2\right)}\right] \tag{3}$$

The parameters r_0 (pulse radius), z_0, and ω_0 $(= 2\pi f_0)$, completely specify the SG pulse.

In comparison, the acoustic potential for the MPS pulse is given by

$$\Phi(r,z,t) = \mathrm{Re}\left[\frac{2R_m z_0}{z_0 + i(z - ct)}\,\frac{\exp(-z_0 s(z,r,t)/w_0^2)}{s(r,z,t) + 2R_m}\right] \qquad s(r,z,t) = \frac{r^2}{z_0 + i(z - ct)} - i(z + ct) \tag{4}$$

where w_0 is the pulse radius, z_0 is the pulse axial halfwidth, and R_m is the range scale length. These parameters are related to Ziolkowski's MPS parameters a, b, and β (ref. 3) by

$$a = \frac{1}{z_0} \quad , b = \frac{2R_m z_0^2}{w_0^2} \quad , \beta = 2R_m z_0 \quad . \tag{5}$$

From the acoustic potential, the normal velocity in the aperture (z = 0) is obtained by $v_z = \partial\Phi/\partial z$.

RESULTS

Two MPS pulses (MPS1 and MPS2) and two superposed Gaussian pulses (SG3 and SG4) were launched from the array. The parameter values for the MPS pulses were R_m = 1000 meters, z_0 = 0.48 cm., and w_0 = 7.5 cm. (MPS1) or w_0 = 21 cm. (MPS2). For the superposed Gaussian pulses we used f_0 = 30 kHz, z_0 = 0.0, and r_0 = 15 cm (SG3) or r_0 = 7.5 cm (SG4). Also, tone bursts of 18, 21, 35, and 50 kHz were launched for comparison. The directivity patterns of MPS2, SG3, and SG4 showed narrower central lobes than the tone bursts and lower side lobes. Pulse MPS1 showed a curious double humped central lobe structure. The low side lobes of the LWs occurred at an angle near ±50°, in agreement with the grating lobes of the 50 kHz tone burst. This was further confirmed by the power spectrum of the received pulse at ±50°. In summary, the behavior of the superposed Gaussian pulses compared well with that of the MPS pulses leading us to include them as another member of the class of localized waves.

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