SAND2024-15982C

Hydrodynamic instability and breakup of liquid-gas interfaces via vibration

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APS DFD 2024

November 25th, 2024

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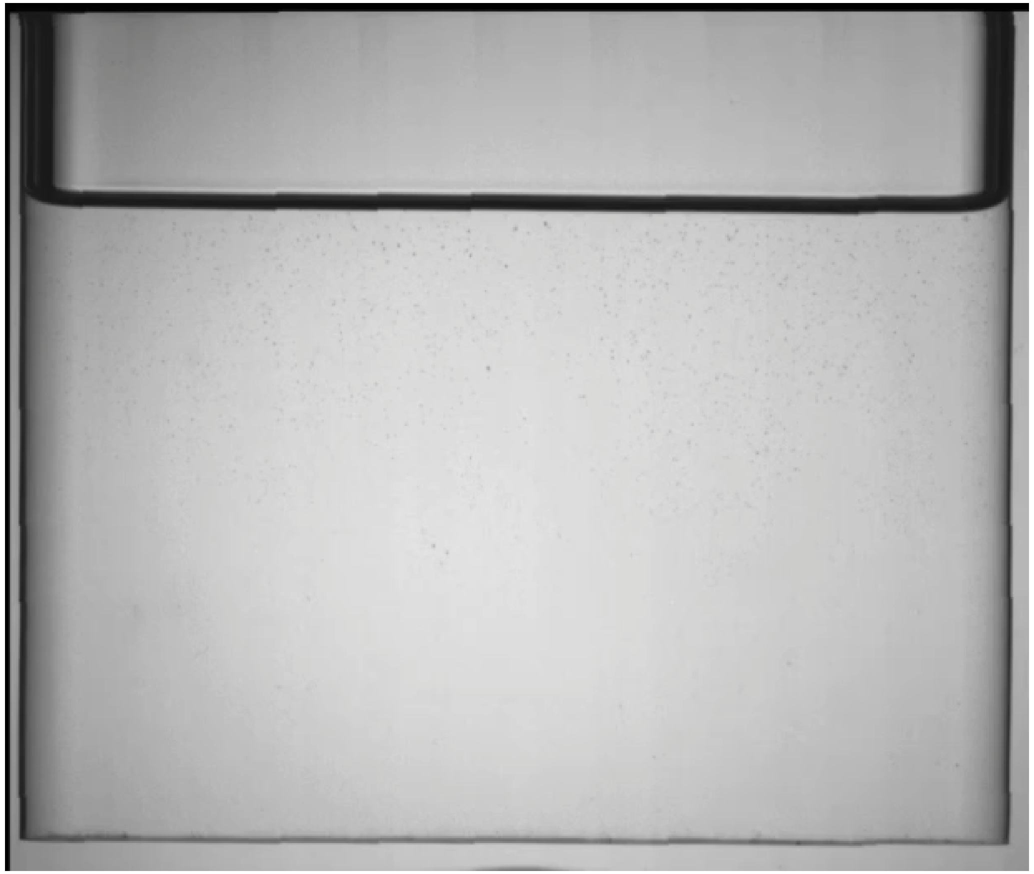
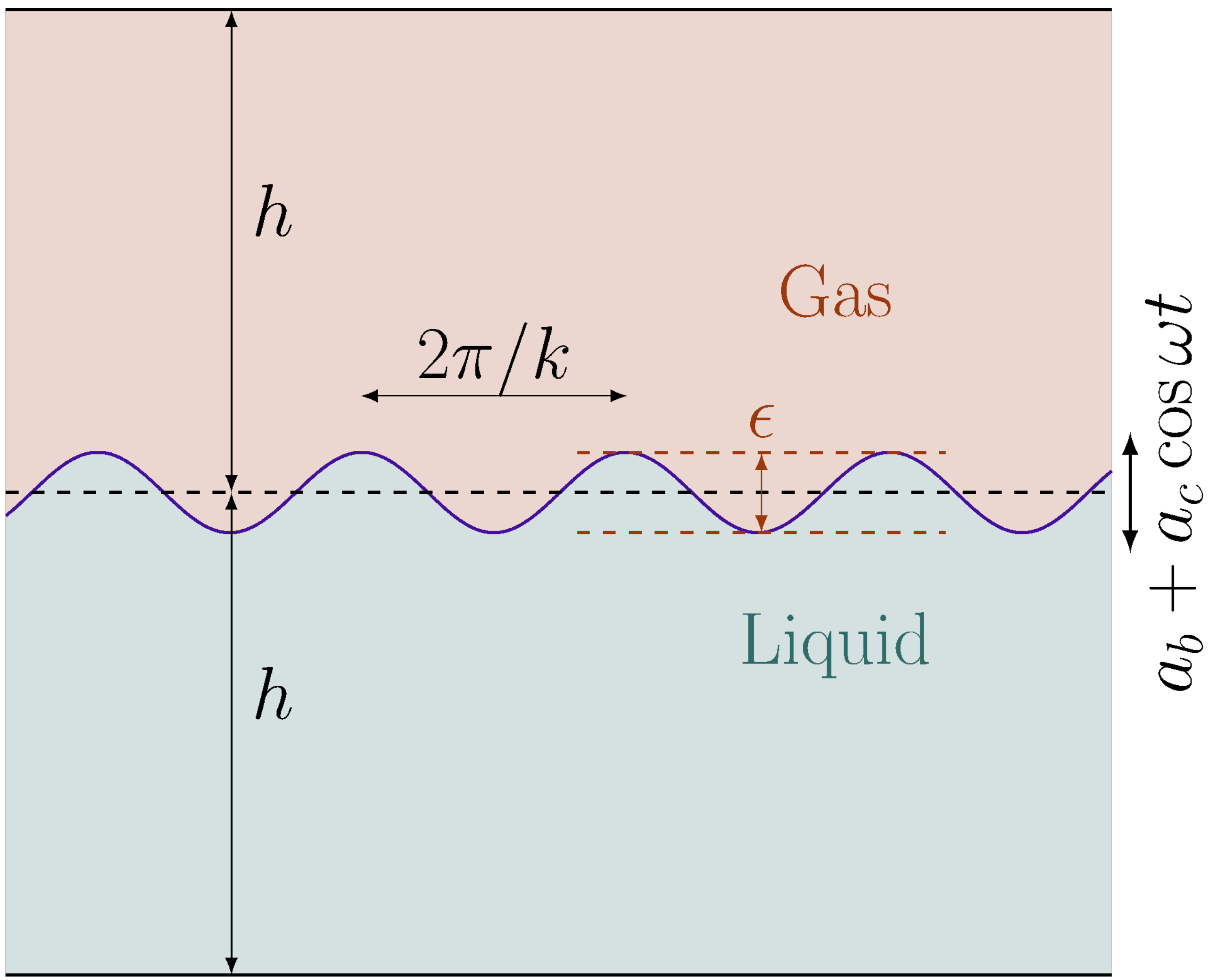
Introduction

- **Gas Injection**
  - Changes in liquid-gas mixture
- **Formation of lower gas region**
  - Changes hydrodynamic properties
  - Deleterious effects in fluid-structure interactions
- **Dependent on Initial Breakup**
  - Not yet well-understood
- **Experimental data limited**
  - Need DNS instead



T. O'Hern, unpublished

Problem description



- An interface with small perturbation initially separates a liquid and gas
- An oscillatory acceleration is applied using a body force in the domain
  - $a^* \in O(10)$
  - $\frac{\omega}{2\pi} \in O(100 \text{ Hz})$
- The oscillatory acceleration either stabilizes or destabilizes the interface

Model

- **Diffuse Interface Method**
  - Baer-Nunziato-like 6-eqn[1]
- **Extra Physics:**
  - Surface-tension[2]
  - Viscosity
  - Body forces

$$\frac{\partial \alpha_i \rho_i}{\partial t} + \nabla \cdot (\alpha_i \rho_i \boldsymbol{u}) = 0$$

$$\frac{\partial \rho \boldsymbol{u}}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u} + p\boldsymbol{I} - \boldsymbol{T} + \boldsymbol{\Omega}) = -\rho \boldsymbol{g}$$

$$\frac{\partial \alpha_1 \rho_1 e_1}{\partial t} + \nabla \cdot (\alpha_1 \rho_1 e_1 \boldsymbol{u}) + \alpha_1 p_1 \cdot \nabla \boldsymbol{u} = -\mu p_I \delta P - \alpha_1 \boldsymbol{T}_1 : \nabla \boldsymbol{u}$$

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$$\frac{\partial \alpha_1}{\partial t} + \boldsymbol{u} \cdot \nabla \alpha_1 = \mu (p_1 - p_2)$$

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Numerics

- **Finite Volume**[1]
  - WENO3[2]
  - Shock/discontinuity capturing
- **Riemann Solver**
  - HLLC[3[
- **Time Stepping**
  - Explicit TVD RK[4]

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[1] V. Coralic & T. Colonius (2014) JCP

[2] C.-W. Shu (1997) Tech. Reprt.

[3] E. F. Torro (1997) Springer Berlin Heidelberg

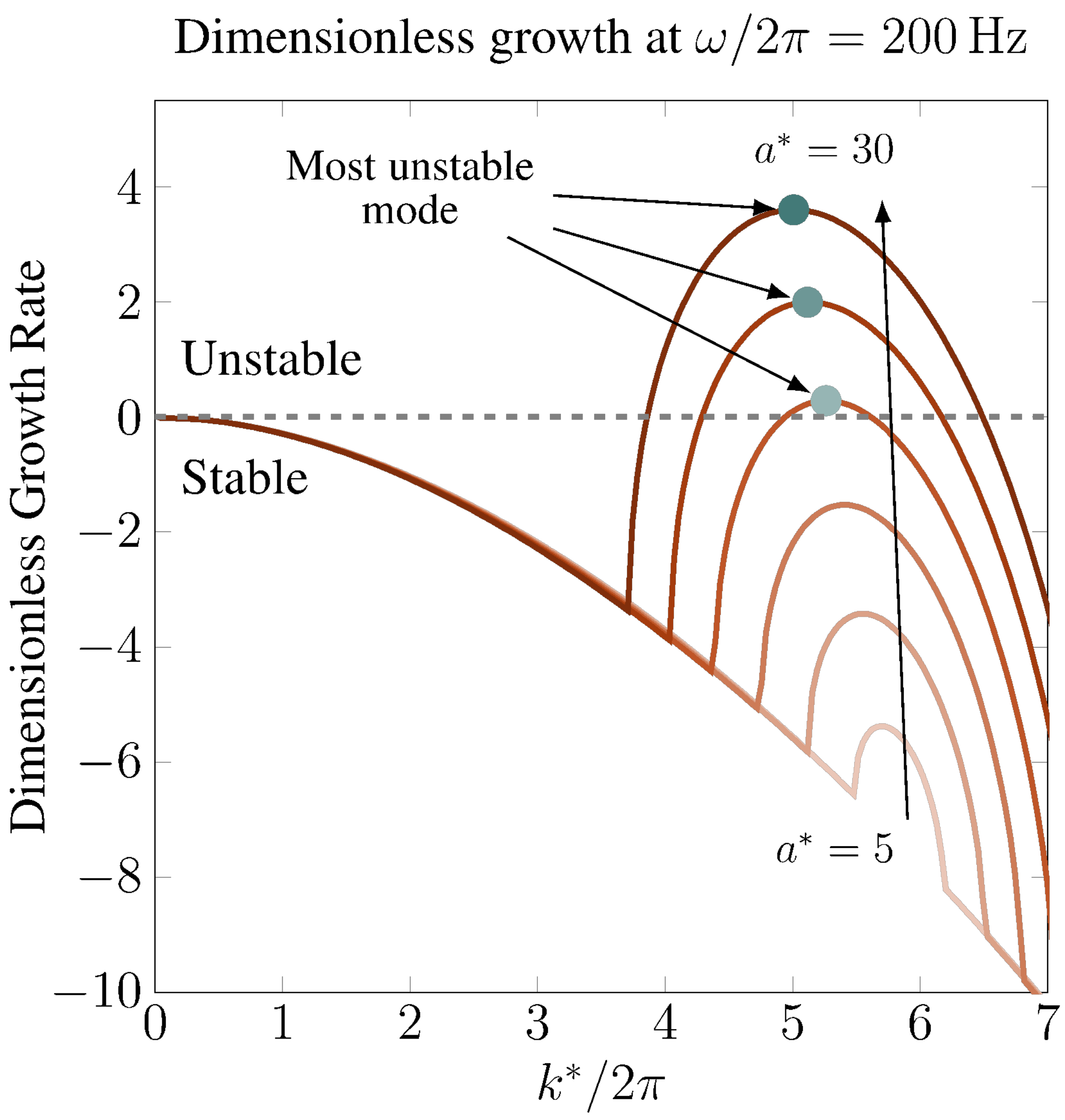
[4] S. Gottlieb & C.-W. Shu (1998) MC

Linear stability analysis

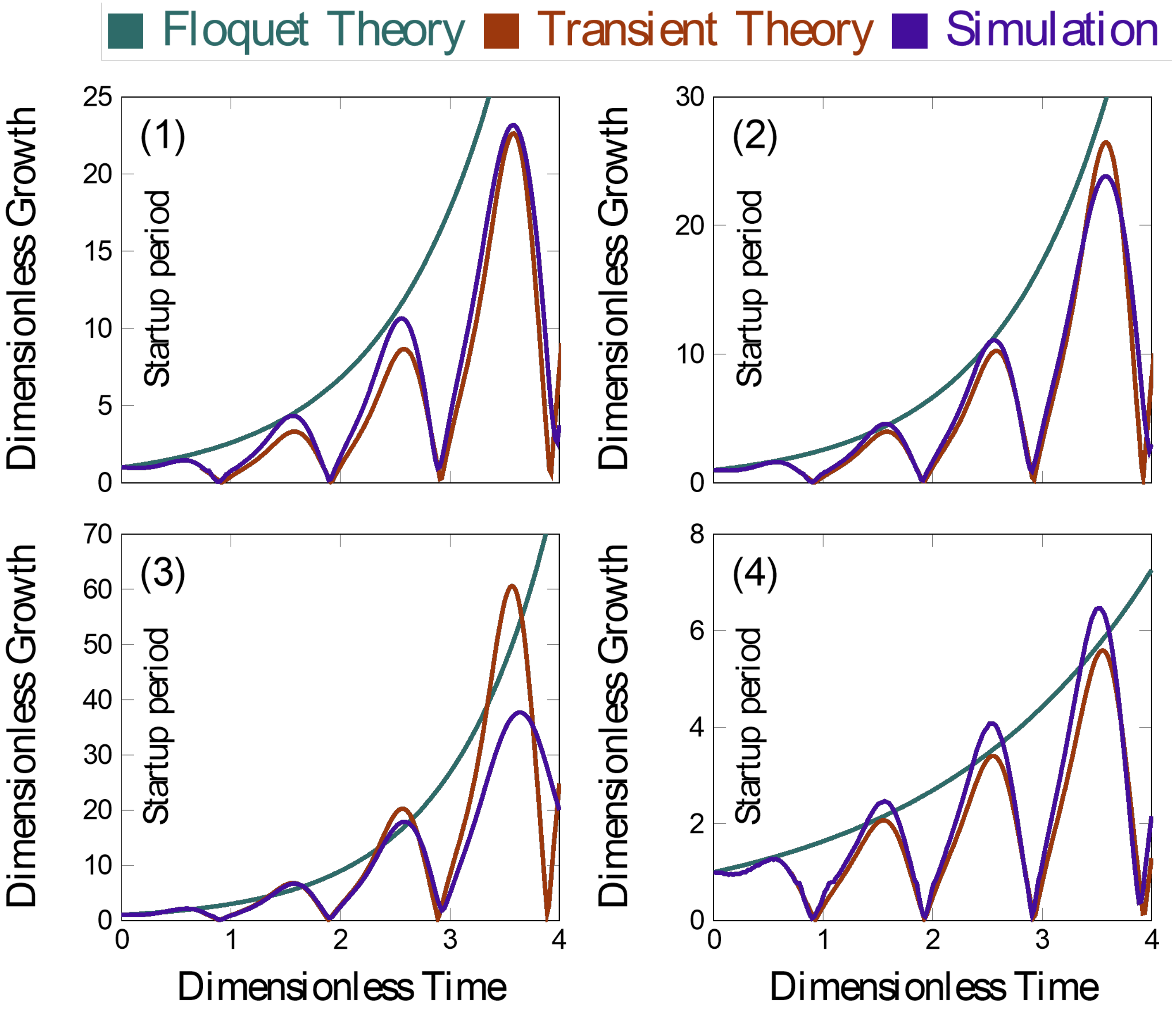
- Floquet analysis allows for the estimation of growth rates in periodic systems
- Allows for the identification of the most unstable mode for a given acceleration and oscillation frequency

$$k^* = kh = \frac{2\pi h}{\lambda}$$

Ratio of interface height to wavelength



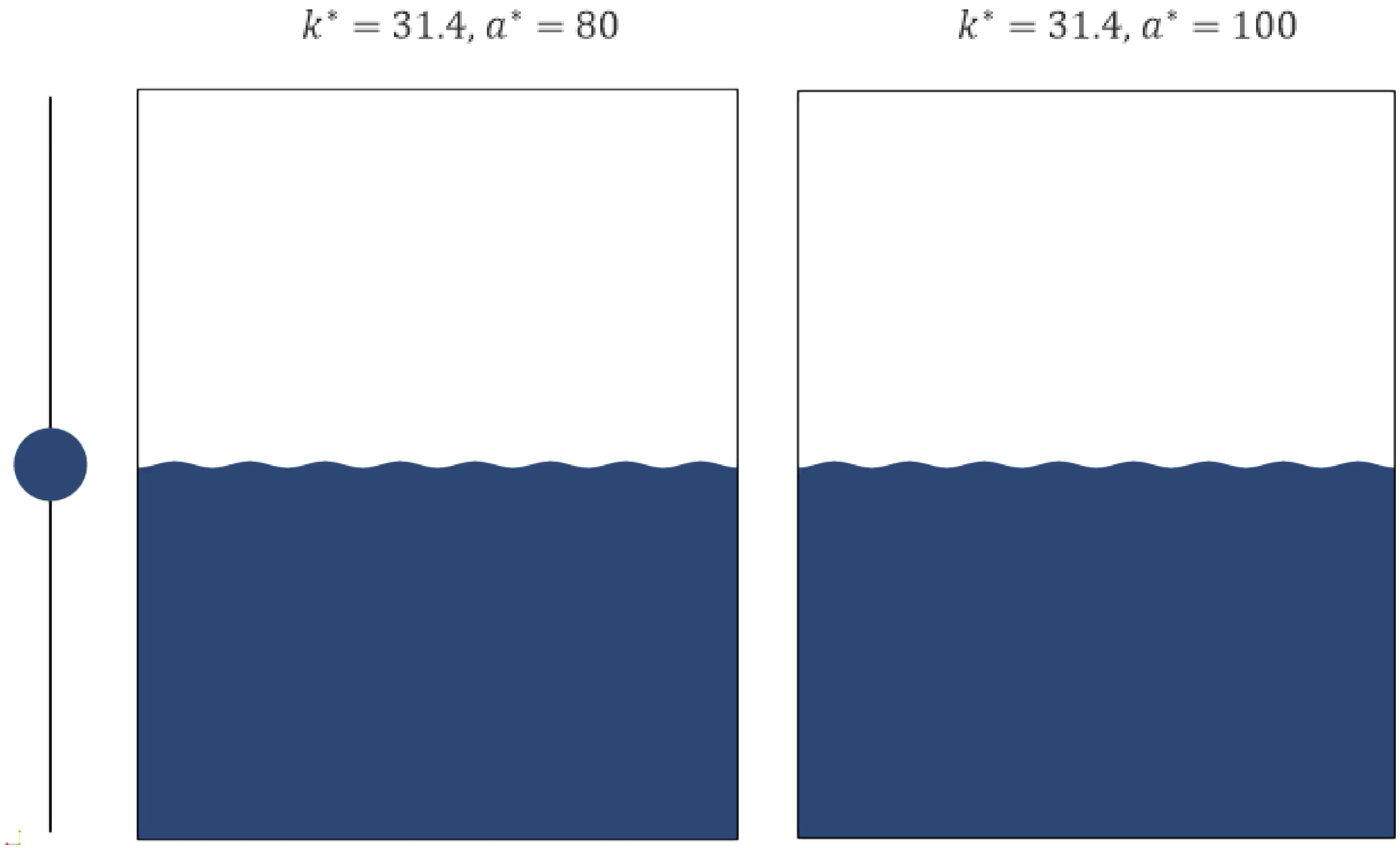
Validation with linear theory



Case	a^*	ω^*	k^*
1	10	3.92	31.4
2	15	4.71	31.4
3	30	6.20	31.4
4	30	7.27	31.4

- Use an epsilon of 1% of the wavelength to avoid non-linear effects
- Show good agreement with theory, particularly in the first and second periods of oscillation

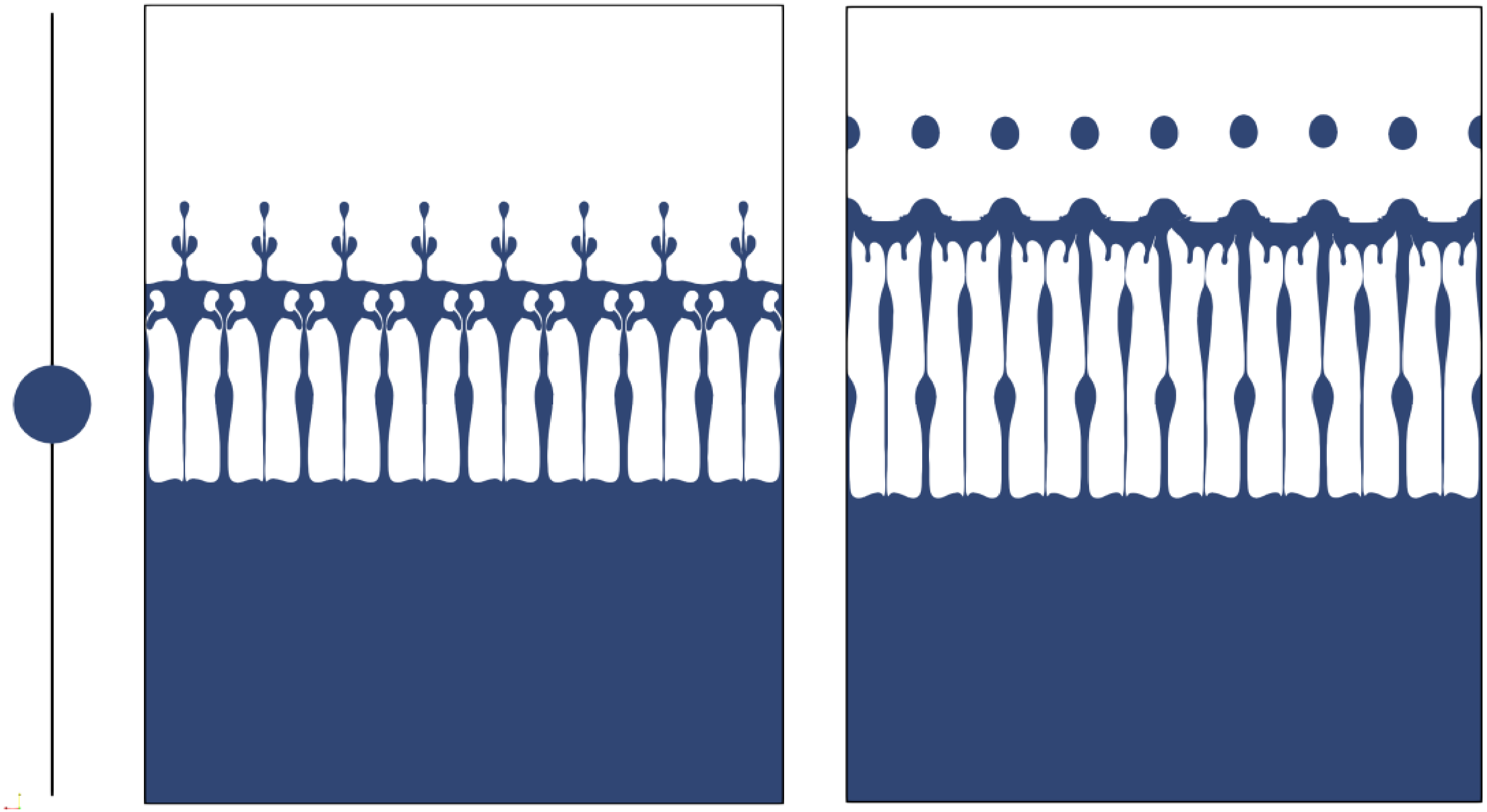
Results at 200 Hz



- Nonlinear effects develop slowly
- Droplet pinch-off and ejection are observed
- Lack of gas injection is suspected to be a result of no droplet impacts happening at the interface

Results at 200 Hz

$k^* = 31.4, a^* = 80$ $\quad$ $k^* = 31.4, a^* = 100$



- Nonlinear effects develop slowly
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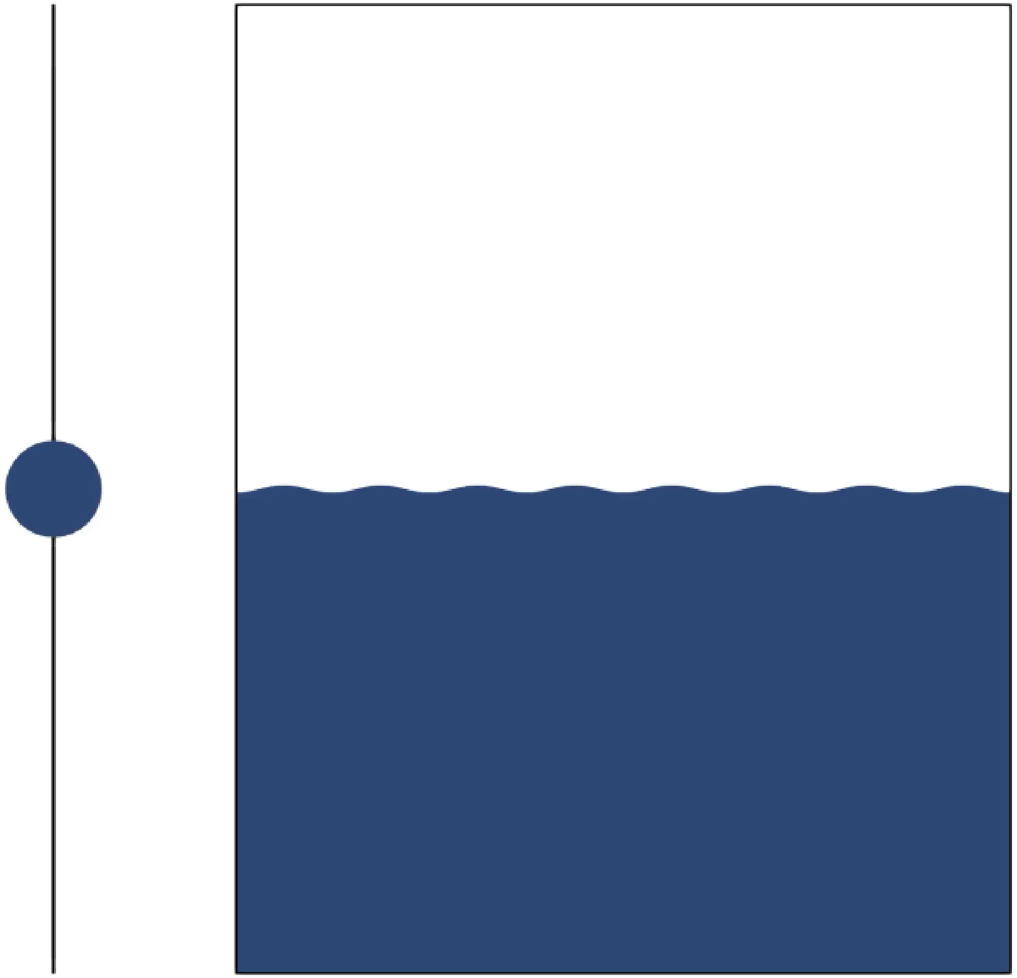
Result at 100 Hz

$k^* = 31.4, a^* = 45$



- Nonlinear effects occur much earlier in the simulation
- Small regions of gas entrapment where ejected fluid impacts interface

Result at 100 Hz

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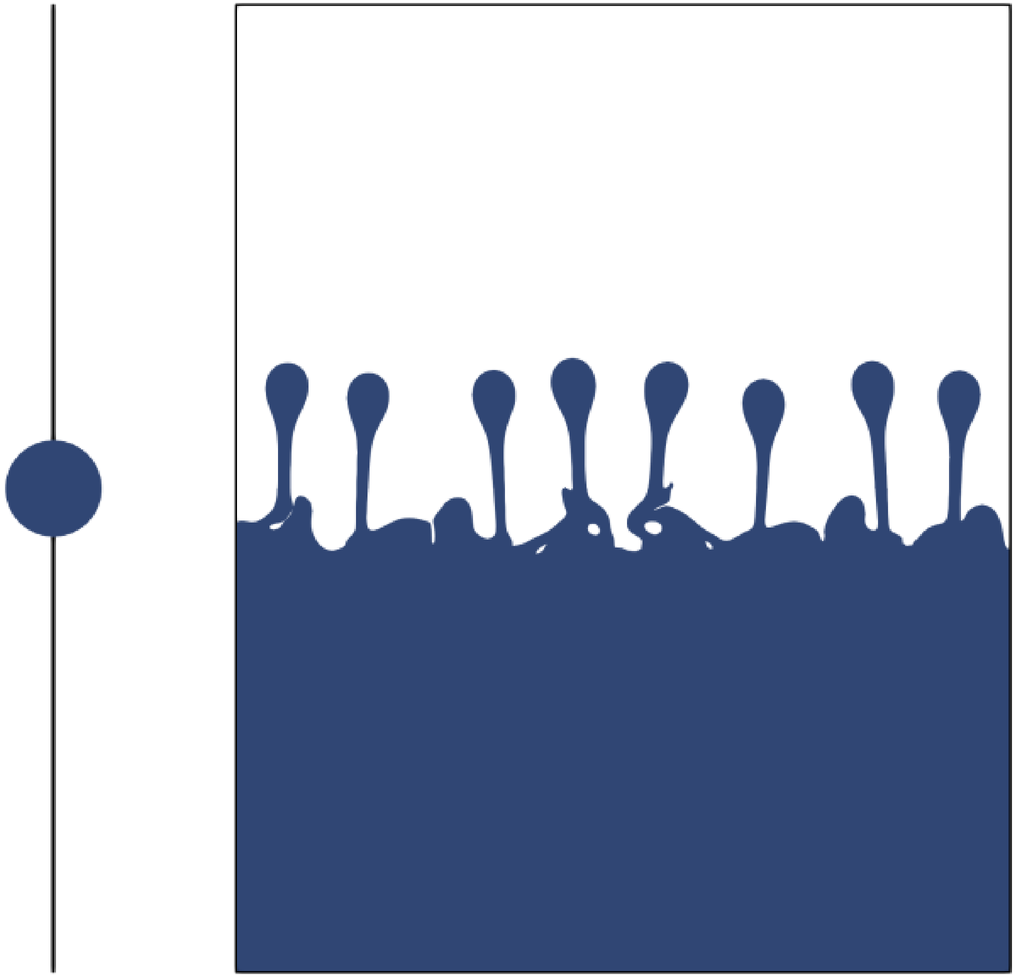

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- Small regions of gas entrapment where ejected fluid impacts interface

Takeaways

- Gas Injection sensitive to early breakup
- Time scales make simulation of interface breakup expensive
- Next steps: Extension to 3D

mflowcode.github.io

Thank you!

Acknowledgments:

Sandia National Laboratories

Oak Ridge National Laboratory | Leadership Computing Facility

NSF | ACCESS

NNSA National Nuclear Security Administration

U.S. Department of Energy

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