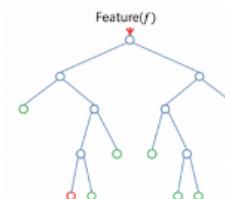
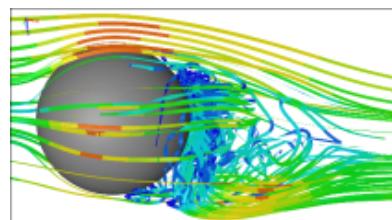




Robust Data-Driven Turbulence Modeling for RANS Closures Using a SciML Approach for Validation



PRESENTED BY: Uma Balakrishnan

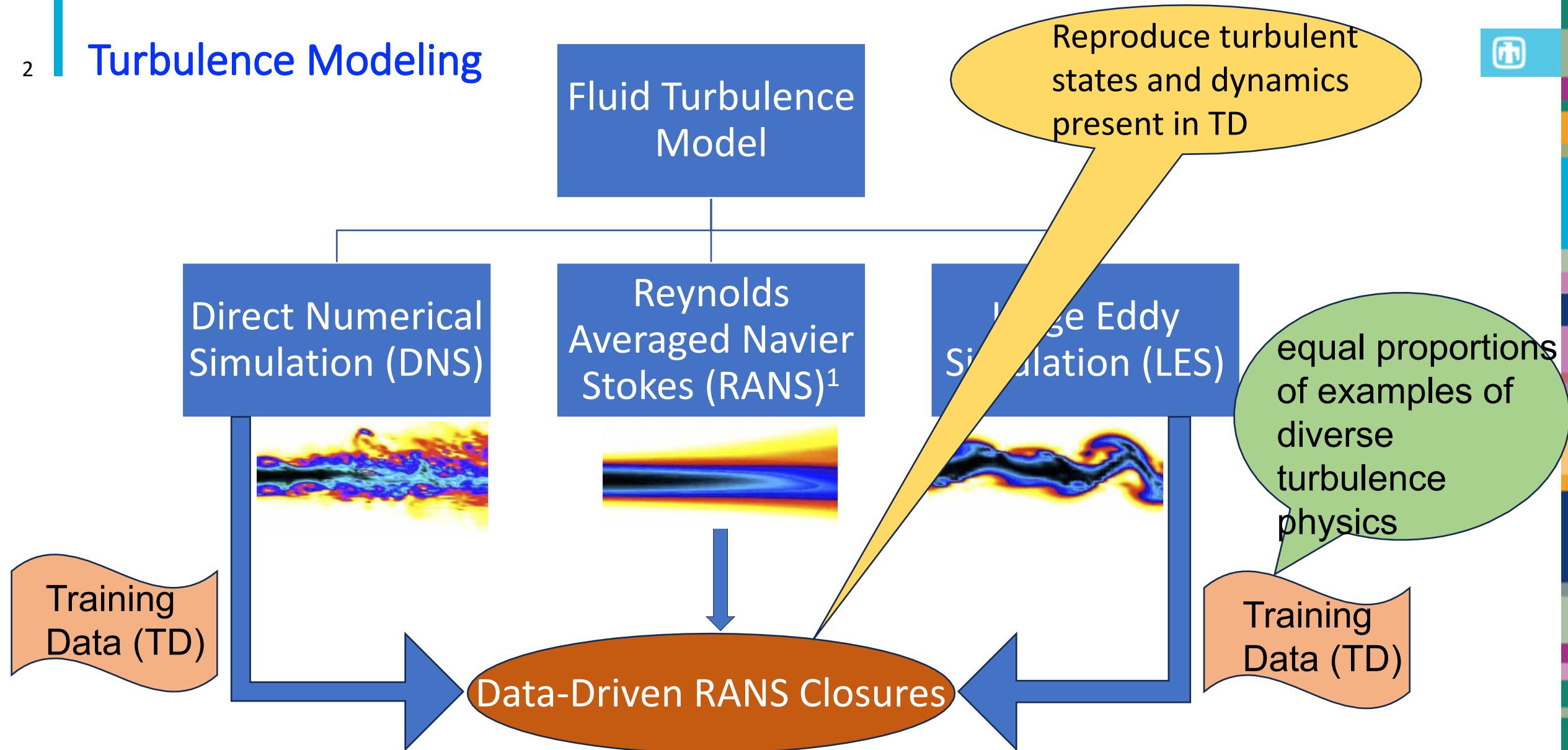
Collaborators: William J Rider, Matthew Barone, Eric J. Parish

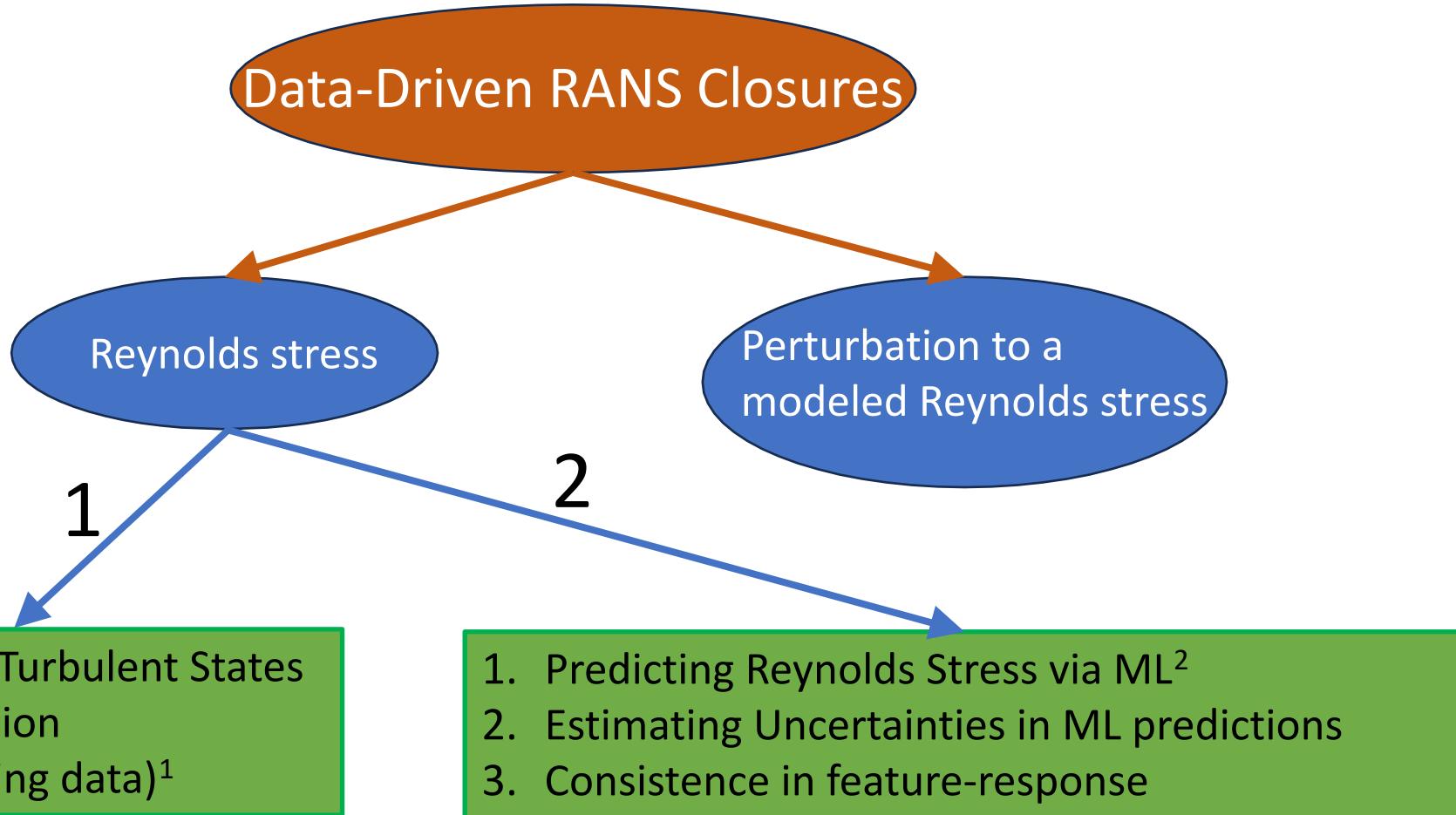
Sandia National Laboratories



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Turbulence Modeling





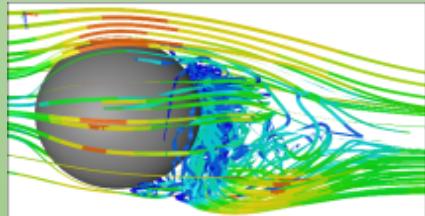
1. M.F.Barone, J.Ray, and S.Domino, [Feature Selection, Clustering, and Prototype Placement for Turbulence Datasets](#), AIAA Journal 2022 60:3, 1332-1346.
2. E. Parish, D.S. Ching, N.E. Miller, S. J. Beresh and M. F. Barone. ["Turbulence modeling for compressible flows using discrepancy tensor-basis neural networks and extrapolation detection,"](#) AIAA 2023-2126. AIAA SCITECH 2023 Forum. January 2023.

Motivating Credibility for Scientific Machine Learning (SciML)

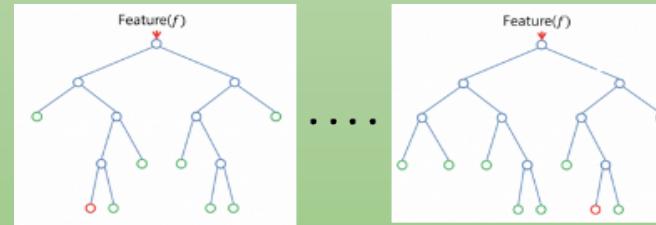


Machine learned models are used in lieu of, complementary to, or as surrogates for science and engineering computational simulation models.

What does VV/UQ/Credibility Mean for Scientific Machine Learning?



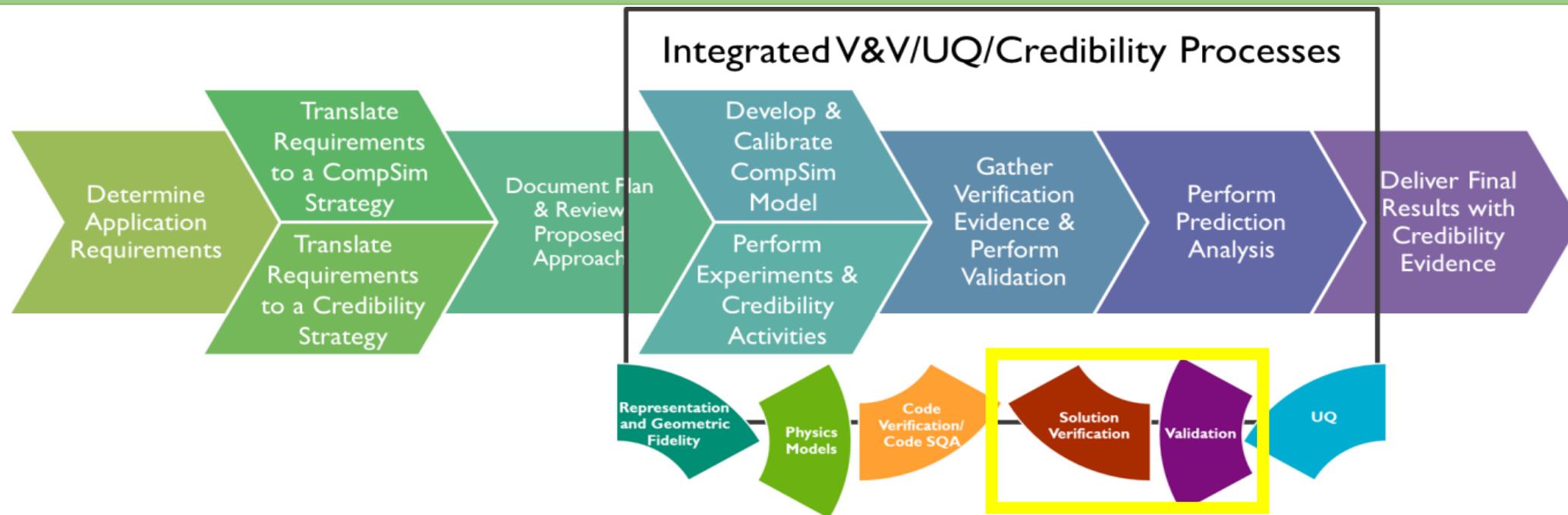
Scientific Computing



Machine Learning

Putting “correct” math methods and physics models into our codes.

Produce “correct” codes and models which leads to “correct” results.



Data-Driven Turbulence Modeling



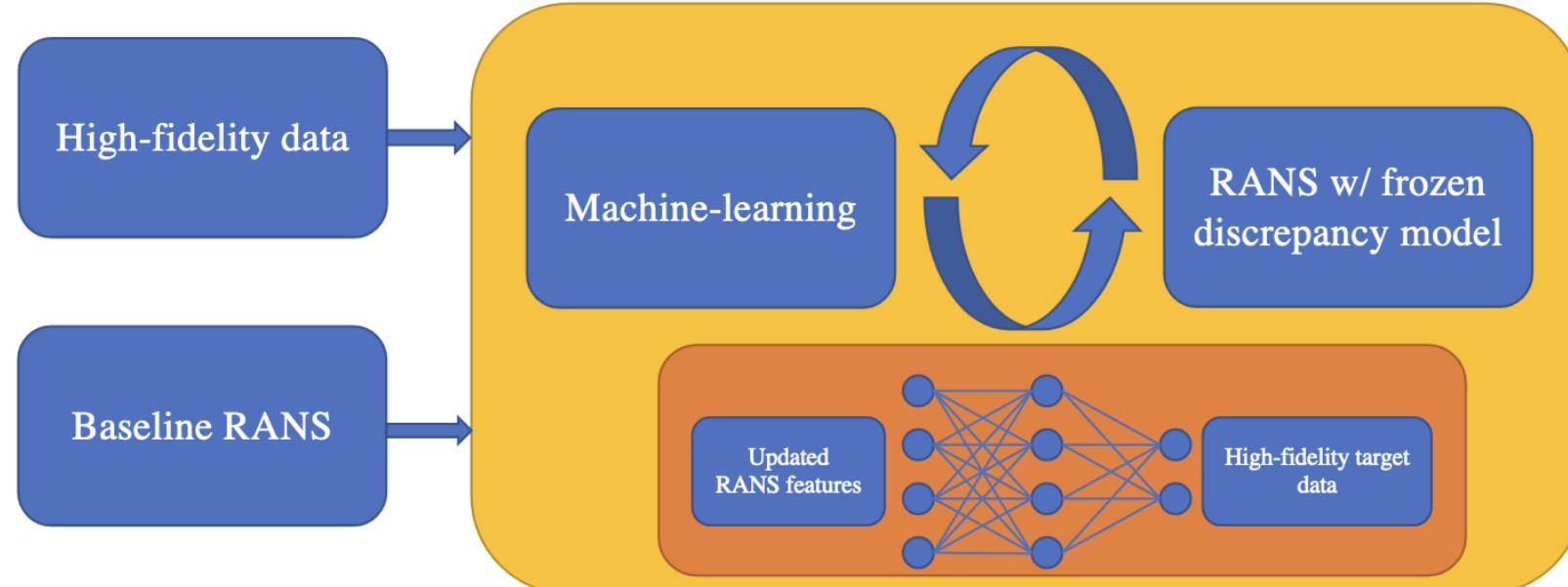
$$a_{ij} \approx a_{ij}^{\text{RANS}} + m_{ij}^{\text{ML}} \quad \text{Anisotropy-based discrepancy term}$$

Anisotropy tensor

$$a_{ij} = \frac{-\tau_{ij}}{\rho u''_k u''_k} - \frac{1}{3} \delta_{ij}$$

$$a_{ij}^{\text{RANS}} = \frac{\tau_{ij}^{\text{RANS}}}{2\bar{\rho}\tilde{k}} - \frac{1}{3} \delta_{ij} \quad \text{Anisotropy tensor predicted by a standard RANS}$$

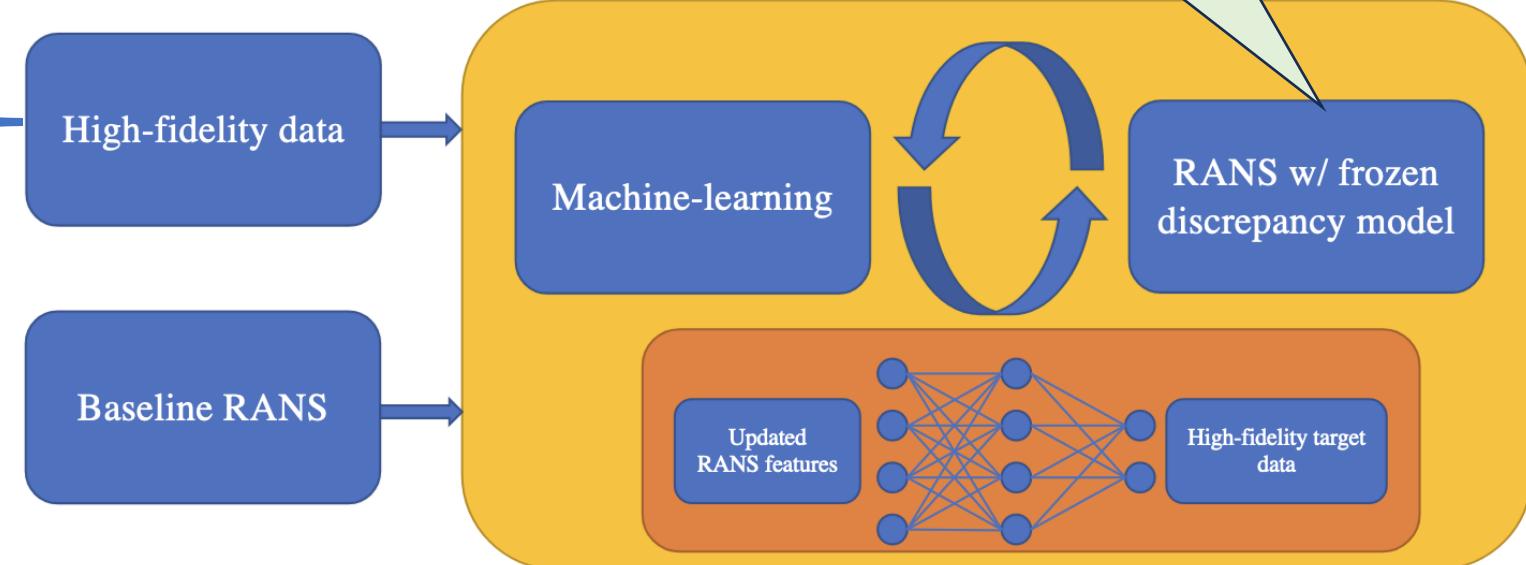
δ_{ij} is the Kronecker delta & m_{ij}^{ML} is an ML correction



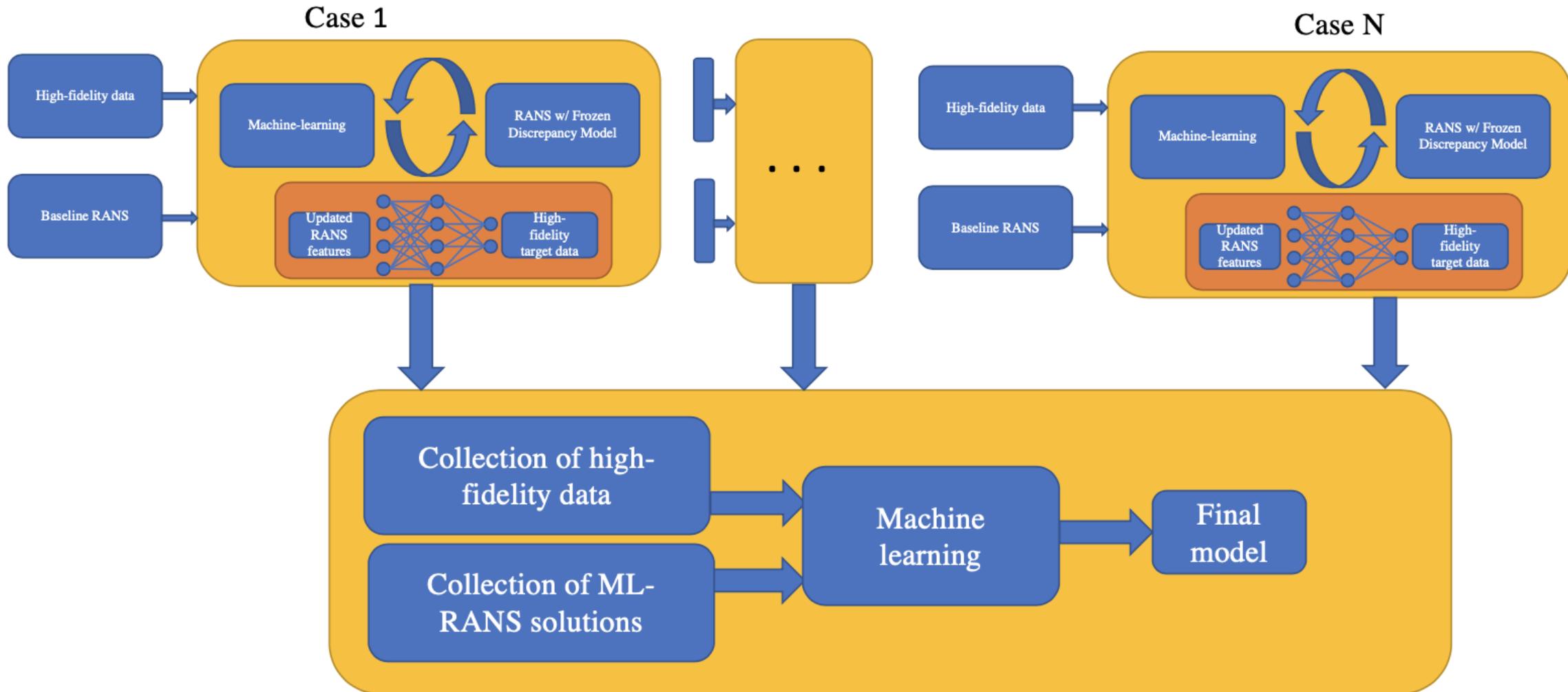
Data-Driven Turbulence Modeling

High Fidelity Datasets for Training

- A. Channel flow with $Re = 180$
- B. Channel flow with $Re = 395$
- C. Channel flow with $Re = 590$
- D. Duct flow at $Re = 3500$
- E. Flow over periodic hill
- F. HS BL at $M = 6, T_w/T_r = 0.25$
- G. HS BL at $M = 6, T_w/T_r = 0.76$
- H. HS BL at $M = 14, T_w/T_r = 0.18$



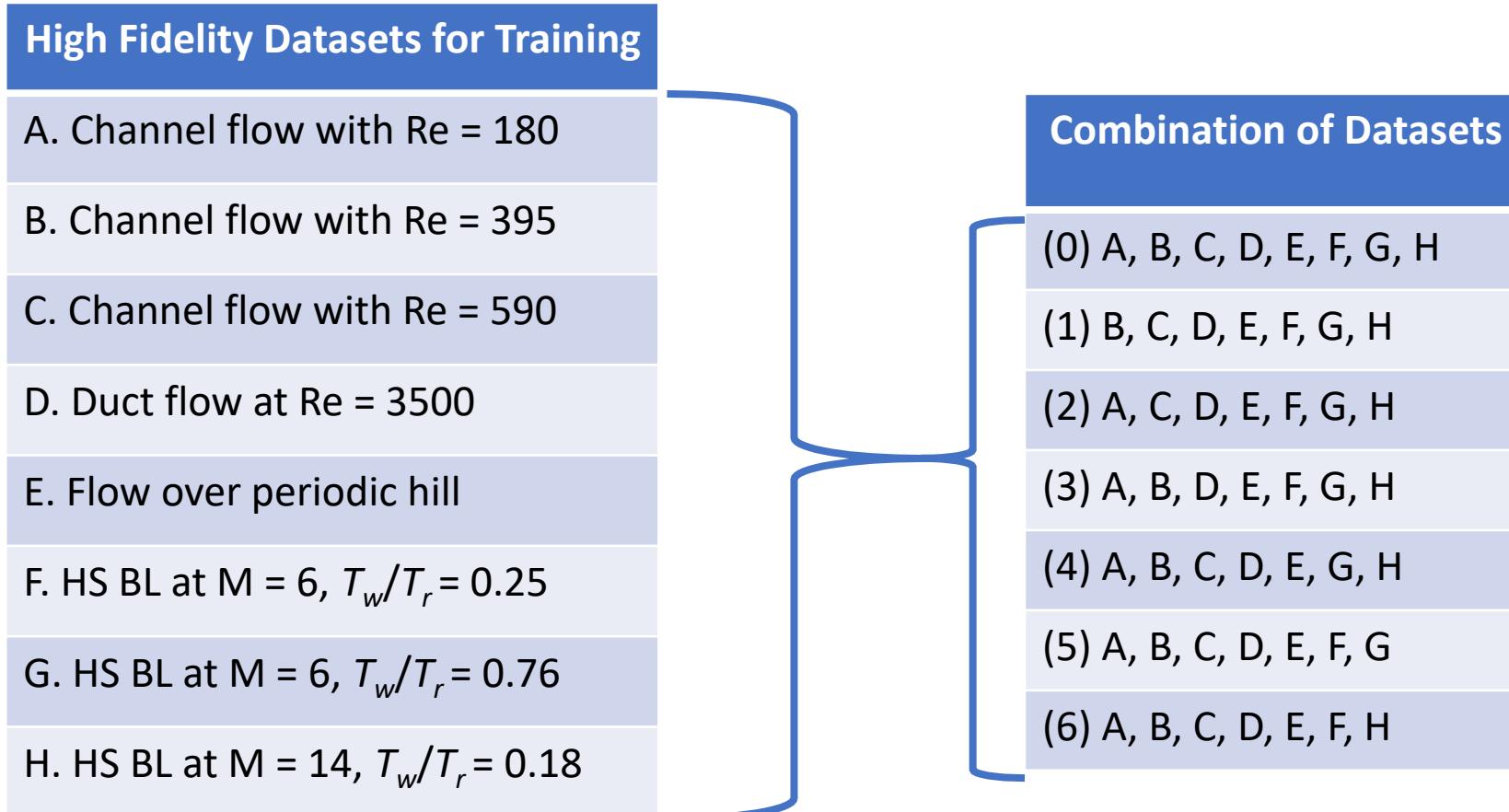
Global Training Process



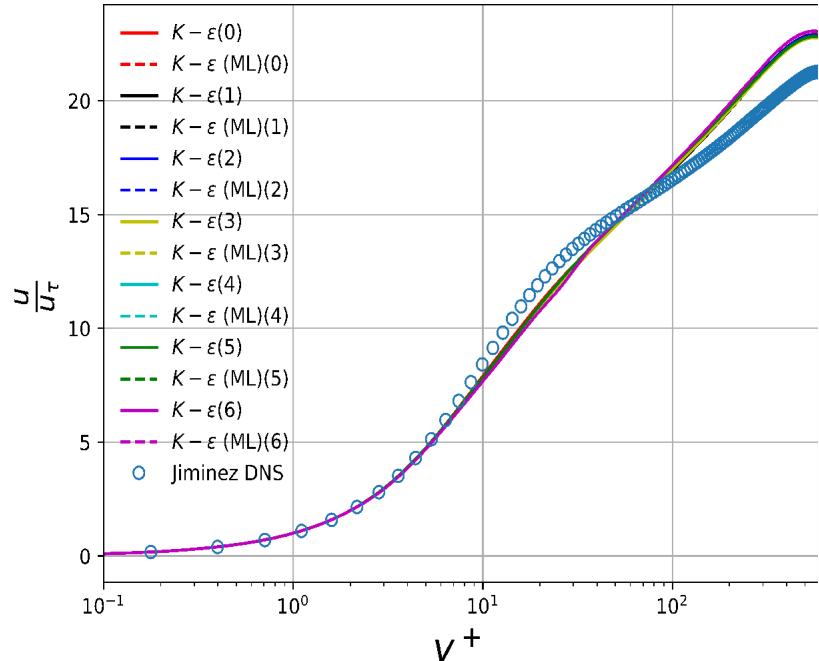


- Global iterative training procedure improves feature consistency.
- Complete consistency in response has not been achieved.
- The goal is to minimize overall inconsistency.
- ML models involve many hyperparameters.
- Considering various combinations of training datasets and testing hyperparameters might help validate and improve the overall response consistency.

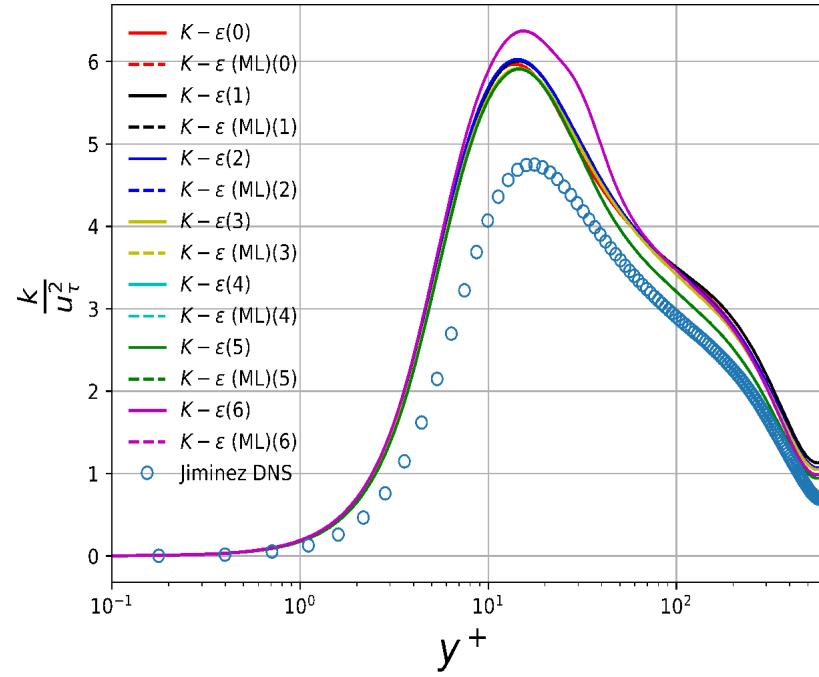
Various Combination of Training Datasets



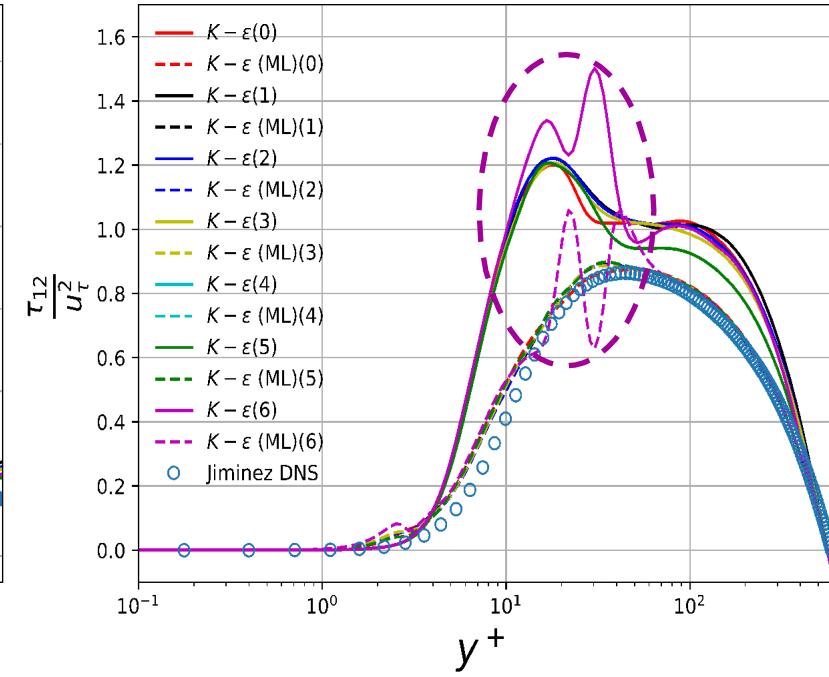
Validating Training Datasets & Testing Channel Flow



Velocity



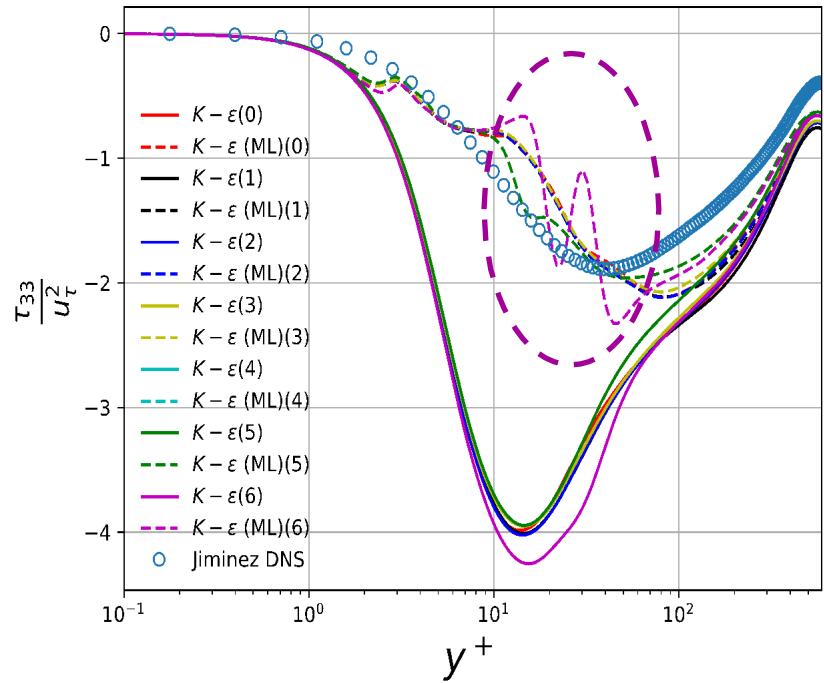
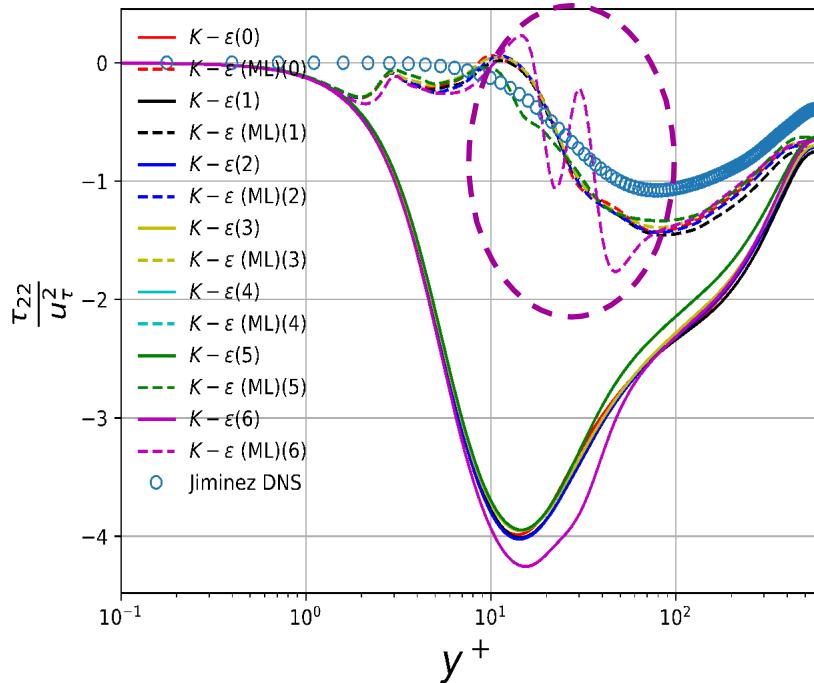
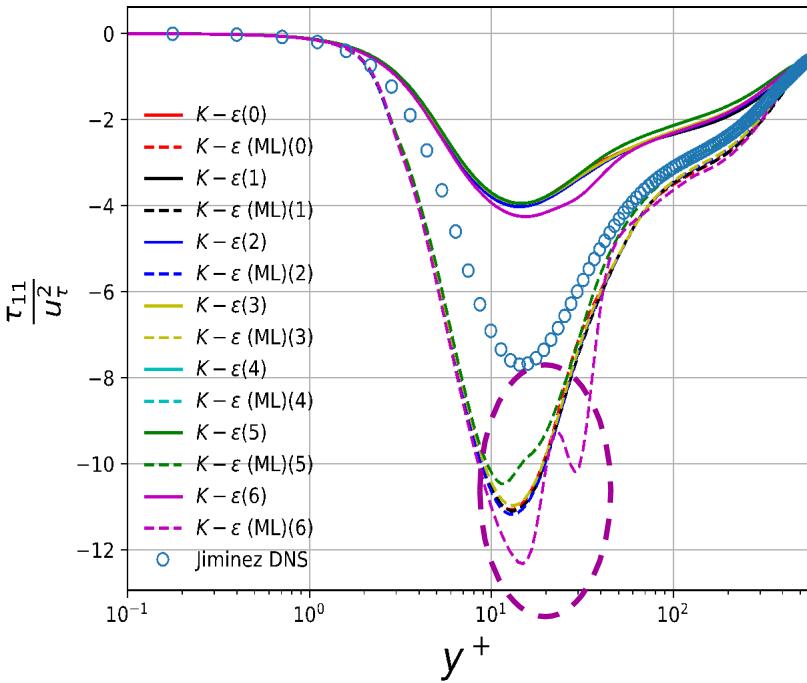
Turbulence Kinetic Energy



Reynolds Shear Stress

- The global iterative procedure was trained on various combinations of datasets as described in the previous slide (w/o changing any hyperparameters).
- It was then tested on the channel flow dataset with $Re = 590$.

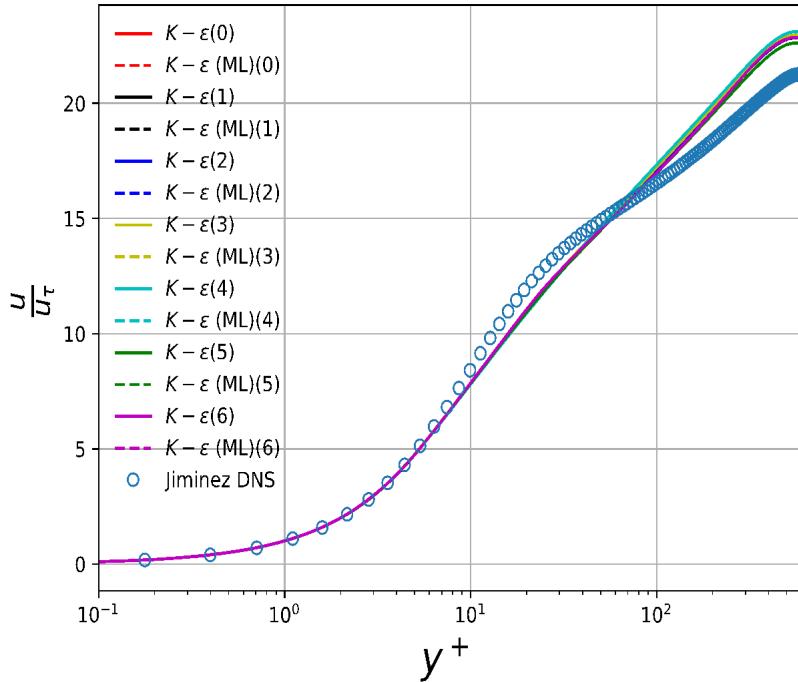
Validating Training Datasets & Testing Channel Flow



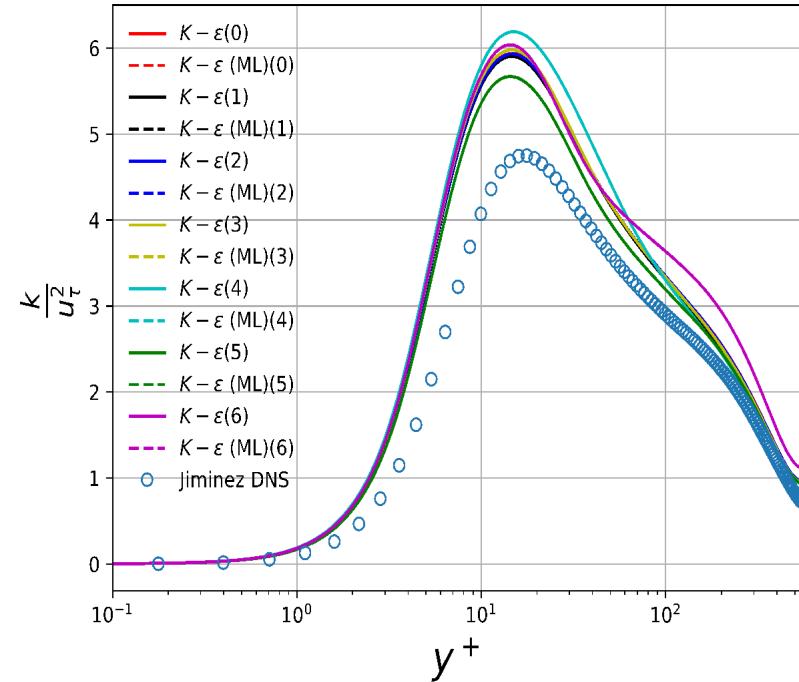
Normal Stress in x, y & z direction

- Figures clearly show that the ML correction term follows the trend of the "true" (DNS) data. However, there is a deviation in the buffer layer, which is consistent across all combinations of training datasets.
- Dataset combination (6) exhibits clear oscillations.

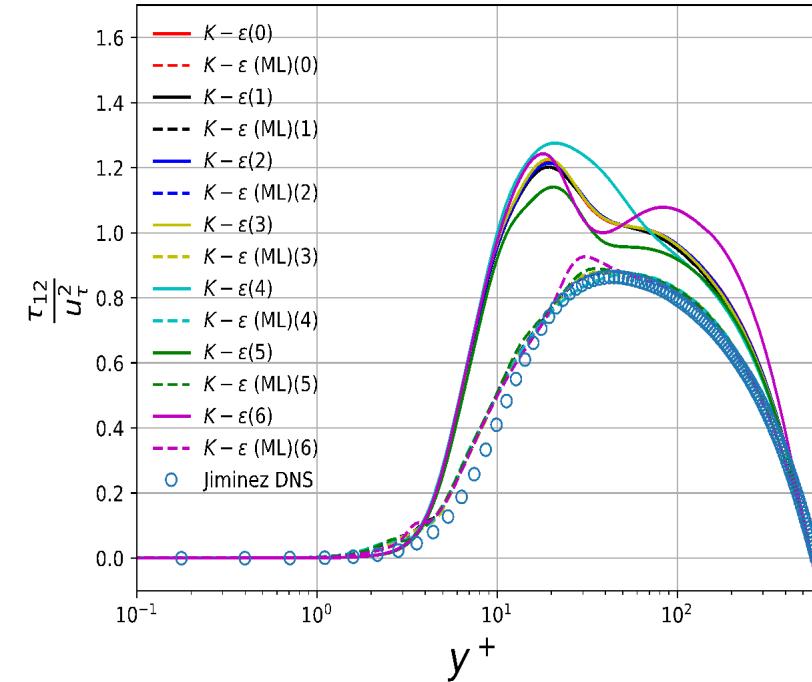
Validating Training Datasets & Testing Channel Flow



Velocity



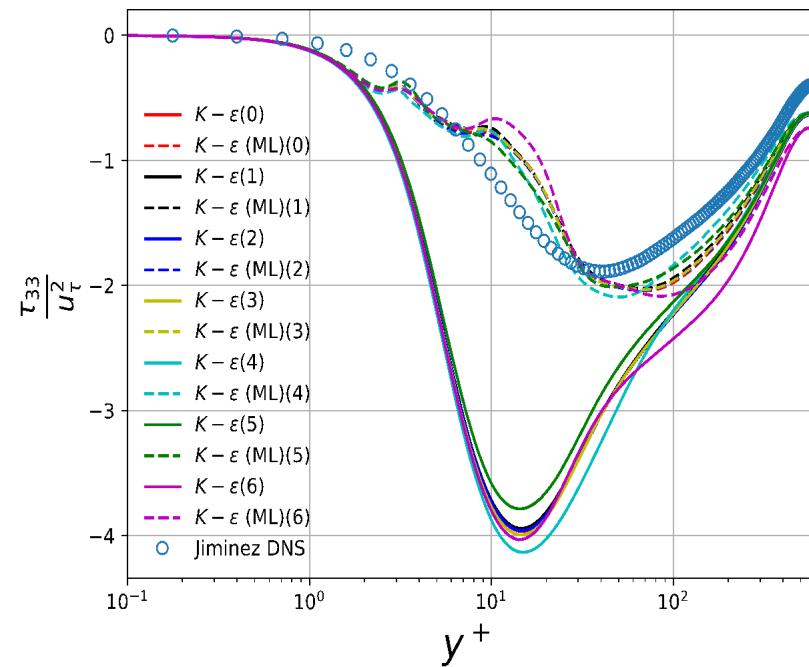
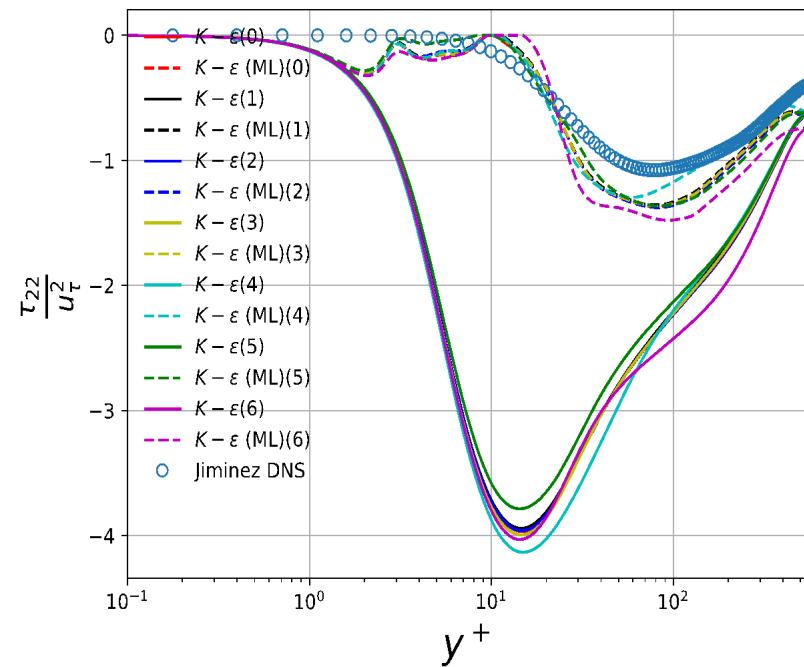
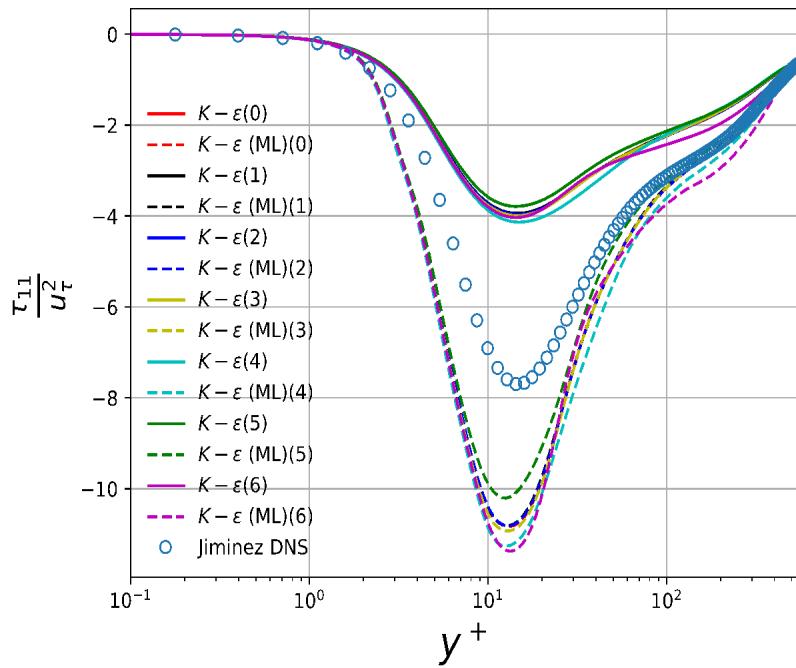
Turbulence Kinetic Energy



Reynolds Shear Stress

- Reducing the depth and width of the neural network along with an optimum epochs clearly reduces the overfitting problem.

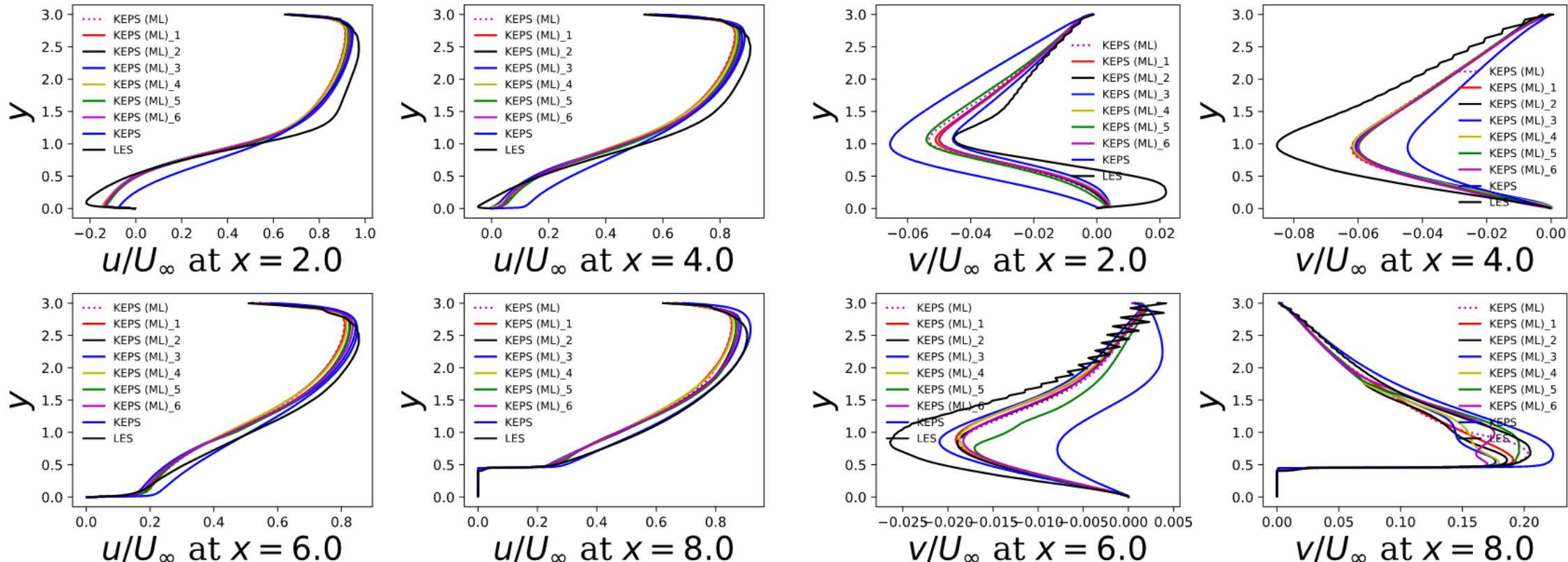
Validating Training Datasets & Testing Channel Flow



Normal Stress in x, y & z direction

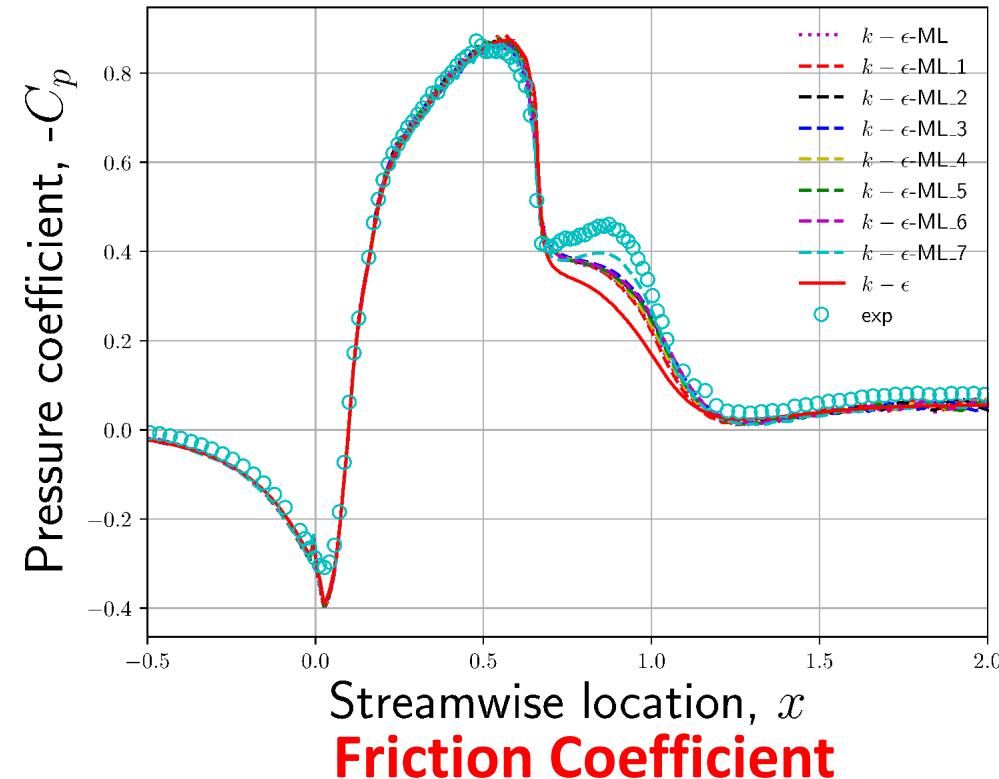
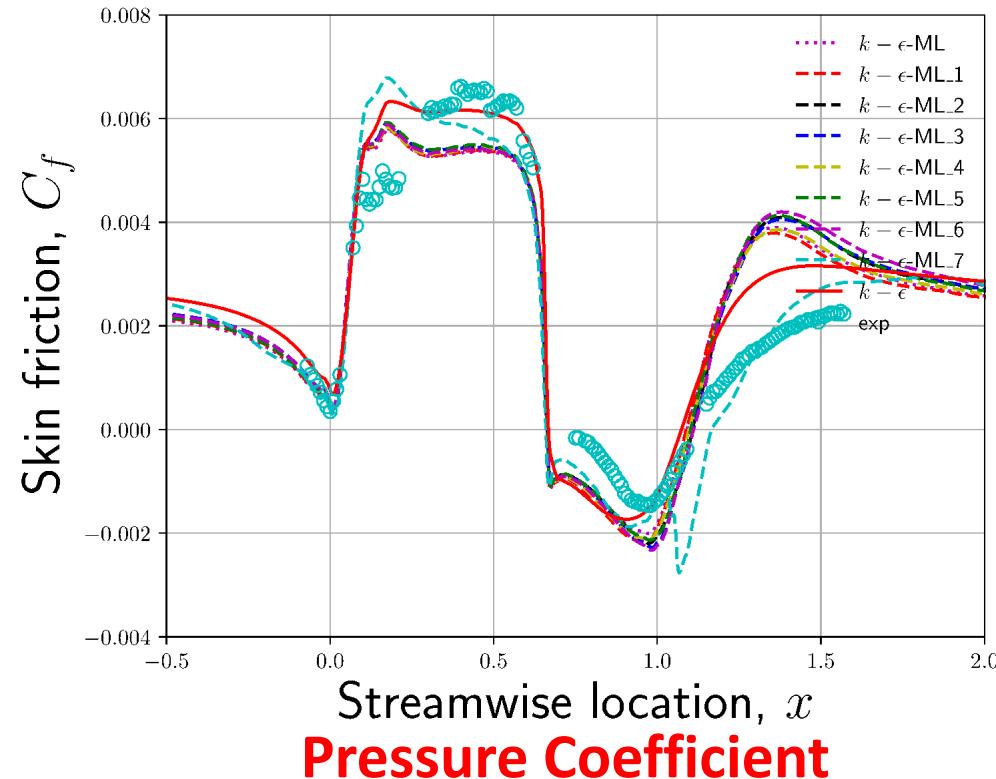
- Reducing the depth and width of the neural network along with an optimum epochs clearly reduces the oscillatory behavior.
- However, there is still a deviation in the buffer layer, which is consistent across all combinations of training datasets.

Validating Training Datasets & Testing Periodic Hill



- We then tested on the Periodic Hill, and the figure shows the velocity profiles as a function of y for various x locations.
- The figures clearly show that the ML correction term performs better than the standard $k-\epsilon$ model.

Validating Training Datasets & Testing NASA Hump (Out-Of-Distribution)



- Figure depicts the pressure coefficient and skin friction as a function of the streamwise location for the $k - \epsilon$ and $k - \epsilon$ -ML models.
- We observe that the $k - \epsilon$ -ML slightly under predicts the peak in pressure and friction coefficient which needs to be improved.

Conclusions and Future Work



- **Performance Consistency:** We conducted rigorous testing on a variety of in-distribution and out-of-distribution datasets using different combinations of training datasets, which demonstrated consistent performance across these combinations.
- **Hyperparameter Tuning:** Some combinations of training datasets highlight the need for hyperparameter tuning to reduce inconsistency in the anisotropy-based discrepancy term.
- **Network Depth Reduction:** Reducing the depth and width of the network effectively along with optimum epochs mitigates oscillation and overfitting, yet there remains inconsistent behavior in the anisotropy-based ML correction with "true" DNS data.
- **Future Work:** We will focus on explainable machine learning models for turbulence closures utilizing SHAP (SHapley Additive exPlanations) & LIME (Local Interpretable Model-Agnostic Explanations) analysis.



Thank You for Your Time and Attention!

For questions or follow-up discussions:

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