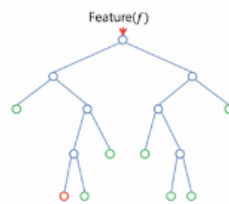
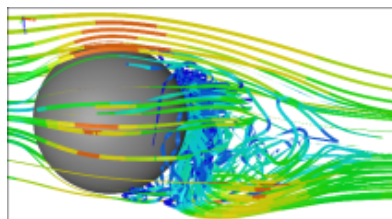




# Robust Data-Driven Turbulence Modeling for RANS Closures Using a SciML Approach for Validation



PRESENTED BY: Uma Balakrishnan

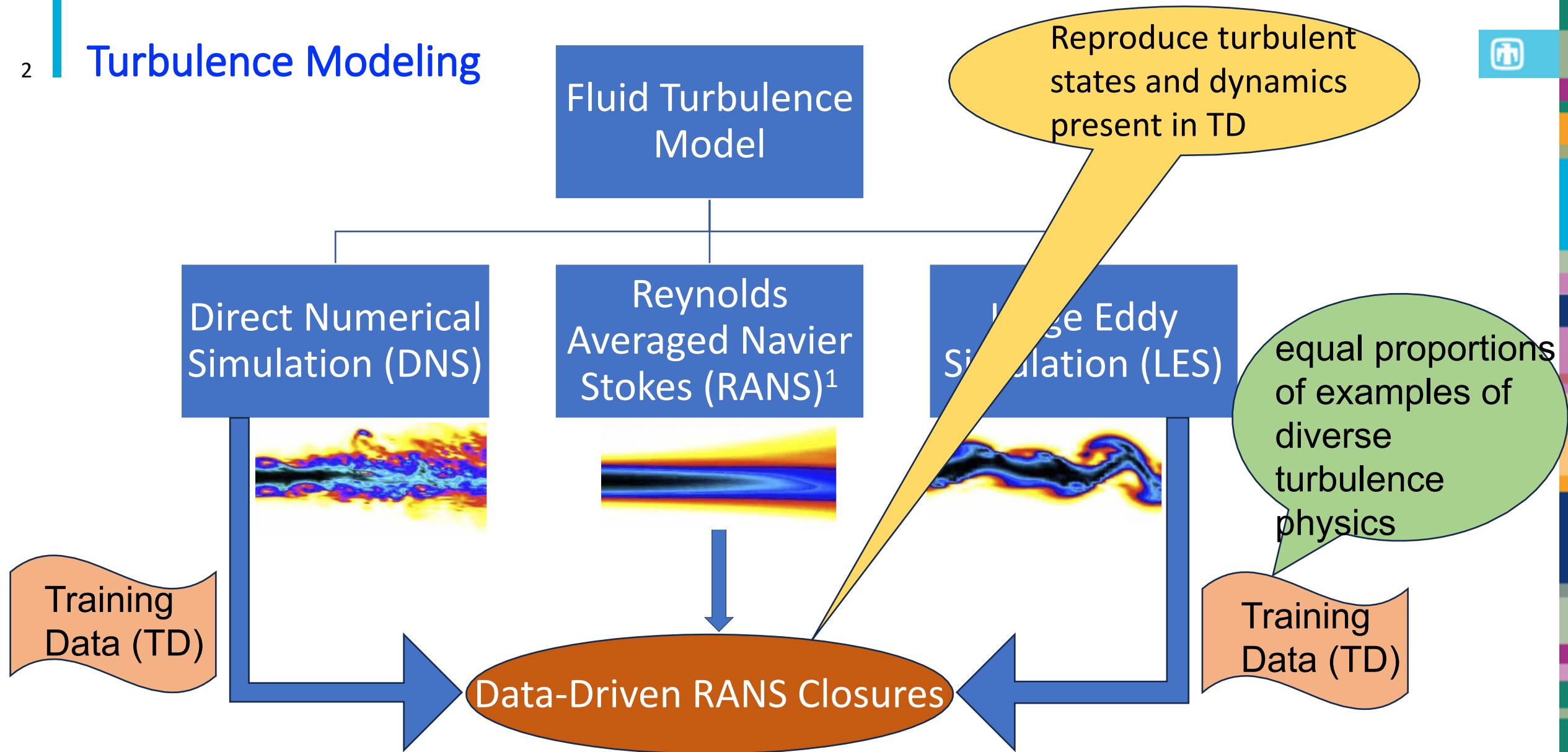
Collaborators: William J Rider, Matthew Barone, Eric J. Parish

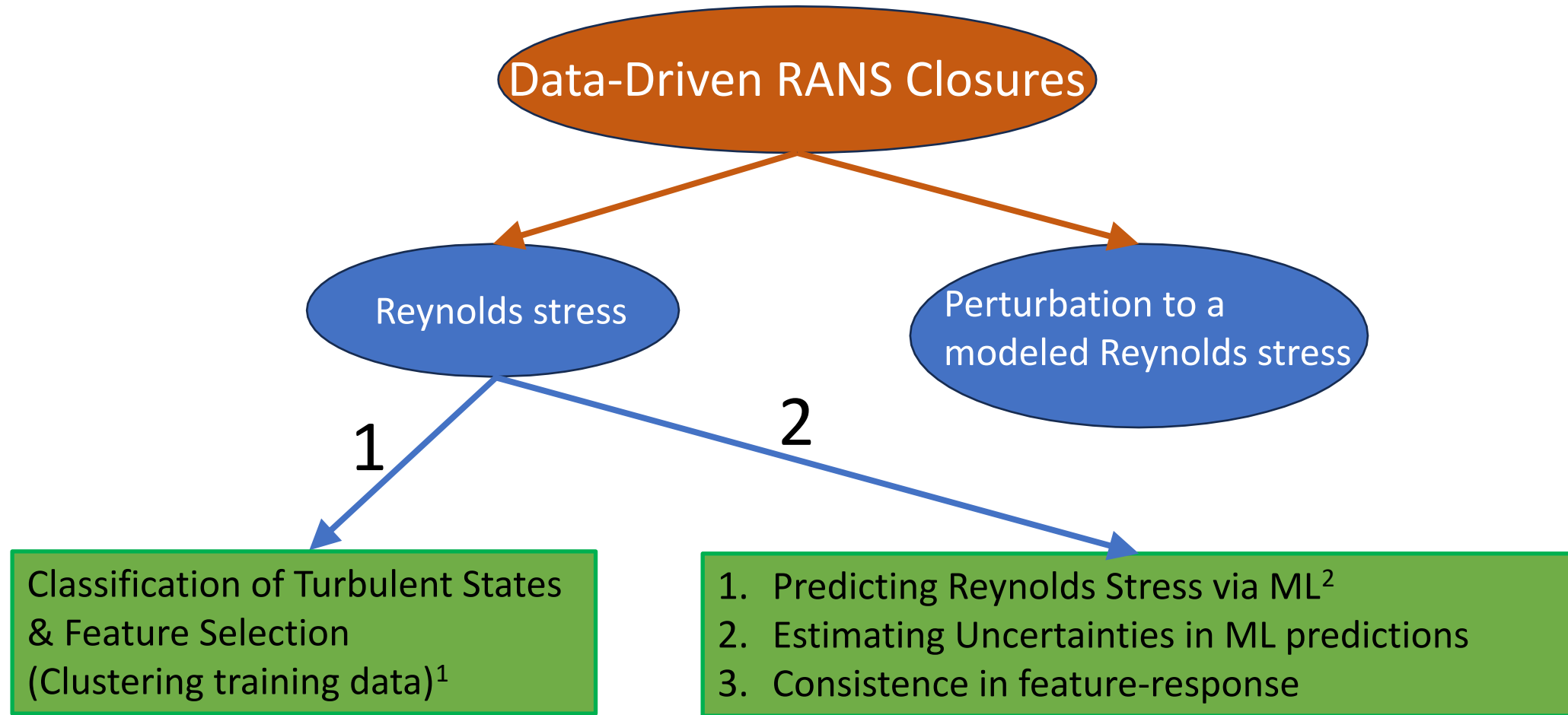
Sandia National Laboratories



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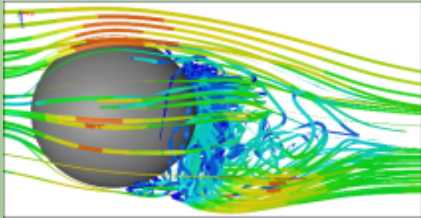
1. M.F.Barone, J.Ray, and S.Domino, Feature Selection, Clustering, and Prototype Placement for Turbulence Datasets, AIAA Journal 2022 60:3, 1332-1346.
2. E. Parish, D.S. Ching, N.E. Miller, S. J. Beresh and M. F. Barone. "Turbulence modeling for compressible flows using discrepancy tensor-basis neural networks and extrapolation detection," AIAA 2023-2126. *AIAA SCITECH 2023 Forum*. January 2023.

# Motivating Credibility for Scientific Machine Learning (SciML)

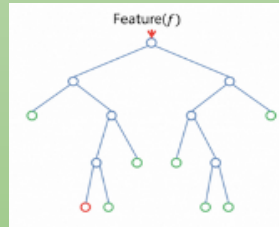


Machine learned models are used in lieu of, complementary to, or as surrogates for science and engineering computational simulation models.

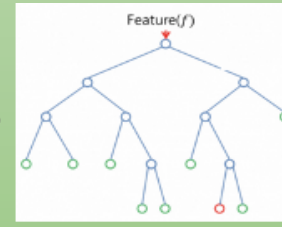
What does VV/UQ/Credibility Mean for Scientific Machine Learning?



Scientific Computing

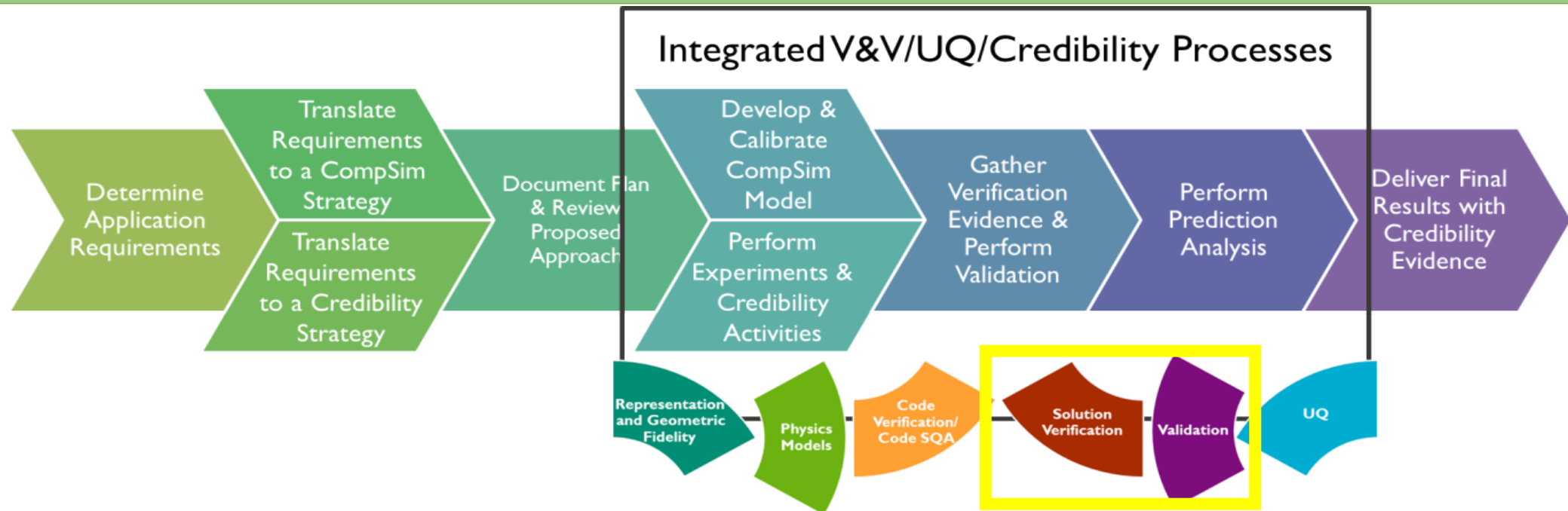


Machine Learning



Putting “correct” math methods and physics models into our codes.

Produce “correct” codes and models which leads to “correct” results.



# Data-Driven Turbulence Modeling



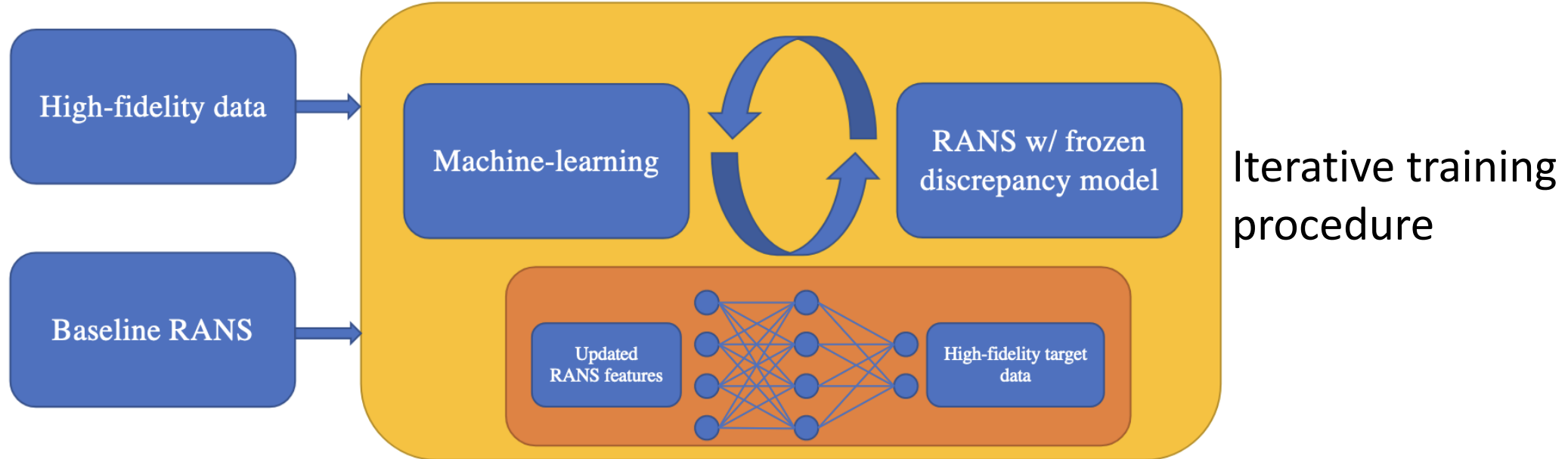
$$a_{ij} \approx a_{ij}^{\text{RANS}} + m_{ij}^{\text{ML}}$$

**Anisotropy-based discrepancy term**

**Anisotropy tensor**  $a_{ij} = \frac{-\tau_{ij}}{\rho u_k'' u_k''} - \frac{1}{3} \delta_{ij}$

**Anisotropy tensor predicted by a standard RANS**  $a_{ij}^{\text{RANS}} = \frac{\tau_{ij}^{\text{RANS}}}{2\bar{\rho}k} - \frac{1}{3} \delta_{ij}$

$\delta_{ij}$  is the Kronecker delta &  $m_{ij}^{\text{ML}}$  is an ML correction





# Data-Driven Turbulence Modeling

## High Fidelity Datasets for Training

- A. Channel flow with  $Re = 180$
- B. Channel flow with  $Re = 395$
- C. Channel flow with  $Re = 590$
- D. Duct flow at  $Re = 3500$
- E. Flow over periodic hill
- F. HS BL at  $M = 6$ ,  $T_w/T_r = 0.25$
- G. HS BL at  $M = 6$ ,  $T_w/T_r = 0.76$
- H. HS BL at  $M = 14$ ,  $T_w/T_r = 0.18$

High-fidelity data

Baseline RANS

Machine-learning

RANS w/ frozen  
discrepancy model

Updated  
RANS features

High-fidelity target  
data

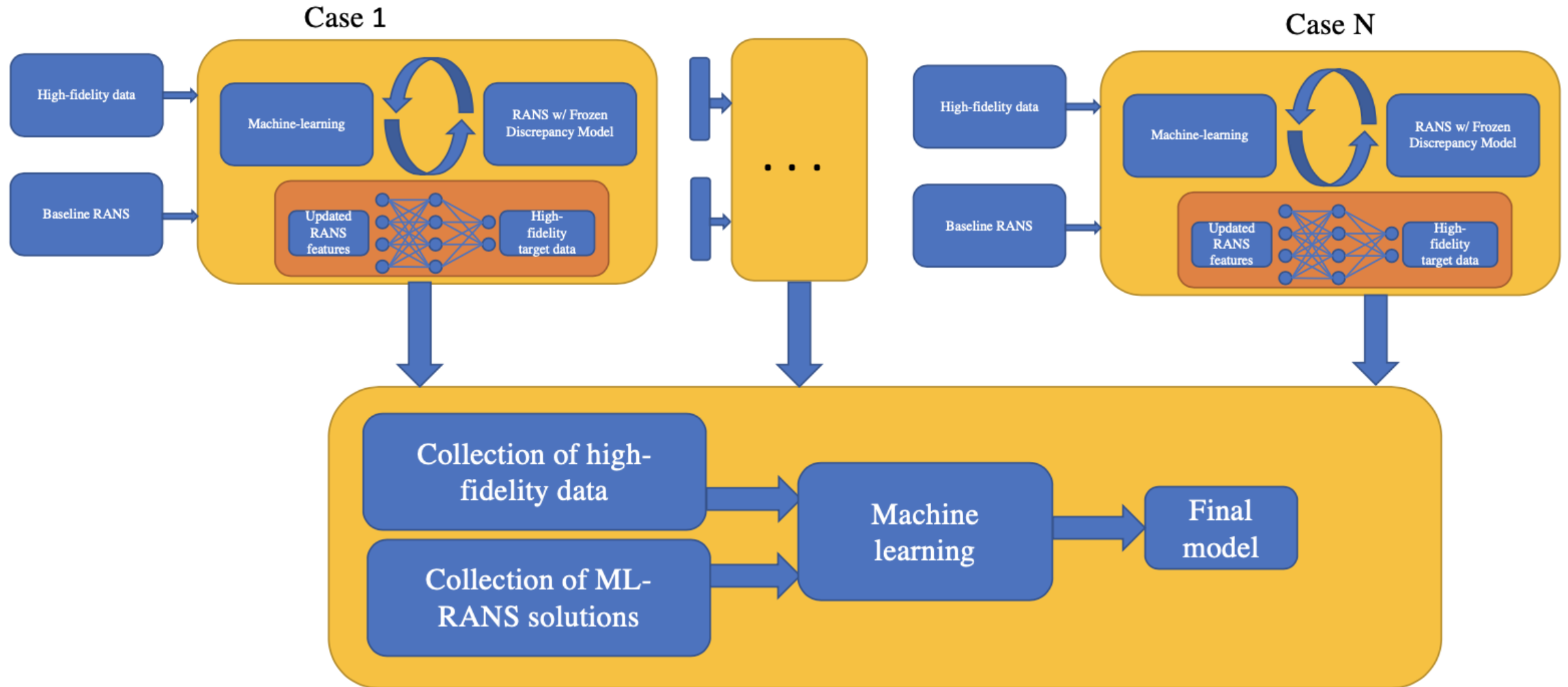
$$a_{ij} \approx a_{ij}^{\text{RANS}} + m_{ij}^{\text{ML}}$$

Discrepancy modes for an anisotropy tensor are implemented in Sandia's Parallel Aerodynamics Re-entry Code (SPARC) which supports various discretization



Validation

# Global Training Process

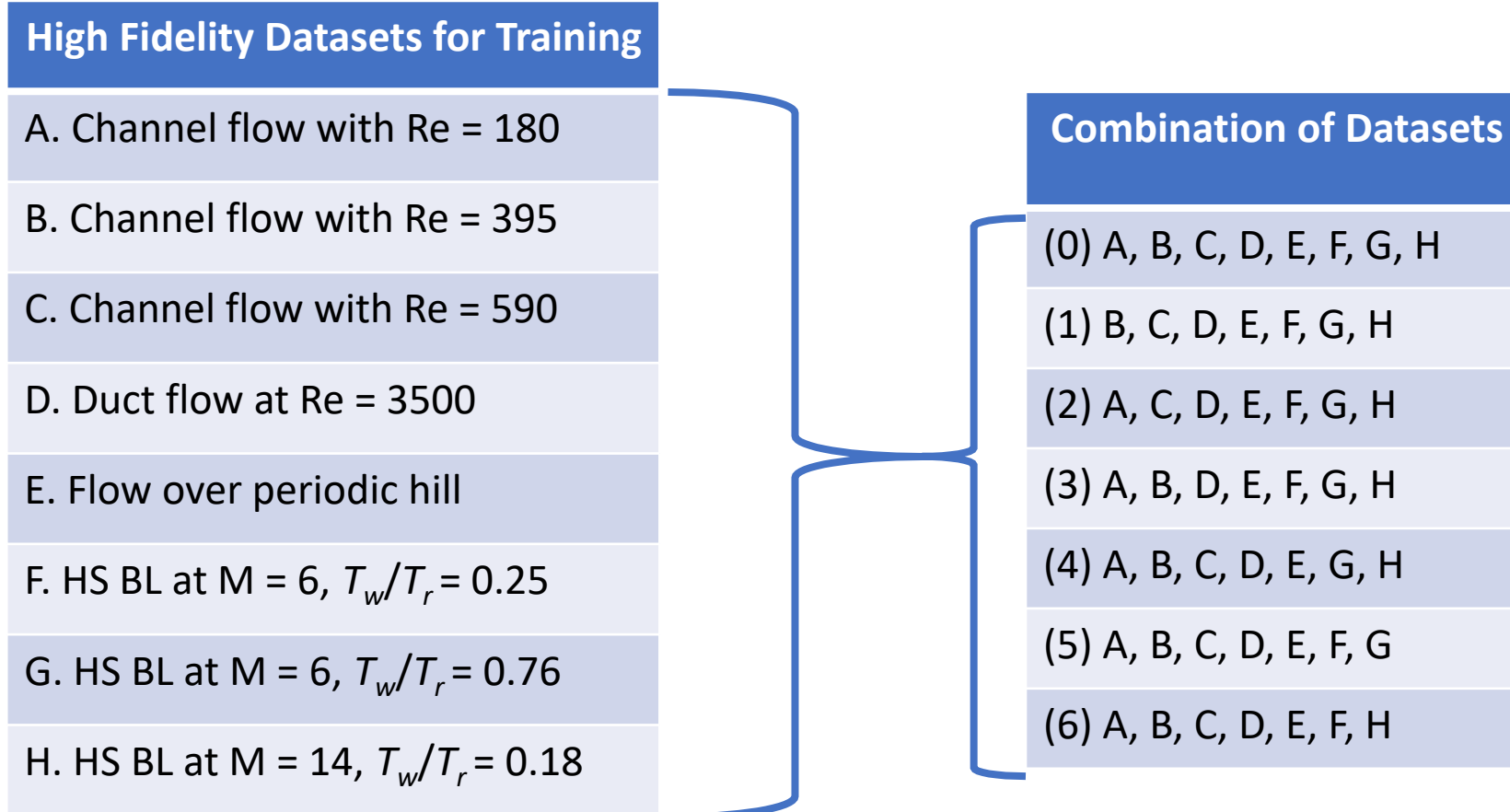




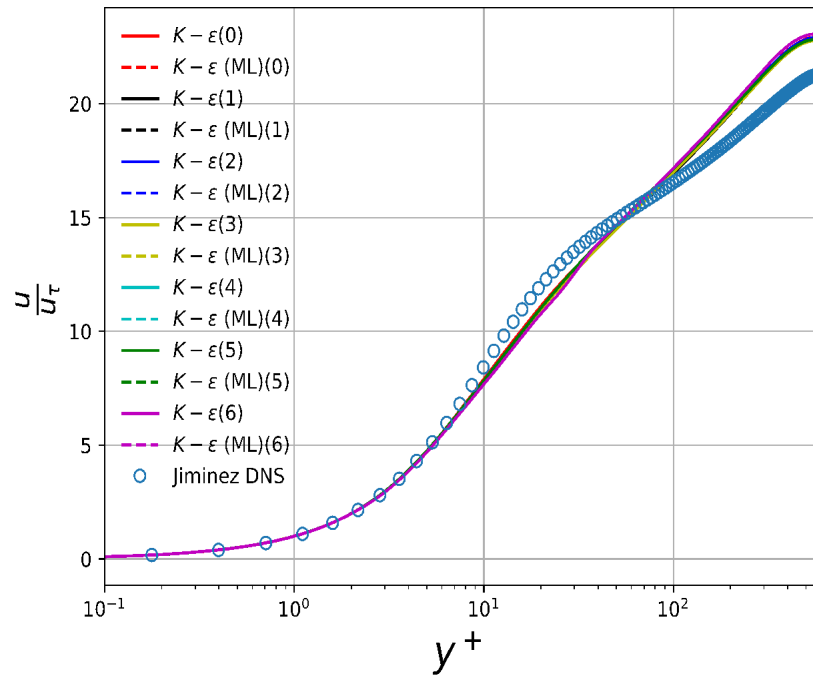
- Global iterative training procedure improves feature consistency.
- Complete consistency in response has not been achieved.
- The goal is to minimize overall inconsistency.
- ML models involve many hyperparameters.
- Considering various combinations of training datasets and testing hyperparameters might help validate and improve the overall response consistency.



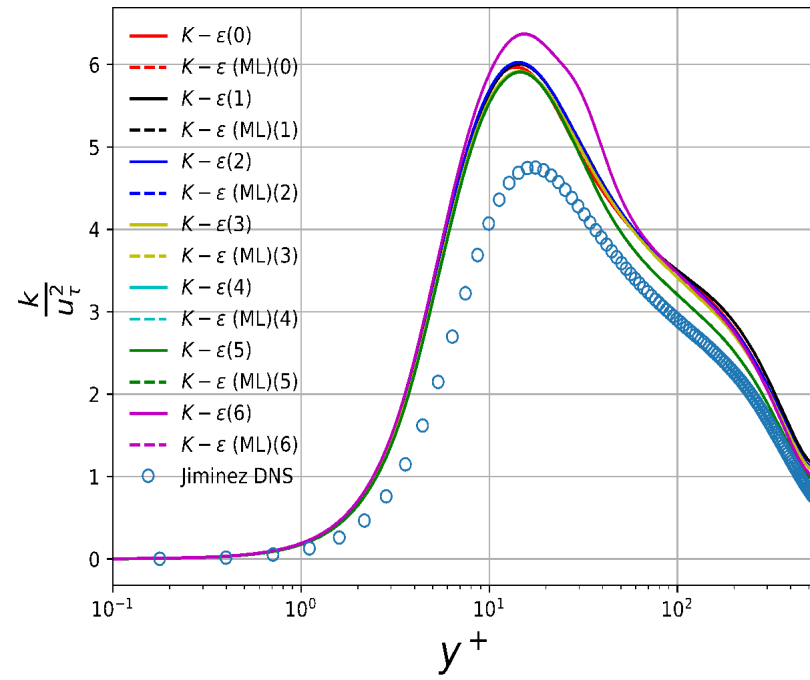
# Various Combination of Training Datasets



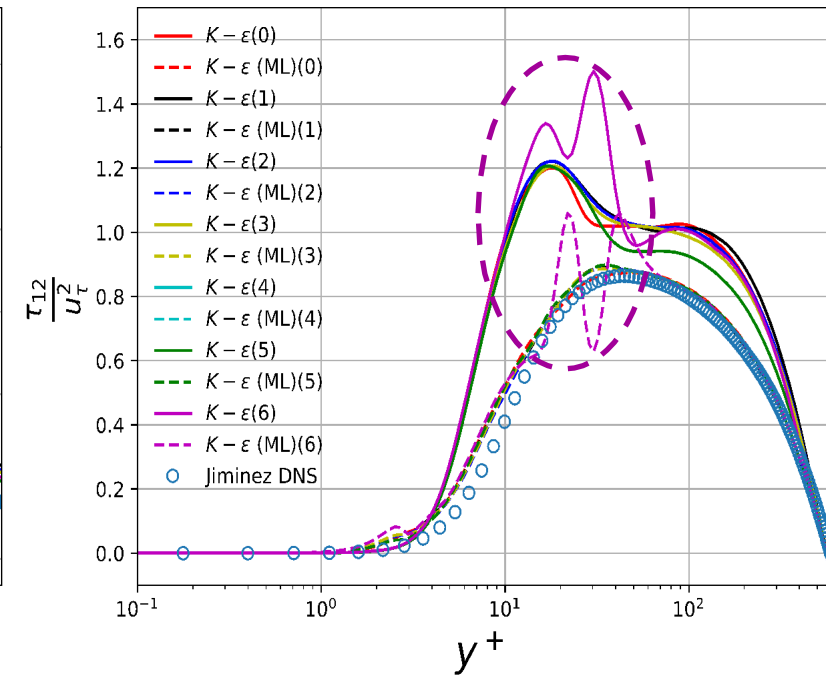
# Validating Training Datasets & Testing Channel Flow



**Velocity**



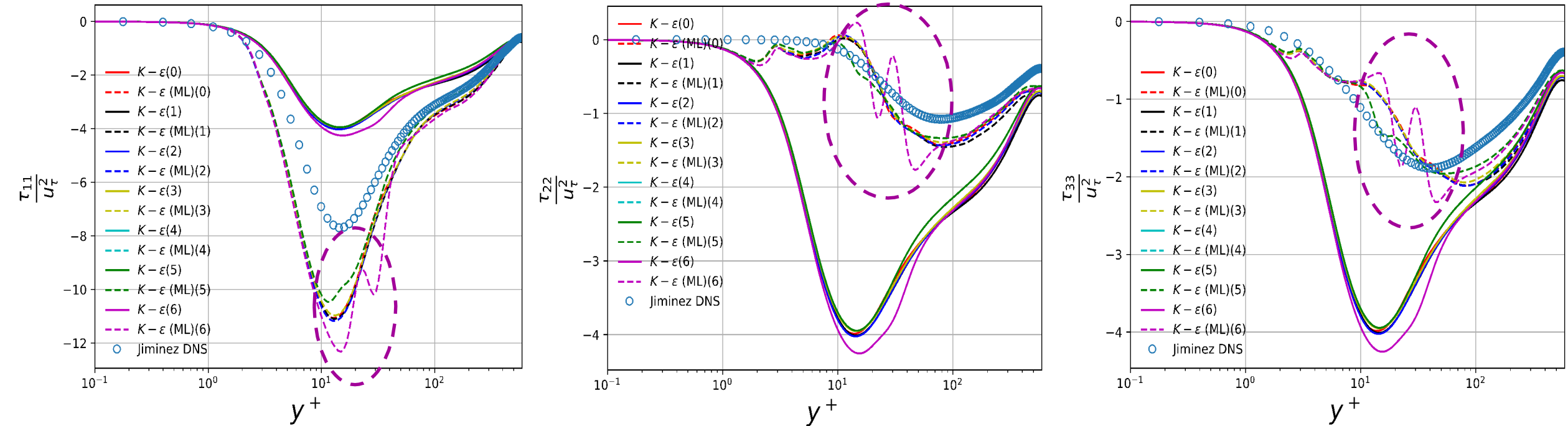
**Turbulence Kinetic Energy**



**Reynolds Shear Stress**

- The global iterative procedure was trained on various combinations of datasets as described in the previous slide (w/o changing any hyperparameters).
- It was then tested on the channel flow dataset with  $Re = 590$ .

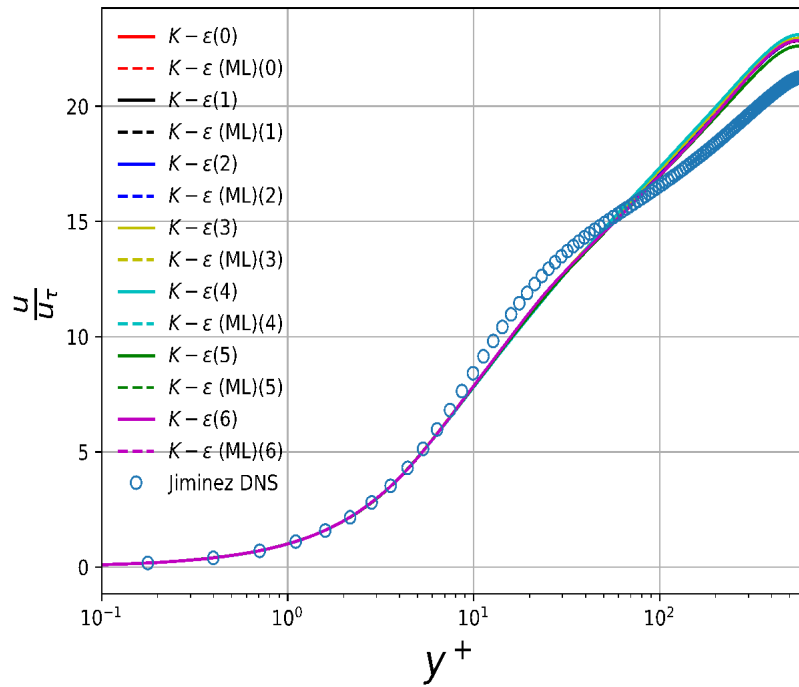
# Validating Training Datasets & Testing Channel Flow



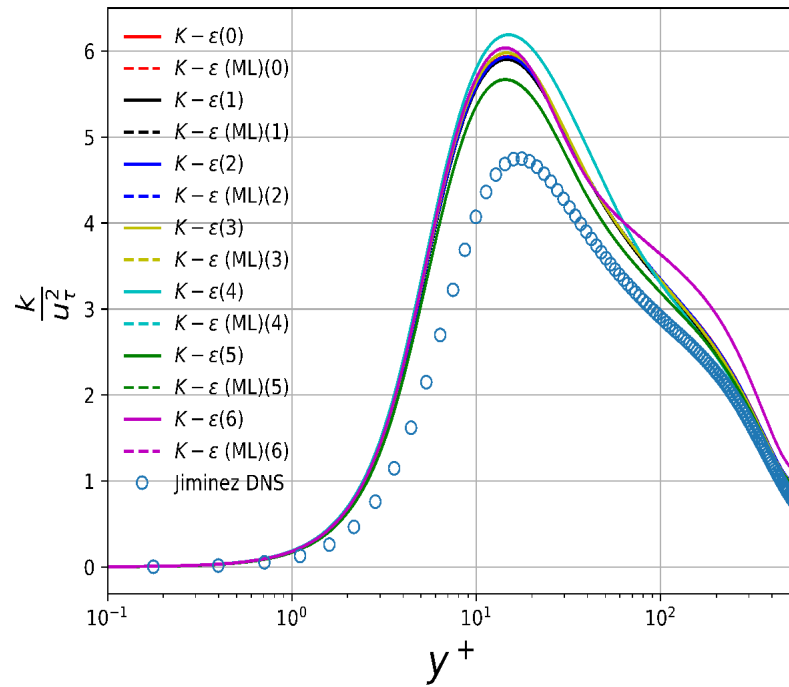
**Normal Stress in x, y & z direction**

- Figures clearly show that the ML correction term follows the trend of the "true" (DNS) data. However, there is a deviation in the buffer layer, which is consistent across all combinations of training datasets.
- Dataset combination (6) exhibits clear oscillations.

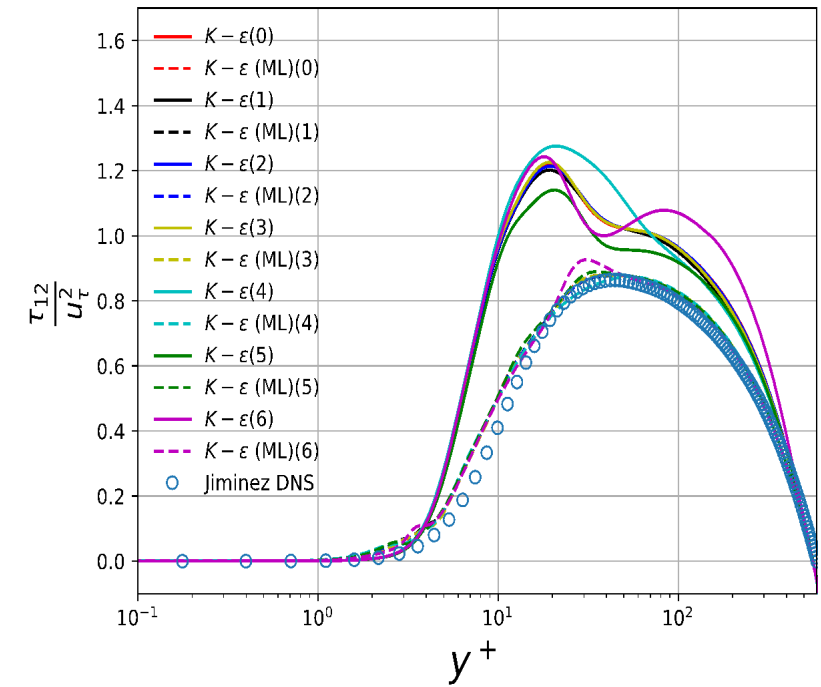
# Validating Training Datasets & Testing Channel Flow



**Velocity**



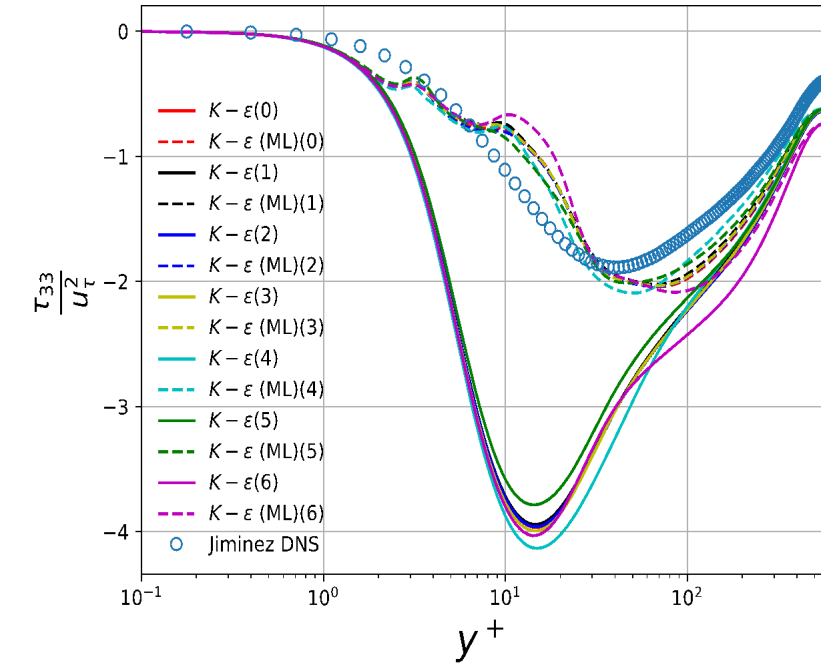
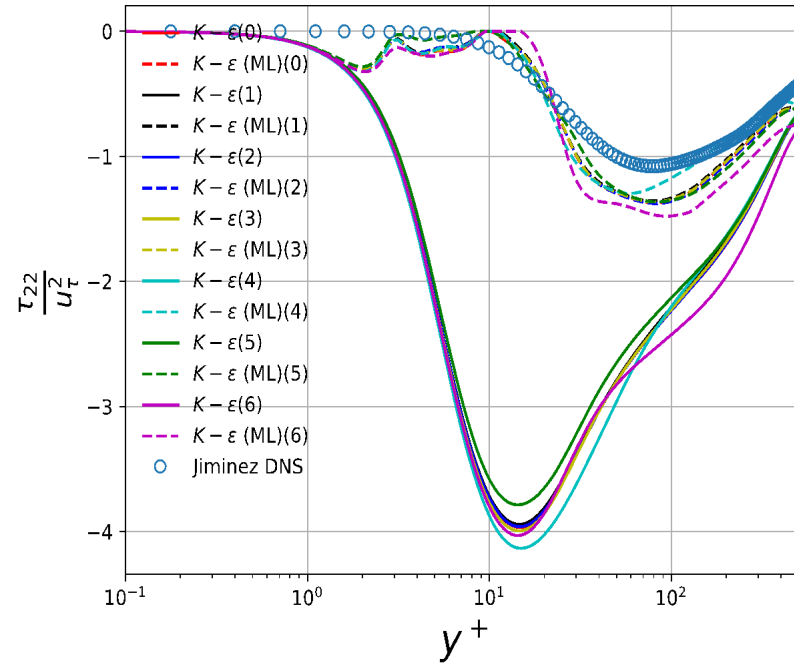
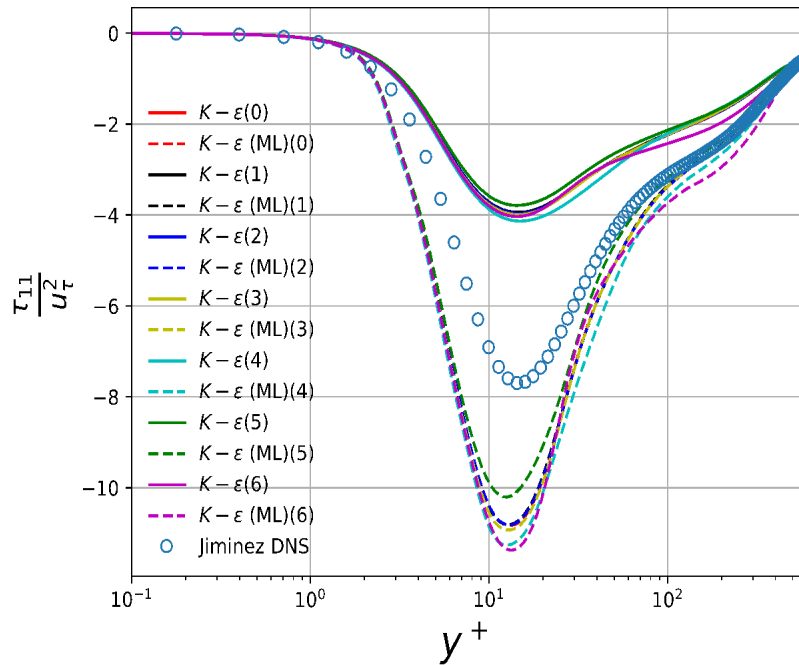
**Turbulence Kinetic Energy**



**Reynolds Shear Stress**

- Reducing the depth and width of the neural network along with an optimum epochs clearly reduces the overfitting problem.

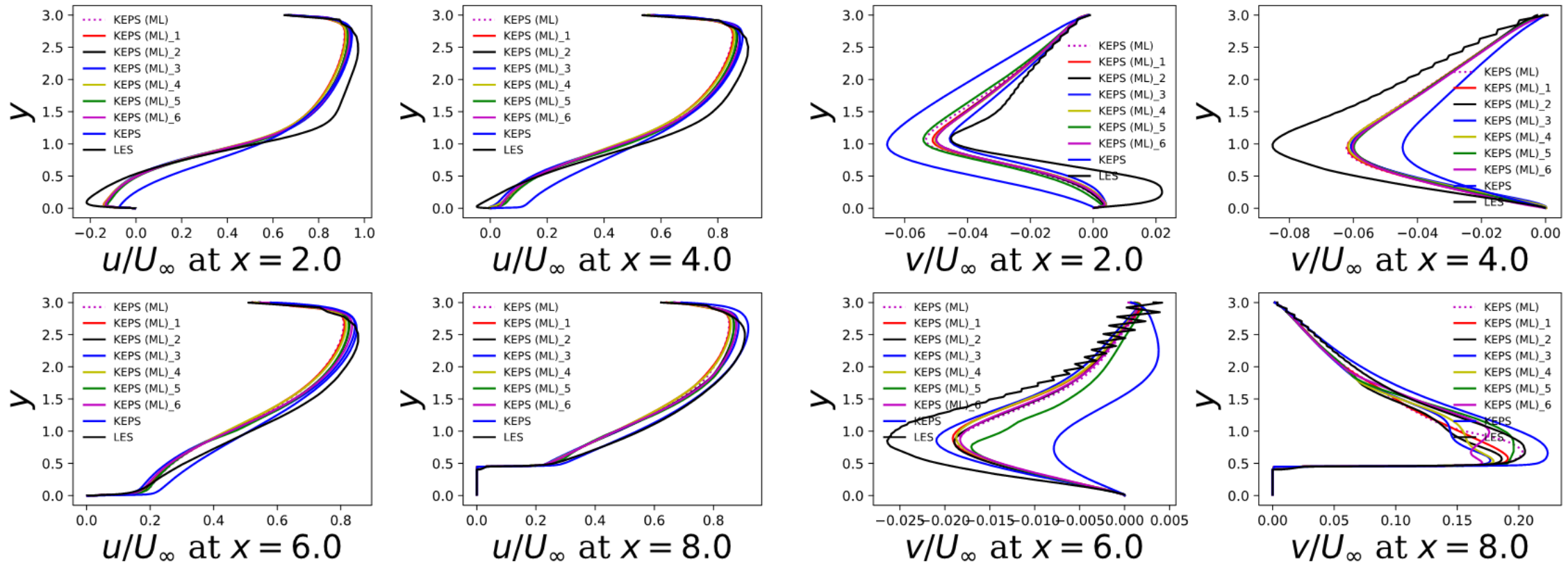
# Validating Training Datasets & Testing Channel Flow



## Normal Stress in x, y & z direction

- Reducing the depth and width of the neural network along with an optimum epochs clearly reduces the oscillatory behavior.
- However, there is still a deviation in the buffer layer, which is consistent across all combinations of training datasets.

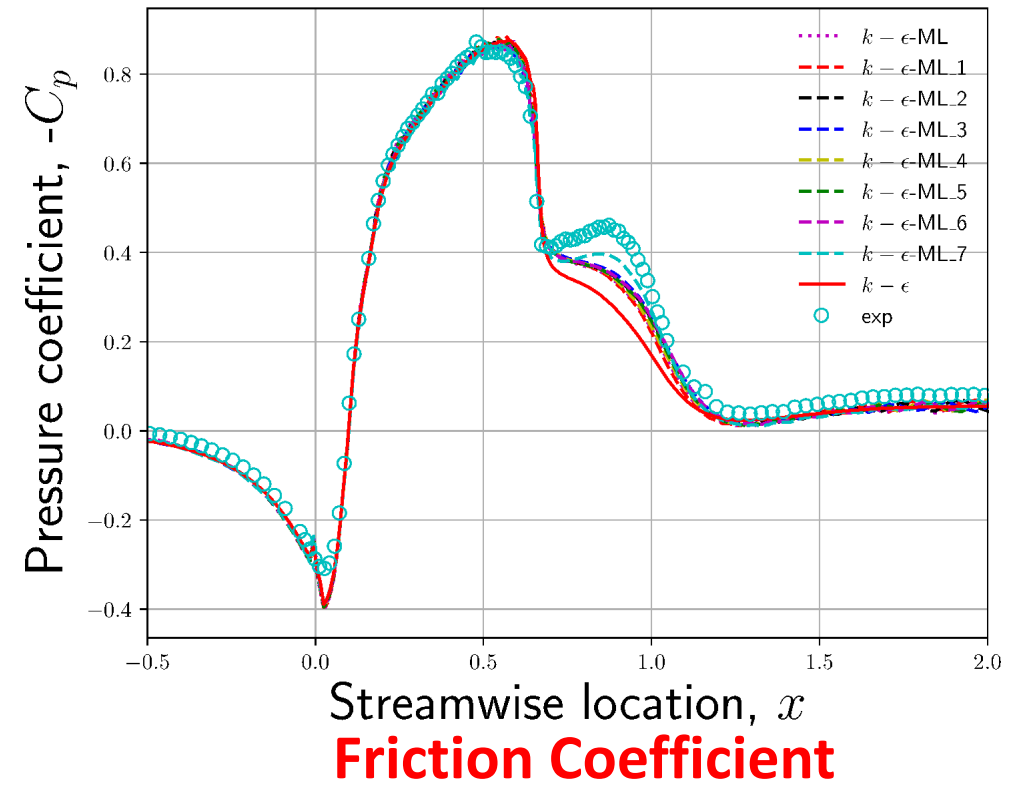
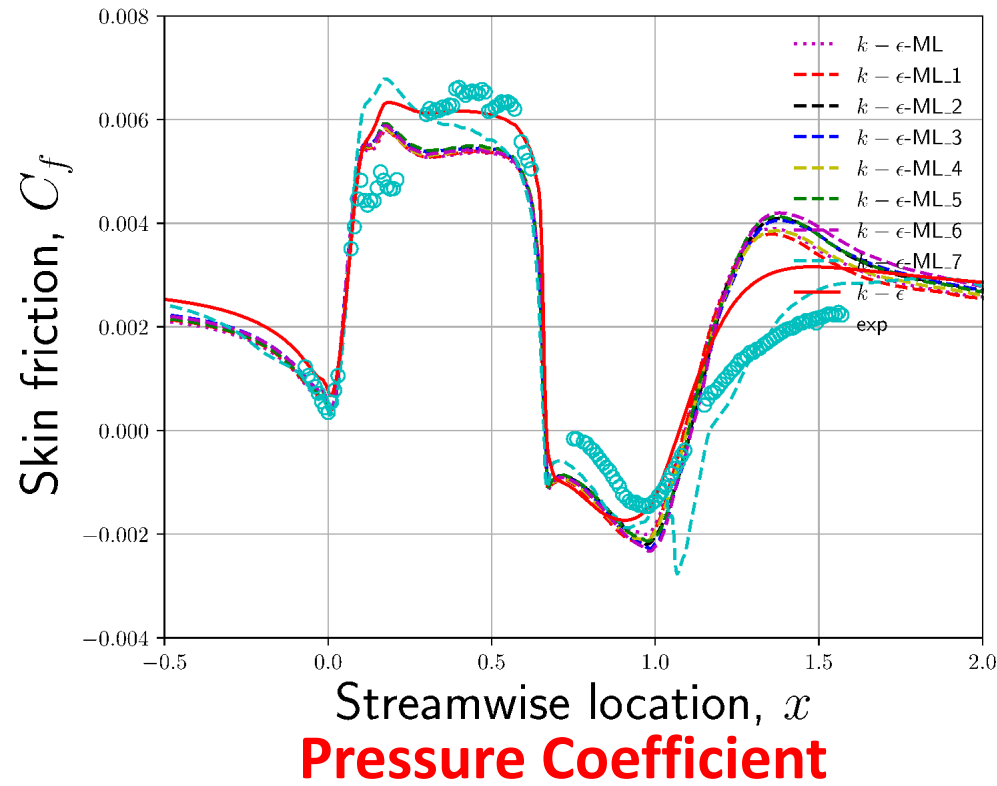
# Validating Training Datasets & Testing Periodic Hill



- We then tested on the Periodic Hill, and the figure shows the velocity profiles as a function of  $y$  for various  $x$  locations.
- The figures clearly show that the ML correction term performs better than the standard  $k-\epsilon$  model.



# Validating Training Datasets & Testing NASA Hump (Out-Of-Distribution)



- Figure depicts the pressure coefficient and skin friction as a function of the streamwise location for the  $k-\epsilon$  and  $k-\epsilon$ -ML models.
- We observe that the  $k-\epsilon$ -ML slightly under predicts the peak in pressure and friction coefficient which needs to be improved.

## Conclusions and Future Work



- **Performance Consistency:** We conducted rigorous testing on a variety of in-distribution and out-of-distribution datasets using different combinations of training datasets, which demonstrated consistent performance across these combinations.
- **Hyperparameter Tuning:** Some combinations of training datasets highlight the need for hyperparameter tuning to reduce inconsistency in the anisotropy-based discrepancy term.
- **Network Depth Reduction:** Reducing the depth and width of the network effectively along with optimum epochs mitigates oscillation and overfitting, yet there remains inconsistent behavior in the anisotropy-based ML correction with "true" DNS data.
- **Future Work:** We will focus on explainable machine learning models for turbulence closures utilizing SHAP (SHapley Additive exPlanations) & LIME (Local Interpretable Model-Agnostic Explanations) analysis.



# Thank You for Your Time and Attention!

For questions or follow-up discussions:

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