Linear Solver for Electromagnetic Simulation of General Distribution Feeders

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***Abstract*—High-fidelity electromagnetic transient (EMT) modeling is required for accurate simulation and analysis of power system dynamics in modern distribution feeders. However, the high-fidelity of EMT models often leads to significant computational challenges, particularly in terms of computational resources and simulation time. This paper investigates the development and application of a detailed EMT model for general distribution feeders, with a focus on improving computational efficiency. A direct linear solver is proposed for a bordered block diagonal (BBD) matrix structure commonly encountered in a EMT model of distribution feeders. The solver integrates the Schur complement method with the block tridiagonal matrix algorithm to enhance the computational performance. The proposed solver is validated using the primary feeder of the IEEE 342-node test system, demonstrating its accuracy and efficiency in EMT simulations. Furthermore, the solver's performance is benchmarked against MATLAB's built-in linear solvers, showing significant improvements in computation time while maintaining high fidelity and accuracy in simulation results.**

***Index Terms*—Block Thomas algorithm, block tridiagonal matrix, bordered block diagonal matrix, Distribution feeder, electromagnetic transient simulation, Schur complement method.**

I. Introduction

The increasing penetration of inverter-based resources (IBRs) into modern power systems has introduced significant challenges in modeling and simulating their behavior accurately with the grid. Recent reports by the North American Electric Reliability Corporation (NERC) have emphasized the necessity for detailed electromagnetic transient (EMT) models to represent IBR dynamics accurately during external disturbances [1], [2]. As power electronics continue to penetrate in this energy landscape, detailed modeling and EMT simulations have become increasingly recommended for analyzing power systems with inverter-based resources.

Performing EMT simulations on distribution feeders with detailed modeling, however, presents substantial computational challenges. The detailed modeling required for accurate representation leads to larger systems with high computational demands to simulate, resulting in extended simulation times and resource constraints. Traditional EMT simulation methods struggle to efficiently handle the complexity and size of these models, necessitating the development of accelerated simulation techniques and algorithms.

Diakoptics, also known as network tearing method, was introduced to simplify and solve large-scale systems by decomposing them into smaller and more manageable subproblems. This technique has been actively utilized for large-scale power grid simulations [3]–[6]. Block LU decomposition has been frequently applied in EMT simulations of large-scale power grids, particularly leveraging parallel computation to efficiently handle block matrices and sparse systems [7]–[9]. Recently, a matrix splitting technique for power electronics-dominated distribution systems have been proposed to further expedite EMT simulations, aiming to capture the dynamics of IBRs with enhanced computational speed [10], [11]. While this method introduces significant speed improvements, it often relies on the inclusion of small artificial capacitors to maintain numerical stability. The Schur complement method has been explored as an alternative to accelerate EMT simulations without relying on artificial capacitors [12], [13]. While all of these methods offer more efficient handling of large-scale systems by reducing the size of the system matrix, they treat the sub-blocks as dense matrices. By ignoring the inherent sparsity or structure within these sub-blocks, these methods miss opportunities to further enhance computational efficiency.

In response to these limitations, this paper proposes a direct linear solver for general distribution feeders by integrating a block tridiagonal matrix algorithm (also known as block Thomas algorithm) with Schur complement method. This novel approach extends the capability of accelerated EMT simulations to distribution systems with arbitrary configurations and parallel computation, enhancing the performance of previous methods. Additionally, this work delves into the detailed modeling of distribution feeders, enhancing the accuracy and reliability of simulation results. The effectiveness of the proposed solver is validated using the primary feeder of the IEEE 342-node test system. Comparative analyses with MATLAB built-in linear solvers demonstrate significant

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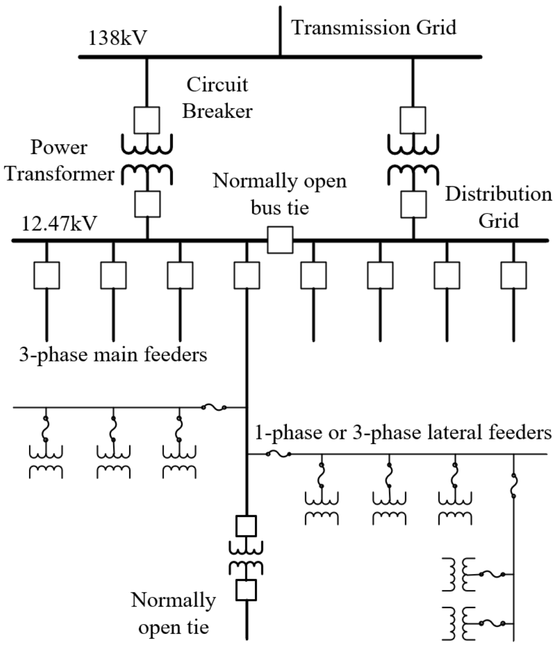

Fig. 1: Illustration of typical distribution substation and feeders [15]

improvements in computational performance, confirming the solver's practicality for large-scale EMT simulations [14].

II. EMT Model of Distribution Feeders

Distribution systems are typically configured with long radial feeders collected in parallel by substation transformers. A typical distribution system generally comprises 1 to 6 station transformers, with each bus accommodating 1 to 8 feeders, and the main feeder length ranging from 2 to 15 miles [15]. A single line diagram for a typical distribution substation and feeders is illustrated in Fig. 1.

A. Transformers

A station transformer is a high-voltage power transformer that boosts up the distribution system medium voltage (MV) to the transmission system high voltage (HV) at distribution substation as shown in Fig. 1. Along with distribution main feeders, there are two types of transformer such as distribution transformers and in-line transformers. Distribution transformers step down MV to low voltage (LV). In-line transformers step down MV within MV range (e.g., 12.47 kV to 2.4 kV). Both types of the transformers can be represented by the classical transformer model as shown in Fig. 2. The dynamics of the transformers can be expressed by the following differential equations (1)−(2).

$$(L_j^{P(s)} + L_m)\frac{di_{j,\text{grid}}^{P(s)}}{dt} + R_j^{P(s)} i_{j,\text{grid}}^{P(s)} - L_m \frac{di_{j,\text{grid}}^{s}}{dt} = -v_{j,\text{grid}}^{P(s)} \quad (1)$$

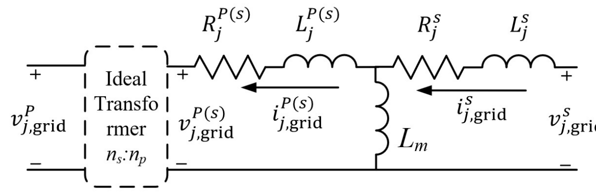

Fig. 2: Classical transformer model

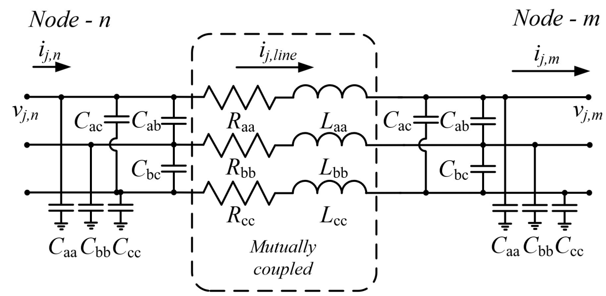

Fig. 3: Three phase PI section line model for feeders

$$L_j^{P(s)}\frac{di_{j,\text{grid}}^{P(s)}}{dt} + R_j^{P(s)} i_{j,\text{grid}}^{P(s)} + L_j^s \frac{di_{j,\text{grid}}^{s}}{dt} + R_j^s i_{j,\text{grid}}^s = v_{j,\text{grid}}^s - v_{j,\text{grid}}^{P(s)} \quad (2)$$

where $v_{j,\text{grid}}^p$ are primary side voltages (j=a,b,c). $v_{j,\text{grid}}^{p(s)}$ are converted primary side voltages in the secondary side (j=a,b,c). $v_{j,\text{grid}}^s$ are secondary side voltages (j=a,b,c). n_s/n_p is turns ratio of the transformer. $L_j^{P(s)}$ and $R_j^{P(s)}$ are equivalent inductance and resistance of the primary side of the transformer converted in secondary side. L_j^s and R_j^s are equivalent inductance and resistance of the secondary side of the transformer. L_m is equivalent inductance for magnetizing currents of the transformer. $i_{j,\text{grid}}^{P(s)}$ are primary side currents converted into the secondary side (j=a,b,c). $i_{j,\text{grid}}^s$ are secondary side currents (j=a,b,c).

B. Feeder Lines

Distribution feeders are typically in radial configuration by overhead lines or underground cables, which interconnect end users with substation transformers as shown in Fig. 1. Distribution systems generally cover large and wide area around 0.5-500 mi^2, resulting in having multiple radial feeders with a long distance. Due to its relatively short lengths between bus (compared to transmission lines), feeder lines can be generally represented by a PI section line model with mutual coupling effects among phases. A three-phase PI section model between nodes *n* and *m* for distribution feeders is illustrated in Fig. 3. The EMT model of the PI section line can be expressed by using differential equations (3)−(5) (for phase a):

$$L_{aa}\frac{di_{a,line}}{dt} + R_{aa} i_{a,line} + L_{ab}\frac{di_{b,line}}{dt} + R_{ab} i_{b,line} + L_{ca}\frac{di_{c,line}}{dt} + R_{ca} i_{c,line} = v_{a,n} - v_{a,m} \quad (3)$$

$$C_{aa}\frac{dv_{a,n}}{dt} + C_{ab}\frac{dv_{b,n}}{dt} + C_{ac}\frac{dv_{c,n}}{dt} = i_{a,n} - i_{a,line} \quad (4)$$

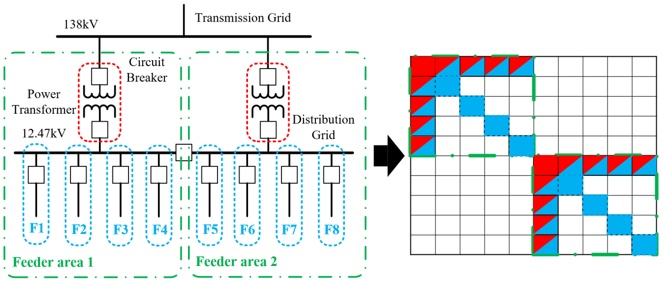

Fig. 4: Typical distribution substation and feeders (left) and corresponding system matrix for EMT simulation (right)

$$C_{aa}\frac{dv_{a,m}}{dt}+C_{ab}\frac{dv_{b,m}}{dt}+C_{ac}\frac{dv_{c,m}}{dt}=i_{a,line}-i_{a,m} \quad (5)$$

where, R_{ii}, L_{ii}, and C_{ii} (i=a,b,c) are self-resistance, self-inductance, self-capacitance of the PI section line model. R_{ij}, L_{ij}, and C_{ij} ($i \neq j$, i,j=a,b,c) are mutual-resistance, mutual-inductance, mutual-capacitance of the PI section line model.

C. System Matrix by Transformer-Feeder Configuration

Configuration of substation transformer and feeders in distribution systems significantly influences the structure and properties of its system matrix used in EMT simulations as presented in Fig. 4. The figure at the left side presents a typical distribution system from Fig. 1, comprising two substation transformers connected to the transmission grid and the distribution grid. This distribution network is divided into two areas: Feeder Area 1 (left) and Feeder Area 2 (right), each containing four feeders (F1 to F4 in Area 1 and F5 to F8 in Area 2). The right-hand side of the figure represents a system matrix that corresponds to the left figure, which is also known as a bordered block diagonal (BBD) matrix. The red cells indicate the station transformers collecting the feeders, while the half red and blue cells represent sections of the network where the feeders are connected with the station transformers. The blue cells represent each radial feeders in the feeder areas. The green lines segment the system matrix into different feeder areas, indicating distinct regions of the distribution network in Fig. 4. These structural properties of the matrix, such as size, sparsity, and complexity, are all determined by the physical arrangement of the network components shown on the left.

An increase in the number of parallel feeders connected to a substation leads to a greater number of diagonal blocks within the system matrix as shown in Fig. 5. In this particular case, the number of parallel feeders increase from 4 to 8 for both areas. Each parallel feeder contributes additional diagonal blocks, thereby expanding the matrix's overall size and complexity. Conversely, extending the length of series feeders increases the size of each diagonal block, as longer feeders incorporate more series elements that must be accurately modeled. In this particular case, the number of bus in the series feeders double up for both areas.

The Schur complement method in [12] is particularly effective for systems with multiple parallel feeders because it efficiently manages the increased number of diagonal blocks without significantly escalating computational demands. However, the method's applicability is limited when dealing with long feeders in series. In configurations where feeders are connected in series, the diagonal blocks become larger and more complex due to the additional components involved. The Schur complement method does not adequately address these larger diagonal matrices, rendering it less effective for series feeder systems.

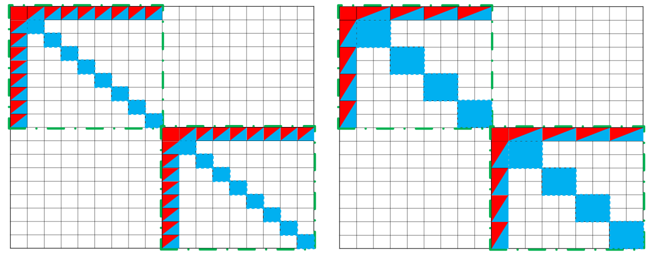

Fig. 5: System matrix: increasing number of feeders in parallel (left) and increasing length of feeders in series (right)

To overcome this limitation, an additional algorithm such as block tridiagonal matrix algorithm is necessary. This solver is specifically designed to handle the increased size and complexity of block tridiagonal matrices resulting from longer series feeders. By incorporating this solver into the EMT simulation process, it becomes possible to maintain numerical stability and computational efficiency, even in distribution feeders with complex series configurations.

III. Direct Linear Solver for General Distribution Feeder

Linear solvers for EMT simulation of distribution networks need to handle large BBD matrices, characterized by both numerous and sizable diagonal blocks. Operating on the large BBD matrix at every simulation time-step can be computationally expensive. To address this, a direct linear solver is proposed by combining the Schur complement method with the block Thomas algorithm, effectively reducing the size of the matrix operated on during simulations [16], [17]. This approach enables efficient management of large BBD matrices, accelerating the simulation of general distribution networks while preserving the accuracy of the results.

A. Schur Complement Method

The $\mathbf{A}$ matrix of distribution network with BBD matrix form can be expressed in (6)

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \dots & \mathbf{A}_{1N} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{0} & \mathbf{0} \\ \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{A}_{N1} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{NN} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_N \end{bmatrix}. \quad (6)$$

Eq. (6) can be expanded into its individual equations as

$$\mathbf{A}_{11}\mathbf{x}_1 + \mathbf{A}_{12}\mathbf{x}_2 + \dots + \mathbf{A}_{1N}\mathbf{x}_N = \mathbf{b}_1 \quad (7a)$$

$$\mathbf{A}_{21}\mathbf{x}_1 + \mathbf{A}_{22}\mathbf{x}_2 = \mathbf{b}_2 \quad (7b)$$

$$\vdots$$

$$\mathbf{A}_{N1}\mathbf{x}_1 + \mathbf{A}_{NN}\mathbf{x}_N = \mathbf{b}_N. \quad (7c)$$

Eqs. (7b) through (7c) can be rearranged for $\mathbf{x}_2$ through $\mathbf{x}_\mathrm{N}$ as

$$\mathbf{x}_2 = \mathbf{A}_{22}^{-1}(\mathbf{b}_2 - \mathbf{A}_{21}\mathbf{x}_1) \tag{8a}$$

$$\vdots$$

$$\mathbf{x}_\mathrm{N} = \mathbf{A}_\mathrm{NN}^{-1}(\mathbf{b}_\mathrm{N} - \mathbf{A}_\mathrm{N1}\mathbf{x}_1). \tag{8b}$$

Substituting $\mathbf{x}_2$ through $\mathbf{x}_\mathrm{N}$ in (7a) with (8a) and (8b) yields

$$\mathbf{A}_{11}\mathbf{x}_1 + \sum_{n=1}^{\mathrm{N}} \mathbf{A}_{1n}\mathbf{A}_{nn}^{-1}(\mathbf{b}_n - \mathbf{A}_{n1}\mathbf{x}_1) = \mathbf{b}_1. \tag{9}$$

To solve for $\mathbf{x}_1$, (9) can be rearranged as

$$\mathbf{x}_1 = \left(\mathbf{A}_{11} - \sum_{n=1}^{\mathrm{N}} \mathbf{A}_{1n}\mathbf{A}_{nn}^{-1}\mathbf{A}_{n1}\right)^{-1} \times \left(\mathbf{b}_1 - \sum_{n=1}^{\mathrm{N}} \mathbf{A}_{1n}\mathbf{A}_{nn}^{-1}\mathbf{b}_n\right). \tag{10}$$

Once $\mathbf{x}_1$ is solved by (10), the rest of $\mathbf{x}_n$ $(n = 2, 3, ..., \mathrm{N})$ can be solved by (8a) through (8b). By this method, the size of the operated A matrix is significantly reduced from that of the complete A matrix to that of the block diagonal sub-matrices. Furthermore, once $\mathbf{x}_1$ is known, it can enable inherent parallelism when solving the rest of $\mathbf{x}_n$ $(n = 2, 3, ..., \mathrm{N})$ due to their independence.

B. Block Thomas Algorithm

When solving (10) for $\mathbf{x}_1$, $\mathbf{A}_{nn}^{-1}$ needs to be calculated for each diagonal blocks for $n = 2, ..., \mathrm{N}$ in (6). This can be a significant computational bottleneck, especially when dealing with a large number and size of diagonal blocks, which commonly occurs in long distribution feeders typically found in distribution systems. When modeling a long distribution feeder in EMT simulation, its system matrix can be represented by a block tridiagonal matrix. Therefore, block tridiagonal matrix algorithm (also known as block Thomas algorithm) can be utilized to accelerate the inverse calculation of the block tridiagonal matrix.

Let's take one of the block tridiagonal matrix $(\mathbf{A}_{nn})$ as an example. The matrix can be represented by

$$\mathbf{A}_{nn} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{C}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{D}_1 & \mathbf{B}_2 & \mathbf{C}_2 & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_2 & \mathbf{B}_3 & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \mathbf{C}_{\mathrm{M-1}} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{D}_{\mathrm{M-1}} & \mathbf{B}_\mathrm{M} \end{bmatrix}. \tag{11}$$

where, $\mathbf{C}_\mathrm{i}$ and $\mathbf{D}_\mathrm{i}$ are upper and lower block matrices for $i = 1, ..., \mathrm{M-1}$. $\mathbf{B}_\mathrm{i}$ is a main diagonal block matrix for $i = 1, ..., \mathrm{M}$. To compute the inverse of this matrix, block Thomas Algorithm can be applied in two main steps called forward elimination and backward substitution.

During forward elimination, the lower diagonal blocks $\mathbf{D}_\mathrm{i}$ can be eliminated for $i = 2, ..., \mathrm{M}$ by modifying the diagonal and the upper block matrices. First, the lower diagonal blocks are updated by solving

$$\mathbf{D}_{\mathrm{i\text{-}1}} = \mathbf{D}_{\mathrm{i\text{-}1}}/\mathbf{B}_{\mathrm{i\text{-}1}}. \tag{12}$$

The main diagonal blocks $\mathbf{B}_\mathrm{i}$ can be updated by solving

$$\mathbf{B}_\mathrm{i} = \mathbf{B}_\mathrm{i} - \mathbf{D}_{\mathrm{i\text{-}1}} \times \mathbf{C}_{\mathrm{i\text{-}1}}. \tag{13}$$

Then, in backward substitution process, the equation for each block can be solved for $i = \mathrm{M\text{-}1}, ..., 1$ from the bottom by

$$\mathbf{X}_\mathrm{M} = \mathbf{I}_\mathrm{M}/\mathbf{B}_\mathrm{M}. \tag{14}$$

where, $\mathbf{I}_\mathrm{i}$ is an identity matrix for each block. The solution for each previous block is updated by

$$\mathbf{X}_\mathrm{i} = (\mathbf{I}_\mathrm{i} - \mathbf{C}_\mathrm{i} \times \mathbf{X}_{\mathrm{i+1}})/\mathbf{B}_\mathrm{i}. \tag{15}$$

This step computes the final solution by utilizing the relationships established in the forward elimination step. After calculating the individual block solutions using both forward elimination and backward substitution, the results should be combined into the full inverse matrix by inserting the computed block components into their appropriate positions.

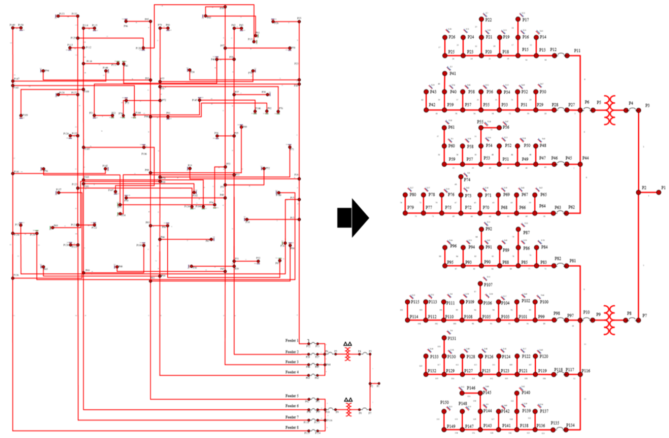

Fig. 6: Single-line diagram of primary distribution feeders in IEEE 342 node test system [18]

IV. Case Study

A. System Description

The IEEE 342-node test system is utilized to evaluate the performance of the proposed solver [18]. This case study focuses on the primary feeders of the test system, which is representative of a large-scale medium voltage distribution network, as depicted in Fig. 6. The system consists of 2 substation transformers and 8 feeders, with each feeder comprising 14 to 17 lines, resulting in a total of 150 nodes connected via station transformers and feeder lines.

The primary feeders are modeled to form the system matrix, yielding a 744 by 744 BBD system matrix. This structure accurately reflects the complexity of large distribution feeders and presents a suitable challenge for validating the computational efficiency and accuracy of the proposed solver in handling large-scale EMT simulations.

TABLE I: Comparison of Linear Solvers for IEEE 342-bus Test Feeder

	linsolve() built-in MATLAB	*mldivide()* built-in MATLAB	Schur complement + Block Thomas (proposed)
Average Time [s]	6.27e-3	6.82e-3	1.02e-3
Maximum errors [%]	0	0	1.78e-15
Speed-up	1x	0.92x	6.17x

B. Simulation Results

The proposed direct solver was implemented in MATLAB and compared with the performance of standard commercial linear solvers provided by MATLAB. The evaluation criteria used for the comparison included average computation time, maximum error, and speed-up factor. To ensure robust and reliable results, over 10,000 computations were performed to generate the results. The simulation results are presented to demonstrate the effectiveness of the proposed direct solver in relation to MATLAB's built-in solvers such as *linsolve()* and *mldivide()* [14].

Table I provides a comparison of the three methods: MATLAB's built-in *linsolve()* and *mldivide()*, as well as the proposed solver, which integrates the Schur complement method with the block Thomas algorithm. The proposed solver achieves a significantly faster average computation time of 1.02e-3 seconds, resulting in a 6.17x speed-up compared to *linsolve()*. By comparison, *mldivide()* shows a slight decrease in performance relative to *linsolve()*, with a 0.92x speed-up. Additionally, the accuracy of the proposed solver is notable, with a maximum error of 0%, compared to 1.78e-15% for *linsolve()* and *mldivide()*. These results indicate that the proposed solver offers both superior computational efficiency and precision when solving BBD matrices for large-scale EMT models, as typically encountered in distribution feeders.

The significant improvements in speed and accuracy demonstrate the effectiveness of the proposed solver in handling BBD matrix with large-scale EMT models of typical distribution feeders, making it a highly efficient and reliable option for large-scale system studies.

V. Conclusion

This paper presents an EMT model of a detailed distribution network, designed for general distribution systems. Key components such as transformers and feeder lines are modeled in detail. The network configuration represents typical distribution systems, incorporating multiple radial feeders connected in parallel through a substation transformer. Additionally, the relationship between the transformer-feeder configuration and the structure of the system matrix is explored, showing how the network design influences the system matrix form.

To enhance the efficiency of EMT simulations for such detailed distribution networks, a direct linear solver is proposed, utilizing the Schur complement and block Thomas algorithm to accelerate matrix operations. The performance of the proposed solver is validated using the primary feeders from the IEEE 342-bus test system, implemented in MATLAB. Comparative analysis is conducted against commercial solvers available in MATLAB, demonstrating that the proposed solver achieves up to a 6.17x speedup while maintaining accuracy. Future work includes implementing and evaluating these algorithms on specialized computing hardware to further enhance computational acceleration.

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