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AUTOMATED MODEL SELECTION WITH FIRST ORDER GAUGE INVARIANT PARAMETERS

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Background

Gate set tomography (GST) is one of the most powerful tools available for the characterization of real-world quantum computers [1]. GST outputs a list of parameters describing the errors of a quantum device, which allows the experimenter to interpret what physical processes are occurring in the lab. But sometimes interpreting this data may pose a challenge.

In practice we observe that in real experiments only a fraction of possible errors are relevant. Automated Model Selection (AMS) is an algorithm used to find a model with the least number of parameters, commensurate with experimental data[2]. A major obstacle for this algorithm, gauge freedom, causes many different models to predict the same physical observations, and thus, these models are equally effective in representing empirical data. In this project, we implement AMS using first-order gauge invariant (FOGI) parameters, which eliminates the gauge freedom problem and simplifies the search.

Gate Set Tomography Models

$$\mathcal{G} = \left\{ \left\{ |\rho^{(i)}\rangle\rangle \right\}_{i=1}^{N_\rho} ; \{G_i\}_{i=1}^{N_G} ; \left\{ \langle\langle E_i^{(m)}| \right\}_{m=1,i=1}^{N_M, N_E^{(m)}} \right\}$$

Above we have a *gate set*, a complete description of a quantum device comprised of states ρ, gates G and measurements with i outcomes E_i. The goal of GST is to find the gate set that is most likely to have produced the data collected from a set of experiments. To do this, we represent each operation as a perfect target operation followed by an error channel. Each error is decomposed into any desired basis, and described by a corresponding set of parameters. By collecting each of these parameters into one list, we create a parameter vector, describing the entire gate set of the quantum device.

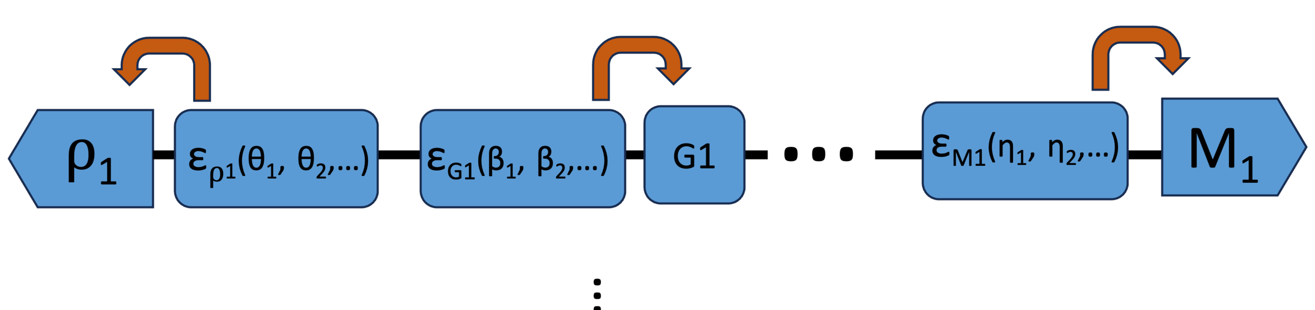

Figure 1: Representation of a circuit consisting of several target operations: state preparation ρ_1, measurement M_1, and unitary gates G_i, followed by their corresponding error channel ε. Each error channel is completely described by a set of parameters.

Automated Model Selection

Interpreting results from a GST model can be challenging, and its difficulty is only increased by the number of parameters contained in the model. As mentioned before, in practice we observe many parameters to be negligible. To take advantage of this, AMS attempts to remove a parameter from the model and evaluate how well it is able to describe the data while sacrificing expressivity due to the parameter loss. The user is able to select a threshold above which a model is thrown away due to its inability to express the data at hand. The threshold we used is the "evidence ratio". In this framework it simplifies to twice the difference in log likelihood of the models

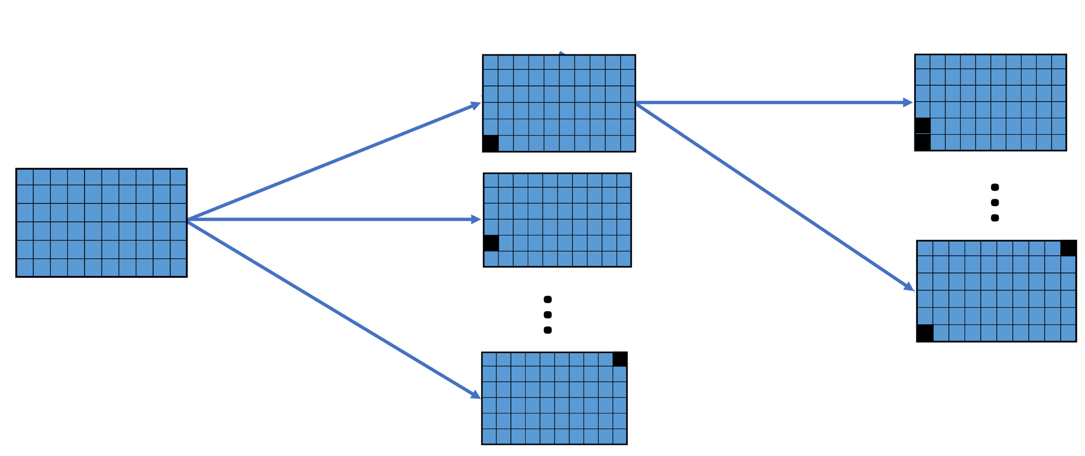

Figure 2: Gate set models generated by removing parameters in each level. Each edge connects a parent model with a reduced model which has one less parameter than the parent.

The Gauge Problem

Let us now analyze why gate sets have this inherent gauge problem. Consider the probability of an outcome E_i of a given circuit:

$$p_i = \langle\langle E_i|\, G\, |\rho\rangle\rangle = \langle\langle E_i|\, \overbrace{T^{-1}T}^{\mathbb{1}} G \overbrace{T^{-1}T}^{\mathbb{1}} |\rho\rangle\rangle$$

As shown above, we are able to insert any invertible T matrix without any effect on the outcome probabilities. Giving us a new gate set, which describes the *same physical device*.

What is the issue with having more ways to describe a quantum device? Consider a model where all the error is attributed to one parameter, such as a unitary gate whose rotation axis is slightly off. Depending on the gauge, as shown in the figure below, this error could be transferred onto a different parameter. Making the exploration of the AMS tree more difficult, as we find several gauge equivalent models.

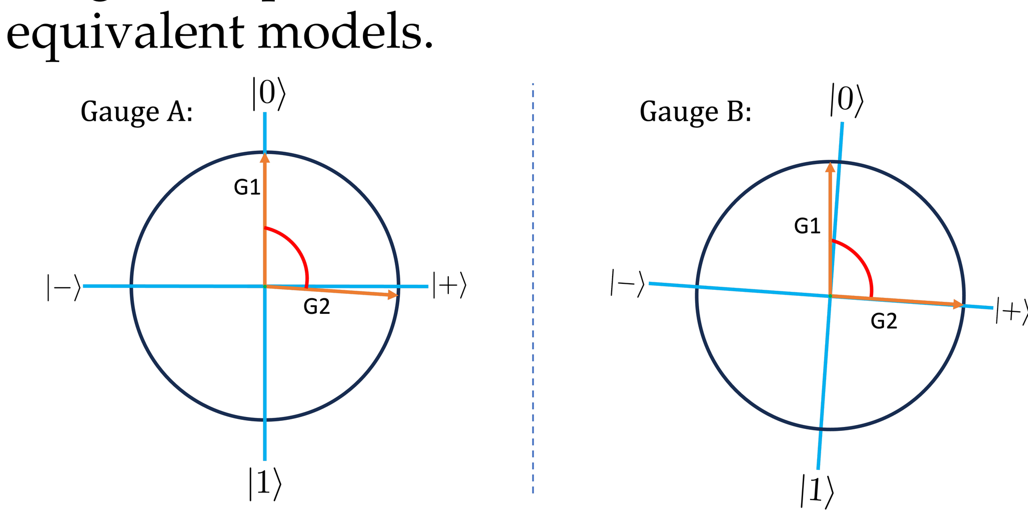

Figure 3: Shown is a slice of the Bloch sphere, represented in two different gauges. The two arrows are the axis around which two different gates, G1 G2, perform rotations. Gauge A would determine that G1 is perfectly aligned while G2 is slightly off the X plane. Gauge B, would determine the opposite. As a consequence, it is harder for an AMS algorithm to determine which parameter is worthwhile to keep.

This motivates the question: **Is there a basis we can describe our gate set in, which is gauge invariant, simplifying the optimization landscape?**
For example, the gate set in Figure 3 could be completely described by one gauge-independent parameter, the angle, depicted in red, between both axis of rotations. However, this is a trivial example. Currently there is no known algorithm to construct a fully gauge-invariant basis to describe an arbitrary model.

First Order Gauge Invariant (FOGI) Basis

A stepping stone towards a fully gauge-invariant basis is the FOGI framework, which is invariant up to *first order* (small) gauge transformations.
We have successfully demonstrated a proof-of-concept implementation of AMS using FOGI-parameterized models. As shown below we can see that we have removed $\tilde{9}0\%$ of all trivial parameters

Full	Reduced	Cost (Evidence Ratio)
0	removed	-8.89E-07
0	removed	4.63E-07
6.86E-19	removed	3.82E-08
-2.41E-18	removed	0.00496409
-2.52E-19	removed	1.47E-06
1.42E-18	removed	-8.94E-08
2.36E-18	removed	0.00471428
-8.32E-20	removed	1.50E-07
3.25E-18	removed	5.62E-06
-2.68E-18	removed	-3.80E-05
-3.64E-18	removed	0.18147445
-1.91E-18	removed	0.39338194
5.08E-19	-6.86E-07	182.167375
2.44E-18	removed	0.05321958
1.10E-18	3.49E-07	454.055133
-6.90E-20	removed	0.38308912
0.002886751	0.00295663	827034.951
-0.006646185	-0.0050864	120.943586
-0.00443079	-0.0035616	62.5068427

Figure 4: Outcome of AMS using FOGI quantities. On the first column we show the parameters of the FOGI model that generated the data with only 3 non-trivial parameters. The middle column shows the parameters output by AMS, which managed to correctly identify and remove 14 trivial parameters.

Although promising, the results above raised some concerns that will be addressed in future work; some trivial parameters are not being removed, and some of the non-trivial parameters are not within error-bars of the true values

[1] Erik Nielsen et al. Gate Set Tomography. *Quantum*, 5:557, 2021.

[2] Stefan Seritan. Automated model selection for gate set tomography. *APS March Meeting*, B72:00013, 2023.

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