SAND2024-14121C

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AUTOMATED MODEL SELECTION WITH FIRST-ORDER GAUGE-INVARIANT PARAMETERS

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American Physical Association March Meeting

03/05/2024

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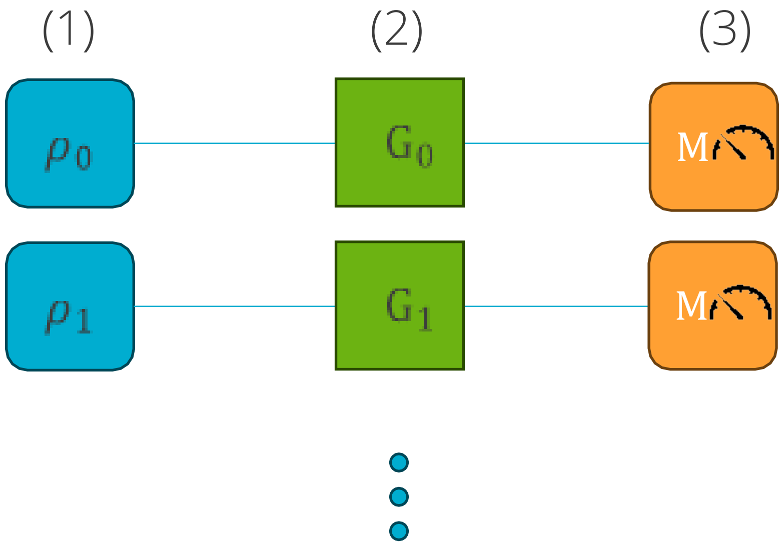
TOMOGRAPHY

- Goal: Construct a comprehensive mathematical model that describes our quantum computer
- Typically we do this by independently describing a set of (1) state preparations, (2) gates, and (3) measurements



$$\langle\langle M_i|\,G_0\,|\rho_0\rangle\rangle$$

$$\langle\langle M_i|\,G_1\,|\rho_1\rangle\rangle$$

$$\vdots$$

STATE/MEASUREMENT/GATE TOMOGRAPHY

State Tomography

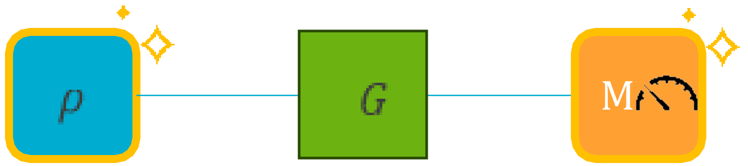
Assumes perfect measurements

Measurement Tomography

Assumes perfect state preparations

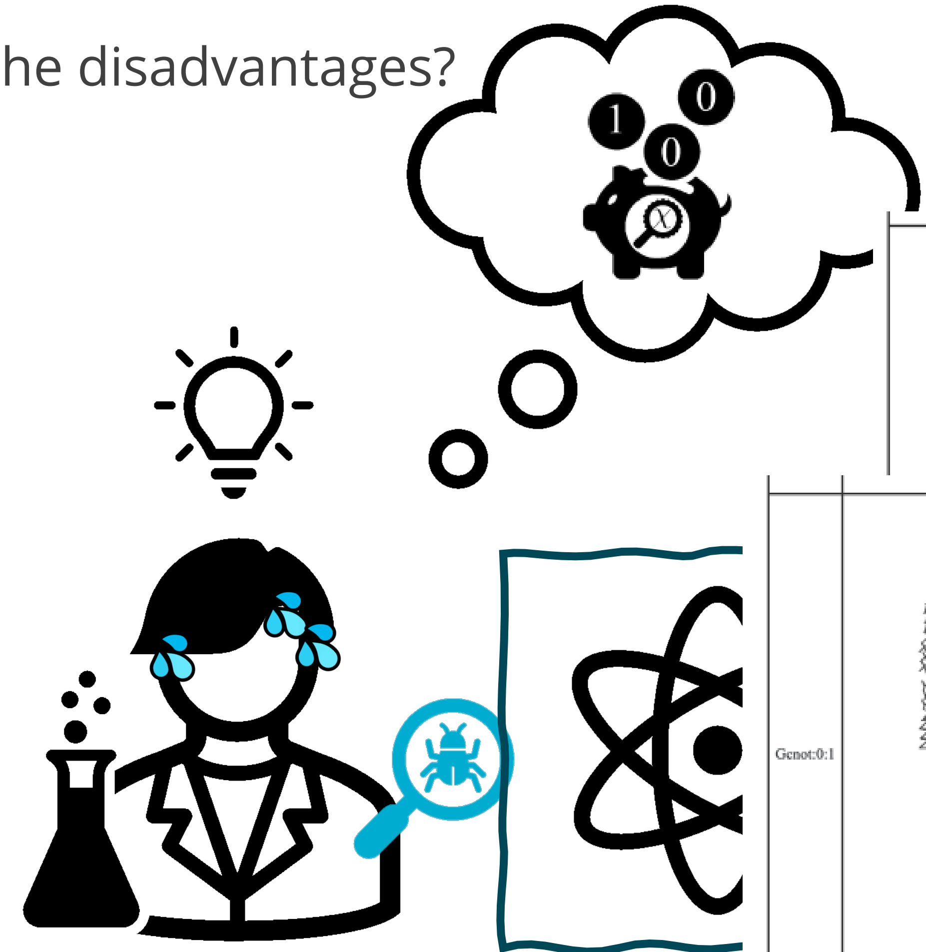
Gate Tomography

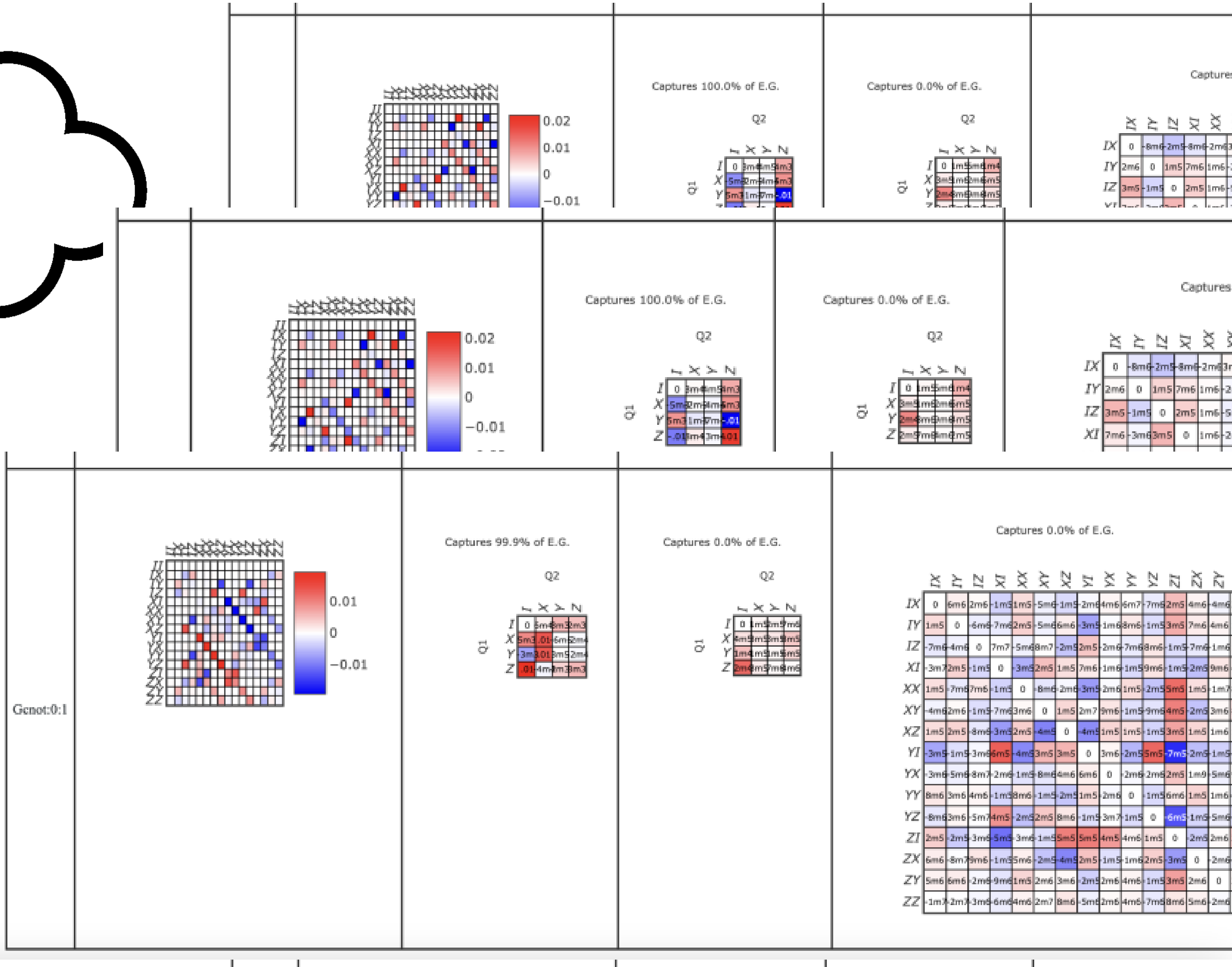
Assumes perfect state preparations and measurements



GATE SET TOMOGRAPHY

- GST assumes no perfect operations! It characterizes noise for states, measurements and gates.
- What are the disadvantages?

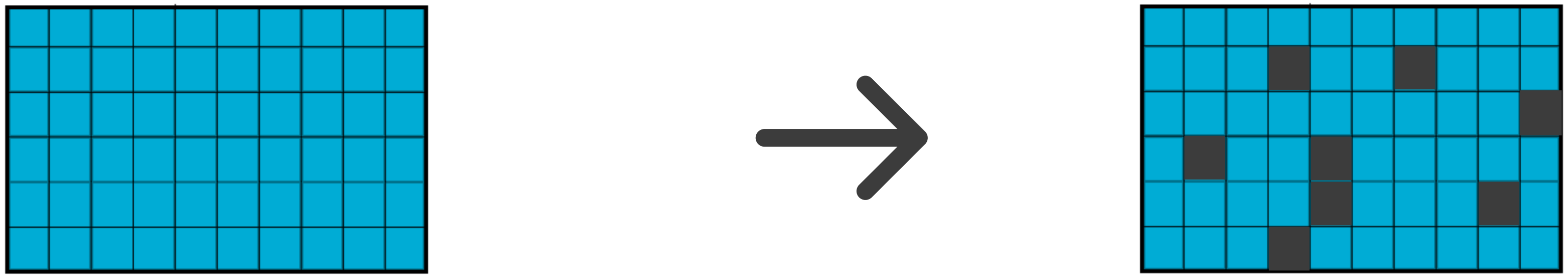




REDUCED MODELS

- In practice, we observe that many parameters are not necessary to describe some systems.

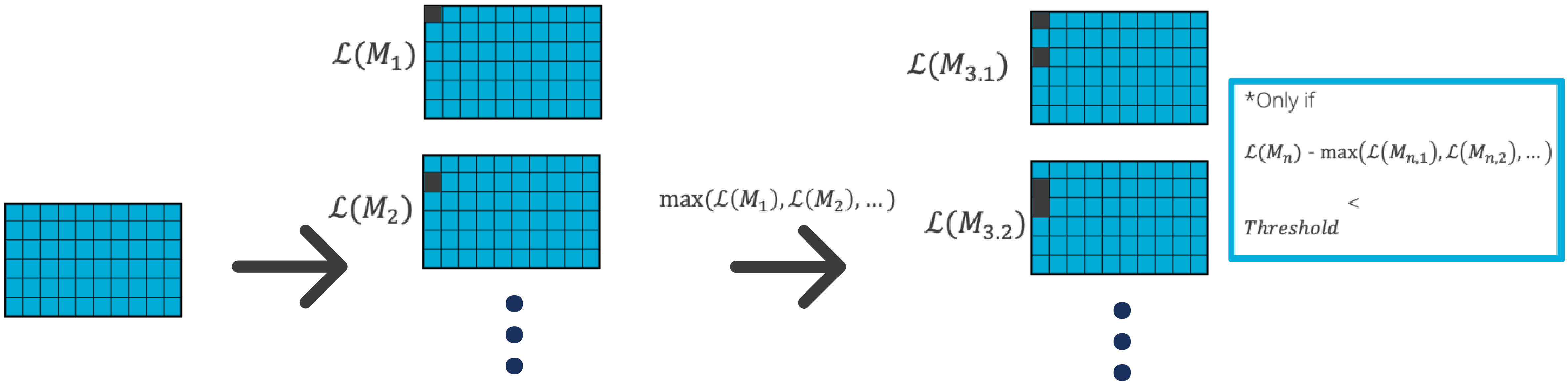
- Can we remove parameters without sacrificing how *well* the model fits experimental data?



- Automated Model Selection (AMS) finds the best bang for your buck

AUTOMATED MODEL SELECTION

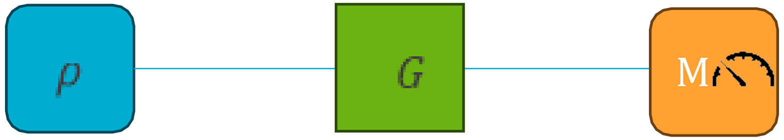
- Goal: Find the best model with the least number of parameters
- In order to quantify how ”good” a model is, we use the likelihood function
- We used a greedy algorithm:
  1. Remove every parameter independently
  2. Run GST on every reduced model. Calculate the likelihood for each reduced model
  3. Pick the one with the best likelihood
  4. Repeat*



GAUGE FREEDOM

- Parameters depend on what basis is chosen to represent a gate set
  - Example, process matrix and vector entries:

$$G = \begin{pmatrix} G_{00} & G_{01} & G_{02} & G_{03} \\ G_{10} & G_{11} & G_{12} & G_{13} \\ G_{20} & G_{21} & G_{22} & G_{23} \\ G_{30} & G_{31} & G_{32} & G_{33} \end{pmatrix}, \quad |\rho\rangle\rangle = \begin{pmatrix} \rho_0 \\ \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix}, \quad \langle\langle M_0| = \begin{pmatrix} E_0 \\ E_1 \\ E_2 \\ E_3 \end{pmatrix}^T \rightarrow \vec{p} = (G_{00}, \dots, G_{33}, \rho_0, \dots, \rho_3, E_0, \dots E_3)$$

$C_1 =$ 

$$P(0|C_1, \vec{p}) = \langle\langle M_0|G|\rho\rangle\rangle = \langle\langle M_0|T\, T^{-1}GT\, T^{-1}|\rho\rangle\rangle = \langle\langle M_0'|G'|\rho'\rangle\rangle = P(0|C_1, \vec{p}')$$

$$\vec{p}' = (G'_{00}, \dots, G'_{33}, \rho'_0, \dots, \rho'_3, E'_0, \dots E'_3) := \vec{p}$$

GAUGE FREEDOM + AMS = GAUGE ***PROBLEM***

- Consider a quantum device with two gates, Gz and Gx
- Let us describe it with a 2-parameter model

$$\vec{p} = (\theta_z, \theta_x)$$

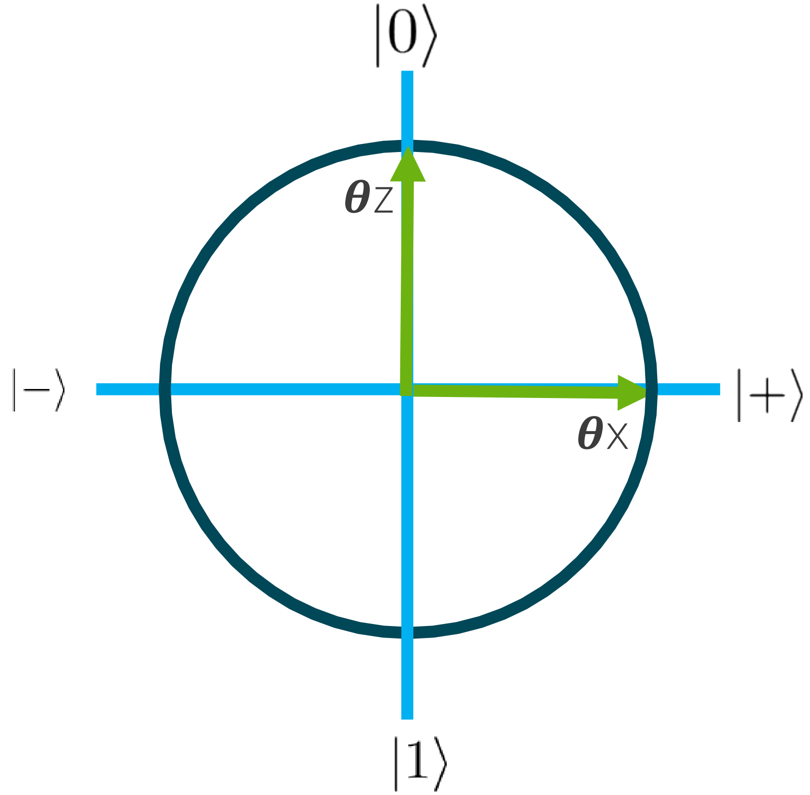
- Let the physical device have 120° between them
- Doing GST on the reduced models will result in:

$$\vec{p}_{r1} = (\blacksquare, 30°) \qquad \vec{p}_{r2} = (30°, \blacksquare)$$

- These two models are gauge equivalent, and thus have the same likelihood value!

$$\vec{p}_{r1} \coloneqq \vec{p}_{r2} \rightarrow \mathcal{L}(\vec{p}_{r1}) = \mathcal{L}(\vec{p}_{r2})$$

- This makes traversing the tree of reduced models much harder



FIRST-ORDER GAUGE-INVARIANT (FOGI) PARAMETERS

- Parameters that are resilient to the gauge-problem should simplify the search landscape
- FOGI comprises a basis that is invariant under *small* gauge transformations

$$\langle\langle M_0 | T\, T^{-1} G T\, T^{-1} | \rho \rangle\rangle \text{ , Where } T = e^K \approx \mathbb{1} + K$$

- For example, expressed in terms of elementary error generators, these are the (first 14) FOGI parameters for a 1 Qubit model with a X and Y gate:

1	$H_x(G_x)$	8	ρ_z + 0.25 M1 + 0.25 Mz
2	$S_y(G\_x) + S_z(G_x)$	9	Hy(Gx) - Hz(Gx) - Cxy(Gx) - Cxz(Gx) - 0.5 Mx
3	$Ayz(Gx)$	10	Axy(Gx) - Axz(Gx) - 0.25 M1
4	$Hy(Gy)$	11	Hx(Gy) + Hz(Gy) - Cxy(Gy) + Cyz(Gy) + 0.5 My
5	$Sx(Gy) + Sz(Gy)$	12	Axy(Gy) - Ayz(Gy) - 0.25 M1
6	$Axz(Gy)$	13	Hy(Gx) + Hz(Gx) + Hx(Gy) - Hz(Gy)
7	$Sy(Gy)$	14	Cxy(Gx) - 0.5 Cxy(Gy) - 0.5 Cxz(Gy) - 0.5 Cyz(Gy)

• • •

RESULTS: AMS ON SIMULATED DATA (NO SAMPLE ERROR)

- Simulated data for a 1 Qubit model with X & Y gates. Modeled with elementary error generators. We selected the following FOGI quantities*:

- In this example we observed FOGI AMS outperform non-FOGI AMS by a reduction of up to **92%** of total parameters in reduced models

	Data Generating Model	AMS Reduced Model
Param 1	0	removed
Param 2	0	removed
Param 3	0	removed
Param 4	0	removed
Param 5	0	removed
Param 6	0	removed
Param 7	0	removed
Param 8	0	removed
Param 9	-1.41e-4	-1.68e-4
Param 10	0	removed

	Data Generating Model	AMS Reduced Model
Param 11	0	removed
Param 12	0	removed
Param 13	0	removed
Param 14	0	removed
Param 15	0	removed
Param 16	0	removed
Param 17	0	removed
Param 18	0	removed
Param 19	-1.49e-8	removed
Param 20	1.00e-2	1.01e-2

*Values below 1e-17 were truncated to 0

RESULTS: AMS ON SIMULATED DATA (WITH SAMPLE ERROR)

- Simulated data for a 1 Qubit model with X & Y gates. Modeled with elementary error generators. We selected the following FOGI quantities*:

- In this example we observed FOGI AMS outperform non-FOGI AMS by a reduction of up to **73%** of total parameters in reduced models

	Data Generating Model	AMS Reduced Model
Param 1	0	removed
Param 2	0	-4.65e-6
Param 3	0	removed
Param 4	0	removed
Param 5	0	removed
Param 6	0	5.29e-6
Param 7	0	removed
Param 8	0	-9.68e-6
Param 9	-1.41e-4	-2.14e-4
Param 10	0	-6.39e-4

	Data Generating Model	AMS Reduced Model
Param 11	0	removed
Param 12	0	removed
Param 13	0	9.06e-6
Param 14	0	removed
Param 15	0	-8.12e-6
Param 16	0	removed
Param 17	0	removed
Param 18	0	removed
Param 19	-1.49e-8	removed
Param 20	1.00e-2	1.01e-2

*Values below 1e-17 were truncated to 0

FUTURE DIRECTIONS

Run AMS on real experimental data

Ongoing collaboration with Sandia neutral atom experimental team

Run AMS “from the top-down”

Save computation instead of adding

Implement in pyGSTi for public use

Stay tuned!

MANY THANKS TO MY COLLABORATORS

Corey Ostrove

Robin Blume-Kohout

Stefan Seritan