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Biased Clustering in an Universe with Hot Dark Matter and Cosmic String

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Summary. The clustering around a cosmic string loop in an universe consisting, at present, of baryons and massive neutrinos has been calculated in linear perturbation approximation. We pay attention mainly on the bias in the clustering. It has been shown that the biased clustering of baryons started just after the decoupling time; the clustering will form an object with structure of a core dominated by baryons in the inner region and a halo dominated by massive neutrinos in the outer region. The present length scale of the cores is about several decades Kpc, depending on the rest mass of neutrinos. The rotation curves of such objects are remarkable flat. The amplitudes of the rotation curves decreases with cosmic time. Comparing the observed rotation velocities with calculated results, it is found that galaxies would be formed in the period of redshift $z \simeq 3 - 10$. The maximum length scale of the bias is about the same as for clusters of galaxies. This result seems to be useful to explain the correlation between the features of rotation curves of cluster galaxies and the galaxy's distance from the center of the cluster.

Key words: cosmology - biased clustering - cosmic string - galaxy formation

1. Introduction

Bias is an important topic in galaxy formation. However, in different articles, the word of bias has sometimes been used in different meaning. From directly observed phenomena, the bias means that the distribution of galaxies are overabundant in the regions of high density (White, 1988). For instance, the systematic growth of mass-to-light ratio, M/L, with the size of the objects shows that galaxies are overabundant relative to dark matter. In rich clusters, galaxies are overabundant relative to the dark matter by about a factor of 5. The mean mass densities given by galaxies found in groups, clusters and even superclusters are always lower than 1. On the other hand, the spatial geometry of the universe seems to be flat $\Omega = 1$ (Loh, 1987). A common hypothesis made to determine the mean density of visible objects is that the distribution of all matter is concentrated in or proportional to visible objects and that there does not exist a more uniform distribution of matter. Therefore, the requirement for Ω to be equal to 1 also implies that the dark matter is not clustered obviously, at least, on scales of less than that of clusters of galaxies (Fang et al, 1982).

This result forces us to search for the following problem: how can such differnces in the distributions of baryons and dark matter come about ? Namely, why do the baryon cluster obviously, especially on smaller scales, while the dark matter has up to now remained rather uniform on these scales ? These problems had been studied by us for a two component universe. It has been found that, in a two component universe, if the densities ρ_1 and ρ_2, the Jeans lengths λ_{1J} and λ_{2J} of the two components satisfy the inequalities $\rho_1 \ll \rho_2$, $\lambda_{1J} \ll \lambda_{2J}$, the developed inhomogeneties with scale less than λ_{2J} in the non-dominant component 1 are always of larger amplitude than those in the dominant component 2, regardless of whether the initial perturbation is in 1 or in 2. This mechanism can, at least qualitively, be used as a possible origin of the biased clustering.

By using the above-mentioned mechanism we show numerically that, in an universe consisting of massive neutrinos and baryons, the clustering of baryons on small scales is always larger than that of neutrinos, regardless of whether the initial perturbations exist in baryons or in neutrinos (Fang, et al. 1984). This means that the clustering of baryons will be biased with respect to neutrinos. However, in this model, the baryonic density perturbations on larger scales get to be nonlinear always earlier than that on the smaller scales. Therefore, it is still not quantatively enough to change the top-down clustering scenario, which is hard to reconcile with observations. This is a common difficulty in hot dark matter model.

In order to overcome the above-mentioned difficulty, we discussed a model of the universe containing two types of dark matter, massive neutrinos as a dominant component and another type of more massive particles as non-dominant component, which plays as the seeds to form small scale objects before the formation of large one. In this case there should be two types of small scale objects: one is related to the formation of large scale structure, while the other is not linked with the large scale structure. These results already gave a plausible picture on the clustering of quasars, especially on the difference between the clusterings of quasars with large-redshift and small-redshift (Chu and Fang, 1987). Comparing with cosmic string model, it can be found that the cosmic string just plays about the same role as that of non-dominant component of dark matters in the considered model. Therefore, the plausible results will also be maintained in a hot dark matter plus cosmic string universe. This is the motivation to study the biased clustering problems in a hot dark matter plus cosmic string universe, namely, we will still consider an universe containing massive neutrinos and baryons, while the cosmic strings play the role of primeval seeds for the density perturbations.

The other reason to study this model is that massive neutrino is still one of the most likely candidates of dark matter in the universe. The neutrions emitted from SN 1987A seems to further strengthen the interest in the role of massive neutrino in the universe. Moreover, the cosmic string model with cold dark matter are unlikely to generate large-scale streaming velocities (Lynden-Bell et al. 1988), and the nonlinear halos accreted by the strings may overlap before the present. All of these difficulties also let people to consider a hot dark matter plus cosmic string model (Bertschinger and Watts, 1988).

2. Qualitative feature of clustering

Let us consider a cosmic string loop with mass M_S , which is surrounded by two components of matters 1 (baryons) and 2 (neutrinos). The two components interact with each other only through gravitation. In a neutrino dominated universe, after the decoupling between baryons and radiation, the mass density ratio of baryons ρ_1 to massive neutrinos ρ_2 is

$$\frac{\rho_1}{\rho_2} \simeq 1.1\cdot 10^{-2}\left(\sum_i m_{\nu i}/100eV\right)^{-1}(\eta_{10}/5). \tag{1}$$

where the sum runs over all neutrino species and $\eta_{10} = (n_B/n_\gamma)\times 10^{10}$ denotes the number density ratio of baryons to photons. After the decoupling, the ratio of Jeans length (or free streaming length) of baryons λ_{1J} to massive neutrinos λ_{2J} is given by

$$\frac{\lambda_{1J}}{\lambda_{2J}} \simeq 0.85\cdot 10^{-2}(m_\nu/30eV)^{3/2}(\eta_{10}/5)^{-1/2}. \tag{2}$$

where $\lambda_{1J} = (v_1^2\pi/G\rho_1)^{1/2}$ and $\lambda_{2J} = (v_2^2\pi/G\rho_2)^{1/2}$ are the Jeans lengths of components 1 and 2, respectively, and v_1 and v_2 being the corresponded velocity dispersions. Equations (1) and (2) shows the system we should considered is

$$\rho_1 \ll \rho_2, \quad \lambda_{1J} \ll \lambda_{2J}. \tag{3}$$

In this case there are following relevant time scales with respect to a density perturbation of length scale ℓ: two damping times are defined by

$$t_{d1} \sim l/v_1, \quad t_{d2} \sim l/v_2 \tag{4}$$

and a growing (collapse) time is defined by

$$t_g \sim (4\pi G\rho + 3GM_S/l^3)^{-1/2} \tag{5}$$

where $\rho = \rho_1 + \rho_2$. The conditions for the growth of density perturbations in 1 and 2 are

$$t_g \leq t_{d1}, \ t_g \leq t_{d2} \tag{6}$$

Therefore, when the clustering dominated by gravity of the string, i.e. $4\pi G\rho \leqslant 3GM_S/l^3$, the clustered lengths of components 1 and 2 around the string M_S are

$$l_1 \leq (3GM_S/v_1^2), \ l_2 \leq (3GM_S/v_2^2); \tag{7}$$

When the clustering dominated by the self-gravity of the two components of matter or $4\pi G\rho \geqslant 3GM_S/l^3$, the clustered lengths of 1 andd 2 are given by

$$l_1 \geq v_1/(4\pi G\rho)^{1/2}, \ l_2 \geq v_2/(4\pi G\rho)^{1/2}. \tag{8}$$

Considering condition (2), we find from eqs.(7) and (8) that in both cases the clustered length scales of component 1 are wider than that of component 2, namely, the clustering of baryons will be easier and more rapid than neutrinos. Even a significant clustering with scale of about λ_{1J} can form in the non-dominant component 1, while not in the dominated component 2. This means the clustering of baryons will be biased with respect to massive neutrionss.

3. Linear perturbation theory

We now calculate the growth of density perturbations around a string loop in an expanding universe. The method used in this calculation is the same as usual linear perturbation theory (Gilbert, 1966; Fang et al. 1984; Bertschinger and Watts, 1988).

We assume that the universe is flat $\Omega = 1$. The present density of massive neutrinos is $\Omega_\nu = 0.9$ and baryons $\Omega_b = 0.1$. The universe became matter-dominated at redshift $1+z_{eq} = 2\times10^4 h^2$. In this case the dimensionless conformal time is given by

$$\xi \equiv \frac{c}{r_{eq}} \int_0^t \frac{dt}{a(t)} = 3[(1+\frac{a}{a_{eq}})^{1/2} - 1] \tag{9}$$

where a(t) is the cosmic scale factor normalized to unity at the present time, and $a_{eq} = 1/(1+z_{eq})$, $r_{eq} = \frac{2}{3} c H_0^{-1} a_{eq}^{1/2}$.

The unperturbed distributions of baryons and neutrinos are described by the Boltzmann and the Fermi distributions, respectively,

$$f_1^0 = \rho_1^{(0)} \frac{1}{(2\pi u_{10})^{3/2}} \exp(-\frac{1}{2}(\frac{u}{u_{10}})^2) \tag{10}$$

$$f_2^0 = \rho_2^{(0)} \frac{1}{6\pi\varsigma(3) u_{20}^3} (\exp(\frac{u}{u_{20}}) + 1)^{-1} \tag{11}$$

where $\rho_1^{(0)}$ and $\rho_2^{(0)}$ are the unperturbed densities of baryons and neutrinos, respectively. $\zeta(3)$ is Reimann ζ function. The comoving velocities u_{i0} are related to the Jeans lengths λ_{iJ} as

$$u_{i0} = (G\rho_i^{(0)}/\pi)^{\frac{1}{2}} \lambda_{iJ}, \tag{12}$$

The density perturbations in both components can be described by

$$\delta_{ik} = \frac{1}{M_S} \int e^{-2\pi i \mathbf{k}\mathbf{x}} (f_i - f_i^0) d\mathbf{x} d\mathbf{u} \tag{13}$$

where f_i are the perturbed distribution functions. It can be shown that, in linear approximation, the density perturbations in baryons and in neutrinos satisfy the following equations (Fang et al. 1984; Bertzchinger and Watts, 1988):

$$\delta_{ik} = \int_0^{\xi} d\xi' \ln\left(\frac{1+6/\xi'}{1+6/\xi}\right) F_i(k\chi_i(\xi',\xi))[\Omega_1\delta_{1k} + \Omega_2\delta_{2k} + G_i(k,\xi')] \tag{14}$$

where F_i are the Fourier transform of the distribution functions f_i^0. χ_i is the free-streaming comoving distances between conformal time ξ' and ξ, namely

$$\chi_i(\xi',\xi) = \alpha_i r_{eq} \ln\left(\frac{1+6/\xi'}{1+6/\xi}\right) \tag{15}$$

where

$$\alpha_1 = (\lambda_{1J}/\lambda_{2J})(\rho_1^{(0)}/\rho_2^{(0)})^{1/2}\alpha_2, \tag{16}$$

$$\alpha_2 = 0.074(100/m_\nu) \tag{17}$$

and m_ν is the rest mass of neutrinos in the unit of eV. The string loop is treated here as a point of mass M_S. The accretion of each component by the string loop are described by functions G_i in eq.(14) defined as

$$G_1 = \begin{cases} 1 - \frac{\sin[kr_{eq}(\xi-\xi_{eq})]}{kr_{eq}(\xi-\xi_{eq})}, & \xi > \xi_{eq}; \\ 0, & \xi \le \xi_{eq}; \end{cases} \tag{18}$$

$$G_2 = 1 - \frac{\sin(kr_{eq}\xi)}{kr_{eq}\xi}. \qquad (19)$$

where ξ_{eq} = 1.24 is the conformal time of the universe becomming matter-dominated. The clustering of baryons can only occur after the decoupling time, which is about the same as ξ_{eq}, so we take G_1 to be zero when $\xi < \xi_{eq}$.

4. Numerical examples

As an example we calculated the linear growth of density perturbations of baryons and neutrinos for several wavenumbers k. For an easy comparison of our results with those obtained in the case of the universe consisting only cosmic strings and massive neutrinos but without baryons (Bertschinger and Watts, 1988), we also used $\ln\xi$ as the integration variable and chose the starting point at $\xi = 10^{-4}$ The numerical results are given in Figs. 1 and 2, in which we take the ratio of baryonic Jeans length to neutrino's free-streaming length as $\lambda_{1J}/\lambda_{2J}$ = 0.01. In fact, our results do not sensitively depend on this ratio as long as that it is much larger than 1.

Fig.1 showed that the clustering in both baryons and neutrinos are no longer to be completely top-down scenario. The macximum of δ_k at present time occurs at the length scale of about $1/K = r_{eq}/1.59$, which is about the same as the scale of clusters of galaxies. Namely, the density perturbations with length scale of about the same as for cluster of galaxies can became nonlinear earlier than that of perturbations with length scale as for superclusters. In the nonlinear stage the clustered objects will quickly be virialized. Therefore, the above-mentioned length scale can just be used to explain why the configuration of objects with scale less than that of clusters of galaxies are often symmetrical or regular, while larger objects are always irregular (Fang and Yan, 1988).

Fig.1 also showed obviously that baryons cluster more strong than neutrinos after the time of ξ_{eq}. When $\xi > \xi_{eq}$, the density perturbations for all wavenumber K always satisfy the following relation

$$\delta_{1k} > \delta_{2k} \tag{20}$$

It can also be seen from Figs. 1 and 2 that for larger K, the correspond larger differences between δ_{1K} and δ_{2K}. This means for smaller length scale of the clustering, correspond higher the ratio of the densities of baryons to neutrinos. This is just the biasing scenario.

Fig.2 plots the growth of density perturbations at the distances of $x = 10^{-4} r_{eq}$ and $10^{-1} r_{eq}$ from the cosmic strings. It shows that there is a remarkable up-jump in the clustering of baryons at ξ_{eq}, namely after the Thomson drag time, the baryonic density perturbation will immediately overcome the density perturbation of neutrinos. This will lead to, in configurational space, a baryonic density around the string loop which overcomes the neutrino density. Therefore, the clustering around a string loop will form an object with a core of baryons and a halo of neutrinos. We define the core radius r_c as the distance from the string loop, at which the baryonic density is equal to the neutrino density

$$\rho_1(r_c) = \rho_2(r_c) \tag{21}$$

Fig.3 shows the evolution of the core radius r_c against redshift. The maximum of radius for reasonable mass of string loops is not larger than about 100 Kpc, which is not strongly dependent upon the rest mass of neutrino. This is also consistent with the observed biased clustering.

5. Application and discussion

In general, the results found from linear perturbation theory cannot be ditrectly compared with observations, because the nonlinear collapse may change the clustering picture in the linear growth stage. However, the spherical collapse model in an expanding universe showed that an accreted mass shell stops to expand and approaches a maximum radius r_{max} when the density perturbation implied by linear theory is 1.5 (Zel'dovich, 1970), namely, the linear model is a reasonable approximation for the clustering until the mass shell drops out from the Hubble flow. After the shell turns around, it collapses to achieve virialization, while the virialized radius r_{vir} differs from r_{max} only by a factor of about 2 (Gunn, 1977; Shu, 1978),

$$r_{vir} \simeq r_{max}/2. \tag{22}$$

Therfore, some features of the clustered objects at present time will not strongly be changed from those existing when the objects just drop out from the Hubble expansion. This can be called a drop picture. Under this picture, some results of linear model can also be used to explain, at least qualitatively, observed phenomena, especially, for the properties which are directly related to the drop out time.

Fig.4 shows the evolution of the profiles of density perturbations around a string loop. It is obviously from Fiq.4 that a core-halo object can be formed by the accretion of the loop. The core is dominated by baryons, and the halo by massive neutrinos. More interesting result might be the flatness of the rotation curves, which are plotted in Fig.5 for several masses of the string loops. The rotation velocity is calculated as

$$v = \sqrt{\frac{GM}{r}}, \quad M = 4\pi \int_0^r \rho(r) r^2 dr \tag{23}$$

It can be seen from Fig.5 that, even at the time of redshift $z \simeq 5$, the rotation curves already, qualitatively, owned the same features as observations. It means that the mass density in the halo is approximately proportional to x^{-2}. This is different from the pure neutrino model, in which, in the linear approximation, the density around the loop shows profile of about x^{-1} (Bertschinger nad Watts 1988). In our model, the biased clustering of baryons contributes a steep density profile, so the total mass density of baryons and neutrinos will, then, distribute as the needs of a flat rotation velocity. If all mass shells obey the relation of eq.(22) in their virialization, the x^{-2} profile and the flatness of the rotation curves will probably be maintained after the virialization.

The amplitudes of the $z = 0$ rotation curves given by Fig.5 are lower than observations. According to the drop out picture, we should not compare the present rotation velocity with $z = 0$ curves in Fig.5, but with that of drop out time. Fig.6 give the rotating velocity at a present distance of 100 Kpc from the loop as a function of redshift. The rotation velocity decreases with the cosmic time. We find from Fig.6 that the values of observed rotation velocities corresponds to the drop out time of about $z \simeq 3 - 10$, which seems to be acceptable as the era of galaxy formatiom.

As mentioned in sec.3, the maximum length scale of the biased clustering is to be about the scale of clusters of galaxies. The accretion of string loops will be able to form an inhomogeneity of the density ratio of baryons to neutrinos on scale as large as the clusters. This result might be used as an explanation of the correlation between the features of rotation curves of cluster galaxies and the galaxy's distance from the center of the cluster. Recently, it has been found (Rubin et al. 1988; Whitmore 1988) that a. the amplitudes of rotation curves of cluster galaxies, especially Sa and Sb, are low compared with field galaxies of equivalent Hubble types and luminosities; b. in clusters, the inner galaxies tend to have falling rotation curves, while the outer galaxies tend to have flat or rising rotation curves.

A most possible cause of these features for cluster galaxy's rotation curves is that the cluster galaxies have different internal dynamics compared with isolated field galaxies. If the environment within the cluster was responsible for the properties of the altered galaxies, then this might reveal itself by a variation in galaxy parameters as a function of distance from the cluster center. The most important environment parameter related to the rotation curves is the density ratio of baryons to dark matter. Both above-mentioned observed results seem to show a deficiency of dark matter in clusters compared with dark matter in field. In fact, observation directely showed that the M/L gradients of galaxies in the inner regions of clusters are flatter than for galaxies in the outer regions. The developed model has, indeed, the required properties: a. the baryon-to-neutrino density ratio in clusters is higher than that in field; b. in cluster, the ratio decreases gradully from the inner regions to the outer region. If this explanation is real, the cluster galaxies should form in the same time or even later than the time of cluster formation.

Finally, we should mention the influence of the motion of string loops on our results. As all models of spherical accretion of loops, our model also neglects the effect of the peculiar velocity of loops. This approximation becomes serious, because, recently, it has been shown (Bennett and Bouchet 1989) that the motion of string loops will, more or less, wash out the correlation of loop initial positions. This means that the one-loop-one-object model seems to be a poor approximation on the clustering of cosmic strings. Nevertheless, some results obtained by this approximation would still be reasonable even considering loop motion. With respect to a perturbation with length scale ℓ , the time scale of loop motion is $t_{1p} \sim \ell / v_{1p}$, v_{1p} being the velocity of loops, and the collapse time t_g is given by eq.(5). Obviously, if $t_{1p} > t_g$, clustering will not strongly be affected by the loop motion. In this case, loop only plays as a initial trigger of clustering. After the very short period of the initial up-jump (Fig.2), clustering will be dominated by self-gravity of neutrinos and baryons. Since baryons starting to collapse at ξ_{eq}, the condition of $t_{1p} > t_g$ is equivalent to a constraint $\ell > (v_{1p}/c) r_{eq}$. When $v_{1p} = 0.5$ c, we have $(v_{1p}/c) r_{eq} \sim 20h^{-1}$ Mpc. Therefore, the conclusions developed here related to length scales of about or larger than that of clusters of galaxies would qualitatively be maintained under the influence of loop motion.

This revised version is done after the Beijing massaccre at June 1989, which forces one of us (Fang) to become a temporary refugee. He wishes to deeply think all persons who help him to still be able to work on astrophysics in this hard time. We also wish to think Dr.Schaeffer for his valuable suggestion.

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Figure Captions

Fig.1 The evolution of Fourier components of density perturbations around a cosmic string loop against conformal time ξ , where the solid line describes the baryonic component and dashed line the neutrino component. The rest mass of neutrinos is taken to be m = 100 eV in (a) and 10 eV in (b). The conformal wavenumbers correspond respectively to kr_{eq}=0.398, 1.59, 6.31, 25.1, 100, 398, 1590.

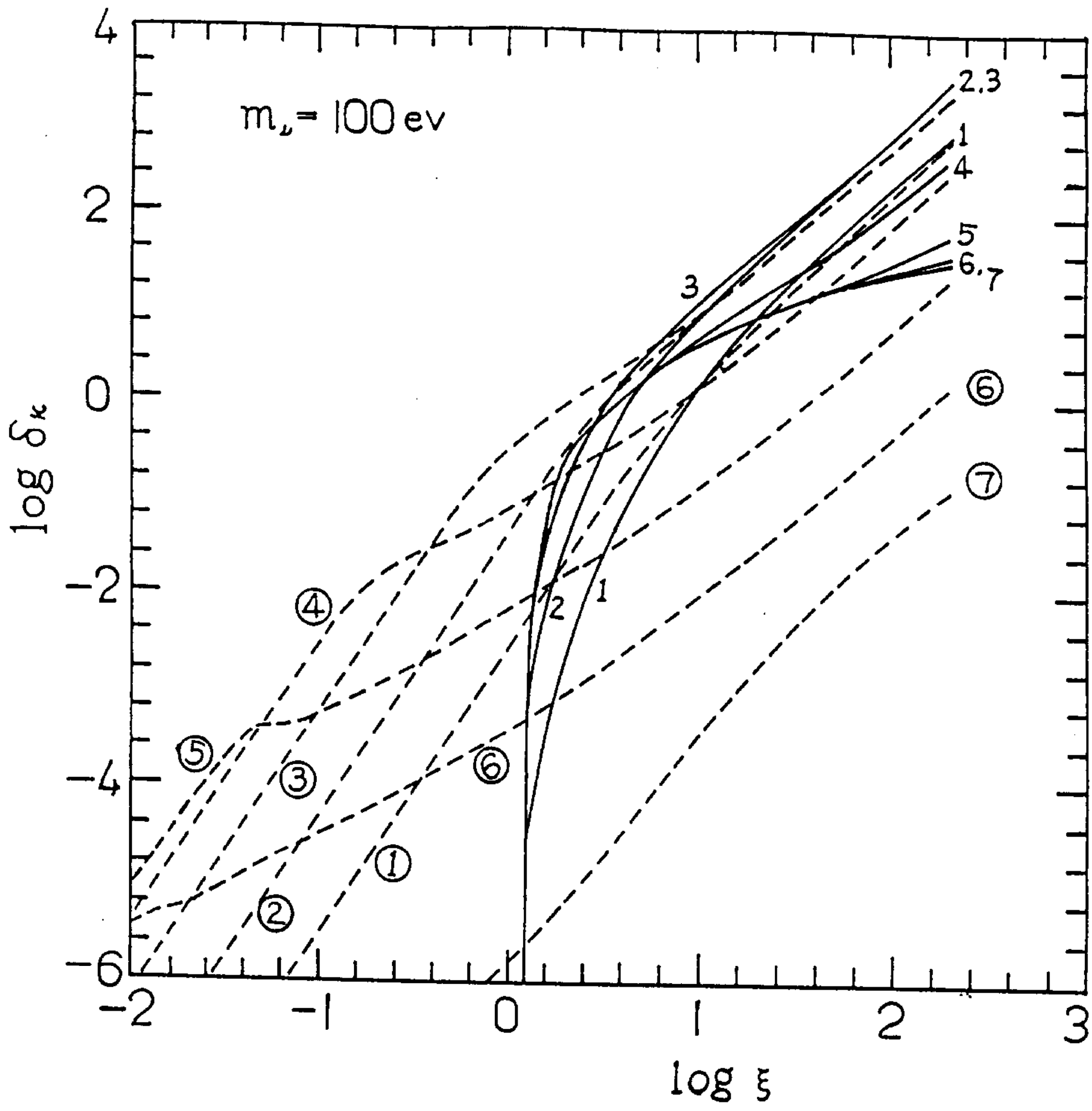
Fig.2 The evolution of the baryonic (solid) and the neutrino (dashed) density perturbations at the distances $x = 10^{-4} r_{eq}$ (subscript 1) and $10^{-1} r_{eq}$ (subscript 2) from the string loop, where A and B denote, respectively, m =100 eV and 10 eV.

Fig.3 The baryonic core radius as functions of redshift. The neutrino mass is taken to be 100 eV (solid) and 10 eV (dashed). The numbers on the lines are the string loop mass in the unit of 10 $M_{\odot}$.

Fig.4 The baryonic (solid) and neutrino (dashed) density perturbation profiles around the string loop at different cosmic times. (a) m_{ν} = 100 eV; (b) m_{ν} = 10 eV.

Fig.5 Rotation curves of the object formed by the accretion of string loop at the times of redshifts $z = 0.04$ (solid) and $z = 5.16$ (dashed). the numbers on the curves are the mass of the loop.

Fig.6 The evolution of rotating velocity at a present distance ~100 Kpc from the loop. The rest mass of neutrinos is taken to be 100 eV (solid) and 10 eV (dashed). The numbers on the curves are the loop mass in unit of 10 $M_{\odot}$.



m_ν = 100 ev
log δ_k
log ξ
4
2
0
−2
−4
−6
−2
−1
0
1
2
3
2,3
1
4
5
6,7
3
2
1
⑥
⑦
④
⑤
③
⑥
①
②

Fig. 1 (a)

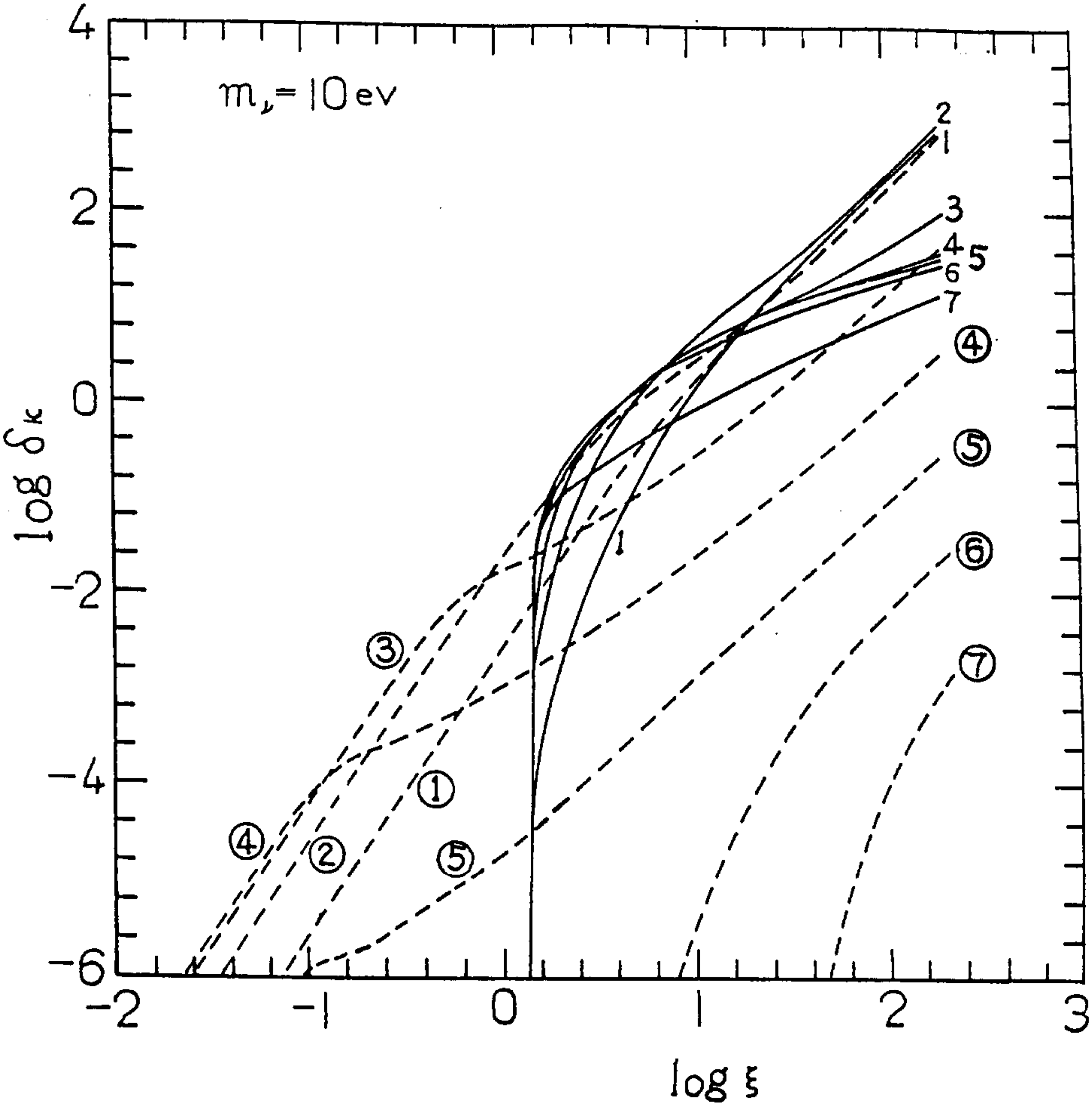


Fig. 1 (b)

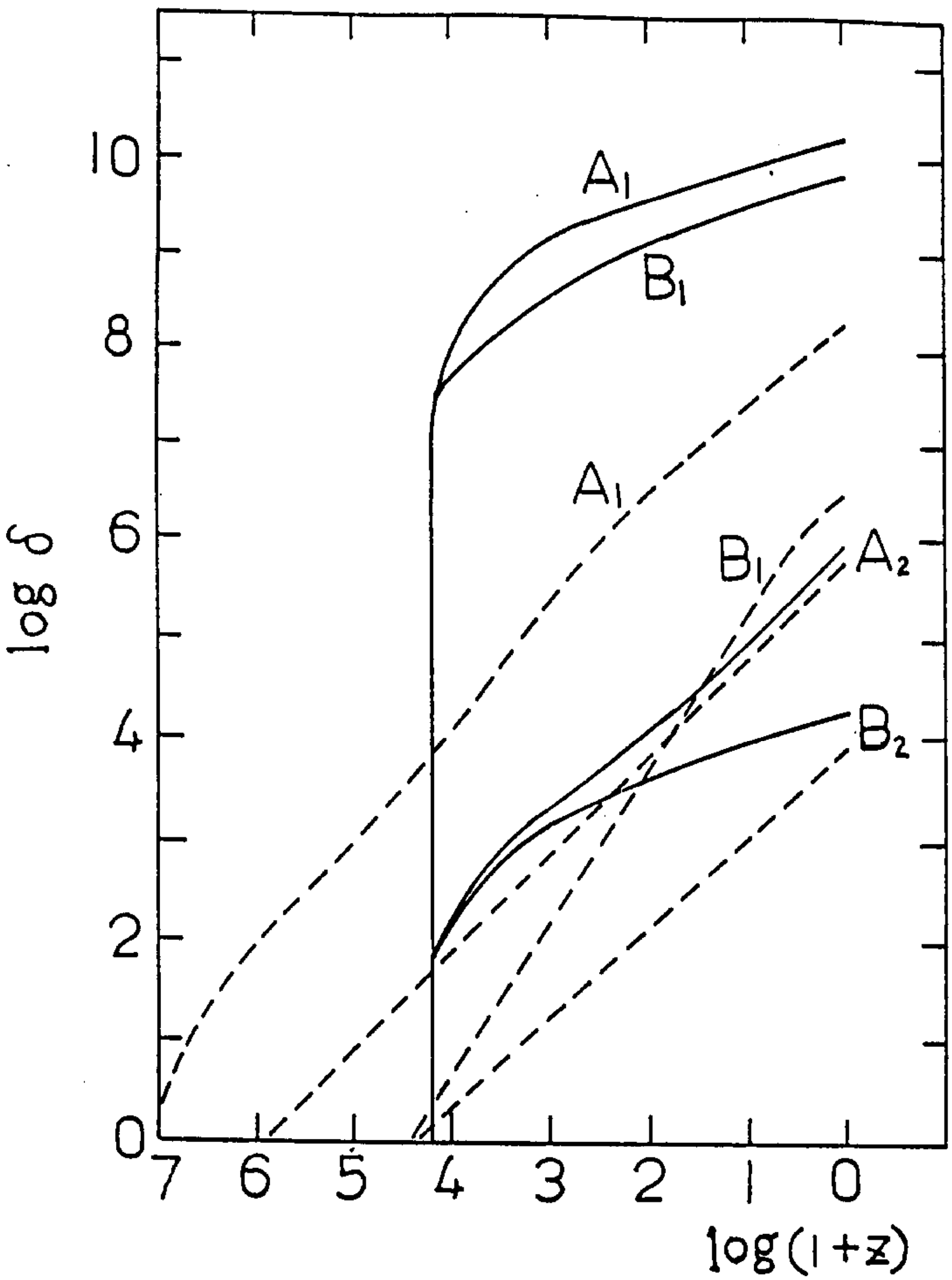


Fig. 2

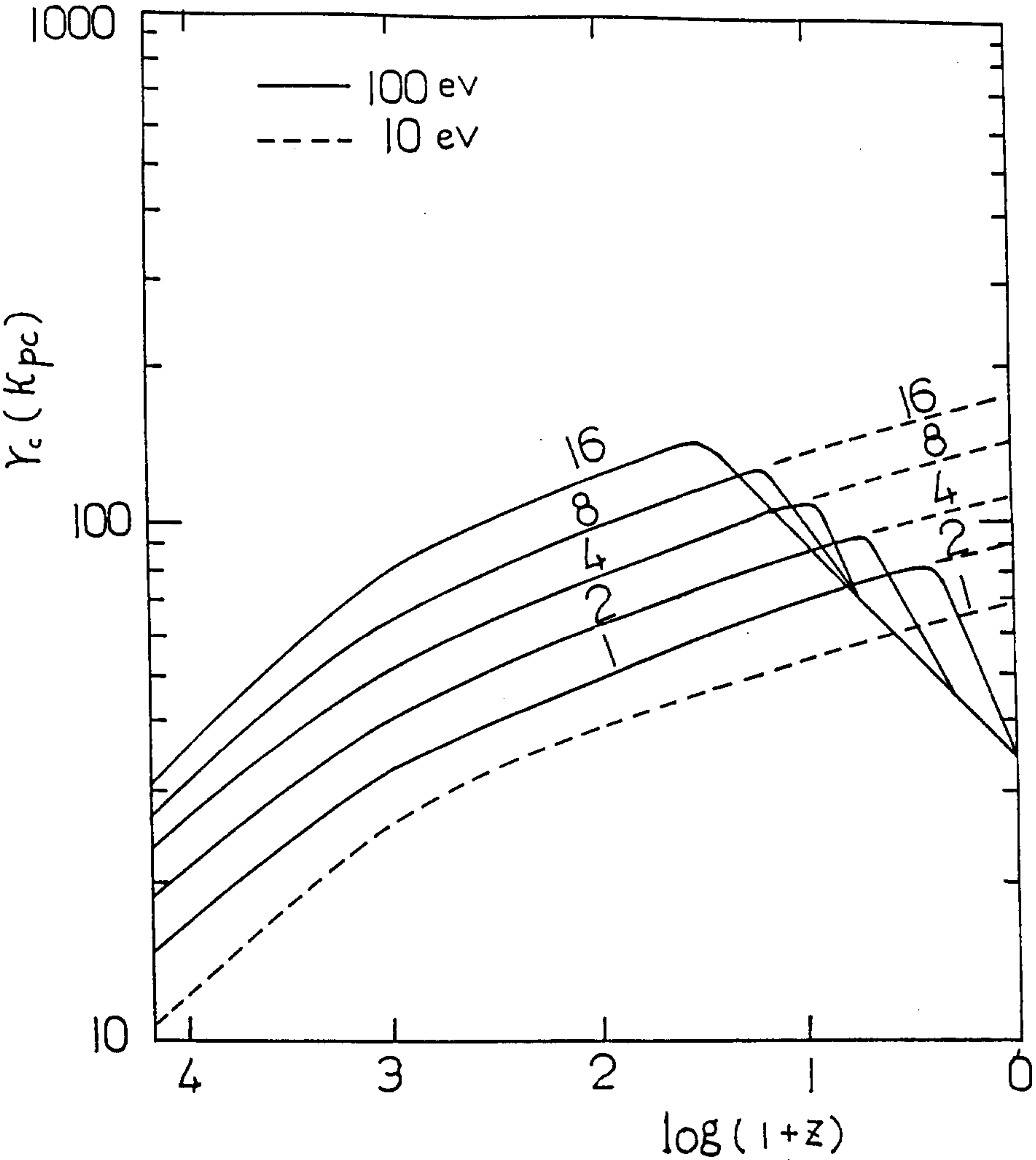
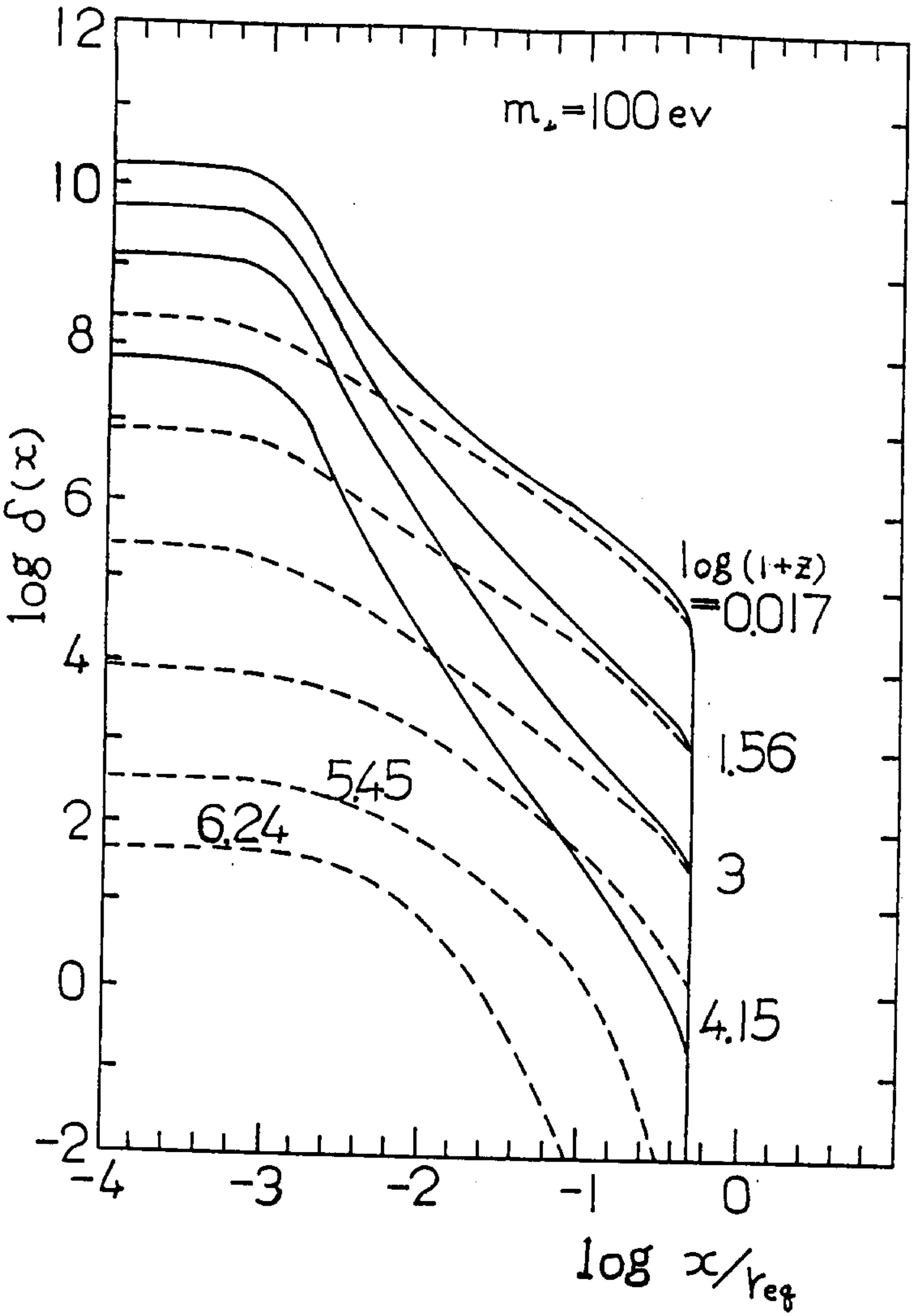


Fig.3



$m_\nu = 100$ ev
log (1+z)
=0.017
1.56
3
4.15
5.45
6.24
log δ(x)
log x/r_{eq}
12
10
8
6
4
2
0
−2
−4
−3
−2
−1
0

Fig. 4 (a)

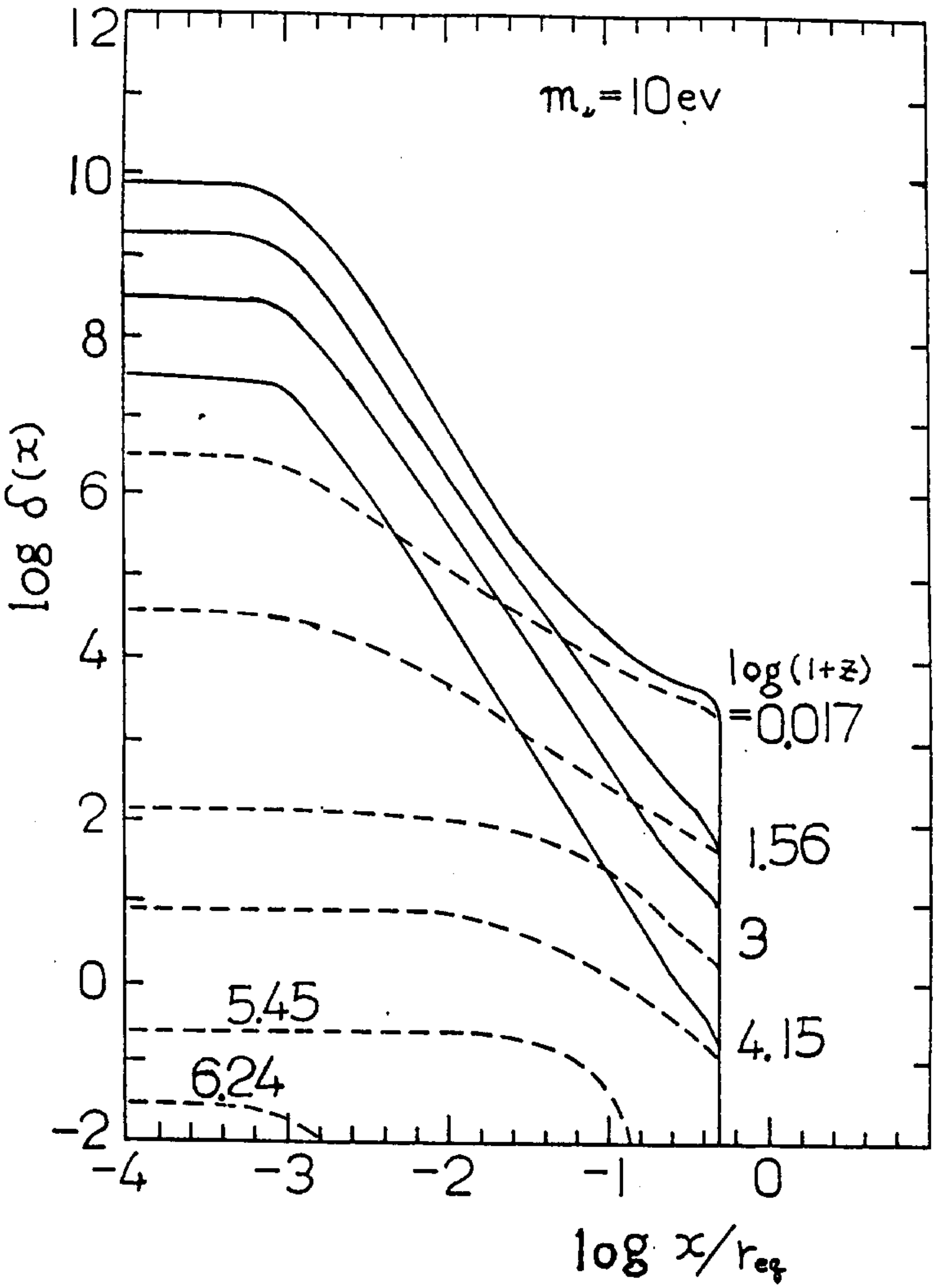

$m_\nu = 10$ ev
log(1+z)
=0.017
1.56
3
4.15
5.45
6.24
log δ(x)
log x/r_{eq}
12
10
8
6
4
2
0
−2
−4
−3
−2
−1
0

Fig. 4 (b)

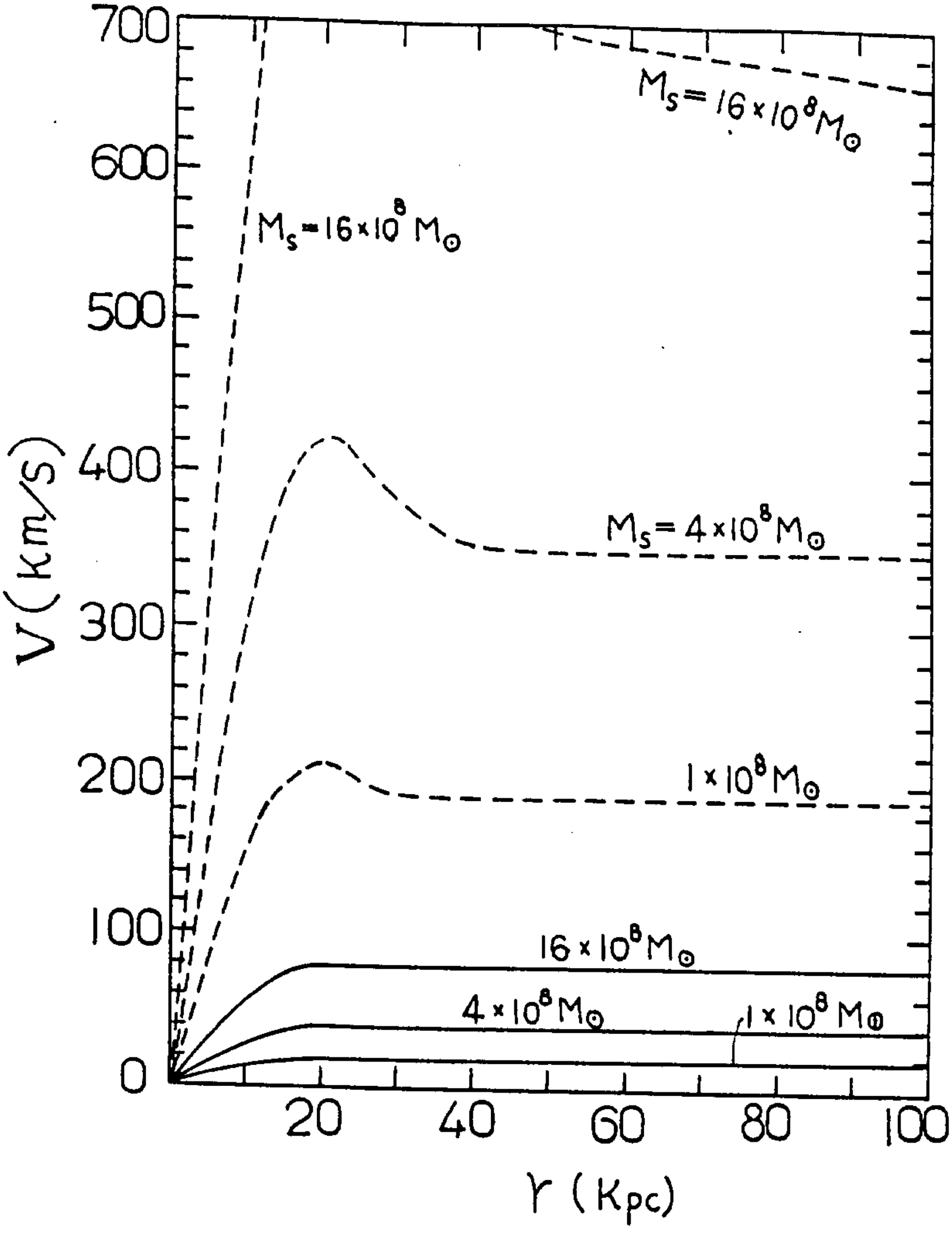

700
600
500
400
300
200
100
0
V (km/s)
20
40
60
80
100
r (Kpc)
$M_s = 16 \times 10^8 M_\odot$
$M_s = 16 \times 10^8 M_\odot$
$M_s = 4 \times 10^8 M_\odot$
$1 \times 10^8 M_\odot$
$16 \times 10^8 M_\odot$
$4 \times 10^8 M_\odot$
$1 \times 10^8 M_\odot$

Fig. 5

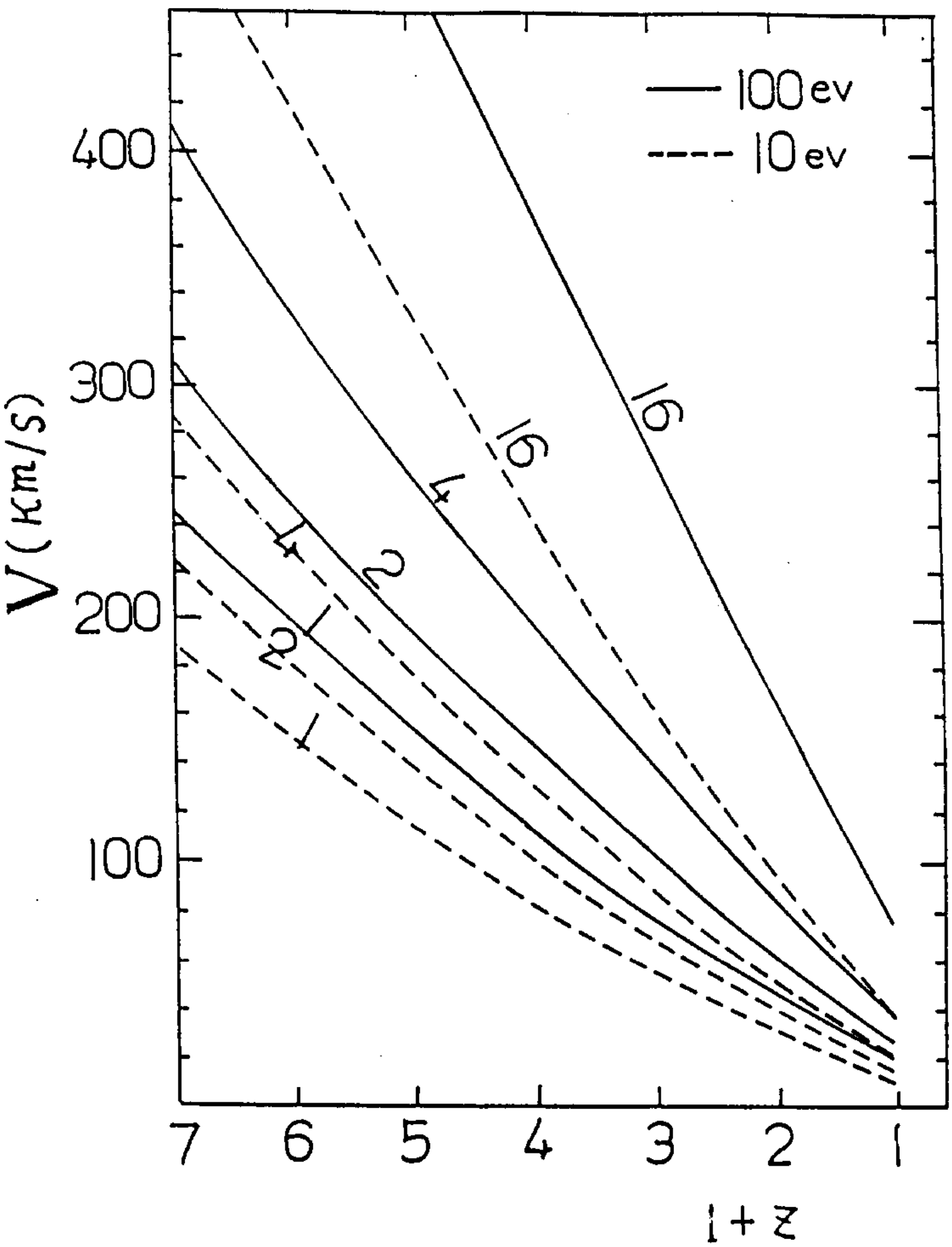


Fig. 6