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Duality in the Presence of Supersymmetry Breaking

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Abstract

We study Seiberg duality for $\mathcal{N} = 1$ supersymmetric QCD with soft supersymmetry-breaking terms. We generate the soft terms through gauge mediation by coupling two theories related by Seiberg duality to the same supersymmetry-breaking sector. In this way, we know what a supersymmetry-breaking perturbation in one theory maps into in its “dual”. Assuming a canonical Kähler potential we calculate the soft terms induced in the magnetic theory and find that some of the scalars acquire negative masses squared. If duality is still good for small supersymmetry breaking, this may imply some specific symmetry breaking patterns for supersymmetric QCD with small soft supersymmetry-breaking masses, in the case that its dual theory is weakly coupled in the infrared. In the limit of large supersymmetry breaking, the electric theory becomes ordinary QCD. However, the resulting symmetry breaking in the magnetic theory is incompatible with that expected for QCD.

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1 Introduction

The constrained structure of supersymmetric (SUSY) field theories provides powerful tools for analyzing their strong dynamics. Using these tools Seiberg gave striking evidence for the existence of “dual” pairs of $\mathcal{N} = 1$ supersymmetric gauge theories that give the same infrared (IR) physics [1]. One is immediately led to ask whether a similar phenomenon exists in the absence of supersymmetry [2]–[5], and in particular, whether theories related by Seiberg duality in the supersymmetric limit still give the same infrared physics when supersymmetry is broken [6]. To answer this question, one obviously starts with a pair of dual $\mathcal{N} = 1$ supersymmetric theories¹. The question of how to introduce supersymmetry breaking into this system is less obvious. In [4], soft supersymmetry breaking masses were added to both theories with the assumption that the scalar masses squared are positive. This assumption was relaxed in [5]. In Refs. [2, 3], soft masses were obtained by promoting some couplings to spurion fields, with frozen supersymmetry-breaking vacuum expectation values (vevs). It is not clear however what an explicit, supersymmetry-breaking perturbation in one theory maps into in the dual theory. In fact, even in the supersymmetric case, only the chiral operator map between the two theories is known in general [1, 9]. Alternatively, one may study theories in which supersymmetry is only spontaneously broken². Then the Lagrangian is manifestly supersymmetric, and one has some confidence in mapping superpotential perturbations between the two theories. This is the approach we take in this paper.

We start with an $SU(N)$ gauge theory with F flavors of matter in the fundamental representation, and imagine coupling it to a sector with dynamical supersymmetry breaking (DSB). Such theories have been extensively studied in the context of gauge-mediated supersymmetry breaking [10, 11, 12]. Some $SU(N)$ supermultiplets couple directly to the DSB sector and become heavy with supersymmetry-breaking mass splittings. Soft masses are then induced for the remaining $SU(N)$ squarks and gluinos through loops involving the heavy fields. In the supersymmetric limit, which is typically attained by setting some superpotential couplings in the DSB sector to zero, the theory has a dual description with gauge group $SU(F - N)$ and with the DSB sector essentially unchanged. We then turn supersymmetry breaking back on. This amounts to adding some superpotential term involving the DSB fields in the electric theory. (We will loosely refer to the original theory as the electric theory, and to the theory obtained by the duality operation as the magnetic theory.) Since supersymmetry is only dynamically broken, we know what this term maps into in the magnetic theory. We then study the effect

¹Refs. [7] considers a softly broken $\mathcal{N} = 2$ $SU(2)$ theory and uses the symmetries of the Seiberg-Witten solution [8] to study the low-energy theory, but only for $N_f \leq 2$.

²A toy model with spontaneous supersymmetry breaking was used in the first paper of [2] to justify the use of spurion fields to generate soft terms.

of the perturbation on the magnetic theory. In this theory, some fields are coupled directly to the DSB sector and become heavy with supersymmetry-breaking mass splittings. As in the electric theory, soft terms are then induced for the remaining $SU(F - N)$ fields, through loops involving the heavy fields. In some cases, and under certain assumptions, we will be able to calculate these masses. We will mainly be interested in the sign of the mass-squared induced for the scalar fields of the magnetic theory. As we will see, these are often negative, resulting in definite predictions for the pattern of global symmetry breaking in the low energy theory.

For small soft masses our analysis is reliable only when the magnetic theory is weakly coupled at low energies. Then the low-energy electric theory is strongly coupled, and we have no direct information about its behavior in the presence of small supersymmetry breaking. If duality continues to hold in the presence of small breaking, our findings for the vacuum of the magnetic theory then give a prediction for the pattern of chiral symmetry breaking in the electric theory.

For large supersymmetry breaking, we compare the global symmetry of the magnetic theory at low energies to what we expect in the electric theory. In many cases the two are incompatible, indicating that the proposed duality (with the assumptions we have made) breaks down.

The duality transformation was shown to withstand different perturbations. The electric and magnetic theories flow to the same IR theory after a mass term is added [1], or when a global symmetry is gauged [13]. As long as supersymmetry is preserved, one does not expect any phase transitions to occur as the size of the perturbation is increased [8, 14]. Once supersymmetry is broken, this is no longer the case. The electric and magnetic theories may undergo a phase transition and cease to be equivalent.

A crucial ingredient in our analysis is the Kähler potential of the dual theory, which is not known even in the supersymmetric case. In that case, the theories are only known to agree at zero energy, and the details of the Kähler potential are irrelevant for this agreement to hold. Here we study a simple class of candidate duals, those with a canonical Kähler potential, up to field renormalizations. These theories may not be the “real” duals. In fact, the correct duals could be ones with complicated Kähler potentials. Our analysis and results thus only apply to this class of simple candidate duals. This problem also plagues previous attempts to study non-supersymmetric duality. However, with the assumption about the Kähler potential in place, we can actually calculate the soft terms in both the electric and the magnetic theory, instead of adding squark masses by hand.

This paper is organized as follows. In section 2 we present the model we study and discuss our assumptions. We then calculate the soft masses induced in the magnetic theory for small supersymmetry breaking in section 3. We consider separately the case that the magnetic

theory is infrared-free, and the case where it is at a Banks-Zaks fixed point. Finally, we calculate the baryon and meson soft masses in the case $N_f = N_c + 1$ through a completely higgsed dual description. In section 4 we study the implications of the soft masses we found for the pattern of chiral symmetry breaking in the vacuum of the magnetic theory. We then move on to the case of large supersymmetry breaking in section 5. Again, we present the soft masses generated in the magnetic theory and the resulting global symmetry. Section 6 contains our conclusions and some final remarks. In appendix A we briefly discuss the magnetic theory with pure matter messengers. Finally, some details of the calculation are summarized in appendix B.

2 Framework

Let us now describe in more detail the model we consider. The theory we start with is an $SU(N)$ gauge theory with $N_f + 1$ flavors: the fields Q^i , H , transform as $SU(N)$ fundamentals, and the fields $\bar{Q}_i$, $\bar{H}$ transform as $SU(N)$ antifundamentals, with $i = 1 \dots N_f$. The theory is coupled to a DSB sector through the superpotential coupling [11]

$$SH \cdot \bar{H} , \quad (2.1)$$

where S is a field of the DSB sector. We do not specify here the details of the DSB sector. Rather, in the following, we will clarify the different requirements on this sector. We return to this point at the end of this section. We would like the full theory to have a stable minimum in which none of the $SU(N)$ fields develop vevs at the tree level. At this minimum, we would like the field S to develop A - and F - type vevs, which we denote by S_0 and F_0 , with $S_0^2 > F_0$. This can be ensured by taking the parameter that induces supersymmetry breaking to be small enough. Then, the squarks and gluinos of $SU(N)$ acquire masses of the order F_0/S_0 times a loop factor [11]. Here we have implicitly assumed that the $SU(N)$ gauge theory is weakly coupled at the scale S_0 . Thus, if $SU(N)$ is asymptotically free, we take its scale Λ to be much smaller than S_0 . If $SU(N)$ is not asymptotically free, we take Λ to be the largest scale in the problem.

In the supersymmetric limit, the theory has a dual description with gauge group $SU(N_f + 1 - N)$, fields q_i , h and $\bar{q}^i$, $\bar{h}$ in the fundamental and antifundamental representations respectively, and gauge singlet fields M_j^i , V^i , $\bar{V}_i$ and P corresponding to the mesons of the electric theory [1]. For the two theories to agree in the infrared, the following superpotential is required in the magnetic theory [1]:

$$W = \frac{1}{\mu'} \left(M_j^i q_i \cdot \bar{q}^j + V^i q_i \cdot \bar{h} + \bar{V}_i h \cdot \bar{q}^i + P h \cdot \bar{h} \right) + PS + W_{SB}(S, \phi) , \quad (2.2)$$

where the scale μ' is required on dimensional grounds, and is related to the $SU(N)$ scale Λ and the $SU(N_f + 1 - N)$ scale $\bar{\Lambda}$ through [1]

$$\Lambda^{3N-N_f-1} \bar{\Lambda}^{2(N_f+1)-3N} \sim \mu'^{N_f+1}. \quad (2.3)$$

The DSB sector remains untouched by the duality transformation. In (2.2) we indicated the superpotential associated with this sector, $W_{SB}(S, \phi)$, and ϕ collectively denotes the fields of this sector apart from S .

Redefining the fields to get a canonical Kähler potential, the superpotential can be rewritten as,

$$W = \lambda \left(M_j^i q_i \cdot \bar{q}^j + V^i q_i \cdot \bar{h} + \bar{V}_i h \cdot \bar{q}^i + P h \cdot \bar{h} \right) + \mu P S + W_{SB}(S, \phi), \quad (2.4)$$

where λ is a dimensionless coupling. The potential is then:

$$\begin{aligned} V = & \lambda^2 \left(\left| M_j^i q_i + \bar{V}_j \bar{h} \right|^2 + \left| M_j^i \bar{q}^j + V^i h \right|^2 + \left| q_i \cdot \bar{q}^j \right|^2 + \left| q_i \cdot \bar{h} \right|^2 + \left| h \cdot \bar{q}^j \right|^2 \right. \\ & + \left. \left| \bar{V}_i \bar{q}^i + P \bar{h} \right|^2 + \left| V^i q_i + P h \right|^2 \right) + \left| \lambda h \cdot \bar{h} + \mu S \right|^2 \\ & + \left| \mu P + \frac{\partial W_{SB}}{\partial S} \right|^2 + V_{SB}(S, \phi). \end{aligned} \quad (2.5)$$

We would like to consider a minimum at which the fields M_j^i , V^i , $\bar{V}_i$, q_i and $\bar{q}^i$ do not get tree-level vevs. Extremizing the potential with respect to P we thus find,

$$\lambda^2 P (|h|^2 + |\bar{h}|^2) + \mu \left(\mu P + \frac{\partial W_{SB}}{\partial S} \right) = 0. \quad (2.6)$$

And for non-zero $\partial W_{SB}/\partial S$, P develops a non-zero vevs. The h derivative then gives,

$$\lambda |P|^2 h^* + (\lambda h \cdot \bar{h} + \mu S)^* \bar{h} = 0. \quad (2.7)$$

Assuming that S develops a vev, there are two qualitatively different possibilities then for a stable minimum. One is that h and $\bar{h}$ do not acquire vevs and the energy, apart from $V_{SB}(S, \phi)$, equals $\mu^2 S^2$. We will refer to this type of minimum as a “matter messenger” minimum. Another possibility, which we will term a “gauge messenger” minimum, is that h and $\bar{h}$ develop vevs to compensate for the S vev. Clearly, when the S vev is large compared to the amount of supersymmetry breaking, this will be the preferred minimum. This is also the minimum that connects smoothly to the supersymmetric case when SUSY breaking vanishes. If the dual description makes sense at all, it had better make sense for small breaking. We will therefore focus on the gauge-messenger minimum. The case of a magnetic theory with matter messengers will be discussed in appendix A.

At the minimum we consider h and $\bar{h}$ acquire vevs. We can choose

$$h^{\bar{N}+1} = -\bar{h}_{\bar{N}+1} = v, \quad (2.8)$$

with $\bar{N} \equiv N_f - N$. The field P also acquires a vev with

$$\lambda P^2 + \lambda v^2 - \mu S = 0. \quad (2.9)$$

The F components of P and h are then,

$$F_h = \lambda P v, \quad F_P = -\lambda P^2. \quad (2.10)$$

The h vevs higgs the group down to $SU(\bar{N})$. This is precisely what one would expect if there were no supersymmetry breaking in the theory. A non-zero vev for S in the electric theory is a mass term for one flavor of that theory. The dual theory is then higgsed by one unit. At low energies, the electric theory is an $SU(N)$ theory with N_f flavors and the magnetic theory has gauge group $SU(N_f - N)$.

As we saw earlier, the squarks of the electric theory obtain soft masses through their gauge interactions with the heavy flavor $H, \bar{H}$. In the magnetic theory, the higgsing is accompanied by supersymmetry breaking. The heavy gauge multiplets corresponding to the broken generators have masses that are split due to the supersymmetry breaking (we give explicit expressions for these masses in appendix B). The remaining squarks couple directly to these heavy gauge multiplets and will therefore receive soft masses starting at the one-loop order. Another consequence of the $h, \bar{h}$ and P vevs is that the “mixed” mesons V^i and $\bar{V}_i$, and the broken components of the squarks, $q_i^{\bar{N}+1}$ and $\bar{q}_{\bar{N}+1}^i$ now obtain tree-level masses through the superpotential. We then have additional heavy multiplets with supersymmetric mass, λv , and splittings proportional to F_h . The remaining squarks and scalar mesons couple to these through Yukawa interactions and receive soft masses, again starting at one-loop. Thus, below the scale v we have an $SU(\bar{N})$ theory with the dual quarks $q_i, \bar{q}^i$, and the mesons M_j^i with $i, j = 1 \dots N_f$. The scalar components of these fields acquire masses through their gauge and Yukawa interactions to the heavy multiplets.

To summarize, at low energies we have constructed two theories related by Seiberg duality and with soft supersymmetry-breaking terms, including scalar masses, gaugino masses and A -terms. In the following sections, we will calculate these terms, starting with the limit of small supersymmetry breaking.

But before doing that, a few comments regarding the source of supersymmetry breaking are in order. We envisage a situation in which the DSB sector has a stable minimum at finite field vevs. We also imagine that the coupling of the DSB sector to the $SU(N)$ and $SU(\bar{N})$ sector does not dramatically alter the qualitative properties of the minimum,

so that there is still a minimum with no runaway fields, although the actual location of the minimum may shift between the electric and magnetic theories. In particular the S vev in the magnetic theory need not be equal to S_0 . Nevertheless, since the DSB sector is only used as a source for providing SUSY breaking, we assume that it does not shift much in coupling to the two theories, (in analogy to the heat bath in thermodynamics,) at least for small SUSY breaking. This leads to some requirements on the different scales in the problem. Essentially, for given (finite) parameters of the $SU(N)$ and $SU(\bar{N})$ theories we need to choose the scales associated with the DSB sector to be large compared to the relevant scales of the $SU(N)$ and $SU(\bar{N})$ sectors. At the same time, such a choice ensures that the $SU(N)$ ($SU(\bar{N})$) theory is perturbative at the scale S_0 (v), so that our analysis is reliable. For example, if both $SU(N)$ and $SU(\bar{N})$ are asymptotically free, then for any given Λ and μ , the typical scale of the DSB sector should be high enough, with $\Lambda, \mu \ll S_0 \sim S$, so that $\bar{\Lambda} \ll v \sim \sqrt{\mu} S$. Though it should be possible to construct a DSB sector (albeit not necessarily an aesthetically pleasing one) that satisfies these requirements, this would not significantly contribute to our investigation and so we do not do so here. Most of our analysis is then qualitatively equivalent to an analysis that treats S as a spurion. From this point of view, one superpotential coupling, the supersymmetric mass for H , $\bar{H}$, is promoted to a spurion field, and the resulting soft supersymmetry breaking induced in each theory by the gauge and/or Yukawa interactions is then calculated.

3 Duality with small SUSY breaking

We first consider the case of small SUSY breaking. In this limit one may reasonably hope that the duality and exact results obtained by Seiberg [1, 15] still hold approximately, with possible corrections higher order in SUSY breaking [4, 2, 3].

In the electric theory, the extra flavor H , $\bar{H}$ get SUSY-preserving and SUSY-breaking masses from the A - and F -type vevs of S , S_0 and F_0 , respectively. Throughout this section, we take N_f to be smaller than, or close to, $3N/2$, so that the magnetic theory is weakly coupled in the IR. The electric theory is then asymptotically free, and we need $S_0 > \Lambda$ in order to be able to perform perturbative calculations.

The electric theory is in the usual “gauge mediation” scenario [11]. At the scale S_0 , gauginos get masses at one loop and squarks get masses at the two-loop order. The results are well known. At leading order in F_0 , they are [11]

$$\tilde{M}_g(S_0) = \frac{\alpha_e(S_0)}{4\pi} \frac{F_0}{S_0}, \quad (3.11)$$

$$\tilde{m}_Q^2(S_0) = \tilde{m}_{\bar{Q}}^2(S_0) = \frac{N^2 - 1}{N} \frac{\alpha_e^2(S_0)}{(4\pi)^2} \left(\frac{F_0}{S_0}\right)^2. \quad (3.12)$$

The squarks obtain positive masses squared. The small SUSY breaking case we consider here corresponds to $\frac{F_0}{S_0^2} \ll 1$ and $\frac{\alpha_e F_0}{4\pi S_0} \ll \Lambda$. Because the gauge coupling α is always accompanied by the loop factor $(4\pi)^{-1}$, we will absorb $(4\pi)^{-1}$ into α and redefine

$$\alpha \equiv \frac{g^2}{16\pi^2} \quad (3.13)$$

for convenience. The expressions with the original definition can be easily recovered by replacing α by $\alpha/(4\pi)$.

We now turn to the magnetic theory. We know that the duality holds when a SUSY-preserving mass S_0 ($F_S = 0$) is added to H and $\bar{H}$: the electric theory flows to a theory with the same gauge group and one less flavor, and the magnetic theory gets higgsed by $h\bar{h} = -\mu S_0$ to a theory with one less color and one less flavor. The resulting low energy theories are still dual to each other. When a small SUSY breaking F_S is introduced, we expect that the vacuum still lies close to the supersymmetric case, ($h\bar{h} \approx -\mu S_0/\lambda$) so that they connect smoothly when $F_S \rightarrow 0$. The gauge symmetry is higgsed from $SU(\bar{N} + 1)$ to $SU(\bar{N})$ by $\langle h \rangle, \langle \bar{h} \rangle$. However, as discussed in the previous section, the masses of the heavy gauge supermultiplets will be split by nonzero F_h and $F_{\bar{h}}$ due to SUSY breaking. Therefore, we have gauge messengers in the magnetic theory. In addition, some matter fields (V^i, q_i^{N+1}) also receive masses from the h and $\bar{h}$ vevs, and they give extra contributions to the soft masses of the light fields through Yukawa couplings. Here we assume that $\langle h \rangle = v > \bar{\Lambda}$ ($< \bar{\Lambda}$) when the magnetic theory is asymptotically (infrared) free, and that the Yukawa coupling is not large, so that we can perform perturbative calculations.

In the magnetic theory, the light scalars can receive one-loop contributions to their soft masses. However, these start at $\mathcal{O}(F_h^4)$ [16, 17] whereas the two-loop contributions start at $\mathcal{O}(F_h^2)$. Therefore, for small SUSY breaking, the two-loop contributions dominate. To calculate them, we can apply the method of Giudice and Rattazzi [17]. In this method, we first derive the wavefunction renormalization of the light field as a function of the heavy fields' threshold, then replace the threshold by $\sqrt{XX^\dagger}$, where X is the field which provides the masses and SUSY breaking for the heavy fields. The soft mass of any light field can then be obtained from the $\theta^2\bar{\theta}^2$ component of the logarithm of the wavefunction to leading order in F_X/X , without any complicated diagram calculation. In the magnetic theory, heavy fields obtain their masses from h and $\bar{h}$, so $X = h, \bar{h}$ in this case³.

³The $(\delta h^{N+1} - \delta \bar{h}_{N+1})$ field also mixes with δP and both receive SUSY breaking effects from F_P . However, the corrections due to F_P should be higher order in F_h , ($F_P \sim F_h^2/v^2$), and are not enhanced by N_f or $\bar{N}$. These fields do not enter at one loop either.

The one-loop renormalization group equations (RGE's) of the dual quark and meson wave-functions, and gauge and Yukawa couplings are

$$\frac{d}{dt} \ln Z_q = 4c_q \alpha_m - 2d_q \alpha_\lambda, \quad (3.14)$$

$$\frac{d}{dt} \ln Z_M = -2d_M \alpha_\lambda, \quad (3.15)$$

$$\frac{d}{dt} \alpha_m = 2b \alpha_m^2, \quad (3.16)$$

$$\frac{d}{dt} \alpha_\lambda = \alpha_\lambda (-4C \alpha_m + 2D \alpha_\lambda), \quad (3.17)$$

where $\alpha_m = g_m^2/(16\pi^2)$, $\alpha_\lambda = \lambda^2/(16\pi^2)$, and $c_q = \frac{(\bar{N}+1)^2-1}{2(\bar{N}+1)} \left(\frac{\bar{N}^2-1}{2\bar{N}} \right)$, $d_q = N_f + 1(N_f)$, $d_M = \bar{N} + 1(\bar{N})$, $b = N_f - 3\bar{N} - 2(N_f - 3\bar{N})$, $C = 2c_q$, $D = 2d_q + d_M$ above (below) the scale v . The method of Giudice and Rattazzi can also give the soft masses of the light fields at low energies directly, including renormalization running effects. We, however, will calculate the masses at the scale v and separate the discussion of the running effects for clarity. The soft masses of the dual gauginos, dual squarks, mesons, and the trilinear scalar coupling A -term generated at the scale v are

$$\tilde{M}_{\tilde{g}}(v) = -2\alpha_m \left(\frac{F_h}{v} \right), \quad (3.18)$$

$$\begin{aligned} \tilde{m}_q^2(v) = \tilde{m}_{\tilde{q}}^2(v) = & \left\{ -\alpha_m^2 \left[2 \frac{\bar{N}^2-1}{\bar{N}} + (N_f - 3\bar{N} - 2) \frac{\bar{N}^2 + \bar{N} + 1}{\bar{N}(\bar{N} + 1)} \right] \right. \\ & + \alpha_m \alpha_\lambda \left[-2 \frac{(\bar{N} + 1)^2 - 1}{\bar{N} + 1} + 2N_f \frac{\bar{N}^2 + \bar{N} + 1}{\bar{N}(\bar{N} + 1)} \right] \\ & \left. - \alpha_\lambda^2 [N_f - \bar{N} - 3] \right\} \left(\frac{F_h}{v} \right)^2, \end{aligned} \quad (3.19)$$

$$\begin{aligned} \tilde{m}_M^2(v) = & \left\{ -\alpha_m \alpha_\lambda \left[2 \frac{\bar{N} - 1}{\bar{N} + 1} \right] \right. \\ & \left. + \alpha_\lambda^2 [2N_f - 2\bar{N} + 3] \right\} \left(\frac{F_h}{v} \right)^2, \end{aligned} \quad (3.20)$$

$$A(v) = \left\{ 2 \frac{\bar{N}^2 + \bar{N} + 1}{\bar{N}(\bar{N} + 1)} \alpha_m - 3\alpha_\lambda \right\} \left(\frac{F_h}{v} \right). \quad (3.21)$$

If we assume that the DSB sector is not drastically altered by the coupling to the electric and magnetic theories, as discussed at the end of section 2, we find, minimizing the potential of the magnetic theory, that for small SUSY breaking and $S_0 \gg \mu$ (as required for perturbative calculations), we have $\frac{F_h}{v} \sim \frac{1}{2} \frac{F_0}{S_0}$. Although we will only concentrate on the sign of the scalar masses, this shows that the soft breaking masses generated in the magnetic theory are about the same order as in the electric theory.

To study the low energy theory, we have to evolve the soft breaking terms down to low scales. We can do so if the couplings remain perturbative at low energies. Therefore, we will consider the following two cases in the duality regime: the magnetically free (MF) case ($N_f > 3\bar{N}$), and the case with a Banks-Zaks fixed point (BZ) [18] for the magnetic theory ($N_f/\bar{N} = 3 - \epsilon$ with large N_f , $\bar{N}$ and $\epsilon \ll 1$). We can also analyze the magnetic theory for $N_f = N + 1$, for which the electric theory confines and the magnetic theory is completely higgsed. In all three cases the electric theory is strongly coupled at low energies, so they are of special interest.

The results in Eqs. (3.19), (3.20) still depend on the unknown couplings in the magnetic theory, especially the relative sizes of the Yukawa coupling and the gauge coupling. If we work in the large N_f , $\bar{N}$ (and also $N_f - \bar{N}$) limit, the results simplify to

$$\tilde{m}_q^2(v) = \tilde{m}_{\bar{q}}^2(v) \approx -(N_f - \bar{N})(\alpha_m - \alpha_\lambda)^2 \left(\frac{F_h}{v}\right)^2, \quad (3.22)$$

$$\tilde{m}_M^2(v) \approx \left\{ 2(N_f - \bar{N})\alpha_\lambda^2 - 2\alpha_m\alpha_\lambda \right\} \left(\frac{F_h}{v}\right)^2. \quad (3.23)$$

We can see that at the scale v the dual squarks generally get negative masses squared, and the mesons get positive masses squared (if α_λ/α_m is not too small). This is already an interesting result.

In the following subsections we consider the RG running effects for each case separately.

3.1 Magnetic free case

In the infrared theory, the couplings get weaker in running down toward low energies, so it is sufficient to use the one-loop RGE's. The one-loop RGE's of the relevant quantities below v are

$$\frac{d}{dt}\alpha_m = (2N_f - 6\bar{N})\alpha_m^2, \quad (3.24)$$

$$\frac{d}{dt}\alpha_\lambda = \alpha_\lambda \left[-4 \frac{\bar{N}^2 - 1}{\bar{N}} \alpha_m + (4N_f + 2\bar{N})\alpha_\lambda \right], \quad (3.25)$$

$$\frac{d}{dt}\tilde{M}_{\tilde{g}} = (2N_f - 6\bar{N})\alpha_m \tilde{M}_{\tilde{g}}, \quad (3.26)$$

$$\frac{d}{dt}A = 4 \frac{\bar{N}^2 - 1}{\bar{N}} \alpha_m \tilde{M}_{\tilde{g}} + (4N_f + 2\bar{N})\alpha_\lambda A, \quad (3.27)$$

$$\frac{d}{dt}\tilde{m}_q^2 = \frac{d}{dt}\tilde{m}_{\bar{q}}^2 = -4 \frac{\bar{N}^2 - 1}{\bar{N}} \alpha_m \tilde{M}_{\tilde{g}}^2 + 2N_f\alpha_\lambda(\tilde{m}_q^2 + \tilde{m}_{\bar{q}}^2 + \tilde{m}_M^2 + A^2), \quad (3.28)$$

$$\frac{d}{dt}\tilde{m}_M^2 = 2\bar{N}\alpha_\lambda(\tilde{m}_q^2 + \tilde{m}_{\bar{q}}^2 + \tilde{m}_M^2 + A^2). \quad (3.29)$$

We should evolve these quantities down to the scale of the soft breaking masses. For sufficiently small SUSY breaking, there will be enough running for these quantities to reach their

asymptotic behavior, which we will discuss below.

The RGE's of the gauge coupling and the gaugino are easy to solve. The solutions are

$$\frac{1}{\alpha_m(p)} = \frac{1}{\alpha_m(v)} + (2N_f - 6\bar{N}) \ln \frac{v}{p}, \quad (3.30)$$

$$\frac{\tilde{M}_{\tilde{g}}(p)}{\alpha_m(p)} = \frac{\tilde{M}_{\tilde{g}}(v)}{\alpha_m(v)}. \quad (3.31)$$

Both α_m and $\tilde{M}_{\tilde{g}}$ get smaller and evolve toward zero at low energies. For the Yukawa coupling, combining Eqs. (3.24) and (3.25), we obtain

$$\frac{d}{dt} \ln \left(\frac{\alpha_\lambda}{\alpha_m} \right) = (4N_f + 2\bar{N})\alpha_\lambda - \left[4 \frac{\bar{N}^2 - 1}{\bar{N}} + 2N_f - 6\bar{N} \right] \alpha_m. \quad (3.32)$$

The ratio α_λ/α_m reaches its fixed point when the right hand side of (3.32) vanishes. So, at low energies,

$$\frac{\alpha_\lambda}{\alpha_m} \rightarrow \frac{N_f \bar{N} - \bar{N}^2 - 2}{(2N_f + \bar{N})\bar{N}}, \quad (3.33)$$

and α_λ approaches zero too. Similarly, combining (3.26) and (3.27), we obtain

$$\frac{d}{dt} \ln \left(\frac{A}{\tilde{M}_{\tilde{g}}} \right) = (4N_f + 2\bar{N})\alpha_\lambda + 4 \frac{\bar{N}^2 - 1}{\bar{N}} \alpha_m \frac{\tilde{M}_{\tilde{g}}}{A} - (2N_f - 6\bar{N})\alpha_m. \quad (3.34)$$

Substituting in the asymptotic ratio of α_λ/α_m , (3.33), we find $\tilde{M}_{\tilde{g}}/A \rightarrow 1$ in evolving to low energies.

For the scalar masses, it is convenient to define the following two linear combinations:

$$X \equiv \tilde{m}_q^2 + \tilde{m}_{\bar{q}}^2 + \tilde{m}_M^2, \quad (3.35)$$

$$Y \equiv \bar{N}\tilde{m}_q^2 - N_f\tilde{m}_M^2. \quad (3.36)$$

Then from (3.28), (3.29) we have

$$\frac{d}{dt} X = -8 \frac{\bar{N}^2 - 1}{\bar{N}} \alpha_m \tilde{M}_{\tilde{g}}^2 + (4N_f + 2\bar{N})\alpha_\lambda X + (4N_f + 2\bar{N})\alpha_\lambda A^2, \quad (3.37)$$

$$\frac{d}{dt} Y = -4(\bar{N}^2 - 1)\alpha_m \tilde{M}_{\tilde{g}}^2. \quad (3.38)$$

Combining (3.37) with (3.26), we obtain

$$\frac{d}{dt} \ln \left(\frac{\tilde{M}_{\tilde{g}}^2}{X} \right) = (4N_f - 12\bar{N})\alpha_m + 8 \frac{\bar{N}^2 - 1}{\bar{N}} \alpha_m \frac{\tilde{M}_{\tilde{g}}^2}{X} - (4N_f + 2\bar{N})\alpha_\lambda - (4N_f + 2\bar{N})\alpha_\lambda \frac{A^2}{X}. \quad (3.39)$$

Substituting in the asymptotic relations between α_λ and α_m , A and $\tilde{M}_{\tilde{g}}$, we find $\tilde{M}_{\tilde{g}}^2/X \rightarrow 1$ in evolving to low energies. Since $\tilde{M}_{\tilde{g}}$ scales toward zero, we obtain the interesting sum rule,

$$X = \tilde{m}_q^2 + \tilde{m}_{\bar{q}}^2 + \tilde{m}_M^2 \rightarrow 0, \quad (3.40)$$

in the extreme infrared. It also tell us that some of the masses squared will be negative! To get the individual masses we integrate (3.38) and the result is

$$(\bar{N}\tilde{m}_q^2(p) - N_f\tilde{m}_M^2(p)) = (\bar{N}\tilde{m}_q^2(v) - N_f\tilde{m}_M^2(v)) + \frac{\bar{N}^2 - 1}{N_f - 3\bar{N}}\tilde{M}_g^2(v) \left[1 - \left(\frac{\alpha_m(p)}{\alpha_m(v)} \right)^2 \right]. \quad (3.41)$$

Since $\alpha_m(p) \rightarrow 0$ as $p \rightarrow 0$, we can solve for the asymptotic scalar masses from (3.40), (3.41),

$$\tilde{m}_q^2 = \tilde{m}_{\bar{q}}^2 = -\frac{\tilde{m}_M^2}{2} = \frac{1}{2N_f + \bar{N}} \left[(\bar{N}\tilde{m}_q^2(v) - N_f\tilde{m}_M^2(v)) + \frac{\bar{N}^2 - 1}{N_f - 3\bar{N}}\tilde{M}_g^2(v) \right]. \quad (3.42)$$

The masses in the infrared are determined by the particular combination of the masses generated at the high scale. As we have seen, the first part, $\bar{N}\tilde{m}_q^2(v) - N_f\tilde{m}_M^2(v)$, is negative for large $\bar{N}$ and N_f . In fact, it is always negative for $N_f > 3\bar{N}$. If there were not the gaugino mass contribution, or if the gaugino mass contribution is small, as for N_f much larger than $3\bar{N}$, the dual squark masses squared are negative and the meson masses squared are positive. The dual squarks will get vevs to break the gauge and global symmetries. We will discuss the resulting vacuum and symmetry breaking pattern in the next section. For N_f close to $3\bar{N}$, the gaugino mass contribution is of the same order as the initial scalar masses, and it may change the signs of the masses squared of the dual squarks and the mesons.

3.2 Magnetic Banks-Zaks fixed point

For $\frac{3}{2}\bar{N} < N_f < 3\bar{N}$, there is a nontrivial fixed point for the gauge coupling [1]. The fixed point is at weak coupling in the limit of large $\bar{N}$ and N_f , with $N_f/\bar{N} = 3 - \epsilon$ fixed and $\epsilon \ll 1$. To lowest order in ϵ it can be obtained by examining the 2-loop RGE of the gauge coupling,

$$\frac{d}{dt}\alpha_m = 2(N_f - 3\bar{N})\alpha_m^2 + 2 \left(-6\bar{N}^2 + 2\bar{N}N_f + 2N_f \frac{\bar{N}^2 - 1}{\bar{N}} \right) \alpha_m^3 - \frac{4N_f}{\bar{N}}\alpha_m^2\alpha_\lambda. \quad (3.43)$$

Taking the large $\bar{N}$, N_f limit and $N_f/\bar{N} = 3 - \epsilon$, the fixed point occurs at

$$\alpha_m = \frac{\epsilon}{6\bar{N}} + \mathcal{O}(\epsilon^2), \quad (3.44)$$

where we have assumed that α_λ/α_m is $\mathcal{O}(1)$, which is justified by checking the RGE of the Yukawa coupling and finding $\alpha_\lambda/\alpha_m \rightarrow 2/7 + \mathcal{O}(\epsilon)$. At this fixed point, the perturbative expansion parameter, $\bar{N}\alpha_m \approx \epsilon/6$, is much smaller than 1, so we can still trust the perturbation theory. Most of the analysis for the magnetic case goes through without modifications, except for the gaugino mass part, which we now discuss.

Similar to the gauge coupling, the one-loop β -function coefficient is $\mathcal{O}(\epsilon)$ for the gaugino mass. We have to include higher loop effects. The 2-loop RGE of the gaugino mass in this limit is

$$\frac{d}{dt}\tilde{M}_g = -2\epsilon\bar{N}\alpha_m\tilde{M}_g + 24\bar{N}\alpha_m^2\tilde{M}_g. \quad (3.45)$$

When the gauge coupling approaches its fixed point, $\alpha_* \approx \frac{\epsilon}{6\bar{N}}$, the β -function is positive for the gaugino mass, so the gaugino mass decreases toward 0 in running down the energy scale. This can also be seen from the exact relation between α_m and $\tilde{M}_{\tilde{g}}$ obtained by Hisano and Shifman [19],

$$\frac{\alpha_m \tilde{M}_{\tilde{g}}}{\beta(\alpha_m)} = \text{RG invariant.} \quad (3.46)$$

As α_m approaches the fixed point, $\beta(\alpha_m) \rightarrow 0$, hence $\tilde{M}_{\tilde{g}} \rightarrow 0$ too. Then following the analysis of the previous subsection, we have similar relations in the extreme infrared,

$$X = \tilde{m}_q^2 + \tilde{m}_{\tilde{q}}^2 + \tilde{m}_M^2 \rightarrow 0, \quad (3.47)$$

$$(\bar{N}\tilde{m}_q^2(p) - N_f\tilde{m}_M^2(p)) = (\bar{N}\tilde{m}_q^2(v) - N_f\tilde{m}_M^2(v)) + M_R^2, \quad (3.48)$$

where

$$\bar{N}\tilde{m}_q^2(v) - N_f\tilde{m}_M^2(v) \approx -2\bar{N}^2\alpha_m^2 + 4\bar{N}^2\alpha_m\alpha_\lambda - 14\bar{N}^2\alpha_\lambda^2 \quad (3.49)$$

in this limit, and

$$M_R^2 \equiv \int_{-\infty}^{\ln v} 4(\bar{N}^2 - 1)\alpha_m \tilde{M}_{\tilde{g}}^2 dt \quad (3.50)$$

is the gaugino mass contribution. We can calculate M_R^2 using (3.46) and (3.43) in the BZ limit. Denoting $\alpha_m(v) \equiv \alpha_0$ and $\alpha(0) \equiv \alpha_* \approx \frac{\epsilon}{6\bar{N}}$,

$$\begin{aligned} M_R^2 &= 4(\bar{N}^2 - 1) \frac{\alpha_0^2 \tilde{M}_{\tilde{g}}^2(v)}{\beta^2(\alpha_0)} \int_{\alpha_*}^{\alpha_0} \frac{\beta(\alpha_m)}{\alpha_m} d\alpha_m \\ &\approx 4(\bar{N}^2 - 1) \frac{\alpha_0^2 \tilde{M}_{\tilde{g}}^2(v)}{\beta^2(\alpha_0)} \int_{\alpha_*}^{\alpha_0} (-2\epsilon\bar{N}\alpha_m + 12\bar{N}^2\alpha_m^2) d\alpha_m \\ &= 4(\bar{N}^2 - 1) \frac{\alpha_0^2 \tilde{M}_{\tilde{g}}^2(v)}{\beta^2(\alpha_0)} [(-\epsilon\bar{N}\alpha_0^2 + 4\bar{N}^2\alpha_0^3) - (-\epsilon\bar{N}\alpha_*^2 + 4\bar{N}^2\alpha_*^3)] \\ &\approx (\bar{N}^2 - 1) \alpha_0^2 \tilde{M}_{\tilde{g}}^2(v) \frac{12\bar{N}}{\epsilon^3} \frac{(1 - 3\frac{\alpha_0^2}{\alpha_*^2} + 2\frac{\alpha_0^3}{\alpha_*^3})}{(\frac{\alpha_0^2}{\alpha_*^2} - \frac{\alpha_0^3}{\alpha_*^3})^2} \\ &\approx (\bar{N}^2 - 1) \alpha_0^2 \tilde{M}_{\tilde{g}}^2(v) \frac{\bar{N}}{18\bar{N}^3\alpha_0^3} \frac{(2 - 3\frac{\alpha_*}{\alpha_0} + \frac{\alpha_*^3}{\alpha_0^3})}{(1 - \frac{\alpha_*}{\alpha_0})^2}. \end{aligned} \quad (3.51)$$

This gaugino mass contribution is generically enhanced by $\bar{N}$ compared with the $\bar{N}\tilde{m}_q^2(v) - N_f\tilde{m}_M^2(v)$ part if $\alpha_0 < \alpha_*$ or $\bar{N}\alpha_0 \ll 1$, and α_λ is not much larger than α_0 . Hence it could make the dual squark masses squared positive, and the meson masses squared negative, in evolving to low energies.

3.3 The completely higgsed magnetic theory ($N_f = N + 1$)

In this subsection we consider the case $N_f = N + 1$ in the electric theory. Without SUSY breaking, the low energy theory is confining without chiral symmetry breaking [15]. The low energy degrees of freedom are described by the baryons B , $\bar{B}$ and the mesons M , with the effective superpotential [15]

$$W_{\text{eff}} = \frac{1}{\Lambda_L^{2N-1}} (M_j^i B_i \bar{B}^j - \det M) . \quad (3.52)$$

It is interesting to see what SUSY breaking masses baryons and mesons receive when SUSY breaking masses are added for the elementary squarks. We follow the same procedure as before. Dualizing the electric theory with one more flavor (which receives SUSY-preserving and SUSY-breaking masses from S), the magnetic theory has an $SU(2)$ gauge symmetry, which is then broken completely by the $h, \bar{h}$ vevs. The low energy dual quarks correspond to the baryons of the electric theory [1]. The $\det M$ term in the superpotential is generated by instantons [1]. We again use the method of Giudice and Rattazzi to calculate the masses in the magnetic theory, which should be sufficient for $N_f > 4$, for which the $\det M$ term is nonrenormalizable and its effect is probably small. The masses of the dual squarks (baryons) and the mesons at the scale v are

$$\tilde{m}_q^2(v) = \tilde{m}_{\bar{q}}^2(v) = -\frac{3N_f - 15}{2} \alpha_m^2 + (3N_f - 3) \alpha_m \alpha_\lambda - (N_f - 4) \alpha_\lambda^2 , \quad (3.53)$$

$$\tilde{m}_M^2(v) = (2N_f + 1) \alpha_\lambda^2 . \quad (3.54)$$

After running to low energies, we have in the extreme infrared,

$$\tilde{m}_q^2 + \tilde{m}_{\bar{q}}^2 + \tilde{m}_M^2 = 0 , \quad (3.55)$$

$$\tilde{m}_q^2(p) - N_f \tilde{m}_M^2(p) \Big|_{p \rightarrow 0} = \tilde{m}_q^2(v) - N_f \tilde{m}_M^2(v) . \quad (3.56)$$

Substituting in (3.53) and (3.54), we obtain

$$\tilde{m}_q^2 = \tilde{m}_{\bar{q}}^2 = -\frac{\tilde{m}_M^2}{2} = \frac{1}{2N_f + 1} \left(-\frac{3N_f - 15}{2} \alpha_m^2 + (3N_f - 3) \alpha_m \alpha_\lambda - (2N_f^2 + 2N_f - 4) \alpha_\lambda^2 \right) . \quad (3.57)$$

For small N_f , the result depends on the relative strength of α_m and α_λ which we do not know. However, for large enough N_f , the dual squark (baryon) masses squared are negative, and baryon number is broken. This may have interesting implications in compositeness models based on $SU(N)$ with $N_f = N + 1$ or analogous theories. For $N = 2$, $N_f = 3$, we do not know how to calculate the masses because the instanton generated superpotential becomes a renormalizable Yukawa interaction. However, there is no distinction between the baryons and

the mesons in this case. Therefore, if they still satisfy the sum rule (3.55), their masses will vanish to leading order⁴.

4 Vacua and symmetry breaking patterns

As we saw in the previous section, some of the fields in the magnetic theory acquire negative masses squared. As a result, some of these fields will develop vevs and break the gauge and/or global symmetries. In this section, we discuss the vacua and the symmetry breaking patterns for the two possibilities we have encountered: $\tilde{m}_q^2 < 0$, $\tilde{m}_M^2 > 0$ and $\tilde{m}_q^2 > 0$, $\tilde{m}_M^2 < 0$.

We start with negative dual-squark masses squared and positive meson masses squared, $\tilde{m}_q^2 < 0$, $\tilde{m}_M^2 > 0$. For small SUSY breaking, this happens in our examples of the magnetic free theories (with $N_f \gg 3\bar{N}$, i.e., $N+1 < N_f \ll \frac{3}{2}N$ in the electric theory), and in the duals of the $N_f = N+1$ theories with large enough N_f .

After integrating out the heavy fields, the potential in the low energy theory is

$$\begin{aligned} V = & \lambda^2 \left(|M_j^i q_i|^2 + |M_j^i \bar{q}^j|^2 + |q_i \cdot \bar{q}^j|^2 \right) + \tilde{m}_q^2 (|q_i|^2 + |\bar{q}^j|^2) \\ & + \tilde{m}_M^2 |M_j^i|^2 + \frac{g_m^2}{2} D^2 + A\text{-terms} . \end{aligned} \quad (4.58)$$

For $\tilde{m}_q^2 < 0$, the potential is unbounded from below along “runaway” directions which correspond to the baryon directions⁵. Up to symmetry rotations and exchange of q and $\bar{q}$, the runaway directions take the following form,

$$q = \begin{pmatrix} u & & 0 \\ & u & \\ 0 & & \ddots \\ & & & u \end{pmatrix}, \quad \bar{q} = 0 . \quad (4.59)$$

The magnetic gauge group is completely higgsed, and the global symmetry is broken to $SU(\bar{N})_L \times SU(N_f - \bar{N})_L \times SU(N_f)_R \times U(1)'$, (or $SU(N_f - 1)_L \times SU(N_f)_R \times U(1)'$ for the case $N_f = N+1$ with large enough N_f), where $U(1)'$ is a linear combination of $U(1)_B$ and a $U(1)$ subgroup of $SU(N_f)_L$.

In the full magnetic theory, the runaway direction (4.59) is stabilized due to the presence of the heavy fields. Recall that the fields h and $\bar{h}$ had vevs of the form $\langle h^{\bar{N}+1} \rangle = -\langle \bar{h}_{\bar{N}+1} \rangle = v$. Hence in the full theory, the direction (4.59) is not D-flat. Instead, the potential has a

⁴Similar spectra of soft breaking masses in the dual and confining theories are also obtained by a different method [20].

⁵In Ref. [5], where the full theory is described by a potential similar to (4.58), the case that all dual squarks have negative masses squared is not considered because of the runaway behavior. In our case however, physics at the scale v stabilizes the runaway.

minimum with the h and $\bar{h}$ vevs slightly shifted from their original values. Then of course our calculation of the light field masses may no longer be valid, and these masses will depend on the shifts. We will ignore this dependence in the calculation, assuming the $h, \bar{h}$ shifts are small. This will prove to be a self-consistent assumption.

Since some of the heavy fields are now relevant, we need to re-examine the full potential (2.5). Extremizing the potential with respect to P we obtain (Eq. (2.6))

$$P = \frac{\mu F_{SB}}{\lambda^2(|h|^2 + |\bar{h}|^2) + \mu^2} , \quad (4.60)$$

where $F_{SB} \equiv -\frac{\partial W_{SB}}{\partial S}$. Letting $\langle h^{\bar{N}+1} \rangle = v + \delta$, $\langle \bar{h}_{\bar{N}+1} \rangle = -v + \bar{\delta}$ and substituting (4.60) into the potential, we have

$$V_{\text{tree}} = \lambda^2 \left[-v^2 - v(\delta - \bar{\delta}) + \delta\bar{\delta} + \mu S \right]^2 - \frac{\mu^2 F_{SB}^2}{\lambda^2 \left[2v^2 + 2v(\delta - \bar{\delta}) + \delta^2 + \bar{\delta}^2 \right] + \mu^2} . \quad (4.61)$$

In addition, there are contributions to the potential coming from D-terms and from soft SUSY breaking squark masses,

$$\begin{aligned} V_D &= \frac{1}{2} g_m^2 \left| \frac{1}{\sqrt{2\bar{N}(\bar{N}+1)}} \left[\bar{N}(v+\delta)^2 - \bar{N}u^2 - \bar{N}(-v+\bar{\delta})^2 \right] \right|^2 \\ &= \frac{\bar{N}}{4(\bar{N}+1)} g_m^2 \left[2v(\delta + \bar{\delta}) + (\delta^2 - \bar{\delta}^2) - u^2 \right]^2 , \end{aligned} \quad (4.62)$$

$$V_{\text{soft}} = -\bar{N} m_1^2 u^2 , \quad (4.63)$$

where $m_1^2 \equiv -\tilde{m}_q^2 > 0$, and we will also ignore its dependence on the scale u to a first approximation. Minimizing the potential (4.61) we find $\bar{\delta} \approx \delta$ to leading order. Expanding (4.61) to the lowest order in δ and simplifying it using Eq. (2.7) and $\bar{\delta} = \delta$, we find

$$V_{\text{tree}} \approx V_{\text{tree}}(\delta = 0) + \frac{4\lambda^2 \mu^2 F_{SB}^2}{(2\lambda^2 v^2 + \mu^2)^2} \delta^2 . \quad (4.64)$$

Combining this with V_D , V_{soft} , and using the relation (2.10), $\frac{F_h}{v} = \frac{\lambda \mu F_{SB}}{2\lambda^2 v^2 + \mu^2}$, the total potential is given by

$$V = V(u = 0, \delta = 0) + \frac{\bar{N}}{4(\bar{N}+1)} g_m^2 (4v\delta - u^2)^2 - \bar{N} m_1^2 u^2 + 4 \left(\frac{F_h}{v} \right)^2 \delta^2 . \quad (4.65)$$

Now we can minimize the potential with respect to u and δ . We find that the minimum occurs at

$$\delta = \frac{\bar{N} m_1^2}{2 \left(\frac{F_h}{v} \right)^2} v , \quad (4.66)$$

and

$$u \approx 2\sqrt{v\delta}. \quad (4.67)$$

Because $m_1^2 \sim \bar{N}\alpha_m^2(\frac{E_h}{v})^2$, δ/v is suppressed by the loop factor $(\bar{N}\alpha_m)^2$. This justifies our assumption that δ is small compared with v .

Below the symmetry breaking scale u , the remaining light fields are $q_{i>\bar{N}}^a$, the phase of u , which correspond to the Goldstone fields of the broken global symmetry, and $M_j^{i>\bar{N}}$. If duality is still good for small SUSY breaking, the electric theory may have the same symmetry breaking pattern. The symmetry breaking scale could be stabilized by strong dynamics or by a nonminimal Kähler potential, and the most natural scale will be the strong coupling scale Λ of the electric theory⁶.

Finally, we note that there is no minimum with the squarks and anti-squarks developing equal vevs with the symmetry broken to $SU(\bar{N})_D \times SU(N_f - \bar{N})_L \times SU(N_f - \bar{N})_R \times U(1)$. Such directions are D-flat even in the full theory, and may be studied using the potential (4.58). However, the only stationary points along these directions are saddle points, and the theory slides towards the “runaway” baryonic directions discussed above. Interestingly, the result that only $SU(N_f)_L$ or $SU(N_f)_R$ is broken has some similarity with the result obtained for $SU(2)$, $N_f = 2$ with small soft breaking in Ref. [7].

We next turn to the case with positive dual squark masses squared and negative meson masses squared, $\tilde{m}_q^2 > 0$, $\tilde{m}_M^2 < 0$. This can happen when the gaugino mass contribution changes the signs of the dual squark and the meson masses squared through RG evolution, as could happen for the magnetic Banks-Zaks fixed point discussed in the previous section⁷. It is also the case in the dual theory of an electric theory with negative squark masses squared, as described in Appendix A.

In the low energy theory, there are also runaway directions which correspond to the meson directions. Classically, M can run away along the directions

$$M = \begin{pmatrix} r_1 & & & 0 \\ & r_2 & & \\ & & \ddots & \\ 0 & & & r_{N_f} \end{pmatrix}. \quad (4.68)$$

⁶Note that in the magnetic theory, δ and u are independent of the SUSY breaking to leading order, so the symmetry breaking scale does not get smaller as SUSY breaking decreases. If this is also true in the electric theory, the only natural scale for symmetry breaking is Λ .

⁷Whether the gaugino mass contribution alters the signs of the scalar masses squared also depends on the relative sizes of the gaugino mass and the scalar masses, in addition to $\bar{N}_f$ and $\bar{N}$. In the examples we studied, the relation between the gaugino and the scalar masses is fixed because a specific messenger sector was chosen. Different ratios of the gaugino and the scalar masses may be obtained from different messenger sectors. For example, increasing the number of messenger flavors will increase the ratio of gaugino to scalar masses in both the electric and magnetic theories.

However, in the supersymmetric case, when $\text{rank}(M) > N_f - \bar{N}$, a nonperturbative superpotential is generated [1]. In the presence of small SUSY breaking, this superpotential still describes the nonperturbative effects at lowest order [2]. For example, if all r_i are equal ($= r$), the nonperturbative superpotential will be $\sim r^{\frac{N_f}{\bar{N}}}$, resulting in a potential $\sim r^{2(\frac{N_f}{\bar{N}}-1)}$. For $N_f > 2\bar{N}$, (true in the cases we considered,) the potential has a minimum along the direction of r at some nonzero r . This is only a saddle point, however, which is unstable in the directions where some r_i increase and others decrease. In fact, for $\text{rank}(M) > N_f - \bar{N}$, the directions in which larger r_i increase and smaller r_i decrease are always unstable when the nonperturbative superpotential is included, so some smaller r_i will slide toward zero.

The vacuum will then run away along the directions

$$M = \begin{pmatrix} r_1 & & 0 & & \\ & r_2 & & & \\ & 0 & \ddots & & 0 \\ & & & r_{N_f-\bar{N}} & \\ & & 0 & & 0 \end{pmatrix}. \quad (4.69)$$

There are $\bar{N}$ flavors of dual quarks left after integrating out the heavy flavors which get masses from r_i . Then, the light dual quarks will confine into “dual mesons” N_k^l , $k, l = N_f - \bar{N} + 1, \dots, N_f$, and baryons $b', \bar{b}'$, with a quantum modified constraint

$$\det N - \bar{b}'b' = \bar{\Lambda}_L^{2\bar{N}}. \quad (4.70)$$

The Yukawa interaction $\lambda M_l^k q_k \bar{q}^l$, $k, l = N_f - \bar{N} + 1, \dots, N_f$, turns into a mass term,

$$\lambda M_l^k N_k^l, \quad k, l = N_f - \bar{N} + 1, \dots, N_f, \quad (4.71)$$

which forces M_l^k, N_k^l , $k, l = N_f - \bar{N} + 1, \dots, N_f$, to be zero. The baryons $b', \bar{b}'$ will get vevs from Eq. (4.70) and break $U(1)_B$ ⁸. The global symmetry is broken, if all r_i are equal, to $SU(N_f - \bar{N})_V \times SU(\bar{N})_L \times SU(\bar{N})_R \times U(1)'$, where $U(1)'$ is a linear combination of $U(1)_B$ and $U(1)$ subgroups in $SU(N_f)_L$ and $SU(N_f)_R$. Interestingly, the symmetry breaking patterns of the runaway vacua exactly correspond to what we expect in an electric theory with negative squark masses squared (see Appendix A). This gives us some confidence in our analysis.

For the theories in which the meson masses squared are positive at high scales, but only turn negative at low energies through the RG evolution, *e.g.*, the magnetic BZ theory discussed in section 3, the mesons can not run away in the full theory because their masses squared are positive at high scales. Their vevs r_i will be stabilized at the scale where their masses squared

⁸As long as the baryons are fixed by the constraint, $\bar{b}'b' = -\bar{\Lambda}_L^{2\bar{N}}$, they do not represent new independent fields along the direction of varying $\bar{\Lambda}_L$, or equivalently, r_i . Therefore, at least to lowest order, there should not be extra independent soft breaking masses and potential along that direction for the baryons.

turn negative after including the one-loop effective potential [21]. Again, we should not trust the symmetry breaking scale since it is large. The natural scale for symmetry breaking in the electric theory is the strong scale Λ .

5 The large SUSY breaking limit

So far we have studied the vacuum of the magnetic theory when this theory is weakly coupled in the IR, and in the presence of very small supersymmetry breaking, such that the soft masses in the electric theory are small compared with its strong coupling scale. While we could analyze the behavior of the magnetic theory at low energies, we had no direct information about the low energy behavior of the electric theory, so we could not compare the two.

We now turn to the other limiting case, that of large supersymmetry breaking. Here, the soft masses generated in each theory are large compared with the scale of the theory (assuming it is asymptotically free). For finite μ , this can be ensured by taking the typical scale of the DSB sector to be much larger than Λ, μ . This limit is interesting for two reasons. First, and most obvious, the electric theory at low energies approaches non-supersymmetric QCD. Second, as we will see, we will be able to confront our findings for the chiral symmetry breaking in the magnetic theory with some known results for the electric theory. In many cases, it will be possible to rule out the proposed duality based on this comparison.

The soft masses in the electric theory are given in Eqs. (3.11), (3.12). When the electric theory is asymptotically free, we take $\Lambda \ll \alpha_e \frac{F_0}{S_0}$. The squarks and gluinos are heavy compared with the scale of $SU(N)$, and the low energy theory is QCD, with N colors and N_f fermion flavors. For sufficiently small N_f , the chiral symmetry of the theory is expected to be broken to $SU(N_f) \times U(1)_B$. This is experimentally known for “real” QCD, and was shown to hold for large N with N_f fixed⁹. For larger values of N_f , with fixed N , the behavior of the theory at low energies is not known. In particular, it is possible that some region of N_f, N exists, where the theory flows to an infrared fixed point with unbroken chiral symmetry, as found by Seiberg for the supersymmetric case [1]. We do know however, that the vector symmetry of the theory, $SU(N_f)_V \times U(1)_B$ remains unbroken [23]. As N_f is increased beyond $11N/2$, the theory becomes IR free, and the full global symmetry group remains unbroken at low energies.

The soft masses in the magnetic theory, to leading order in F_h , are given by the two-loop contributions (3.19), (3.20). They are of the order $\sqrt{N}\alpha f v$ with $f \equiv F_h/v^2$, and α corresponds to either the gauge or the Yukawa coupling constant. When this theory is asymptotically free, we take $\bar{\Lambda} \ll \sqrt{N}\alpha f v$. This can be achieved by taking μ' to be sufficiently small.

As discussed in section 3, these masses also receive one-loop contributions which are of the

⁹More recent work claims this is the case for $N_f < 4N$ [22].

order $\frac{1}{4\pi}f^2 v$, to leading order in f . (For some details of the calculation and the explicit one-loop expressions, see appendix B). The two-loop contributions dominate for $f \ll \sqrt{\bar{N}}\alpha$, and the one-loop contributions are more important for $f > \sqrt{\bar{N}}\alpha$. For the large SUSY breaking mass limit, there are two possibilities: We can either increase both F_h and v while keeping f small so that two-loop contributions still dominate, or increase F_h only, so that at some point the one-loop contributions become more important. We will consider both possibilities in the following.

We first consider the region where the two-loop contributions dominate. As before, we are mainly interested in the sign of the masses. Unlike in the case of small supersymmetry breaking, which we considered before, the scale of the soft masses is not very different from v and we neglect running effects. The squark and scalar meson masses (3.19) and (3.20) are functions of N_f , $\bar{N} = N_f - N$, α_m and α_λ . In the limit $N_f, N \gg 1$ with either $N_f - N$ fixed or N_f/N fixed we find generally $\tilde{m}_q^2 < 0$ and $\tilde{m}_M^2 > 0$ ($\tilde{m}_M^2$ may change sign if $\alpha_m/\alpha_\lambda > N$). As discussed in section 4, either the scalar quarks or the scalar antiquarks then develop vevs along a baryonic direction, and the gauge symmetry is completely higgsed. The theory then has a stable minimum with the chiral symmetry broken to $SU(N_f) \times SU(N_f - \bar{N}) \times SU(\bar{N}) \times U(1)'$. However, the electric theory is $SU(N)$ with N_f flavors of fermion fields, where we do not expect vector-like symmetries to be broken [23]. Thus, the magnetic theory we consider does not seem to give a valid dual description of the electric theory in the infrared in this case.

We now go on to discuss the one-loop contributions to the soft masses, which dominate when f is large compared to the gauge and Yukawa couplings. The explicit one-loop expressions are given in appendix B. In this case, the scalar meson mass squared (B.74) is always negative. The sign of the squark mass squared, $\tilde{M}_q^2$ (Eqs. (B.75)-(B.77)), depends on $\bar{N}$, f/λ , and g_m/λ . We will therefore only give a qualitative description of the behavior of this sign, keeping the Yukawa coupling λ fixed. Roughly speaking, the Yukawa contribution is positive and the gauge contribution is negative, so that $\tilde{M}_q^2$ is large and positive for small g_m and decreases as g_m increases. For large values of $\bar{N}$, say $\bar{N} > 50$, $\tilde{M}_q^2$ is positive for most relevant values of g_m and f (We have varied g_m/λ between 0.05 and 200, and f/λ between 0.1 and 0.95.). For lower values of $\bar{N}$, $\tilde{M}_q^2$ becomes negative below a certain value of f . For example, for $\bar{N} = 10$, we get a negative mass squared for $g_m/\lambda > 0.05$ and $f/\lambda < 0.6$. For $\bar{N} = 3$, $\tilde{M}_q^2 < 0$ in the entire region $0.05 < g_m/\lambda < 200$, $0.1 < f/\lambda < 0.95$.

To summarize, at one-loop, the scalar meson masses squared are always negative, and the sign of the squark mass squared can be either positive or negative. For large $\bar{N}$, it is almost always positive, and for $\bar{N} = 3$ it is almost always negative. For intermediate $\bar{N}$, it is positive for large f , and becomes negative as f is decreased.

We immediately see that there is no region where the full chiral symmetry remains unbro-

ken. Thus, for large N_f , such that the electric theory is IR free, the two theories are clearly different in the IR. Furthermore, the magnetic theory can not correspond to an IR fixed point with the full chiral symmetry unbroken.

For small $\bar{N}$, the global symmetry of the electric theory at low energies is known to be $SU(N_f)_V \times U(1)_B$, while in the magnetic theory, the squark mass squared is almost always negative. If, as a result, some squarks develop vevs, we get different patterns of global symmetry breaking in the two theories. It is still possible however that only the mesons develop vevs, thus generating positive squark masses and driving the squarks to the origin. If the meson vevs are all equal, we may then obtain the same pattern of symmetry breaking as in the electric theory. This is also possible when $\tilde{m}_M^2 < 0$ and $\tilde{m}_q^2 > 0$, as is the case for intermediate values of N_f with f sufficiently large. However, there is no tree-level term to stabilize the potential along this direction. The meson vevs may be stabilized by a nonminimal Kähler potential and/or nonperturbative effects, but because SUSY is badly broken, we do not know what the nonperturbative potential is and there is no small expansion parameter proportional to the SUSY breaking for the Kähler potential. Furthermore, nonperturbative effects will presumably be relevant only for meson vevs larger than the soft masses, which are not much smaller than the scale v in the case at hand. Hence, we cannot determine whether a stable minimum exists with the symmetry broken to $SU(N_f)_V \times U(1)_B$. In fact, if we estimate the nonperturbative potential by $\sim \Lambda_L^4(M)$, where $\Lambda_L(M)$ is the strong coupling scale after integrating out the quark fields which obtain masses from the meson vevs, then much like in the case discussed in the previous section, the potential is lifted at large scales along the direction $M \propto I$ for a certain range of N_f , but the vacuum will slide away, with meson fields obtaining different vevs. Some of the vector symmetries of the theory will then be broken, in contradiction to what we expect for the electric theory.

6 Discussion and Conclusions

In this work, we studied the infrared behavior of theories related by Seiberg duality in the presence of supersymmetry breaking. The difficulty of not knowing what the soft SUSY breaking terms in one theory map into in its dual is overcome by generating the soft breaking terms in both theories from the same SUSY breaking source, *i.e.*, by coupling them to the same sector which breaks supersymmetry spontaneously. In the duality of $\mathcal{N} = 1$ SUSY QCD theories, giving a mass term to a flavor of quark superfields corresponds to higgsing the dual gauge group to a smaller one. Similarly, as we saw here in the presence of SUSY breaking, generating soft breaking masses in the electric theory by heavy *matter* messengers corresponds to generating soft breaking masses in the magnetic theory by heavy *gauge* messengers. Assuming a

canonical Kähler potential, we found that the soft breaking scalar masses squared generated in the magnetic theory are often negative, leading to symmetry breaking in the magnetic theory.

If duality still holds approximately for small SUSY breaking masses (much smaller than the strong coupling scale) and our analysis is valid, it may be used for studying strongly coupled SUSY QCD with small SUSY breaking masses. In particular, for a range of $N+1 \leq N_f < N_f^0$ in the electric theory, where N_f^0 is close to but somewhat larger than $\frac{3}{2}N$, the dual magnetic theory is weakly coupled and we can analyze it. Our results can be roughly summarized as follows: We obtain an interesting sum rule, $\tilde{m}_q^2 + \tilde{m}_{\bar{q}}^2 + \tilde{m}_M^2 = 0$ in the deep infrared, which means that either the masses squared of the dual squarks or the mesons are negative. Whether $\tilde{m}_q^2 < 0$ or $\tilde{m}_M^2 < 0$ depends on the relative sizes of a certain combination of the scalar masses at the high scale where they are induced, and the RG contribution from the gaugino mass. These in turn depend on N , N_f , and the ratio of Yukawa to gauge coupling. In the region $N_f \ll 3N/2$, the magnetic theory is very weakly coupled at low energies and hence the gaugino contribution is small, and we find that $\tilde{m}_q^2 < 0$, $\tilde{m}_M^2 > 0$ in the deep infrared. The theory has a stable minimum with the symmetry broken to $SU(N_f - N)_L \times SU(N)_L \times SU(N_f)_R \times U(1)'$, or with L and R exchanged. When the gaugino mass contribution is large, which could happen because of a larger gaugino mass or a stronger gauge coupling at low energies in the magnetic theory, the signs of the scalar masses squared may be altered due to the RG contribution from the gaugino mass, so that $\tilde{m}_q^2 > 0$, $\tilde{m}_M^2 < 0$ in the infrared. Including the nonperturbative effects in SQCD [1, 15], we find that the symmetry is then broken to $SU(N)_V \times SU(N_f - N)_L \times SU(N_f - N)_R \times U(1)'$.

The relative size of the gaugino and squark mass is fixed in our models because we have only considered a specific messenger sector. However, different ratios of gaugino to scalar masses can be obtained from different messenger sectors, (e.g., more flavors of the messenger fields). Because the sign of the scalar mass depends on the gaugino mass and on the strength of the gauge coupling in the magnetic theory at low energies, we expect the following behavior. For fixed N , N_f , with N_f/N within the range we studied, there will be a critical $\tilde{M}_{\tilde{g}}^c$ such that for $\tilde{M}_{\tilde{g}} < \tilde{M}_{\tilde{g}}^c$, the dual squark masses squared are negative, resulting in the symmetry breaking pattern $SU(N_f - N)_L \times SU(N)_L \times SU(N_f)_R \times U(1)'$ (up to exchanging L and R), and for $\tilde{M}_{\tilde{g}} > \tilde{M}_{\tilde{g}}^c$, the meson masses squared are negative and the symmetry is broken to $SU(N)_V \times SU(N_f - N)_L \times SU(N_f - N)_R \times U(1)'$. We can also turn this statement around. For fixed $\tilde{M}_{\tilde{g}}$ and N , there will be a critical N_f^c such that the symmetry is broken to $SU(N_f - N)_L \times SU(N)_L \times SU(N_f)_R \times U(1)'$ for $N_f < N_f^c$, (where the magnetic theory is more weakly coupled at low energies,) and to $SU(N)_V \times SU(N_f - N)_L \times SU(N_f - N)_R \times U(1)'$ for $N_f > N_f^c$, (with N_f/N within the range we studied). N_f^c will depend on the relative sizes of the gaugino and the scalar masses in such a way that it decreases as the gaugino mass

increases.

We also consider the large SUSY breaking limit. Below the soft SUSY breaking mass scale the squarks and gluino in the electric theory decouple. The theory becomes ordinary nonsupersymmetric QCD for which there are some known results. We can then compare them with the results we obtain for the magnetic theory. In the magnetic theory we typically find that either the mesons or the squarks or both obtain negative masses squared, depending on the values of N_f , N and the gauge and Yukawa couplings. As a result, the magnetic theory has no stable minimum with unbroken vector-like symmetries within the minimal framework we assumed. This is in contradiction to what we expect for non-supersymmetric QCD. The candidate duals we considered therefore do not describe the same low-energy physics as ordinary QCD.

Throughout our analysis we have assumed a minimal Kähler potential in the dual theory. The Kähler potential may contain higher-dimension terms inversely proportional to Λ , the strong coupling scale of the electric theory. These terms may not be neglected, especially when SUSY breaking is large. Our results also depend on the Yukawa coupling of the magnetic theory, and particularly on the assumption that it remains perturbative at the scale where we perform calculations. Still, the fact that these simple duals fail for large supersymmetry breaking is intriguing. We do not know whether including a more complicated Kähler potential or nonperturbative effects would modify this result, because there is no systematic way of studying them. It is possible that the theory undergoes a phase transition after supersymmetry is broken, and duality breaks down.

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A Soft SUSY breaking masses from matter messengers

In this appendix we consider the case where we have only matter messengers in the magnetic theory, *i.e.*, h , $\bar{h}$ do not get vevs but receive a large mass from the vev of P , and $F_P/P^2 \ll 1$. This can be a dual description of an electric theory which has gauge messengers, and therefore negative masses squared for squarks [17]. Again, we can use the method of Ref. [17] to calculate the 2-loop masses for the dual squarks and the mesons in the magnetic theory, and the results

are

$$\begin{aligned}\tilde{m}_q^2(P) = \tilde{m}_{\bar{q}}^2(P) &= \left[\alpha_m^2 \frac{\bar{N}'^2 - 1}{\bar{N}'} - 2\alpha_m \alpha_\lambda \frac{\bar{N}'^2 - 1}{\bar{N}'} + \alpha_\lambda^2 (\bar{N}' + 2) \right] \left(\frac{F_P}{P} \right)^2 \\ &= \left[\frac{\bar{N}'^2 - 1}{\bar{N}'} (\alpha_m - \alpha_\lambda)^2 + (2 + \frac{1}{\bar{N}'}) \alpha_\lambda^2 \right] \left(\frac{F_P}{P} \right)^2, \quad (\text{A.72})\end{aligned}$$

$$\tilde{m}_M^2(P) = -2\bar{N}' \alpha_\lambda^2 \left(\frac{F_P}{P} \right)^2, \quad (\text{A.73})$$

where the magnetic gauge group is $SU(\bar{N}')$. We see that the dual squark masses squared are positive and the meson masses squared are negative. The RG contribution from the gaugino mass will not alter these signs in evolving to low energies.

As we saw in section 2, the magnetic theory we discussed throughout the paper (corresponding to an electric theory with matter messengers and positive squark masses squared) could in principle have a minimum with matter messengers. However, such a minimum does not connect smoothly to the supersymmetric case when SUSY breaking vanishes. Therefore, for small SUSY breaking, this is probably not the true vacuum of the dual theory. For large SUSY breaking, this minimum could have lower energy than the gauge messenger minimum we discussed so far, and a phase transition in the magnetic theory could occur. There is, however, no good reason to expect that this vacuum gives the dual description of the electric theory after the phase transition.

B One-loop contributions to the scalar masses

This appendix describes the calculation of the one-loop contributions to the scalar masses. It also contains some explicit expressions for these masses.

Let us first review in more detail the spectrum of the dual theory at tree level. The gauge group $SU(\bar{N} + 1)$ is higgsed down to $SU(\bar{N})$ by the expectation values of h and $\bar{h}$. Thus there are $2\bar{N} - 1$ heavy gauge multiplets, corresponding to the broken generators. These consist of vectors of mass kgv , scalars of mass $v\sqrt{kg^2 + 2\lambda^2 f^2}$, and fermions of mass $\pm v\left(\sqrt{kg^2 + \lambda^2 f^2/4} \pm \lambda f/2\right)$, where $f \equiv F_h/v^2$, and k is a group theory factor: $k = 1$ for $2(\bar{N} - 1)$ of the vector multiplets, and $k = 2(\bar{N} - 1)/\bar{N}$ for one multiplet, corresponding to the broken “ $U(1)$ ” generator. Additional fields become heavy through their superpotential coupling to h and $\bar{h}$. These include the mesons V^i , $\bar{V}_i$ and the “broken” component of the dual quarks and antiquark. Together, these combine to form $2\bar{N}$ chiral multiplets, $V_\pm^i$, $\bar{V}_{i\mp}$, with scalars of mass $m_\pm^2 = \lambda^2 v^2 (1 \pm \frac{f}{\lambda})$, and fermions of mass λv . Note that for $f > \lambda$, these fields obtain vevs, but we will not study this possibility here. We assume that some coupling of the DSB sector can be chosen small enough so that $f < \lambda$.

As mentioned above, the scalar masses receive contributions starting at the one-loop level. The one-loop contributions start at order $\mathcal{O}(f^4)$ for scalar masses squared, whereas the two-loop contributions start at order $\mathcal{O}(f^2)$, with $f \equiv F_h/v^2$.

Before going into details, it is instructive to understand the vanishing of the leading-order one loop contributions [17]. As the authors of [17] point out, to leading order in the supersymmetry-breaking parameter, the scalar masses are given by the *second* derivative of the relevant wave-function renormalization with respect to the logarithm of the threshold scale. Since the one-loop contribution only involves *single* logs, it vanishes to leading order in the supersymmetry breaking.

The scalar mesons then get masses at one-loop through their superpotential couplings to $V_{\pm}^i, \bar{V}_{i\mp}$,

$$\tilde{M}_M^2 = \frac{1}{16\pi^2} \lambda^4 v^2 \left[\left(2 + \frac{f}{\lambda}\right) \ln\left(2 + \frac{f}{\lambda}\right) + \left(2 - \frac{f}{\lambda}\right) \ln\left(2 - \frac{f}{\lambda}\right) \right], \quad (\text{B.74})$$

which is always negative.

This contribution dominates the two-loop contribution (3.20) for $f > \sqrt{\bar{N}}\alpha_\lambda$. Thus, there can be some intermediate region of f , namely, $\lambda > f > \sqrt{\bar{N}}\alpha_\lambda$, for which this contribution is relevant.

The dual squark masses are more complicated. These receive contributions both through their superpotential couplings to the fields $V_{\pm}^i, \bar{V}_{i\pm}$, and through their gauge and superpotential couplings to the heavy vector multiplets. The resulting masses can be written as,

$$\tilde{M}_q^2 = M_{U(1)}^2 + M'^2, \quad (\text{B.75})$$

where $M_{U(1)}^2$ is induced by the vector multiplet corresponding to the broken “ $U(1)$ ” generator, and is given by

$$M_{U(1)}^2 = \frac{1}{16\pi^2} \frac{1}{\bar{N}^2} g_m^4 v^2 \left[\ln(1 + 8f_n^2) - \frac{8f_n}{\sqrt{1 + f_n^2}} \ln(\sqrt{1 + f_n^2} + f_n) \right], \quad (\text{B.76})$$

where $f_n = \sqrt{\bar{N}/(2(\bar{N} - 1))} f/g_m$.

Interestingly, $M_{U(1)}^2 < 0$ for all values of f_n . Note that this contribution is a pure gauge contribution, and is suppressed for large $\bar{N}$.

The remaining piece of the squark mass, M'^2 , depends on both the gauge coupling and Yukawa coupling, but is $\bar{N}$ -independent. It is given by a rather lengthy expression

$$M'^2 = \frac{1}{16\pi^2} \lambda^4 v^2 \times \left[g_\lambda^4 (f_\lambda (-g_\lambda^4 + 3f_\lambda^4 + 15f_\lambda^2 g_\lambda^2 - 3f_\lambda^2 g_\lambda^4 - g_\lambda^6) \ln(g_\lambda) \right. \quad (\text{B.77})$$

$$\begin{aligned}
& \times \frac{(f_\lambda + \sqrt{4g_\lambda^2 + f_\lambda^2})/2 - 2g_\lambda^6 + 3f_\lambda^4 g_\lambda^2 + 12f_\lambda^2 g_\lambda^4 - 3f_\lambda^2 g_\lambda^6 + 2g_\lambda^8}{f_\lambda (5f_\lambda^2 g_\lambda^2 + 5g_\lambda^4 + f_\lambda^4 - 3g_\lambda^6 - f_\lambda^2 g_\lambda^4) (f_\lambda + \sqrt{4g_\lambda^2 + f_\lambda^2})/2 + g_\lambda^2 (4f_\lambda^2 g_\lambda^2 + 2g_\lambda^4 + f_\lambda^4 - 2g_\lambda^6 - f_\lambda^2 g_\lambda^4)} \\
& + \frac{(1 + f_\lambda) (-2 - f_\lambda + 2f_\lambda^2 + 2g_\lambda^2 - f_\lambda g_\lambda^2 + f_\lambda^3 + f_\lambda^2 g_\lambda^2)}{2(-1 - f_\lambda + g_\lambda^2 + 2f_\lambda^2)} \ln(1 + f_\lambda) \\
& + \frac{(1 - f_\lambda) (-2 + f_\lambda + 2f_\lambda^2 + 2g_\lambda^2 + f_\lambda g_\lambda^2 - f_\lambda^3 + f_\lambda^2 g_\lambda^2)}{2(-1 + f_\lambda + g_\lambda^2 + 2f_\lambda^2)} \ln(1 - f_\lambda) \\
& + \frac{(g_\lambda^2 + 2f_\lambda^2) (g_\lambda^6 - 2g_\lambda^4 + 2f_\lambda^2 g_\lambda^4 + g_\lambda^2 + 3f_\lambda^2 g_\lambda^2 - 6f_\lambda^2 + 6f_\lambda^4)}{2(-1 - f_\lambda + g_\lambda^2 + 2f_\lambda^2) (-1 + f_\lambda + g_\lambda^2 + 2f_\lambda^2)} \ln(g_\lambda^2 + 2f_\lambda^2) \\
& + 4 \ln(C) \\
& \times \frac{f_\lambda^2 (f_\lambda^2 + 2g_\lambda^2) (-g_\lambda^8 - g_\lambda^4 + 3f_\lambda^2 g_\lambda^2 + 2g_\lambda^6 + f_\lambda^4) C - f_\lambda g_\lambda^2 (g_\lambda^2 + f_\lambda^2) (g_\lambda^8 + g_\lambda^4 - 3f_\lambda^2 g_\lambda^2 - 2g_\lambda^6 - f_\lambda^4)}{(-f_\lambda - 1 + g_\lambda^2) (f_\lambda - 1 + g_\lambda^2) ((2g_\lambda^4 + 4f_\lambda^2 g_\lambda^2 + f_\lambda^4) C + g_\lambda^2 f_\lambda (f_\lambda^2 + 3g_\lambda^2))} \Big]
\end{aligned}$$

where $C = (f_\lambda + \sqrt{4g_\lambda^2 + f_\lambda^2})/2$, and $f_\lambda = f/\lambda$, $g_\lambda = g_m/\lambda$.

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