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# Effects of Stator and Rotor Core Ovality on Induction Machine Behavior

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**Abstract**—Asymmetries in the air gap of induction motors produce additional harmonics in the flux density and force waves. A complete transient finite element model analyzes the harmonics produced from two possible asymmetries, a stator core ovality and a rotor ovality. The analysis of the air gap flux density and magnetic force waves determined by the finite element model shows unique harmonic frequencies due to the ovality of the air gap.

## INTRODUCTION

Induction motor air gap asymmetries, including static and dynamic eccentricity, have been discussed in [1, 2] and others. The introduction of an oval stator or rotor is presented in [3]. The use of a complete, transient finite element model of an induction motor to study these asymmetries provides a means to quantify the effects of stator core and rotor ovality on induction machine behavior.

## ANALYTICAL DISCUSSION

### Oval Stator

When there is an ovality in the stator core, the air gap length,  $g$ , can be written as a function of the mechanical angle,  $\theta$ , as

$$g(\theta) = g_o [1 - a/g_o \cos(2\theta)] \quad (1)$$

where  $g_o$  is the average air gap length and  $a$  is the peak deviation from the average. This variation in the air gap length produces specific harmonics in the permeance of the air gap. The permeance is expressed as

$$\mathcal{P}(\theta) = \frac{\mu \times (2\pi r_m l)}{g_o \left[ 1 - \frac{a}{g_o} \cos(2\theta) \right]} \quad (2)$$

where the product  $(2\pi r_m l)$  is the surface area of the air gap. The permeance can be rewritten in terms of a geometric series because the term  $|\frac{a}{g_o} \cos(2\theta)| < 1$ .

$$\mathcal{P}(\theta) = \frac{\mu \times (2\pi r_m l)}{g_o} \left( 1 + \frac{a}{g_o} \cos(2\theta) + \right.$$

$$\left. \left( \frac{a}{g_o} \right)^2 \cos^2(2\theta) + \left( \frac{a}{g_o} \right)^3 \cos^3(2\theta) + \dots \right) \quad (3)$$

Then, using trigonometric identities, the permeance can be written as

$$\mathcal{P}(\theta) = \frac{\mu \times (2\pi r_m l)}{g_o} \left( 1 + \frac{\epsilon_o^2}{2} + \left( \epsilon_o + \frac{3\epsilon_o^3}{4} \right) \cos(2\theta) + \right. \\ \left. \frac{\epsilon_o^2}{2} \cos(4\theta) + \frac{\epsilon_o^3}{4} \cos(6\theta) + \dots \right) \quad (4)$$

where  $\epsilon_o$  has been defined as  $a/g_o$ . This could be considered a measure of the "relative ovality."

The air gap flux density is proportional to the product of air gap MMF and permeance waves. We might expect that the most influential ovality-induced flux density waves are those formed by combining the fundamental component of air gap MMF with the permeance waves due to the oval stator. The fundamental MMF combines with the first cosine term in (4) to produce a unique harmonic due to the ovality.

$$\mathcal{M} \cos(P\theta - \omega t) \times \mathcal{P}_{so,1} \cos(2\theta) \Rightarrow$$

$$B_{so,1,2} \cos[(P \pm 2)\theta - \omega t] \quad (5)$$

where the arrow ( $\Rightarrow$ ) does not represent an equality, but rather shows what follows from the combination of MMF and permeance. The subscript *so* signifies that this harmonic is due to stator ovality.

According to [4], under these conditions, other principal harmonics of the air gap flux density field include the fundamental ( $B_1$ ) and the two slot harmonics ( $B_2$  &  $B_3$ ).

$$B_{gap} = B_1 \cos(P\theta - \omega t) + \dots$$

$$+ B_2 \cos[(S - P)\theta + \omega t] + B_3 \cos[(S + P)\theta - \omega t] + \dots$$

$$+ B_{so,1} \cos(P + 2\theta - \omega t) + B_{so,2} \cos(P - 2\theta - \omega t) + \dots \quad (6)$$

where  $P$  is the number of pole-pairs of the motor,  $S$  is the number of stator slots,  $\omega$  is the radian frequency of the supply,  $\theta$  is the radian displacement along the stator bore,  $t$  is time in seconds, and the  $B$ 's are the amplitudes

of the various flux density harmonics. The direction of the travelling waves is indicated by the sign in front of the time frequency value: a "-" sign indicating a forward travelling wave rotating in the direction of the fundamental; a "+" indicating a backward travelling wave rotating opposite to the fundamental.

### Rotor Ovality

The air gap length of an oval rotor can be described in a similar manner to (1) with the motion of the rotor added. The air gap length now depends on both position and time. The mechanical rotation of the rotor is  $\omega_{rot}$  radians per second.

$$g_{rot}(\theta) = g_o [1 - a/g_o \cos(2\theta - 2\omega_{rot}t)] \quad (7)$$

Using (7) as the air gap length, the same procedure outlined above is followed in order to obtain specific harmonics in the permeance wave due to the oval shape of the rotor. Specifically, the permeance wave for an oval rotor is

$$\begin{aligned} \mathcal{P}_{rot}(\theta) = \frac{\mu \times (2\pi r_m l)}{g_o} \left( 1 + \frac{\epsilon_o^2}{2} + \right. \\ \left. \left( \epsilon_o + \frac{3\epsilon_o^3}{4} \right) \cos(2\theta - 2\omega_{rot}t) + \right. \\ \left. \frac{\epsilon_o^2}{2} \cos(4\theta - 4\omega_{rot}t) + \frac{\epsilon_o^3}{4} \cos(6\theta - 6\omega_{rot}t) + \dots \right) \quad (8) \end{aligned}$$

Combining the MMF and permeance waves will produce flux density harmonic components that are unique to the rotor ovality. One example is the fundamental MMF and the first harmonic term of (8).

$$\begin{aligned} \mathcal{M} \cos(P\theta - \omega t) \times \mathcal{P}_{ro,1} \cos(2\theta - 2\omega_{rot}t) \Rightarrow \\ B_{ro,1,2} \cos[(P \pm 2)\theta - (\omega \pm 2\omega_{rot})t] \quad (9) \end{aligned}$$

where the subscript *ro* represents the rotor ovality.

### Force Harmonics

The air gap magnetic force per unit area in the radial direction can be determined using  $4.0B_{gap}^2 \times 10^5$  newtons per square meter with (6) used for  $B_{gap}$ . The primary ovality-induced force harmonics should result from combining the fundamental flux density with the flux density harmonics due to the oval stator.

$$\begin{aligned} B_1 \cos(P\theta - \omega t) \times B_{so,1} \cos[(P + 2)\theta - \omega t] \Rightarrow \\ \begin{cases} F_{so,1} \cos(2\theta + 0\omega t) \\ F_{so,2} \cos[(2P + 2)\theta - 2\omega t] \end{cases} \quad (10) \end{aligned}$$

$$\begin{aligned} B_1 \cos(P\theta - \omega t) \times B_{so,2} \cos[(P - 2)\theta - \omega t] \Rightarrow \\ F_{so,3} \cos[(2P - 2)\theta - 2\omega t] \quad (11) \end{aligned}$$

These results agree with [3], where a different approach is used to find these force harmonics.

The rotor ovality produces force harmonics by a similar combination.

$$\begin{aligned} B_1 \cos(P\theta - \omega t) \times B_{ro,1} \cos[(P + 2)\theta - (\omega + 2\omega_{rot})t] \Rightarrow \\ \begin{cases} F_{ro,1} \cos(2\theta + 2\omega_{rot}t) \\ F_{ro,2} \cos[(2P + 2)\theta - (2\omega + 2\omega_{rot})t] \end{cases} \quad (12) \end{aligned}$$

$$\begin{aligned} B_1 \cos(P\theta - \omega t) \times B_{ro,2} \cos[(P - 2)\theta - (\omega - 2\omega_{rot})t] \Rightarrow \\ F_{ro,3} \cos[(2P - 2)\theta - (2\omega - 2\omega_{rot})t] \quad (13) \end{aligned}$$

### NUMERICAL RESULTS

To test the magnitude of these ovality-induced harmonics, a 4 pole induction motor with 48 stator slots and 36 rotor slots was used in the finite element simulation. The stator core was modified to exhibit an ovality as described by (1). The relative ovality,  $\epsilon_o$ , was set at 30%.

Table I shows the significant terms in a two-dimensional Fourier transform expansion of the air gap flux density field and corresponding air gap force field in the radial direction as analyzed by the finite element model. Table I shows that one of the unique harmonics due to the oval stator is present, that is,  $B_{so,1}$ . The induction machine used in this example has 2 pole-pairs, so the frequency of  $B_{so,2}$  has zero poles. For most induction motors, a unidirectional flux cannot flow from the stator to the rotor because of the extremely high reluctance return path through the rotor shaft, bearings, and stator frame.

The numerical analysis reveals two additional ovality-induced harmonics, labelled  $B_{so,3}$  and  $B_{so,4}$  in Table I. The source of these harmonics is most likely the combination of the MMF slot harmonics with the first harmonic term in (4).

$$\begin{aligned} \mathcal{M} \cos[(S \mp P)\theta \pm \omega t] \times \mathcal{P} \cos(2\theta) \Rightarrow \\ B_{so,3,4} \cos[(S \mp P - 2)\theta \pm \omega t] \quad (14) \end{aligned}$$

While the analytic work can predict this frequency component, the relative magnitude can be judged from the two-dimensional Fourier transform.

Table I also shows that the force harmonics due to the oval stator appear in two of the three expected components. As seen in (11), the force harmonic  $F_{so,3}$  is dependent upon the flux density component  $B_{so,2}$ . This flux density harmonic is non-existent for the particular example motor used. In general, this flux density and force harmonic would be expected to appear.

The force wave  $F_{so,2}$  has a relatively low, non-zero pole number with an oval stator and, therefore, has the capability to produce motor vibrations at twice the supply frequency. The magnitude of this component, though,

TABLE I: 2D-FFT Analysis of Induction Motor With Round/Oval Stator

No.	space pole pairs	time freq. (Hz)	Amplitude in % of Fundamental		Origin
			Round Stator	Oval Stator	
$B_2$	46	-60	4.6	3.9	$(S-P)\theta + \omega t$ Stator Slot Permeance Harmonic
$B_3$	50	+60	4.3	3.6	$(S+P)\theta - \omega t$ Stator Slot Permeance Harmonic
$B_{so,1}$	4	+60	0.0	0.9	$(P+2)\theta - \omega t$ Ovality Harmonic
$B_{so,2}$	0	+60	0.0	0.0	$0\theta - \omega t$ Ovality Harmonic
$B_{so,3}$	44	-60	0.0	0.9	$(S-P-2)\theta + \omega t$ Ovality Harmonic
$B_{so,4}$	48	+60	0.0	0.9	$(S+P-2)\theta - \omega t$ Ovality Harmonic
$F_{so,1}$	2	0	0.0	2.1	$2\theta - 0\omega t$
$F_{so,2}$	6	+120	0.0	1.6	$(2P+2)\theta - 2\omega t$
$F_{so,3}$	2	+120	0.0	0.0	$(2P-2)\theta - 2\omega t$

TABLE II: 2D-FFT Analysis of Induction Motor With Round/Oval Rotor

No.	space pole pairs	time freq. (Hz)	Amplitude in % of Fundamental		Origin
			Round Rotor	Oval Rotor	
$B_2$	46	-60	4.6	4.5	$(S-P)\theta + \omega t$ Stator Slot Permeance Harmonic
$B_3$	50	+60	4.3	4.2	$(S+P)\theta - \omega t$ Stator Slot Permeance Harmonic
$B_{ro,1}$	4	+119.33	0.0	3.8	$(P+2)\theta - (\omega + 2\omega_{rot})t$ Ovality Harmonic
$B_{ro,2}$	0	+0.67	0.0	0.0	$0\theta - (\omega - 2\omega_{rot})t$ Ovality Harmonic
$F_{ro,1}$	2	-59.33	0.0	0.0	$2\theta + 2\omega_{rot}t$
$F_{ro,2}$	6	+179.33	0.0	6.5	$(2P+2)\theta - (2\omega t + 2\omega_{rot})$
$F_{ro,3}$	2	+60.67	0.0	8.4	$(2P-2)\theta - (2\omega t - 2\omega_{rot})$

may indicate that this will not present a problem in the dynamics of the motor.

The rotor ovality harmonics are shown in Table II. The results from the finite element analysis verify that the expected harmonics appear with the rotor asymmetry and are not present when the rotor is perfectly round.

#### CONCLUSIONS

It is important to recognize that deformities and asymmetries in the induction motor can result in unwanted noise and vibration. By studying the combination of the MMF and permeance waves, one can predict some of the primary harmonics produced by air gap asymmetries, such as the oval stator core. The complete, transient finite element model provides quantitative results that can be used to judge the relative effect of the oval stator core on the behavior of induction motors.

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