An Integral Equation Based Computer Code for High-Gain Free-Electron Lasers

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Abstract

A computer code for gain optimization of high-gain free-electron lasers (FELs) is described. The electron motion is along precalculated period-averaged trajectories, and the finite-emittance electron beam is represented by a set of thin "partial" beams. The radiation field amplitudes are calculated at these thin beams only. The system of linear integral equations for these field amplitudes and the Fourier harmonics of the current of each thin beam is solved numerically.

The code is aimed for design optimization of high-gain short-wavelength FELs with nonideal magnetic systems (breaks between undulators with quadrupoles and magnetic bunchers; field and steering errors). Both self-amplified spontaneous emission (SASE) and external input signal options can be treated. A typical run for a UV FEL, several gain lengths long, takes only one minute on a Pentium II personal computer (333 MHz), which makes it possible to run the code in optimization loops. Results for the Advanced Photon Source FEL project are presented.

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1. Introduction

The short-wavelength high-gain free-electron lasers [1,2] offer the possibility to extend the FEL operation to the x-ray energy range. The advantage of such a device is that mirrors are not needed for the operation; however, tight requirements for the quality of the electron beam and the undulator magnetic fields are essential. An electron beam with high peak current, low transverse emittance and small energy spread is necessary for successful operation. The undulator needs to be very long (typically tens of meters) and carefully aligned with respect to the magnetic elements and to the beam. To aid in the technical design, it is useful to have a fast and versatile computer code that calculates the signal growth for a multisectional undulator with brakes between the sections, taking into account quadrupoles, magnetic bunchers, steering coils, undulator magnetic field errors, and beam-steering errors at the entrance.

However there are few working codes, especially in the 3-D case. This article describes a computer code that calculates the linear time-independent growth rate of radiation in a single pass FEL for a multisegmented system. It was applied to the optimization of design parameters of the FEL under construction at Argonne National Laboratory [3,4].

2. Basic equations

We use a mathematical model based on solving a set of integral equations to describe the process of coherent radiation of the beam in the undulator field [5]. In this model, the internal structure of the beam is represented by a set of N thin beams with different initial conditions $x_q(0), \dot{x}_q(0), y_q(0), \dot{y}_q(0)$. (The point is used to denote a derivative with respect to the longitudinal coordinate z, and q is the number of the beam.) Using these initial conditions we calculate the trajectories $x_q(z), y_q(z)$, neglecting the influence of the radiation field. (We consider the motion averaged over the undulator period.) Thus one can consider the motion of the electrons in the transverse direction as if they were small beans strung on a thin rigid wire (trajectory). The problem is therefore reduced to one-dimensional motion along precalculated trajectories. A planar undulator with a vertical magnetic field given by $B_0(z)\sin[k_w z + \varphi(z)]$ is considered. It is convenient to use the z coordinate as the independent variable, the relative energy deviation Δ as the canonical momentum ($E = \gamma mc^2(1 + \Delta)$ is the electron energy, m is the mass of the electron and c is the speed of light), and the time delay $\tau = t - t_1$ with respect to the equilibrium unperturbed particle time

$$t_1 = \int_0^z \left[1 + \frac{1}{2\gamma_{\parallel}^2} + \frac{\dot{x}_q^2 + \dot{y}_q^2 + k_x^2 x_q^2 + k_y^2 y_q^2}{2} \right] \frac{dz'}{c},$$

as the canonical coordinate, where $\gamma_\parallel^2 = \dfrac{\gamma^2}{1+K^2/2}$, $K = \dfrac{eB_o}{k_w mc^2}$ is the deflection parameter, and k_x^2 and k_y^2 the rigidities of horizontal and vertical undulator focusing, respectively. In these variables, the projection of the "velocity" in the phase space to the energy axis is simply the projection of the force (caused by the radiation electric field) on the particle velocity. The longitudinal distribution function $F^q(z,\Delta,\tau)$ of each beam obeys the Liouville equation

$$\frac{\partial F^q}{\partial z} - \frac{\Delta}{\gamma_\parallel^2 c}\frac{\partial F^q}{\partial \tau} + \frac{eK(JJ)}{\gamma^2 mc^2}\mathrm{Im}\left[A^q(z,t)e^{ik_w z + i\varphi - ik_0 \int_0^z\left(\frac{x_q'^2+y_q'^2+k_x^2 x_q^2+k_y^2 y_q^2}{2}+\frac{1}{2\gamma_\parallel^2}\right)dz' - ik_0 c\tau} \right]\frac{\partial F^q}{\partial \Delta} = 0 \quad (1),$$

where the electric field of the radiation on the q-th beam is represented by

$E_x^q = A^q(z,t)e^{ik_0(z-ct)} + complex\ conjugate$, $\dfrac{2\pi}{k_0}$ is the wavelength near the fundamental harmonic of the undulator radiation, (JJ) is the standard combination of Bessel functions, and e is the charge of the electron.

In our case, the radiation field may be calculated in the paraxial approximation. As the relativistic electron emits in the forward direction, one can neglect the backward radiation. This allows us to rewrite the system of N partial differential equations as integral equations in which the integration is carried out from the undulator entrance to the current longitudinal point in z, like in the Volterra equations. This feature allows us to construct a simple explicit numerical algorithm. The obtained equations are nonlinear as the radiation field depends on the beam current, which is derived from integration of the distribution function. To simplify the calculations, we consider only the linear regime when the distribution function can be written as a sum of a time-independent part and a small perturbation. By making a Fourier transformation and carrying out integration by energy, we obtain the equations for the beam current harmonics

$j_\omega^q(z) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} F^q e^{i(k_0 c+\omega)\tau}d\Delta\,d\tau$ for all N thin beams. The final system of $2N$ equations can be written as,

$$j_\omega^q(z) = \int\limits_{-\infty}^{\infty} f_\omega^q(\Delta,0)e^{-ik_0\Delta\int_0^z\frac{dz'}{2\gamma_\parallel^2}}d\Delta + \frac{ie}{2\gamma^2 mc^2}\int_0^z \Phi^q\left(k_0\int_{z'}^z\frac{dz''}{\gamma_\parallel^2}\right)A^q(z')$$

$$\times e^{-ik_0\int_0^{z'}\frac{x_q'^2+y_q'^2+k_x^2 x_q^2+k_y^2 y_q^2}{2}dz''}\, e^{ik_w z' - i(k_0+\frac{\omega}{c})\int_0^{z'}\frac{dz''}{2\gamma_\parallel^2}}\,K(JJ)e^{i\varphi}\,dz'$$

$$A^q(z') = A_{0\omega}^q(z')e^{-i\frac{\omega}{c}z'} - \frac{k_0 I}{2\gamma c}\int_0^{z'}\sum_p \frac{1}{z'-z''}e^{-ik_w z''+i\left(k_0+\frac{\omega}{c}\right)\int_0^{z''}\frac{dz'''}{2\gamma_\parallel^2}}$$

$$\times e^{ik_0\frac{[x_q(z')-x_p(z'')]^2+[y_q(z')-y_p(z'')]^2}{2(z'-z'')}+ik_0\int_0^{z''}\frac{x_p^2+y_p^2+k_x^2 x_p^2+k_y^2 y_p^2}{2}dz'''}\, j_\omega^p(z'')K(z'')(JJ)e^{-i\varphi(z'')}dz''$$

(2)

where $f_\omega^q(\Delta,0)$ is the harmonic of the initial distribution function, I is the beam current,

$A_{0\omega}^q(z) = \int_{-\infty}^{\infty} A_0^q(z,t)e^{i\omega t}dt$ describes the external wave (input signal), and

$\Phi^q(x) = \int_{-\infty}^{\infty}\frac{\partial F_0^q}{\partial \Delta}e^{-ix\Delta}d\Delta$, where $F_0^q(\Delta)$ is the unperturbed distribution function.

3. Description of the code

To solve the system of integral equations, we use a trapezoidal estimation for the integrals. This leads to the following set of equations for the discrete values of $J(q, n)$ and $A(q, n)$ at $z = \sum_{m=1}^{n-1}h(m)$, where h is the step length,

$$A(q,n) = \sum_{m=1}^{n}\sum_p G_{qpnm}\cdot J(p,m) + A_0(q,n)$$

$$J(q,n) = \sum_{m=1}^{n-1}H_{qnm}\cdot A(q,m) + J_0(q,n),$$

(3)

where A_0 and J_0 corresponds to the inhomogeneous terms of Eq. (2). To calculate $J(q, n)$, we need only the preceding values of $A(q, m)$ $(1 \le m < n)$. Thus, solving this system becomes trivial.

The magnetic system is described by the following input parameters: the deflection parameter K, the phase φ of the undulator field, the sextupole focusing parameter along each step, the optical strength of the thin quadrupole lens, and the vertical and horizontal angle deflections (kicks) at the entrance of each step.

We use a unit monoenergetic excitation of one of the thin beams to calculate the self-amplified spontaneous emission case, which corresponds to the situation of a single particle moving into the beam without initial density fluctuations. The output intensity will be the sum of the contributions from the different particles because we consider only the linear regime and the initial noise is random.

4. Results

The code has been used for optimization of the UV FEL at Argonne National Laboratory. The dashed curve in Fig. 1 shows the results for a homogeneous undulator with sextupole focusing. The normalized growth rate is

$$F = \frac{1}{2k_w\,D}\frac{d}{dz}\ln\left|\sum_{q=1}^{N} e^{ik_0\int_0^z \frac{x_q^2+y_q^2+k_x^2x_q^2+k_y^2y_q^2}{2}dz'}\,j_\omega^q(z)\right|^2 \ , \text{where } D = 2\sqrt{\frac{e\,I}{\gamma\,m\,c^3\left(1/2+K^{-2}\right)}}(JJ). \text{ It}$$

is clear that after approximately two gain lengths L_g the growth rate becomes constant indicating that only one eigenmode is prevalent. The normalization was chosen such that $F(\infty) = \dfrac{1}{2k_w DL_g}$ will be equal to the scaling factor for this eigenmode, which was calculated earlier (see e.g., [2,5,6]). It was seen from many different runs, that this steady-state growth rate did not depend on the initial noise.

The solid curve in Fig. 1 shows the results for the real project with breaks and horizontally focusing quadrupoles [7]. The lower but nonzero growth rate in the breaks corresponds to the rudimental bunching that take place there. The small growth rate reduction in undulators in comparison with the homogeneous case is, probably, due to beating of beta-functions, caused by the inhomogeneity of the focusing.

5. Discussion

According to Eq. (3), the calculation time is proportional to the square of the number of the partial beams N and to the square of the longitudinal number of steps n, which makes it feasible to quickly study very long systems (a typical run for the APS UV FEL takes only one minute on a 333 MHz Pentium II personal computer). Commonly, it is sufficient to use about 10 steps per undulator and 3 steps for the breaks with the quadrupole inside, however, for simulations using magnetic field errors from magnetic measurements the number of steps tends to be larger - typically one step per period.

The typical number of the partial beams for the APS runs was 49, which proved to be sufficient. We also found that the results for the APS project parameters were insensitive to different initial distributions of the partial beams. For shorter wavelengths (x-rays), when the diffraction is less significant, the number of partial beams needs to be increased, and more attention to the actual initial beam distributions must be given. The code was used to define tolerances for the undulator alignments and to optimize the break length and focusing strength for the APS project [7,8].

In spite of the fact that the general integral equation [5] is nonlinear and time dependent, the corresponding extension of this code for nonlinear phenomena seems unrealistic because it uses linearity and stationarity in several places. However, since the

major part of the length of a SASE FEL operates with a small signal, the code is a convenient design tool to make an optimal system for beam bunching. The last section of the superradiant FEL has to be considered separately from both theoretical and engineering points of view.

The angle distribution of the radiation intensity was added recently but has not yet been extensively tested. The calculation of the spectrum of the radiation intensity is underway and will be tested in the near future.

6. Acknowledgment

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Figure Caption

Fig. 1. The dimensionless scaling factor versus distance along the undulator for a homogeneous undulator (dashed curve) and for an inhomogeneous undulator (solid curve).