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MOBILIZATION OF TRAPPED FOAM
IN POROUS MEDIA

TOPICAL REPORT

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Mobilization of Trapped Foam in Porous Media

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ABSTRACT

Usually, foam in a porous medium flows through a small and spatially varying fraction of available pores, while the bulk of it remains trapped. The trapped foam is under a pressure gradient corresponding to the pressure gradient imposed by the flowing foam and continuous wetting liquid fractions. The imposed pressure gradient and coalescence of the stationary foam lamellae create periodic flow channels in the trapped foam region. Each of these channels flows briefly, but it eventually plugs, while another flow channel opens elsewhere, only to be plugged again by the finer bubbles pushed into the trapped region. This results in a cycling of flow channels that open and close throughout the trapped foam.

The dynamic behavior of foam trapped in porous media has been modeled here with a pore network simulator. The simulator also predicts the magnitude of the pressure drop along a trapped foam region necessary to generate a flow channel through it. The mobilization pressure drop depends only on the number of lamellae in the flow path and on the geometry of the pores that make up this path.

A predictive model of foam flow in porous media requires the knowledge of the foam fraction that traps under an imposed pressure gradient and for a given distribution of pore sizes. Here we present the first analytic expression for the trapped foam fraction as a function of the pressure gradient, and the mean and standard deviation of the pore size distribution. This expression provides the final missing piece of the continuum foam flow models based on the moments of the volume-averaged population balance of foam bubbles .

Key Words: foam, mobilization, network simulation, pressure gradient

INTRODUCTION

Foam is an excellent candidate for improving oil recovery from porous media because of its low mobility and blocking ability. A significantly greater pressure gradient is required to mobilize foam than to maintain flow of either the gas or liquid phases in a porous medium. When compared to continuous gas flow, foam lowers the mobility by factors as high as 5000 (Kovscek, 1994). Experimental observations have been done where the pressure gradient is significantly greater than 100 psi per foot in a bead pack at gas flow rates of meters per day (Fagan, 1992).

In applications such as enhanced oil recovery (EOR), it is important to understand the flow properties of foam in a porous medium. Tracer studies reveal that a large fraction of foam under flowing conditions in a porous medium remains trapped (Holm, 1968; Gillis, 1990; Falls, 1989). In fact, as much as 80% of the foam volume may be trapped at any time during flow (Gillis, 1990). Existing models for foam flow lack an understanding of the physics of the trapped fraction. Population-balance models (Patzek, 1988; Falls *et al.*, 1988; Friedmann *et al.*, 1991) for foam in porous media treat the trapped foam fraction as an adjustable constant (Kovscek and Radke, 1994; Kovscek *et al.*, 1995). Trapped regions exert a tremendous influence on foam mobility because they reduce the gas-phase relative permeability.

Kovscek and Radke (1994) recognized the dynamic nature of the regions of trapped foam. Stationary foam undergoes coarsening and eventually remobilizes while some flowing foam becomes trapped. In this way, the fraction of trapped gas remains nearly constant at all times and, on average, the foam texture, or number of gas bubbles per unit volume, is approximately constant throughout the entire system. Constant foam texture is assumed because the dynamic nature of the system results in trapped and

flowing bubbles being interchangeable. Here, we show that the texture of foam in the trapped region does in fact change with time.

Hanssen and Dalland (1991) and Hanssen and Haugum (1991) generated foam in a porous medium under constant pressure gradient rather than at constant flow rate. Foams which they refer to as "strong" eventually totally plug the porous medium and block gas flow through the porous medium. Gas flow reduces to a trickle, and this "pseudo steady state" can remain for weeks or months. We describe here a mechanism by which, under a constant pressure difference, foam remains in a stable configuration, and gas transports through the system by diffusion. As the pressure drop across the porous medium increases, different static configurations of foam are achieved, until finally the pressure drop is high enough to mobilize the foam.

Significant effort has been directed to predict the pressure drop required to mobilize a bubble train in a porous medium. This mobilization pressure drop is important because an extremely high mobilization pressure gradient limits the success of foam in EOR applications. If the required mobilization pressure gradient is too large, the foam is not an effective drive fluid for EOR, because there is a limit to what injection pressures can be used in the field, especially in places where the well-to-well distance is very long (Rossen and Gauglitz, 1990; Friedmann *et al.*, 1991).

Different predictions for required mobilization pressure drop are presented in the literature. The highest estimate was given by Falls *et al.* (1989). They assumed that all lamellae start in the pore throats; it takes a large mobilization pressure gradient to move them out of the throats. Their estimates for the mobilization pressure gradient are on the order of several MPa/m. Flumerfelt and Prieditis (1988) assume an initial random distribution of lamella positions, some curved in the direction of the pressure gradient and some curved against the pressure gradient. Thus, on average, they concluded that the mobilization pressure drop of a foam in a porous medium approaches zero for long bubble trains.

Rossen (1990) predicted a mobilization pressure gradient between the values of Falls *et al.* and Flumerfelt and Prieditis. He correctly asserts that most lamellae are curved in the direction that resists flow, and therefore there is not a complete balance of curvatures that results in a mobilization pressure drop of zero. Rossen defines pores to be conical and derives the pressure drop which must be exceeded in order for a lamella to mobilize. This value is greater than zero solely because of the discontinuity that occurs at the pore bodies, where there are corners. He concludes that if the pores were rounded into sinusoidal shapes, the mobilization pressure drop would approach zero. In water-wet porous media the aqueous phase congregates in any corners, smoothing the edges. Thus, according to Rossen's argument, foams mobilize at pressure drops that are very close to zero.

Only the analysis of Falls *et al.* (1989) explains why such a high fraction of foam is trapped in porous media under flow conditions with significant imposed pressure gradients. None of these available theories can explain the dynamic nature of foam trapping; i.e., why foam that is stationary in some channels eventually mobilizes while foam in other channels remains trapped. Here we examine the time evolution of trapped foam, and discuss the conditions which lead to the mobilization of foam through specific channels.

Percolation models have employed the theories described above to predict the propagation of foams in porous media. Rossen and Gauglitz (1990) hypothesized that gas flow begins in the channel with the fewest blocking lamellae. Out of all possible pathways, the one with the minimum mobilization pressure drop must be found. The mobilization pressure gradient is the maximum pressure drop an individual lamella can withstand divided by the average distance between lamellae in the direction of flow along the particular path. de Gennes (1992) considered a train of lamellae resting in the pore throats and roughly defined percolation regimes for mobilization as a function of foam density and pressure gradient. He speculated that the system remains close to the

mobilization threshold because trapped regimes coarsen while bubble density increases in flowing regimes as more lamellae are formed.

All the above theories conceptually impose a pressure gradient across a region containing trapped lamellae and increase that pressure gradient until flow begins. However, as a pressure gradient is imposed, foam lamellae must continually realign so that every channel sustains the pressure drop. In other words, the Young-Laplace equation dictates which configurations of lamellae are admissible for a given imposed pressure gradient. Thus, the premise of Flumerfelt and Prieditis (1988) that lamellae are randomly distributed, is only possible when there is no pressure gradient across the channel. On average, the lamellae are in positions to resist flow, as conjectured by Rossen.

More importantly, when foam is trapped under a pressure gradient, gas does in fact flow, but by diffusion. As long as there is a net curvature of lamellae in a foam channel, which is true under an imposed pressure gradient, a gas flux occurs through the channel. The flow rate of gas resulting from diffusion is a small fraction of the total flow of gas in mobilized channels, but can be predicted using the techniques we present here.

Lamellae in regions of trapped foam translate somewhat due to gas diffusion. In fact, diffusion causes coarsening of foam with time. As foam coarsens, the pressure required to mobilize it decreases. This effect confirms the hypothesis of Rossen and Gauglitz (1990) that the mobilization pressure is a function of the number of lamellae in a channel in the direction of flow. This also confirms the conclusion of de Gennes (1992) that coarsening of lamellae leads to mobilization of higher fractions of foam.

In this paper, we model stationary foam in a porous medium under an imposed pressure gradient. We predict the flux of gas that passes through the trapped region in order for the pressure gradient to remain constant. The gas flux varies with time reflecting rearrangements of lamella configurations. For pressure gradients below the mobilization pressure gradient at all times, the system settles at a steady state

configuration. For higher pressure gradients, the model predicts how long it takes for the stationary foam to mobilize and through what path that mobilization occurs. We illustrate the process of foam coarsening under an imposed pressure gradient. Coarsening results in configurations that withstand different pressure gradients and, therefore, may result in the mobilization of foam.

The model here is a variation on our earlier network simulation (Cohen, Patzek, and Radke, 1996) where much of the physics involved in the rearrangement of lamellae and change in pressures as a result of diffusion is presented. However, that model was subject to nearly random initial conditions and a pressure drop could not be imposed initially. In fact, previously, the pressures were nearly uniform in all the pore bodies, with slight variations. Thus, we first present a technique for finding an initial lamella configuration that sustains a pressure difference imposed across a region of a porous medium. Then, a model is discussed which assures that this pressure drop remains the same at all times during the diffusion process, by allowing gas to flow through the system.

MODEL FRAMEWORK

The current model is similar to a two-dimensional network model described elsewhere (Cohen, Patzek, and Radke, 1996). Each size network is specified by a pair of integers describing the number of pore bodies in each dimension of the network, $m_y \times m_z$. The current model uses a coordination number of 4, where 4 pores connect to each pore body. The pores are hourglass shapes with adjustable characteristic lengths, R_g , and pore throat diameters, r_t . A 5×2 network and the number scheme are illustrated in Figure 1. The "upstream" and "downstream" pressures correspond to the leftmost and rightmost pores in the network, respectively. The "downstream" side of the network, which is at lower pressure, has pore bodies represented by the index $(k-1)m_y+1$, where k represents the row of the network. The "upstream" side pore bodies are represented by the index

km_y. The pressures in the pore bodies decrease as one moves from left to right in the network.

The initial pressure conditions are specified by two input values. One is the equilibrium pressure, P_{eq} , which is the pressure achieved if the pressure gradient is allowed to relax to the equilibrium configuration, in which all the lamellae sit in pore throats and pressure is uniform everywhere (Cohen, Patzek, and Radke, 1996). The second input value is the pressure gradient. This is the difference between the "upstream" and "downstream" pressures. In the 5×2 network illustrated in Figure 1, the pressure gradient defines the identical difference in pressure between pores (5,1) and (1,4) and between pores (10,1) and (6,4).

A configuration of lamellae must be found so that the pressure gradient is satisfied and the upstream pressures are the same in every row. The initial lamella configuration is reported as a vector of individual lamella positions. Each half-pore in the system is identified by a pair of indices (i,j), where i is the pore body number and j represents the specific half-pore in question. An individual lamella position is defined as $x_{i,j}$, representing the fraction of the way the lamella sits between the pore throat and the pore body. So $x_{i,j}=0$ when the lamella is at the pore throat. If the lamella is located on the opposite side of the pore throat from pore body i, its position is negative in sign, by definition.

An acceptable initial configuration is one in which the sum of the pressure drop across all the lamellae in every row sum to the imposed system pressure drop. There is a maximum pressure drop that any lamella can withstand. It is defined from the Young-Laplace equation to be

$$\Delta P_{\max} = \frac{2\sigma}{r_{\text{crit}}} , \quad [1]$$

where r_{crit} is the radius of curvature of a lamella sitting at the critical position. The critical position is the position in a pore with curved walls at which the radius of curvature of a lamella is a minimum, and the lamella correspondingly sustains a maximum pressure drop (Cohen, Patzek, and Radke, 1996). When there is one lamella on each side of a pore throat, the maximum pressure increase allowed between adjacent pore bodies occurs when the lamella on the lower pressure (downstream) side of the pore throat is at the critical position and the other lamella (upstream) sits at the pore throat. The maximum pressure drop across the entire network is then

$$(\Delta P_{\text{max}})_{\text{total}} = \left(\frac{2\sigma}{r_{\text{crit}}} \right) (m_y + 1) . \quad [2]$$

If the pressure drop desired by the initial conditions exceeds $(\Delta P_{\text{max}})_{\text{total}}$, then some pores must have both lamellae on the same side of the pore throat. Figure 2 shows an example of a pore with both lamellae on the same side of the pore throat. The actual upper bound for the maximum pressure drop between adjacent pore bodies is double the value of $(\Delta P_{\text{max}})_{\text{total}}$ given in Eq. [2], because this is where all lamellae sit at the downstream critical position. However, if this pressure difference were achieved, each pair of lamellae would quickly merge together into one, and the resulting configuration could no longer withstand the imposed pressure drop. At this pressure drop or higher, the foam is mobilized. As we discuss in the next section, for foam to remain trapped in a porous medium, it must approach a configuration where there is only one lamella between pore bodies.

Equation [2] assumes that r_{crit} is a constant. The network model allows for each pore to have its own unique geometry. Therefore, r_{crit} is function of the critical lamella position x_{crit} and the pore shape parameter $B_{i,j}$, defined as

$$B_{i,j} = \left(1 + \frac{r_{t,i,j}}{R_{g,i,j}} \right) \quad [3]$$

for an hourglass pore, where $r_{t,i,j}$ is the pore throat radius of the pore in question, and $R_{g,i,j}$ is the characteristic length of that pore. Both $B_{i,j}$ and x_{crit} are functions of the individual pore geometry. So the more accurate expression for the maximum pressure drop across a row of hourglass pores with one lamella between each pair of pore bodies is

$$\Delta P_{lim} = \sum_{i=1}^{y+1} \left(\frac{2\sigma(x_{crit})_{i,1}}{[B_{i,1} - (1 - (x_{crit})_{i,1}^2)^{1/2}]R_{g,i,1}} \right) . \quad [4]$$

The pressure drop defined by Eq. [4] is known as the limiting mobilization pressure drop for a channel in a porous medium filled with foam.

The technique used to find an acceptable initial configuration is discussed in Appendix A. Once the initial configuration has been established, a constant pressure drop is imposed across the network, allowing us to find how much gas diffuses through the network in order to maintain the imposed pressure gradient. At a high enough pressure gradient, bubble coalescence eventually leads to configurations that can no longer withstand the imposed pressure gradient, and hydrodynamic flow begins, i.e. mobilization occurs.

FOAM COARSENING

A lamella residing in a pore with curved walls is always curved so that the gas pressure is higher on the throat side than on the body side. As a result, gas diffuses across the lamella, with the pressure difference acting as the driving force. Gas in the bubbles on each side of a lamella is assumed to be well mixed, and thus has a uniform pressure. The only resistance to mass transfer from one bubble to another is the lamella.

Mass transfer through a lamella is limited by the diffusion of gas through the film, and not by the initial rate of dissolution into the film (de Vries, 1958). The rate of gas diffusion is described by the mass transfer equation,

$$\frac{dn}{dt} = -kA\Delta P; \quad k = \frac{SD}{h}, \quad [5]$$

where A is the surface area of one interface of the lamella, ΔP is the pressure drop across the lamella, defined by the Young-Laplace equation, and k is the mass transfer coefficient, which is a function of the gas solubility S , its liquid phase diffusivity D , and the lamella thickness h .

As gas diffuses through a lamella, the lamella slowly translates towards the pore throat (Cohen, Patzek, and Radke, 1996). With no imposed pressure gradient on the system, the lamellae continue to move until they all rest in the pore throats. This is a stable equilibrium configuration.

The initial configuration in our model has two lamellae in each pore, usually on opposite sides of the pore throat. Diffusion drives the two lamellae closer and closer together until they merge, leaving just one lamella in the pore. Hence, the bubble density of foam (texture) in the porous medium decreases. As the total number of lamellae decreases, so does the total number of bubbles, and the foam coarsens.

Figure 3 shows the sequence of two lamellae merging together in a pore with a pressure drop from left to right. As gas diffuses from the bubble that spans the pore throat across the two lamellae into the two pore bodies, the lamellae move toward one another. The lamella on the right continues to move past the pore throat and has a negative position just before the merge. Our model assumes that the lamella with the negative position ruptures as the two lamellae merge together.

The minimum allowable distance between two lamellae before they coalesce is a parameter set as the input to the model. Merging causes a disturbance to the system, as

there must be a sudden redistribution of gas. Any gas that remains between the two lamellae expands into the pore body. In Figure 3, the gas between the two lamellae expands to the right of the remaining lamella. This causes the remaining lamella to jump away from the pore throat as the pressures equilibrate. In frame d of Figure 3, the resulting single lamella is to the right of the pair of lamellae in frame c.

The sudden jump during the merge can be explained mathematically by analyzing the Young-Laplace equation. Before the merge, there are two lamellae and the pressure difference between pore bodies 1 and 2 is the sum of the two corresponding Young-Laplace equations. Since the lamellae have almost the same curvature, we approximate this pressure difference in the two-dimensional system as

$$\Delta P = 4 \sigma / r_c . \quad [6]$$

After the merge, there is only one lamella remaining, so the pressure difference between pore bodies 1 and 2 is now

$$\Delta P = 2 \sigma / r_d . \quad [7]$$

Since the two ΔP values are nearly the same, the radius of curvature after the merge r_d must be smaller than the radius of curvature before the merge r_c . Near the pore throat, the radius of curvature decreases as the lamella moves away from the throat. Hence the lamella is farther from the throat after the merge.

The mathematical argument shows why making the merge distance smaller and thus forcing fewer moles to be displaced by the merge does not necessarily reduce the magnitude of the disturbance to the system. In addition, the merge can not be ignored by allowing the two lamellae to come vanishingly close together for two reasons, one physical and the other mathematical.

As two lamellae get very close together, their Plateau borders overlap and the lamellae come in contact with one another. This draws the films together. Also, as the number of moles of gas in the bubble between the lamellae approaches zero, the numerical system approaches a singularity because the pressure is calculated by dividing by the number of moles.

In a system with an initial distribution of lamellae, each pair of lamellae undergoes a merge until each pore contains only one lamella. This is the irreducible foam texture; no further coarsening occurs. When there is an imposed pressure gradient, the steady-state configuration of lamellae is for all of them to be away from the pore throat towards the downstream side of the system. In this manner, the imposed pressure gradient is sustained, and gas continues to migrate through the trapped foam by diffusion.

CONSTANT PRESSURE GRADIENT

When a pressure gradient is imposed across a region of trapped foam, the foam lamellae either mobilize or configure themselves in a way that resists bulk flow. If the latter occurs and there is a source of gas upstream of the region, gas diffuses through the system from bubble to bubble toward the downstream end of the system. If the region containing trapped foam is adjacent to a mobilized channel, the flowing gas has an associated pressure drop, and this pressure drop must also be satisfied across the region of stationary foam. Nearby flowing gas bubbles are a source and sink for gas, which seeps through the stationary foam by diffusion. It is also possible that the entire medium contains only stationary foam, as in the constant-pressure-drop experiments of Hanssen and Haugum (1991).

Figure 4 is a schematic diagram showing foam flowing through one channel and foam trapped in the other channels. Not shown is the aqueous wetting phase occupying the smallest flow channels. The pressure difference across the diagram is the same in all the pore channels, and the pressure difference is determined by the flow conditions of the

foam bubbles in the larger channels. The prediction of the pressure difference has been addressed by other investigators (Kovscek and Radke, 1994).

The result of a pressure gradient imposed across the network of Figure 1 is a diffusive flux of gas through the system from left to right, with gas flowing in through pores (5,1) and (10,1) and leaving through pores (1,4) and (6,4). The net curvature of lamellae is in the direction of the pressure gradient. This means that on average, the lamellae must be concave to the left. This is why the pores on the right side of the network in Figure 1 are extended past their pore throats; all of the lamellae finally reside on the downstream side of the pore throats.

There are two ways to handle the top and bottom boundaries of the network. One is to assume that they consist of dead-end pores, with no loss or addition of gas across the pore throats. In this case, the lamellae in these pores always end up at the pore throat. We can also demand periodic boundary conditions, so that the pore throats on the bottom of the network connect to the ones on the top. Doing this does not change the fundamental result or time scales, but it does allow one to approximate the behavior of larger porous media without requiring extremely large networks.

To start a network simulation, the initial conditions must be satisfied as described in the previous section. Once found, the pressure drop across in the system is held constant at all times,

$$\Delta P = P(y,1) - P(1,4) = P(2y,1) - P(y+1,4). \quad [8]$$

The goal of the simulation is to track the evolution of the lamellae-ensemble from an initial configuration to its steady-state configuration under the influence of the imposed pressure drop ΔP , which remains constant at all times during the process. As a result of diffusion, the foam texture coarsens, and one of two things happens. Either there is a steady-state configuration in which the lamellae no longer move and gas continues to

diffuse through the stationary region, or one of the channels mobilizes before steady-state can be reached. The variables that must be found at each time step are the lamella positions (\mathbf{x}), the number of moles of gas in each bubble (\mathbf{n}), the number of moles of gas entering each row on the left per unit time ($\dot{\mathbf{n}}_{\text{in}}$), and the amount of gas leaving each row on the right per unit time ($\dot{\mathbf{n}}_{\text{out}}$). Each of these quantities is a vector array of variables. The dimension of \mathbf{x} is the number of lamellae in the system. The dimension of \mathbf{n} is the total number of half-pores and pore bodies in the system, and the dimension of the two flow rate $\dot{\mathbf{n}}$ vectors is the number of rows z in the network.

For each cluster, a pore body and the four half-pores that surround it, there are 9 equations that must be satisfied (Cohen, Patzek, and Radke, 1996). All governing equations are nondimensionalized by using \bar{R}_g as the characteristic length, σ/\bar{R}_g as the characteristic pressure, and $\sigma\bar{R}_g^2/\mathcal{RT}$ is the characteristic number of moles, where \mathcal{R} is the gas constant and T is temperature. Each lamella (i,j) must satisfy the Young-Laplace equation,

$$\frac{\eta_{i,j}}{\gamma_{i,j}(x_{i,j})} - \frac{\eta_i}{\gamma_i(x_{i,k=1,4})} - \frac{2}{\rho_{i,j}(x_{i,j})} = 0, \quad [9]$$

where pressures have been expressed using the ideal gas law. Dimensionless volumes are indicated by γ ; η represents the dimensionless number of moles; and ρ is the dimensionless radius of curvature of a lamella. Single subscripts refer to a pore body and double subscripts refer to a particular half-pore. The volume of a pore body is a function of the positions of all the lamellae around it.

In addition to Eq. [9], each pair of lamellae must satisfy a pair of constraints. The first constraint ensures conservation of mass in each bubble spanning a pore throat. Since each one of these bubbles, other than those on a boundary, is surrounded by two lamellae, the equation of mass conservation is made up of the sum of two mass transfer relations, Eq. [5],

$$\frac{d\eta_{i,j}}{d\tau} + \frac{d\eta_{k,l}}{d\tau} + 4\arccos(1-x_{i,j}^2)^{1/2} + 4\arccos(1-x_{k,l}^2)^{1/2} = 0, \quad [10]$$

which have been nondimensionalized after substituting expressions for A and ΔP as a function of x . Two adjacent half-pores are also related by the second constraint:

$$\frac{\eta_{i,j}}{\gamma_{i,j}(x_{i,j})} - \frac{\eta_{k,l}}{\gamma_{k,l}(x_{k,l})} = 0, \quad [11]$$

which guarantees that their pressures are equal. In Eqs. [10-11], half-pore (k,l) is the one that connects to half-pore (i,j) .

Each pore body in the network also obeys conservation of mass,

$$\frac{d\eta_i}{d\tau} - \sum_{m=1}^4 4\arccos(1-x_{i,m}^2)^{1/2} = 0, \quad [12]$$

which ensures that the amount of gas transferred into a pore body is equal to the amount of gas leaving each of the pores around it.

Equations [9-12] are grouped together to constitute a residual vector \mathbf{R} ,

$$\mathbf{R}[\mathbf{u}(t)] = 0, \quad [13]$$

where \mathbf{u} is the solution vector made up of all the x and η values. Dimensionless time τ is defined as

$$\tau = \frac{tD}{\overline{R_g^2}\beta}. \quad [14]$$

The parameter β is equal to the film thickness divided the product of the gas constant, temperature, solubility, and characteristic length. Its inverse is the conductivity of the lamella to gas transport.

In addition to the equations summarized above, there is one equation for each row in the network defining the constant pressure drop;

$$\frac{\eta_{iy,1}}{\gamma_{iy,1}(x_{iy,1})} - \frac{\eta_{(i-1)y+1,4}}{\gamma_{(i-1)y+1,4}(x_{(i-1)y+1,4})} - \Delta\Pi = 0, \quad [15]$$

where i is the row number, and Π is dimensionless pressure. Also, all upstream pores must have equal pressures at all times, so

$$\frac{\eta_{iy,1}}{\gamma_{iy,1}(x_{iy,1})} - \frac{\eta_{(i-1)y,4}}{\gamma_{(i-1)y,4}(x_{(i-1)y,4})} = 0, \quad i > 1. \quad [16]$$

There are two unknown \dot{n} variables per row. All but one of these flow rate variables are linearly independent. We find $\dot{n}_{out,1}$ by recognizing that the total amount of gas transferring out of all the right hand pores must be equal to the total inflow on the left,

$$\sum_i \dot{n}_{in,i} = \sum_i \dot{n}_{out,i}. \quad [17]$$

The inflow and outflow of gas are taken into account by altering the mass transfer equations for the pores at the ends of each channel. In addition to gas transferred by diffusion across each lamella, gas is removed from the pores on the far right, and gas is added to the pores on the far left. For example, the equation for pore (1,4) in Figure 1 becomes

$$\frac{d\eta_{1,4}}{d\tau} + 4\arccos(1 - x_{1,4}^2)^{1/2} + \dot{n}_{out,1} = 0. \quad [18]$$

When all the proper equations are collected together, the unknown variables (x , η , \dot{n}) are found at incremental times by marching forward using a finite difference approximation and solving at each time step using Newton-Raphson iteration. The computer code which solves the problem is listed elsewhere (Cohen, 1996).

MOBILIZATION PRESSURES

Foam in a porous medium can remain stationary under an imposed pressure gradient. The more lamellae there are in a potential flow channel, the higher the pressure drop that the channel can withstand. Once the imposed pressure gradient is higher than the maximum allowable gradient, foam lamellae begin to flow. The path of the flow is the one of least resistance. So foam may not mobilize through a straight channel, but may take a tortuous path. Which path is taken depends on the size of the pores along all possible paths and the texture of foam, or lamella density, along these paths.

The size of the pores is an important consideration in determining when foam mobilizes in a channel. In smaller pores, lamellae resist higher pressure drops. This can be seen from Eq. [1], which shows that the maximum pressure a lamella can resist is inversely proportional to the radius of curvature of a lamella at the critical position. The radius of curvature of a lamella at a given value of x is smaller in a smaller pore, so the maximum allowable pressure is larger for these pores.

In addition to pore size, foam texture is an important consideration in determining the path of least resistance for foam. The fewer lamellae there are in a particular channel, the easier is the mobilization of the bubble train. Thus, prediction of mobilization depends on both the size of the pores in a channel and the number of lamellae in the channel.

The term mobilization pressure drop describes the maximum allowable pressure drop prior to initiation of hydrodynamic flow through a path filled with stationary foam.

Above the mobilization pressure drop, a lamella train translates. For one lamella to flow, all lamellae around it must mobilize as well. Thus, unless the imposed pressure drop is high enough to mobilize all the lamellae in a path, the entire train of lamellae remains stationary. For a given flow path, the mobilization pressure drop, $\Delta\Pi_{\text{mob}}$, can be calculated from

$$\Delta\Pi_{\text{mob}} = \sum_{i=1}^{\ell} \left(\frac{2(x_{\text{crit}})_i \bar{R}_g}{[B_i - (1 - (x_{\text{crit}})_i^2)^{1/2}] R_{g,i}} \right), \quad [19]$$

where $(x_{\text{crit}})_i$ and B_i are pore geometry values for the pore which corresponds to a particular lamella and the summation is over the total number of lamellae, ℓ , in the path.

To illustrate the dependence of $\Delta\Pi_{\text{mob}}$ on the number of lamellae in a channel, we investigate a single, 5-cluster long channel with uniform pore sizes. Table 1 shows the mobilization pressure drop as a function of the number of lamellae in the channel. The mobilization pressure drops are scaled by surface tension divided by the characteristic pore size (σ/\bar{R}_g). Starting with 2 lamellae per pore, there are 10 lamellae initially. After all coalescence events are complete, 6 lamellae remain. Assuming that the limiting capillary pressure for lamella coalescence is not exceeded in the system (Khatib *et al.*, 1988), these 6 lamellae remain indefinitely in their steady-state positions. Diffusion continues along the channel, but as long as the mobilization pressure drop is not exceeded, the 6 lamellae are stationary.

Table 1. Dimensionless mobilization pressure drop in a 5-cluster channel as a function of number of lamellae.

<u>no. of lamellae</u>	<u>mobilization pressure drop</u>
10	30.15
9	27.14
8	24.12
7	21.11
6	18.09

Clearly, for a system with a uniform pore-size distribution, the mobilization pressure drop is simply the number of lamellae multiplied by the mobilization pressure drop per lamella. The dimensionless mobilization pressure drop per lamella in Table 1 is 3.015. The magnitude of this limiting mobilization pressure drop in pores with $R_g=23\text{ }\mu\text{m}$ throats is about 0.045 atm per pore. So when there are 1000 pores in a row, the foam remains trapped until the applied pressure difference is 45 atm! But when $R_g=100\text{ }\mu\text{m}$, the limiting mobilization pressure drop is only 10 atm for 1000 pores.

The discrete mobilization pressure drops shown in Table 1 are analogous to energy levels; we call them mobilization levels. For a given number of lamellae in a channel, there is a mobilization level which, if exceeded, results in flow through the channel. For example, if the imposed pressure drop across the 5-cluster channel in Table 1 is $\Delta\Pi=22$, the channel does not mobilize at the initial configuration of 10 lamellae, because the mobilization level is 30.15. But as diffusion proceeds, the lamellae translate toward the throats and eventually two lamellae in a single pore approach one another and coalesce. After each merge, the mobilization level decreases. When there are 8 lamellae remaining, the mobilization level is 24.12. The imposed pressure drop of 22 is still below this level, so the system does not mobilize yet. As diffusion continues, another pair of lamellae merge together. Now, the mobilization level drops to 21.11, which is below the imposed pressure drop. Immediately, the lamella train in the channel mobilizes.

This process is illustrated graphically in Figure 4, as a history of dimensionless pressure drop versus dimensionless time. The heavy dashed line delineates the imposed pressure drop, which remains constant at all times. The remaining lines show the mobilization levels as a function of time. This level is constant for a constant number of lamellae, but decreases each time a lamella merges with its neighbor. Each mobilization level is labeled with the number of lamellae that remain in the channel. When the system

goes from 8 to 7 lamellae, the mobilization level drops below the imposed pressure gradient, and hydrodynamic flow results.

Of course, different imposed pressure drops result in different behavior. For example, Figure 6 shows the 5-cluster, equal-pore size channel as it behaves under an imposed pressure drop of 19 instead of 22. Not only does this change the number of lamellae which must merge before flow begins, but it changes the time scales of the events during the process. This is because the different imposed pressure drop results in a different initial configuration, so the lamellae do not merge together at the same time, and the diffusion rates are changed. Note that the length of the lines which represent the lifetimes of the lamellae in Figure 6 are different than the corresponding ones of Figure 5.

The coalescence events happen fairly quickly in these systems. For a porous medium with a characteristic length of 100 μm , it takes 484 s for the first merge to occur, corresponding to a dimensionless time of 600 in Figure 6. The second merge occurs less than 60 s later. So, the channel remains stationary for about 650 s, or just over 10 minutes, before mobilizing.

Clearly, foam mobilization is not as simple as some of the previous works imply (Falls *et al.*, 1989; Flumerfelt and Prieditis, 1988; Rossen and Gauglitz, 1990). If the imposed pressure drop is above the mobilization pressure drop for the number of lamellae in a path, then flow begins immediately. Otherwise, it is possible that flow starts after a period of diffusion-driven coarsening. Mobilization cannot be thought of as a sudden on/off switch. There is a period of time under an imposed pressure gradient when the foam remains trapped until its coarsening texture results in the next discrete mobilization level being exceeded. At this time, foam flows out of the mobilized channel and is replaced by flowing foam from upstream. Depending on its texture, this new foam becomes trapped, and the process begins again. In this way, the trapped region undergoes intermittent pulses, with foam flowing out in bursts and then trapping again as the inflowing bubble size decreases and the mobilization pressure is no longer exceeded.

In addition, there are imposed pressure drops below all mobilization pressure drops, and the foam never mobilizes. For the 5-cluster systems of Figures 5 and 6, the pressure drop below which foam remains trapped in a channel is $\Delta\Pi=18$. This is the limiting mobilization pressure drop as defined in Eq. [4]. The foam undergoes diffusion-driven coarsening until there are 6 lamellae, and then the system remains at steady-state with a net diffusive flow of gas through the channel. Figure 7 shows a system of lamellae at steady-state under a pressure gradient in a system with uniform pore sizes. All of the lamellae are at the same position in their corresponding pores, and are curved to withstand the pressure gradient and to allow gas to diffuse through. In a single row network, \dot{n}_{in} and \dot{n}_{out} are the same at steady state.

Changing the imposed pressure gradient across the channel in Figure 7 results in different steady-state lamella positions. As the pressure drop increases, the steady-state lamella positions approach the critical position. At $\Delta\Pi_{mob}$ for 6 lamellae, the lamellae all reside at the critical positions. Above $\Delta\Pi_{mob}$, the lamella train mobilizes. Conversely, increasing the molar diffusion flow rate \dot{n} through the channel results in steady-state lamella positions that are farther away from the pore throat. In fact, the diffusive flow through the channel can continue to increase above the value corresponding to the lamellae at their critical positions. But the pressure drop that is required across the system decreases above this flow rate. Flow through a lamella, as defined by Eq. [5], is a function of the product of the curvature of the lamella and its surface area. The surface area is a stronger function of lamella position than is curvature, and thus the diffusion flow rate continues to increase as the lamella moves away from the pore throat. The maximum diffusive flow that can pass through a lamella in this geometry occurs when the lamella is at $x=1$, which is where the pore intersects the pore body.

Table 2 shows the how steady state depends on the diffusive flow rate of gas through the channel. The table gives the steady-state lamella positions and the pressure drop in a 5-cluster channel as a function of diffusive gas flow. Remember that the

mobilization pressure drop for this system is 18.09, so all the diffusive flow rates in Table 2 result in a stable stationary foam. In the table, a diffusive flow of 2.35 is the dimensionless flow rate which occurs when all the lamellae are at the critical position and the steady-state pressure drop equals the mobilization pressure drop.

Table 2. Steady-state lamella positions and actual pressure drops for increasing values of diffusive gas flux through a 5-cluster channel.

<u>\dot{n} (dimensionless diffusive flow rate)</u>	<u>Steady-state lamella positions</u>	<u>Channel $\Delta\Pi$</u>
0.5	.125	7.20
1.0	.247	12.85
1.5	.366	16.31
2.0	.479	17.84
2.35	.554	18.09
2.5	.585	18.05
3.0	.635	17.83
3.5	.768	16.48
4.0	.841	15.31
4.5	.902	14.08
5.0	.949	12.87
5.5	.981	11.71
6.0	.997	10.60

Table 2 emphasizes the fact that there are two steady-state lamella configurations possible for many imposed pressure gradients. As diffusion proceeds in a real system, the lamellae migrate toward the pore throats, and neighboring lamellae coalesce near the throats. Therefore, the more likely steady-state configurations are ones where x is smaller than the critical value. Therefore, the steady-state lamella positions are assumed to be a monotonically increasing function of pressure drop.

The fraction of the total gas flow through a porous medium due to diffusion through stationary foam depends on the magnitude of all the system variables. Converting $\dot{n}=1.0$ in Table 2 to typical flow rate units in a porous medium with an average pressure of 1 atm and at ambient temperature gives diffusive flow rates ranging from 0.15 m/day for pores with $R_g=23\text{ }\mu\text{m}$ to 0.025 ft/day for $R_g=500\text{ }\mu\text{m}$. The flow rates for the smaller pores seem high, but in reality, the average pressure in these systems

is on the order of 10 atm or more. A factor of 10 in pressure lowers the dimensional flow rate by a factor of 10. So at 10 atm average pressure, the diffusive flow rate in pores with $R_g=23\text{ }\mu\text{m}$ is 0.015 m/day. Thus the magnitude of gas flow resulting from diffusion through stationary pores can be significant under specific conditions, but is most often small compared to gas convection. For better comparison, Kavscek and Radke (1994) reported a Darcy velocity for gas of 0.43 m/day at an exit pressure of 47.4 atm. Using that pressure as the average pressure in the system, which is a conservative estimate, the gas flow velocity resulting from diffusion through a pore with a $5\text{ }\mu\text{m}$ throat is about 0.003 m/day.

After completing the discussion of mobilization of lamellae in a single channel, it is important to study the behavior of systems described by multiple row networks, which model more realistically the behavior of foam in porous media. We start with two-row networks. The only interesting behavior occurs when all the pores in the network have different dimensions. For example, if a 5×2 network has 10 clusters of the same size, the behavior of stationary foam is the same in each row as it would be for a single channel system with the same dimensions.

Figure 8 shows a two-row network where each row is made up of uniform pore sizes, but the pores connecting the pore bodies in the top row are smaller than the pores in the bottom row. The pressure drop across each row is the same, and as a result the diffusive gas flow into it is different in general. The same is true for the flow out of each row. The flow through the narrower channel has to be smaller in order for it to have the same pressure drop as in the wider channel. Figure 8 illustrates the behavior of the system when the imposed pressure drop is below the mobilization pressure drop for either channel. This system results in the solution vectors \dot{n}_{in} and \dot{n}_{out} being the same at steady state. The lamellae in the pores connecting the two rows merge together and situate themselves in the pore throats. Thus, the five pore bodies in the top row have the same pressures as those in the bottom row, there is no gas flow between the rows, and the

pressure drop across each lamella in the system is the same. The lamellae in the bottom row are farther from the throats than those in the top row, and the lamellae in neither row exceed the critical position. Since a lamella residing in a smaller pore has a smaller radius of curvature than one at the same position in a larger pore, a lamella in a larger pore must be farther away from the throat in order to achieve the same curvature.

As the imposed pressure drop across the system in Figure 8 increases, mobilization eventually occurs. Clearly, the mobilization pressure for the narrower channel is higher than that for the wider channel, so it would seem that the wider channel always mobilizes first. However, initially the lamellae in the narrower channel are closer to the pore throats in order to maintain the same pressure gradient as that across the wider channel. Therefore, they may merge together sooner and cause mobilization in the narrower channel before it occurs in the wider one. In order to determine when and where the lamellae mobilize, two effects must be balanced. Which channel mobilizes depends first on the relative sizes of the two channels and second on the imposed pressure drop. Figure 9 gives a history of mobilization levels for the two rows of the network, similar to the analysis done in Figures 5 and 6, for a system where the pores in the narrower channel are 56% as big as those in the wider channel. The problem is scaled based on the larger pore size, and the dimensionless mobilization pressure drop is 32.06 in the narrow channel as opposed to 18.09 in the wider channel. For a pressure drop $\Delta\Pi=22$ across the system, the wider channel mobilizes after 2 lamellae have merged with their neighbors in the system.

When the narrower channel contains pores which are 75% of the size of the pores in the wider channel and the imposed pressure drop is the same, the narrower channel mobilizes first, after the fourth merge occurs. This is because the lamellae in the wider channel are farther from the pore throats and take longer to merge than the ones in the narrow channel. This result is illustrated in Figure 10, in which the channels have limiting mobilization pressure drops of 23.94 and 18.09, respectively. So an applied

pressure drop between 18 and 24 results in mobilization of only the wider channel, while any pressure drop above 24 results in the narrow channel mobilizing first.

The most important system to study is one with a distribution of pore sizes. To illustrate the effect of a pore-size distribution, a 5×3 network simulation has been performed with a Gaussian distribution used to define the characteristic size of each pore. The average pore size is $\bar{R}_g = 500 \mu\text{m}$, with a standard deviation, $\text{std} = 50 \mu\text{m}$. In the simulation, the pore-size distribution is defined by the parameter

$$\psi = \frac{\bar{R}_g + \text{std}}{\bar{R}_g}, \quad [20]$$

where $\psi = 1$ for a uniform pore size distribution. The distribution used here is $\psi = 1.1$.

After the initial pore configuration has been defined, the system is subjected to a dimensionless pressure drop of 10, which is well below the limiting mobilization pressure drop for any row of the network. The system reaches steady state when all the lamellae have coalesced with their neighbors and the foam has reached its minimum texture. The steady-state configuration for this 5×3 system is shown in Figure 11. Due to the nonuniformity of pore sizes in each row, there now is cross-flow between the rows. This is evidenced by the steady-state curvatures of the lamellae in the pores that are not directly in the line of flow.

In Figure 11, all pores are drawn to be the same size, even though they are not. The bottom row has a dimensionless limiting mobilization pressure drop of 18.25; the middle row 17.22, and the top row 20.10. In this situation, the rate of gas diffusing out of each row is not the same as that diffusing in. In fact, the bottom row has more gas diffusing out the right than in the left, while the middle row has more diffusing in the left than out the right. The pressure drop across each entire row is the same, but the pressure drop across each pore is not uniform.

If the imposed pressure drop in the above 5×3 system is increased so that it is above at least one of the limiting mobilization pressure drops, we can learn much about the mobilization of real systems. For example, we can find out whether or not mobilization always occur in straight channels, through a minimal number of pores, or perhaps some systems mobilize through more tortuous paths. Figure 12 shows a plot of mobilization levels during the coalescence process for the network of Figure 11 under an imposed pressure drop of 20. This type of analysis only takes into consideration the mobilization pressures through horizontal channels. If the mobilization level for any of these channels falls below the imposed pressure drop, the lamellae in the horizontal row mobilize. Vertical spacing between mobilization levels is not uniform anymore because each pore is of a different size and thus adds a unique contribution to the mobilization pressure drop.

In the case illustrated by Figure 12, the system mobilizes after the merge of lamella (10,4) with its neighbor at $\tau=1530$. But the mobilization level of the second straight channel is still above the imposed pressure drop. The plot of mobilization levels does not take into account more tortuous flow paths. Here, a path which contains pore (10,4) is mobilized, but not simply along the second row of the network. In order to determine which path mobilizes, it is necessary to calculate the mobilization levels for all possible paths containing pore (10,4). This is done by adding the mobilization pressure drops of all lamellae that are in the flow direction. In other words, there is no contribution from lamellae in the vertical pores. If one of these paths has a level below the imposed pressure drop, then it mobilizes. In this particular example, the path is found by summing up the contributions of pores (10,1), (9,1), (13,1), (12,1), (11,1), and (11,4). At the point of mobilization, the mobilization level of this path is 19.09, which is below the imposed pressure drop of 20. The channel also includes pore (9,2), but this pore is perpendicular to the flow direction, so it is not included when summing up the contributions to the mobilization level. Figure 13 shows the lamella configuration at the

time of mobilization. The figure only indicates the presence of two or one lamellae in a pore; it does not reflect the exact positions of the lamellae. The long arrow shows the channel which opens up when the foam mobilizes. The location of the final event which triggers flow is also marked.

FOAM TRAPPING

The physical understanding gained here into the mobilization of foam in porous media is useful in completing the effort to model foam flow. In their continuum models, Patzek (1988), Kovscek and Radke (1994), Friedmann *et al.* (1991), and Kovscek *et al.* (1995) did not describe accurately the trapping characteristics of foam under varying conditions. The previous discussion describes the conditions that need to be met in order for foam to remain stationary in a region of a porous medium. If the pressure drop across a region is below the limiting mobilization pressure drop, the foam does not undergo hydrodynamic flow. The network simulator used here investigates the transient behavior of foam of a given texture in a finite size region of pores of a particular geometry and under a fixed pressure gradient. If the limiting mobilization pressure drop is exceeded, foam eventually mobilizes. When mobilization occurs, the simulation run is complete, and it is possible to determine which channel opens to flow.

Under a given pressure drop, foam of a given texture flows through a certain fraction of pores. Other channels contain foam which is trapped and intermittently flows as its texture coarsens to below the critical texture, corresponding to the limiting mobilization pressure drop. Finally, there is a fraction of the pores which contain foam that is trapped at all times. Given the information presented earlier, it is possible to calculate this fraction as a function of applied pressure gradient and for various pore size distributions and entry capillary pressures.

Since the mobilization pressure drop is calculated by adding up the pressure drop sustained by each lamella were it located at the critical position, there is a critical number

of lamellae, corresponding to a critical foam texture, below which foam mobilizes through a channel. For any single channel, the critical texture depends on the geometry of each of the pores between the two ends. If the initial texture is above the critical texture, but the equilibrium texture, where all lamellae sit at the pore throats, is below the critical texture, foam mobilizes through the channel after a period of diffusion-driven coarsening.

If the equilibrium texture is above the critical texture for the given pressure drop, foam remains trapped in the channel at all times. Our goal is to predict the fraction of foam that remains trapped at any given pressure drop. To do this, a porous medium is defined as a collection of flow channels connecting one end of the medium with the other. Each channel is made up of a linear combination of pores of various sizes and has about the same total length. Each channel then has a corresponding pressure drop which would be required to move foam of a given texture through it. Some channels, those with more large pores, mobilize at relatively low pressure drops, while others require higher pressure drops. By sampling a sufficiently large number of these channels, a representative fraction of channels that are mobilized at any given pressure gradient can be calculated.

The maximum equilibrium foam texture has one lamella in each pore. Any denser texture eventually coarsens to this one as a result of diffusion. This equilibrium texture results in the highest mobilization pressure and thus the smallest mobilized fraction for a given pressure. This is the texture we use in our subsequent calculations.

The result of this calculation depends on the pore size distribution selected. It is generally accepted that a skewed distribution, such as the lognormal distribution (Aitchison, 1957) or the Weibull distribution (Mohanty, 1982) well describes pore-size distributions in a porous medium. For simplicity, we choose the lognormal distribution here, with a mean characteristic pore size, \bar{R}_g , and a given standard deviation. Randomly generated pores are placed in a row until the predetermined length is exceeded. Each

pore and surrounding pore body is assumed to have a length of $4\overline{R}_g$. The predetermined length of the channel is the length taken up by 20 pores with the average characteristic length. Therefore, the minimum channel length is $80\overline{R}_g$. Some channels are made up of fewer than 20 pores, and some have more than 20 pores.

Each channel has an entry capillary pressure. This is the pressure that must be exceeded for gas to displace completely the liquid in the channel, and open it for a foam lamella. Liquid drainage out of a pore is controlled by the pore throats (Wardlaw, 1987). Therefore, in order for foam to occupy a channel, the capillary pressure must overcome that required to displace water from the smallest pore in the channel or

$$P_c = \frac{2\sigma}{r_{t,\min}}, \quad [21]$$

where $r_{t,\min}$ is the radius of the smallest pore throat. In our geometry, the pore throats are one-fifth of the characteristic pore size, $r_t=0.2\overline{R}_g$.

The calculations that follow are for a lognormal distribution with $\overline{R}_g = 100 \mu\text{m}$ and a standard deviation of $50 \mu\text{m}$. The complete pore-size distribution is shown in Figure 14. We proceed by creating 1000 channels, just over 8 mm in length, consisting of a random sample of the pores. For all the pores to be occupied by foam, a capillary entry pressure of 0.182 atm must be exceeded. The limiting mobilization pressure drop of each channel is calculated and divided by the length of the channel, in order to represent a pressure gradient. The computer program used to calculate these mobilization pressure gradients is reproduced in Cohen's thesis (1996). Then a plot is made which presents the percentage of pores that are mobilized at increasing imposed pressure gradients. The result is shown in Figure 15 as a normal probability plot. The shape of the plot indicates that at low mobilization pressures, an increase in pressure only generates a small increase in the number of channels mobilized. The same is true at high mobilization pressures. Most of the channels make the transition from trapped to flowing

at intermediate values of mobilization pressure. Half of the foam is mobilized at an imposed pressure gradient of 26 atm/m.

The data in Figure 15 are for a system which is at a capillary entry pressure such that all the pores are invaded by foam. If the capillary entry pressure is below this value, only those pores with pore throats above a corresponding value can be invaded by gas. Therefore, to find out how the mobilization curve varies as a function of capillary entry pressure, only channels which contain pores that are large enough to be invaded at the particular capillary pressure can be used. As pores are randomly selected to be placed in channels, a pore is only selected if it can be invaded under the capillary pressure conditions. The same pore size distribution (Fig. 14) is used, but only pores above a given cutoff are used to make the channels. Figure 16 shows the same data as Figure 15, but for 5 different values of capillary entry pressure. These values correspond to using pores larger than 250 μm , 200 μm , 150 μm , 100 μm , 50 μm , and 0 μm , respectively, ordered from left to right on the plot. The 0 μm case is the situation when all pore sizes are allowed.

It is clear that for lower capillary entry pressures, the mobilization pressure gradient needed to overcome trapping is reduced. From Figure 16, we see that for a capillary entry pressure of 0.011 atm, all the channels that contain foam are mobilized under a pressure gradient of 2.5 atm/m, as compared to about 50 atm/m needed to mobilize all the channels at the highest capillary entry pressure. In order to mobilize foam in a porous medium when a limited pressure driving force is available, either a small fraction of the total porous medium must be invaded, i.e., capillary entry pressure is low, or the foam texture must be very coarse, i.e., only one lamella exists for every 10 or more pores in the system. The results in Figure 16 are for the highest equilibrium foam texture, which is one lamella per pore.

It is possible to represent the mobilization curves such as the ones in Figures 15 and 16 analytically. To do this, we need to know the mean pressure gradient, a , at which

half of the channels in the system are mobilized and the standard deviation, b , of the calculated pressure gradients. The plot gives flowing fraction, X_f , as a function of mobilization pressure gradient,

$$X_f = \frac{1}{2} \operatorname{erfc} \left(\frac{a - |\nabla P_{\text{mob}}|}{b\sqrt{2}} \right). \quad [22]$$

The expression

$$X_t = 1 - \frac{1}{2} \operatorname{erfc} \left(\frac{a - |\nabla P_{\text{mob}}|}{b\sqrt{2}} \right) \quad [23]$$

gives the fraction of foam which is trapped, $X_t (=1-X_f)$, and can be used in modeling foam flow in porous media. Given the pore size distribution and pore geometry, a simple program finds the mean and standard deviation of mobilization pressure gradients, which then can be used in Eq. [23] to find the fraction of foam which is trapped under a given pressure gradient. This result can in turn be used in existing continuum models.

Figure 17 shows the same information as Figure 15, but plotted on a linear scale. Shown are the data and the fit using Eq. [22]. The data for the 0.182 atm capillary pressure case have a mean mobilization pressure gradient of 26 atm/m and a standard deviation of 4.9 atm/m.

SUMMARY

A network model, first developed elsewhere (Cohen, Patzek, and Radke, 1996), has been used here to show how stationary foam in a porous medium adjusts under the influence of a constant pressure gradient. At a constant pressure drop across a pore network, gas diffuses through the network. Ultimately, the lamellae arrange themselves so as to resist the imposed pressure drop and allow only a net diffusive flow of gas from

the high pressure side of the network to the low pressure side. Our results can be summarized as follows:

- There is a maximum pressure drop that a train of lamellae can withstand in a channel of pores, called the mobilization pressure drop. This maximum pressure drop is a function of the number of lamellae in the channel and is calculated by noting that each lamella can withstand a pressure drop limited by its critical position. All possible flow paths must have mobilization pressure drops that are not exceeded in order for the entire region to remain trapped.
- Our model assumes a minimum lamella density of one lamella per pore. The initial lamella density is twice this value, but diffusion leads to coarsening, leaving one lamella near each pore throat. The mobilization pressure drop corresponding to the minimum lamella density is called the limiting mobilization pressure drop. For pressure drops below this value, mobilization cannot occur.
- The steady-state configuration in this case is one which has a constant diffusive flow rate of gas passing through the stationary foam region and lamellae which remain at positions away from the pore throats. This steady-state behavior was observed experimentally in the constant-pressure foam flow experiments of Hanssen and Haugum (1991).
- Above the limiting mobilization pressure drop, foam eventually mobilizes through a particular path in the porous medium after a series of lamella merges. As each lamella merges with its neighbor, the mobilization level for each path which contained the lamella decreases, and if one of these levels falls below the imposed pressure drop, mobilization begins. This process is illustrated by mobilization level history plots.
- When the mobilization level is exceeded and mobilization begins through a particular channel, foam flows out of that channel and is replaced by finer-textured foam from upstream. This foam then becomes trapped in the open

channel and the process begins again until another channel, or possibly the same one, has its mobilization level decrease below the imposed pressure drop. For systems with average pore sizes of about 100 μm , this entire cycle occurs in about 10 minutes.

- Based on the conclusions of this model, it is possible to predict the fraction of foam which is trapped in a porous medium under a given pressure gradient, and given a representation of the pore size distribution. We present an analytic expression for the fraction of foam which is trapped at any imposed pressure gradient. Such information is necessary for predictive modeling of foam flow in porous media.

Symbols

a	-	pressure gradient at which half the channels are mobilized (atm/m)
A	-	lamellar surface area (m^2)
b	-	standard deviation of calculated mobilization pressure gradients (atm/m)
B	-	pore characteristic $[1+r_l/R_g]$
D	-	diffusivity of gas through liquid phase (m^2 / s)
h	-	lamella thickness (m)
k	-	mass transfer coefficient (mol / N s)
m_y	-	number of clusters in the network in the flow direction
m_z	-	number of clusters in the second dimension of the network
n	-	moles of gas (mol)
\dot{n}_{in}	-	dimensionless moles of gas entering the network per unit time (nondimensionalized by the same factor as η)
\dot{n}_{out}	-	dimensionless moles of gas leaving the network per unit time (nondimensionalized by the same factor as η)

P	-	gas pressure (Pa)
P_c	-	capillary pressure (Pa)
P_{eq}	-	equilibrium gas pressure (Pa); pressure if all lamellae were at pore throat
P_u	-	upstream pressure (Pa)
P_d	-	downstream pressure (Pa)
r_{crit}	-	radius of curvature of a lamella at the critical position (m)
r	-	radius of curvature (m)
r_t	-	pore throat radius (m)
\mathbf{R}	-	residual vector
R_g	-	radius of cylinder used to generate an hourglass pore [grain radius] (m)
\bar{R}_g	-	characteristic length [average grain radius] (m)
\mathcal{R}	-	gas constant (8.314 N m / mol K)
S	-	solubility of gas in liquid phase (mol / N m)
t	-	time (s)
T	-	temperature (K)
\mathbf{u}	-	solution vector made up of all lamella positions and mole values
V	-	bubble volume (m ³)
x	-	dimensionless distance of a lamella from the pore throat [$0 \leq x \leq 1$]
x_{crit}	-	critical position
X_f	-	fraction of foam which is flowing
X_t	-	fraction of foam which is trapped

Greek letters

β	-	solubility parameter [$h/\mathcal{R}T\bar{R}_g$]
γ	-	dimensionless volume [V/\bar{R}_g^3]
η	-	dimensionless number of moles of gas [$n\mathcal{R}T/\sigma\bar{R}_g^2$]
Π	-	dimensionless pressure [$P\bar{R}_g/\sigma$]

$\Delta\Pi_{\text{mob}}$	-	dimensionless mobilization pressure
ρ	-	dimensionless radius of curvature [r/\bar{R}_g]
σ	-	liquid/gas interfacial tension (N/m)
τ	-	dimensionless time [$tD/\bar{R}_g^2\beta$]
ψ	-	grain size distribution factor [$(\bar{R}_g+\text{std})/\bar{R}_g$]

Subscripts

i	-	cluster index $\{i \in (1, m_y m_z)\}$
j	-	pore index $\{j \in (1, 4)\}$

Pressure Drops

ΔP	-	imposed pressure drop on network (Pa)
$\Delta\Pi$	-	dimensionless imposed pressure drop on network
ΔP_{max}	-	maximum pressure drop allowed across a single lamella (Pa)
ΔP_{lim}	-	limiting mobilization pressure drop (Pa)
$\Delta\Pi_{\text{mob}}$	-	dimensionless mobilization pressure drop
∇P_{mob}	-	mobilization pressure gradient (Pa/m)

APPENDIX A. INITIAL CONDITIONS

The first challenge in solving the constant pressure-drop problem is to find an initial lamella configuration that satisfies the initial condition parameters. Imposing a specific pressure drop on the network severely restricts the possible lamella positions. A robust algorithm has been developed (Cohen, 1996) to find an acceptable initial configuration, given the imposed pressure drop on the network.

Given the upstream and downstream pressures and the lamella positions in the upstream and downstream pores, the pore body pressures on the upstream and

downstream sides of network are found. Then the initial condition generator sweeps the network one pore body at a time and finds acceptable lamella positions assuming that there are two lamellae between each pair of bodies.

The first row of the network has a uniform decrease in pressure from one pore body to the next. After calculating the pressures in the upstream and downstream pore bodies, the remaining pressure drop is divided into equal portions to find the uniform decrease. The fact that the pore body pressures are set puts constraints on the positions of the two lamellae in the pore connecting the adjacent bodies. Given the pore body pressures and the position of one of the lamellae between them, the position of the other lamella is fixed. The radius of curvature of this lamella is calculated from the Young-Laplace equation and the corresponding lamella position is calculated from

$$x_{i,j} = \frac{B_{i,j} \rho_{i,j} R_{g,i,j} \bar{R}_g \pm \sqrt{R_{g,i,j} - B_{i,j}^2 R_{g,i,j}^4 + \rho_{i,j}^2 R_{g,i,j}^2 \bar{R}_g^2}}{(\rho_{i,j}^2 \bar{R}_g^2 + R_{g,i,j}^2)}, \quad [A.1]$$

where $B_{i,j}$ is the pore shape parameter, $\rho_{i,j}$ is the dimensionless radius of curvature of the lamella, $R_{g,i,j}$ is the characteristic length of the pore in question, and \bar{R}_g is the average characteristic pore length for the network. It is possible that there is no solution to Eq. [A.1] for a given set of conditions. In this case, the other lamella must be moved around until a solution is found.

Subsequent rows in the network are set so that the upstream and downstream pressures are the same in every row. In each row, other than the first, the pore bodies do not have uniform pressure differences between them. The initial configuration in each row is calculated one pore body at a time. For each pore body, there are four lamellae that must be positioned, two between the current pore body and the one in the row below and two between the current pore body and the upstream pore body. These lamellae are placed so as to give an appropriate value of the pore body pressure. The only constraint

for this pressure is that the remaining lamellae must be able to withstand enough of a pressure gradient so that the last downstream pore body has the proper pressure.

Once a satisfactory configuration of lamellae has been achieved to handle the desired pressure drop, the pressures and mole contents of each bubble are calculated. First, the pressure in one bubble is arbitrarily set and all the other pressures are calculated from the Young-Laplace equation. For these pressures, the number of moles in each bubble is calculated, given the volume of each bubble. The moles and pressures are then adjusted in order to satisfy the prescribed equilibrium pressure. This is done by calculating from the equilibrium pressure, P_{eq} , the number of moles, N ,

$$N = \frac{P_{eq} V_t}{\mathcal{R} T}, \quad [\text{A.2}]$$

where V_t is the total volume of the system. The number of moles of gas in bubble i is adjusted by the equation

$$n_{i,\text{new}} = n_{i,\text{old}} + c V_i, \quad [\text{A.3}]$$

where V_i is the volume of the bubble i and $N = \sum_j n_{j,\text{new}}$. The constant c is

$$c = \frac{N - \sum_k n_{k,\text{old}}}{V_t}. \quad [\text{A.4}]$$

The initial configuration so obtained is one of the allowable configurations for the network under the imposed pressure gradient. The initial conditions are much less random than they were for the systems studied in the previous work (Cohen, Patzek, and Radke, 1996). The lamella configuration portrays a net curvature that is convex toward the downstream side of the network. In other words, lamellae arrange themselves to sustain an imposed pressure drop. The convex configuration resists flow under the

imposed pressure drop until the foam in the channels coalesces and fewer lamellae remain to resist flow.

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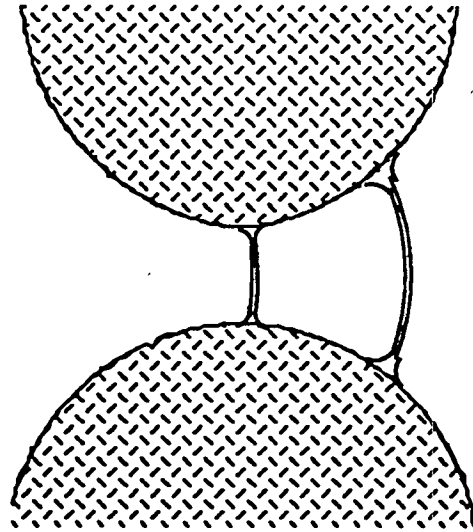


Figure 2.

Figure 2. Pore with a pressure drop across it larger than ΔP_{\max} given by Eq. [1]. The lamella on the right is at the critical position and the lamella on the left is at a position such that $x < 0$.

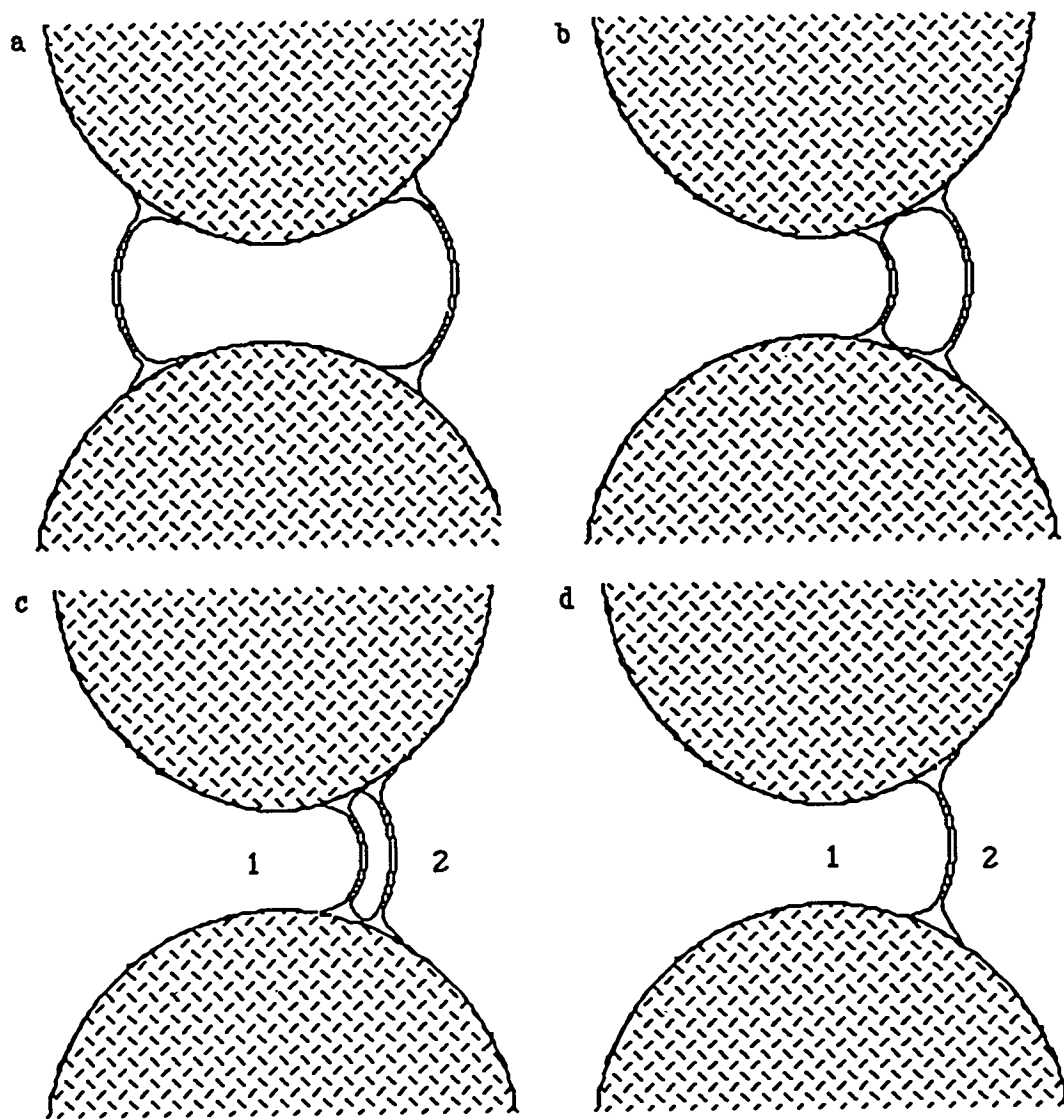


Figure 3.

Figure 3. Schematic of the sequence of two lamellae merging into one. Two lamellae are moving together as a result of diffusion from the interior to the pore bodies. When they get close enough together, they merge into one. The resulting single lamella is slightly farther away from the pore throat than either "parent" immediately before the merge.

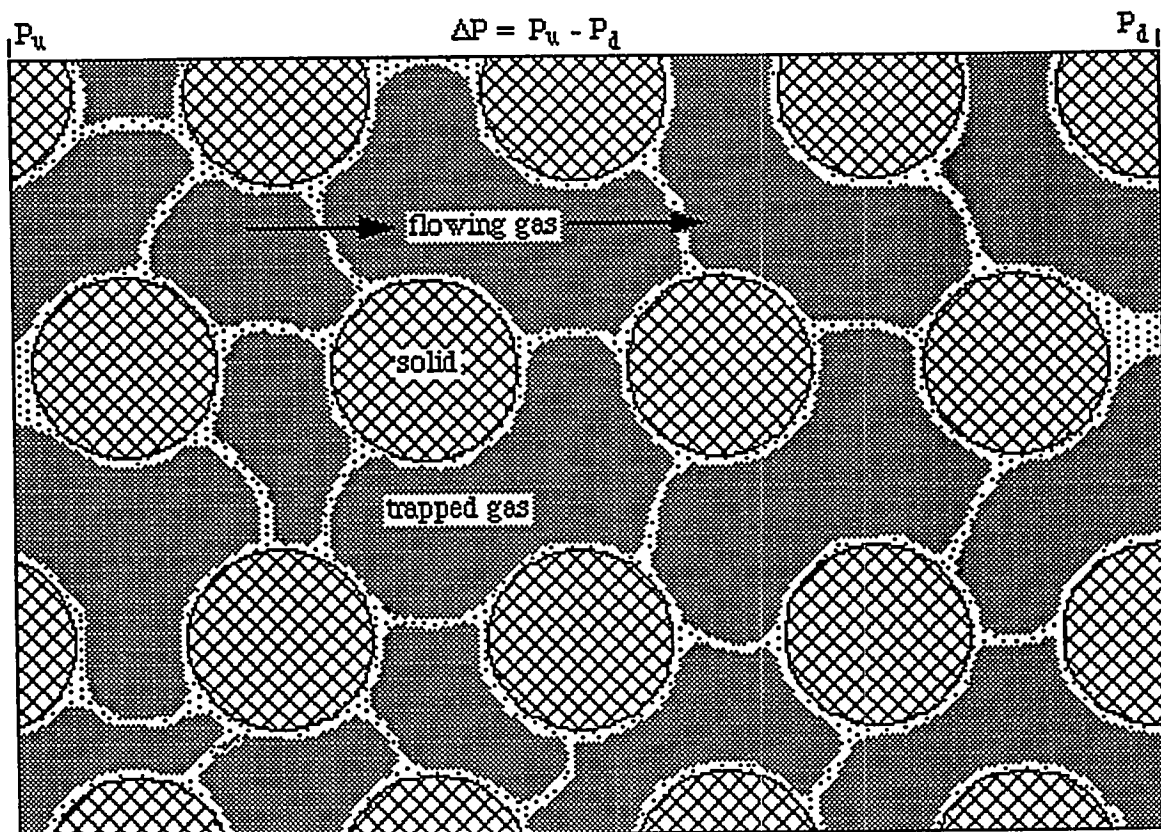


Figure 4.

Figure 4. Foam is mobilized through the largest pore channel at a given injection pressure. Due to the flow, a pressure difference, ΔP , is maintained, between the upstream and downstream pressures (P_u and P_d). All the foam in the figure is under the influence of the same pressure gradient, and there is a resulting diffusive flux of gas through the trapped foam region.

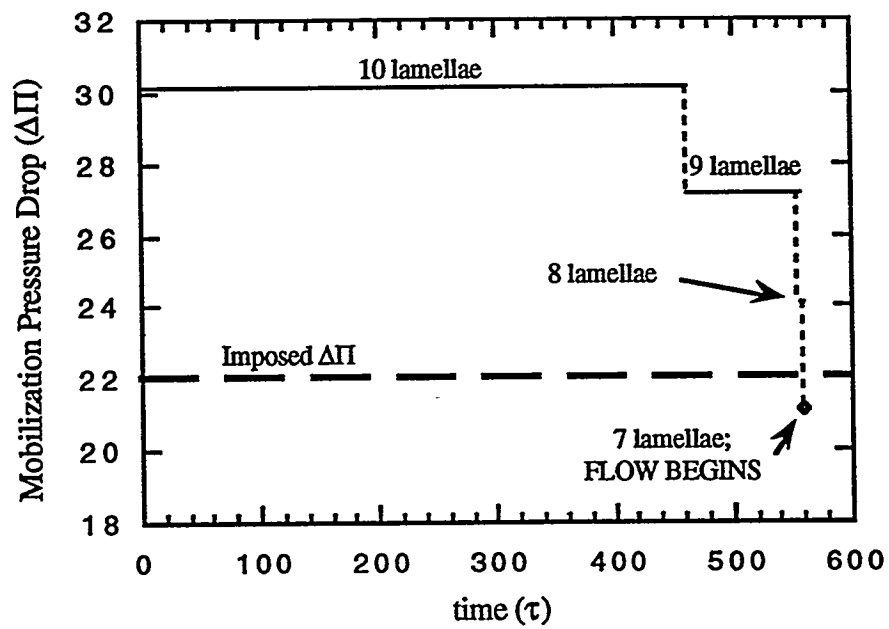


Figure 5.

Figure 5. Mobilization levels in a 5-cluster, single channel system. After 3 of the lamellae merge with their neighbors, flow begins, because the imposed pressure drop is greater than the mobilization pressure drop for 7 lamellae in the channel.

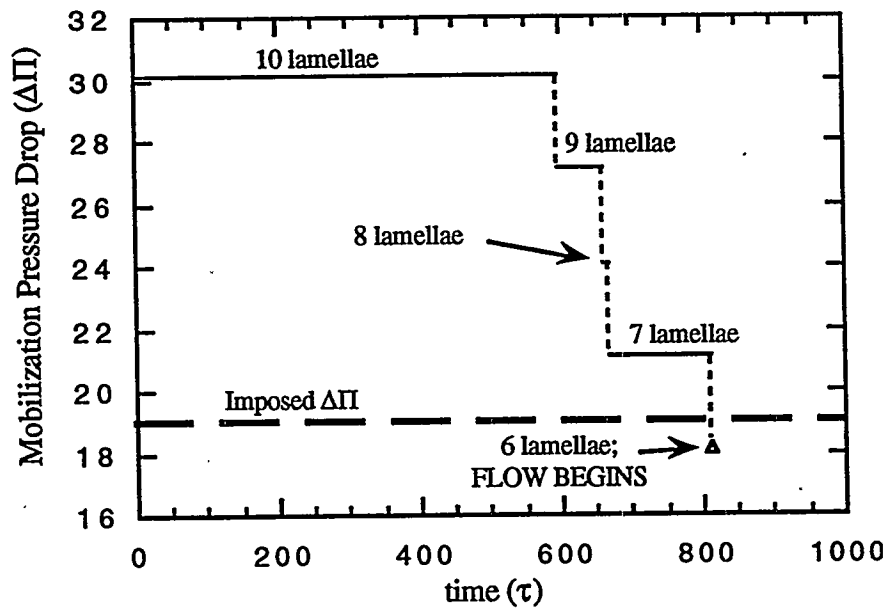


Figure 6.

Figure 6. Mobilization levels in a 5-cluster, single channel system for lower imposed pressure drop than in Figure 4. Flow begins after 4 lamellae merge with their neighbors.

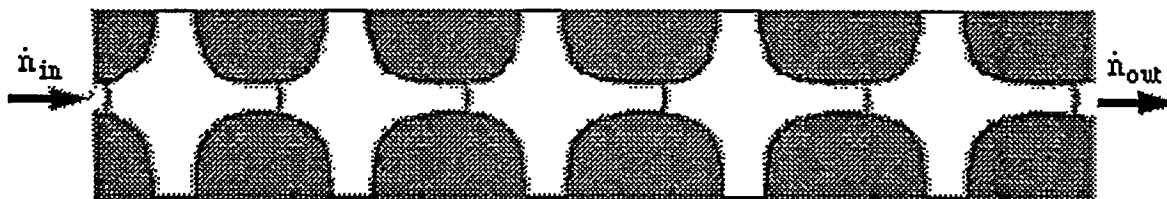


Figure 7.

Figure 7. A 5-cluster channel under an imposed pressure gradient less than the mobilization pressure drop for 6 lamellae. At steady state, there is a diffusive flux of gas through the channel, but no hydrodynamic flow.

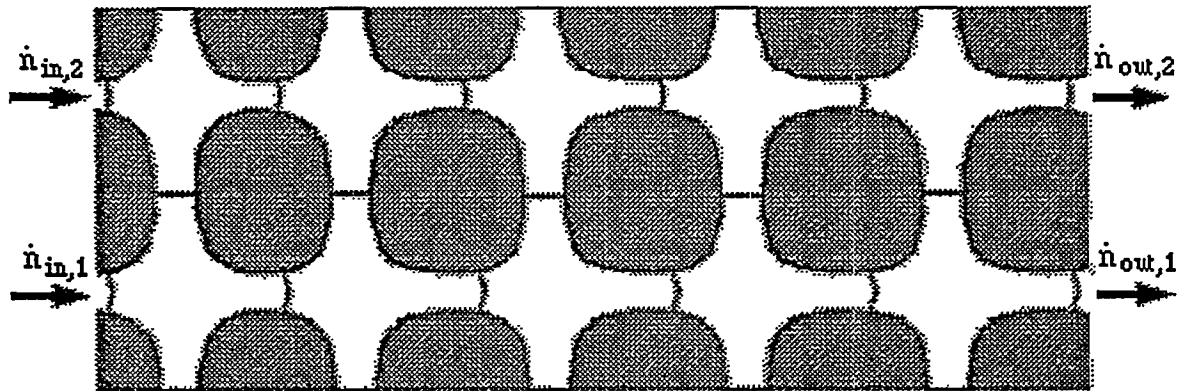


Figure 8.

Figure 8. Steady-state configuration of a system with two 5-cluster rows under an imposed pressure gradient less than the mobilization pressure drop for either row. The top row is narrower than the bottom one, so the lamellae in the bottom row are farther away from the pore throats.

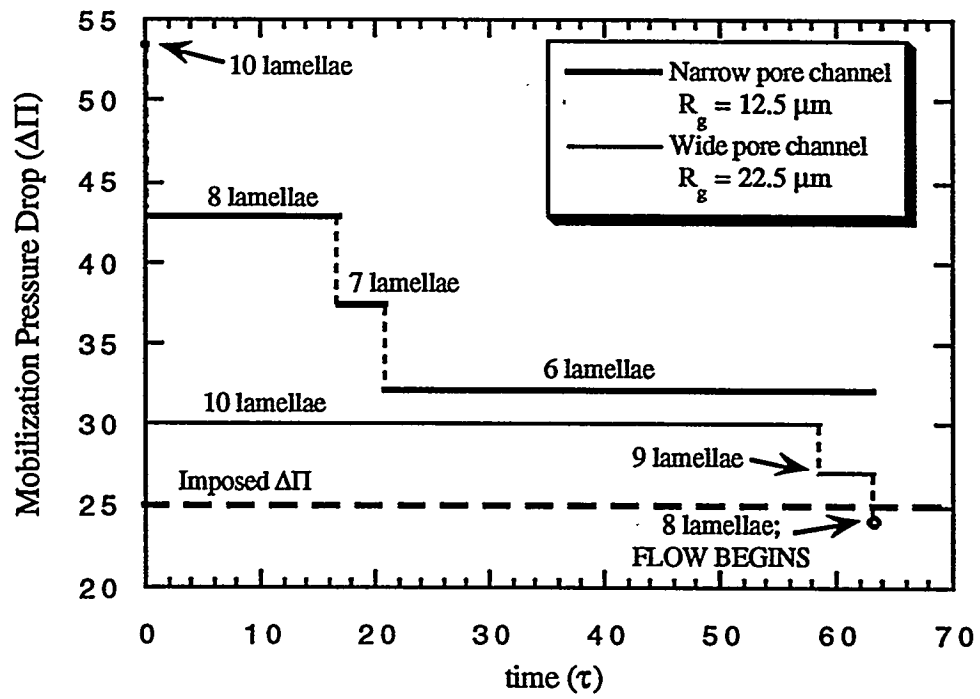


Figure 9.

Figure 9. Mobilization levels in a 5×2 system under an imposed pressure drop of 25. Flow begins after 2 lamellae in the wider channel merge with their neighbors.

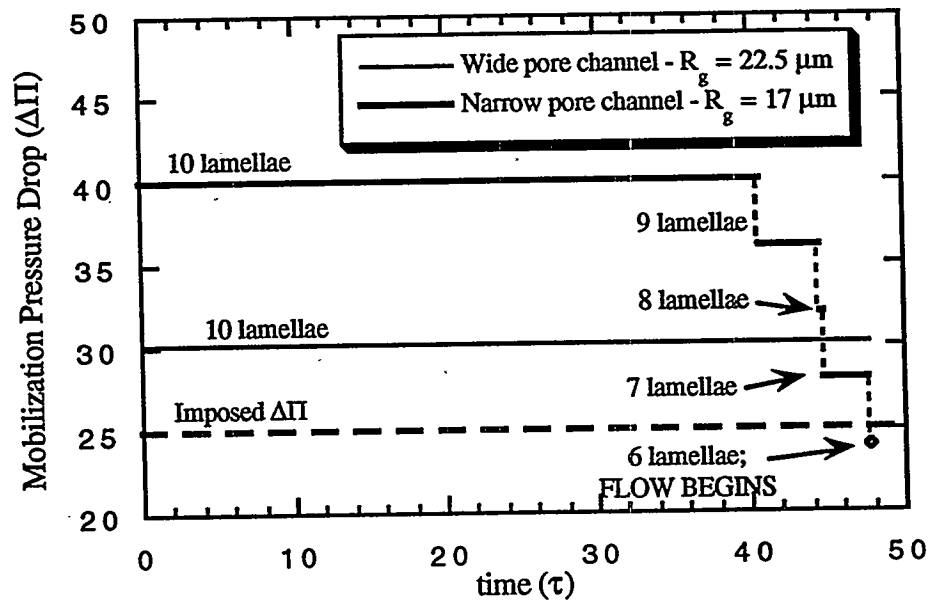


Figure 10.

Figure 10. Mobilization levels in a 5×2 system under an imposed pressure drop of 25. Flow begins after 4 lamellae in the narrower channel merge with their neighbors.

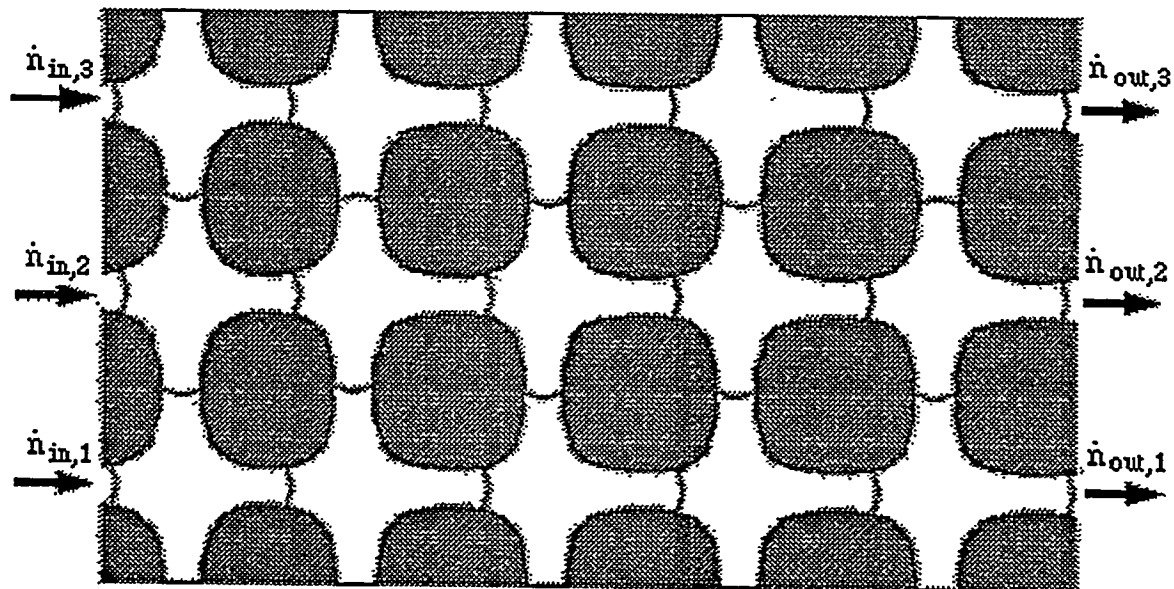


Figure 11.

Figure 11. Steady-state configuration of a 5×3 system with $\psi=1.1$ under an imposed pressure gradient less than the mobilization pressure drop for any channel. There is a diffusive flux of gas between the rows.

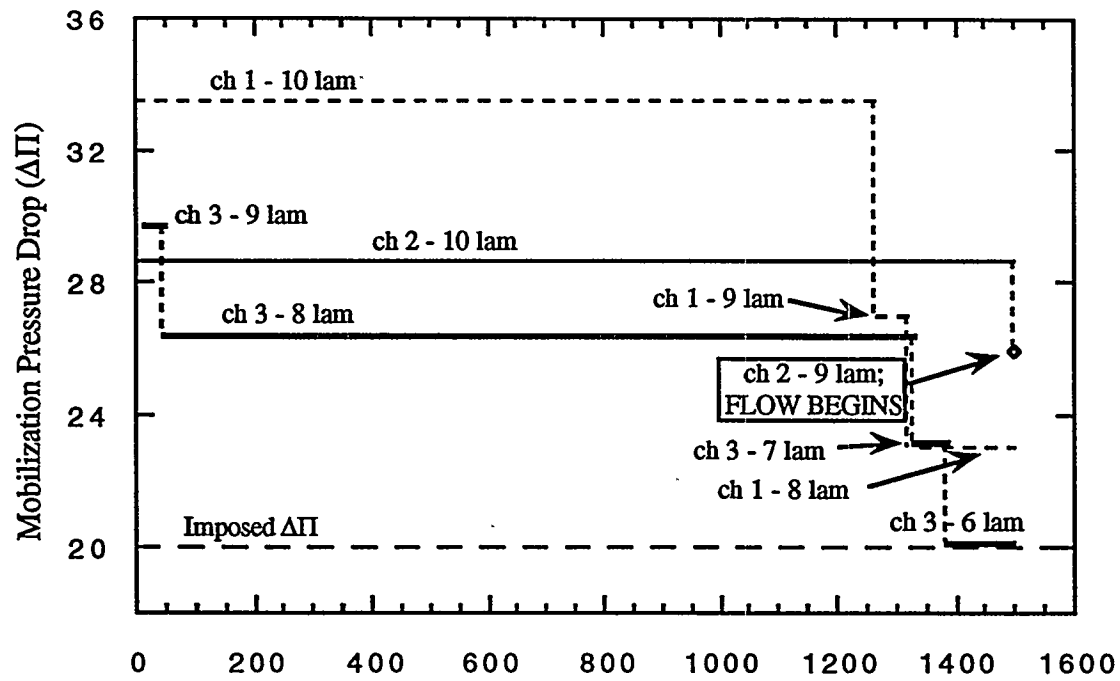


Figure 12.

Figure 12. Mobilization levels in a 5×3 system with $\psi=1.1$. In this case, mobilization occurs when the mobilization levels in all three straight channels are higher than the imposed pressure drop.

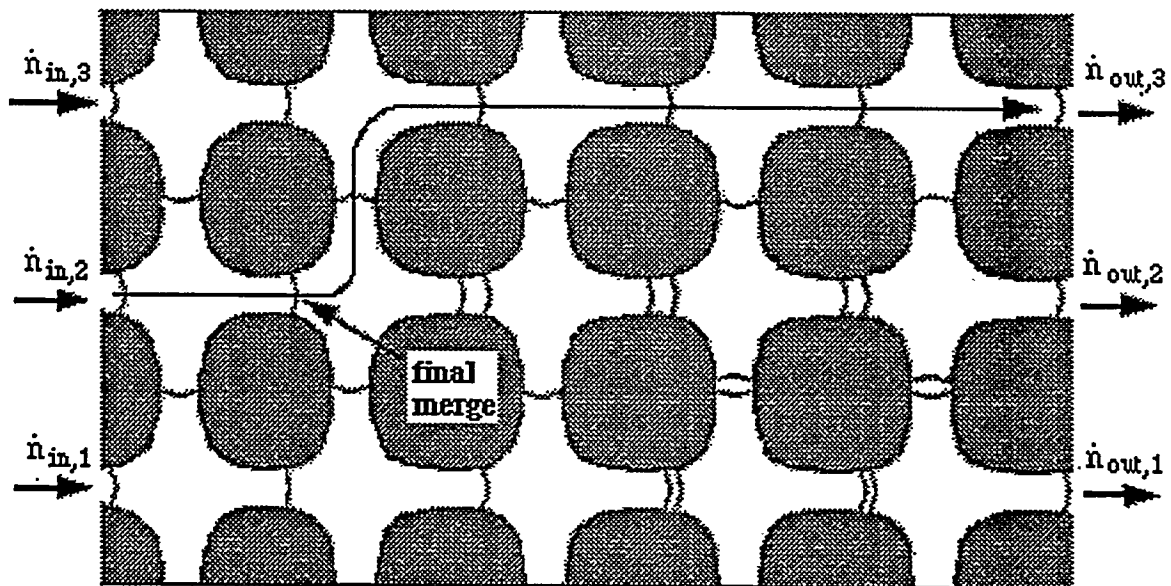


Figure 13.

Figure 13. Lamella configuration at the point of mobilization in a 5×3 network with $\psi=1.1$. Upon the coalescence of two lamellae at the labeled position, the path marked with a long arrow mobilizes through the system.

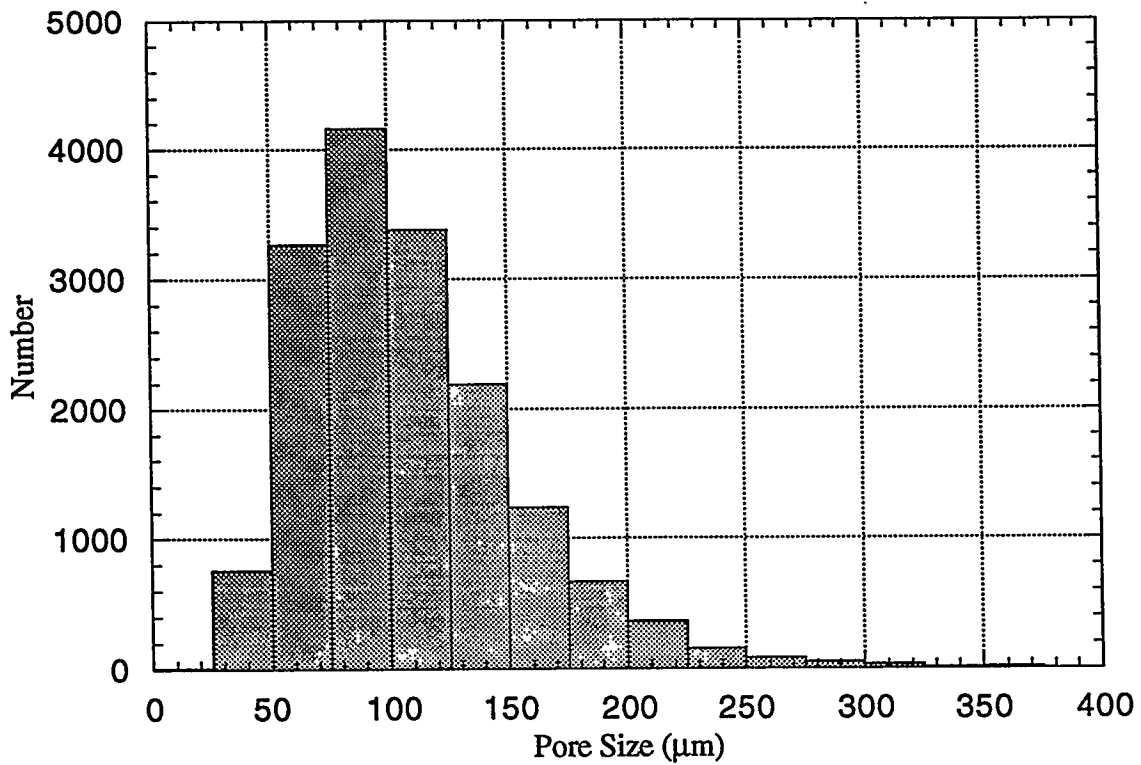


Figure 14.

Figure 14. Lognormal pore size distribution used for trapped fraction calculated. The mean characteristic pore size is 100 μm.

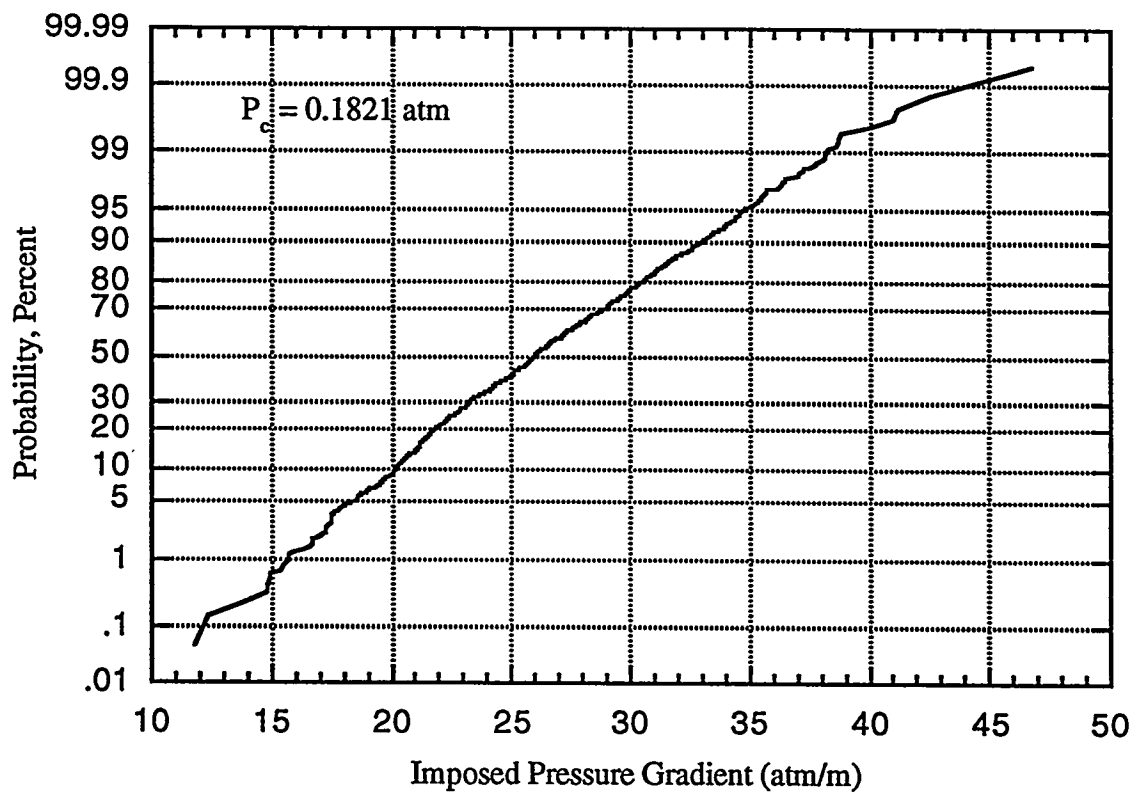


Figure 15.

Figure 15. Percentage of flowing foam as a function of mobilization pressure gradient with a capillary entry pressure of 0.182 atm in a porous medium made up of pores distributed as in Figure 14.

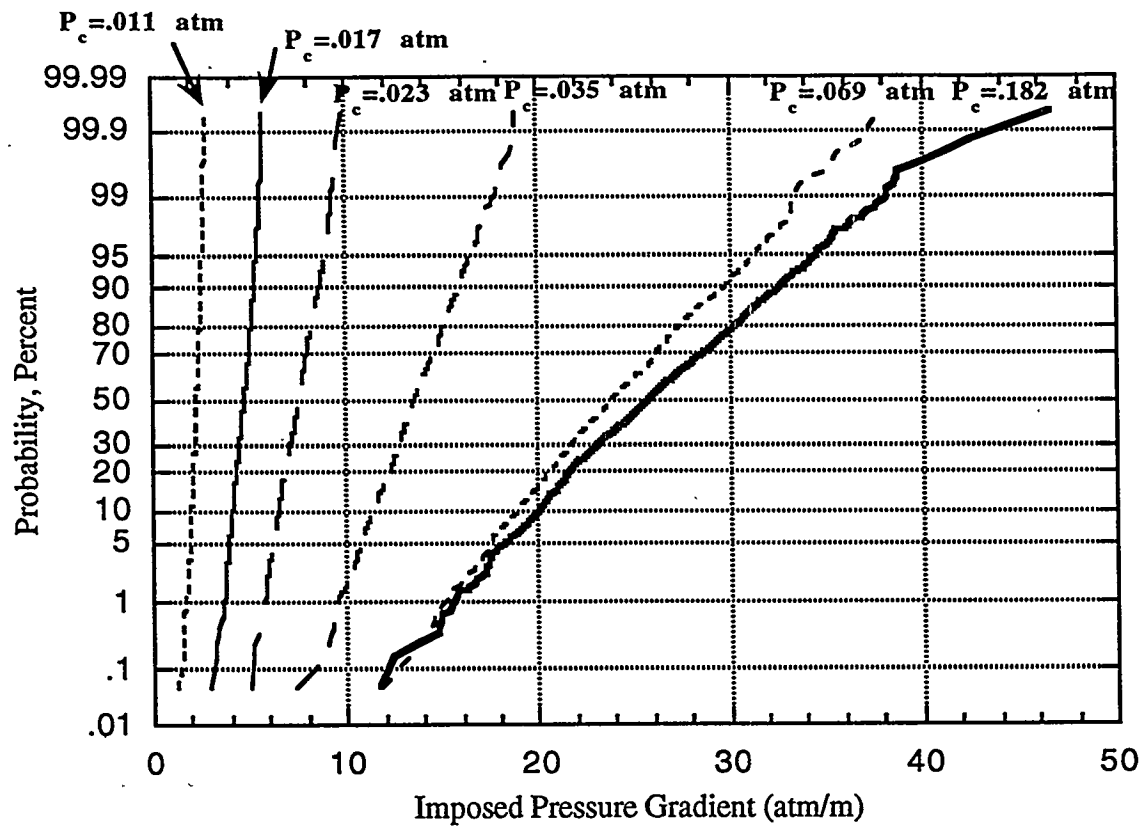


Figure 16.

Figure 16. Percentage of foam in porous media that is mobilized as a function of mobilization pressure gradient for various capillary entry pressures. A fraction of the total number of pores is occupied by foam at a given capillary pressure. All of the pores are occupied by foam at $P_c = 0.182$ atm.

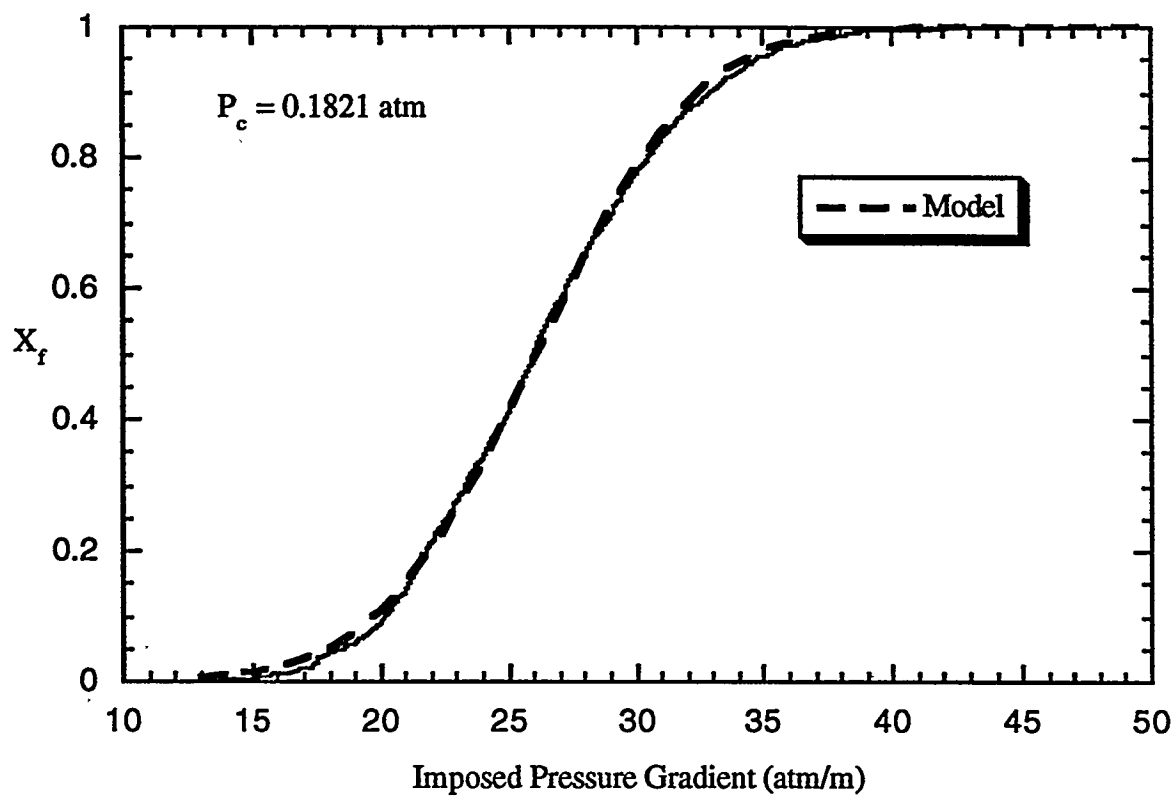


Figure 17.

Figure 17. Flowing foam fraction as a function of mobilization pressure gradient with a capillary entry pressure of 0.182 atm, plotted using a linear scale. Also included is the model curve; $a=26.141$ and $b=4.94$.