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HANDLING UNCERTAINTY IN QUANTITATIVE ESTIMATES IN INTEGRATED RESOURCE PLANNING

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LIST OF ACRONYMS

CAA - Clean Air Act

DSM - Demand-Side Management

DSR - Dempster-Shafer Rule

IRP - Integrated Resource Planning

LCDF - Lower cumulative distribution function

MW - Mega-watt

PUC - Public Utility Commission

UCDF - Upper cumulative distribution function

EXECUTIVE SUMMARY

This report addresses uncertainty in Integrated Resource Planning (IRP). IRP is a planning and decisionmaking process employed by utilities, usually at the behest of Public Utility Commissions (PUCs), to develop plans to ensure that utilities have resources necessary to meet consumer demand at reasonable cost. IRP has been used to assist utilities in developing plans that include not only traditional electricity supply options but also demand-side management (DSM) options.

Uncertainty is a major issue for IRP, as is shown in Section 2. Future values for numerous important variables (e.g., future fuel prices, future electricity demand, stringency of future environmental regulations) cannot ever be known with certainty. Many economically significant decisions are so unique that statistically-based probabilities cannot even be calculated. The entire utility strategic planning process, including IRP, encompasses different types of decisions that are made with different time horizons and at different points in time. Because of fundamental pressures for change in the industry, including competition in generation, gone is the time when utilities could easily predict increases in demand, enjoy long lead times to bring on new capacity, and bank on steady profits.

The purpose of this report is to address in detail one aspect of uncertainty in IRP: Dealing with Uncertainty in Quantitative Estimates, such as the future demand for electricity or the cost to produce a mega-watt (MW) of power. A theme which runs throughout the report is that every effort must be made to honestly represent what is known about a variable that can be used to estimate its value, what cannot be known, and what is not known due to operational constraints. Applying this philosophy to the representation of uncertainty in quantitative estimates, it is argued that *imprecise probabilities* are superior to classical probabilities for IRP. Section 3 contains mathematical definitions of each.

Deciding how to represent uncertainty quantitatively is only one part of the challenge. Section 4 discusses how to manipulate two or more uncertain quantitative estimates. Methods such as combination, conditionalization, and consensus are defined and illustrated. In most IRPs it is necessary, at some point, to calculate expected values for important variables like electricity demand, cost of electricity production, and prices of competing fuels. Section 5 presents methods for calculating expected values using imprecise probabilities. The method based upon nonlinear optimization can be considered a major technical achievement attributable to this research. Lastly, it is also very important to understand factors underlying uncertainty in quantitative estimates. Section 6 presents a qualitative framework utilities can use to accomplish this task. The report concludes with a discussion of issues for future research and deliberations.

1. INTRODUCTION

Decisionmakers in the electric utility industry must deal with uncertainty in an efficient and rational manner to ensure the economic survival of utilities and meet the expectations of the public. This is an important challenge because many decisions have significant impact upon the ability of the utility to compete (e.g., cost of new capacity) and satisfy public expectations (e.g., siting and construction of new transmission lines). Utility-based decisionmaking is quite difficult because many such decisions are one-of-a kind—meaning that uncertainties are difficult if not impossible to ascertain statistically—irreversible, and characterized by complex and complicated outcomes (Hirst and Schweitzer, 1988).

This report focuses on uncertainty and IRP in the electric power industry. IRP is a process conducted by utilities, typically at the behest of PUCs, to build plans to acquire power resources. In practice, integrated resource plans include a mixture of traditional supply resources (e.g, coal, oil, natural gas, hydro) and DSM options (e.g., conservation, time-of-day pricing). Good IRPs, for instance, are based upon forecasts of energy prices and electricity demand (Hirst 1992), only two of many aspects of IRP that entail significant uncertainty.

Accordingly, utilities have begun to address uncertainty in IRP. Section 2. summarizes utilities' attempts, to date, to accomplish this task. For example, IRPs often represent uncertainty qualitatively through the use of cases (e.g., base case, high energy demand growth case, etc.). Fewer integrated IRPs use quantitative techniques to represent uncertainty (e.g., probabilities). Fewer still employ quantitative methods to manipulate quantitative uncertainties.

The motivation for this report is our strong belief that utilities should increase their use of quantitative techniques both to represent and manipulate uncertainties in integrated resource plans. Four reasons are offered in support of this contention.

(1) Quantitative techniques are very appropriate given the preponderance of uncertain quantitative variables and estimates in the plans. (2) The process of quantifying uncertainty about an estimate, yields a deeper appreciation into complexities surrounding the estimate. (3) Quantitative techniques, if used properly, represent explicit and rigorous statements of uncertainty that can be readily communicated and evaluated by others. (4) Quantitative representations of uncertainty (e.g., expected values) are required inputs for quantitative decision analytic methods, which should be at the foundation of important resource planning decisions.

This report is expressly written to assist utilities and analysts incorporate quantitatively uncertainty in IRPs. To provide a further focus, the report only deals with one general quantitative paradigm: probability. However, as Section 3. indicates, the report takes a broad view of probability. The section contains a brief history of the concept (Section 3.1) and provides a mathematical overview of "classical probability (Section 3.2)."

A theme which runs throughout the report is that one should honestly represent what one knows and no more using the probabilistic paradigm. Oftentimes, and we would argue in the preponderance of cases in IRP, classical probability is too restrictive, forcing one to overstate one's knowledge about the value for an important variable. Thus, we argue for the use of imprecise (e.g., upper and lower) probabilities. Sections 3.3 and 3.4 present definitions of upper and lower probabilities, and upper and lower probability distributions, respectively. The balance of this section discusses how to construct imprecise probabilities and provides examples.

Section 4 addresses the manipulation of quantitative uncertainties in general and imprecise probabilities in particular. The section begins by presenting a conceptual model that encompasses different situations where one would need to synthesize two or more imprecise probabilities (Section 4.1). The conceptual model is needed to define quantitative methods such as combination, conditionalization, and consensus.

As the reader peruses this report, it will be evident that we have tried to synthesize general discussions of uncertainty, detailed mathematical presentations, and straightforward examples. Section 4. exemplifies the approach. Following the discussion of the conceptual model is a short note on the mathematics of conditionalization and imprecise probability (Section 4.2). Then several examples are presented (Section 4.3).

In many ways, the most important quantitative challenge facing IRP analysts is the calculation of expected values. Such calculations are straightforward using classical probabilities. Section 5. demonstrates that there are well-understood methods to calculate expected values using lower and upper probabilities. One such method, known as Choquet Expected Values, is discussed in Section 5.1. There are also opportunities to extend these methods to more general imprecise probabilities and to frameworks of importance to utility analysts. Section 5.2 illustrates this by introducing a new method to calculate upper and lower expected values via nonlinear optimization for decision-tree applications. A detailed example of this method appears in Section 6.3.

Quantitative estimates of uncertainty, especially those based on subjective judgments, are challenging to develop. Section 6. provides assistance to those needing to specify imprecise probabilities. Presented first, in Section 6.1 is a presentation of a qualitative frame (i.e., checklist) one can use to describe why there is uncertainty in a quantitative estimate. For example, uncertainty may arise due to factors under the analyst's control, such as data quality or the application of appropriate estimation techniques. On the other hand, a large degree of the uncertainty may be inherent in the estimation problem itself and therefore beyond the control of the analyst. In those cases, it is appropriate to represent uncertainty using imprecise probabilities.

In any case, the frame can indicate to utilities what can be done, if anything, to reduce the uncertainty associated with key planning variables. Implications of this realization on the cost and value of information are discussed in Section 6.2. The section concludes with a discussion of elicitation issues and examples of using the frame.

It is well known in the physical sciences that most of the easy problems have been solved. The remaining problems are much more challenging conceptually and oftentimes require more sophisticated and expensive equipment and experiments. An analogy can be made to uncertainty and decisionmaking. As utilities and PUCs strive to make better decisions, to fine tune utility investments and operations to reduce costs and increase service to the public, the problem of decisionmaking gets progressively, if not exponentially, more difficult.

As this report points out, dealing with uncertainty is not just an exercise in identifying what features in the utility environment cause uncertainty. To fully appreciate the topic, one needs to master powerful but oftentimes subtle concepts and understand mathematical presentations and methods. We believe that the effort is well worth it.

2. OVERVIEW OF UNCERTAINTY AND IRP

2.1 ASPECTS OF UNCERTAINTY IN IRP

IRP is a process by which utilities and PUCs work to establish mutually acceptable plans for meeting the public's need for utility services. Specifically, integrated resource plans detail how utilities will supply electricity services to meet forecast demands. In the electric utility industry, supply options traditionally focused on building new power plants. IRP has evolved to include DSM, renewables, and other energy sources in the list of supply options (Schweitzer, Hirst, and Hill 1991).

The art of IRP has evolved to where suggestions can be offered about how to develop good integrated resource plans. Hirst (1992) states that plans need to: be technically competent; present adequate, detailed, and consistent (with long-term plans) short-term action plans; incorporate the interests of various stakeholders; and be clear and comprehensive in presentation. Technically competent plans address: energy and demand forecasts; supply and demand resources; resource integration; and uncertainty, which is the topic of this report.

Indeed, a strong argument can be made that uncertainty dominates every aspect of IRP. Hirst and Schweitzer (1990) surveyed numerous plans and found that uncertainties abound (Table 2.1). The uncertainties pertain to issues internal to utilities and relate to factors external to the utility, which are beyond utility control. Important uncertainties, for example, involve the utility's cost of providing power and forecasting load growth.

To that list should be added uncertainty concerning the future of the utility industry. Numerous factors are pressuring the industry to change (Tonn and Schaffhauser, 1994; Dasovich et al., 1993). These include increasing competition in generation and potential competition in the form of retail wheeling. It is unclear

Table 2.1. Key Uncertainties in Integrated Resource Planning

Uncertainties Internal to Utilities
- Type, availability, and/or costs of new generating facilities
- Availability and/or costs of existing generating facilities
- Availability and/or costs of power from life-extension projects
- DSM capability
- Availability of renewable energy resources

Uncertainties External to Utilities
- Load Growth
- Fuel Prices
- Availability and/or costs of purchased power
- Actual savings from DSM and related efforts
- Regulatory policies
- Inflation and interest rates
- Environmental constraints

(Adapted from Hirst and Schweitzer, 1990, p.139)

whether these forces will result in: a substantially decentralized, vertically deintegrated industry; a substantially more centralized, vertically integrated industry; or an industry little changed from the situation today, which is still dominated by utilities in the areas of generation, transmission, and distribution, although less so in generation than in the past.

The important point for this research is that the need to handle uncertainty is even more important as the utility industry heads toward the next century. Irregardless of whether PUCs continue to mandate IRP, utilities will have an increased need to conduct their own strategic planning exercises to ensure organizational survival. PUCs and other governmental bodies will also have an increased need for analysis to ensure that current government regulations are appropriate for the utility industry of the future and to predict the consequences of proposed regulations. Central to strategic planning, policy analysis, and IRP is the representation and management of uncertainty.

2.2 UNCERTAINTY AND IRP: CURRENT PRACTICE

Approaches currently used to handle uncertainty in IRP are summarized in Hirst (1992), Hirst and Schweitzer (1990), and Hirst et al. (1990). Five approaches have been explicitly found to be used in IRPs: scenario analysis, sensitivity analysis, portfolio management, probabilistic methods, and worst-case analysis. Table 2.2 summarizes these techniques. Hirst (1992) presents examples of actual plans that use each technique.

Table 2.2. Approaches currently used to handle uncertainty in IRP

Scenario	Alternative, internally consistent, futures are constructed, and then resource options are identified to meet each future. Best options can then be combined into a unified plan.
Sensitivity	Preferred plan (combination of supply options) is identified. Key factors are then varied to see how the plan responds to these variations.
Portfolio	Multiple plans are developed, each of which meets different corporate goals. Often, these plans are then subjected to sensitivity analysis.
Probabilistic	Probabilities are assigned to different values of key uncertain variables, and outcomes are identified that are associated with the different values of the key factors in combination. Results include the expected value and cumulative probability distribution for key outcomes, such as electricity price and revenue requirements.
Worst-Case	Utility creates a plan to meet an extreme set of conditions (e.g., high load growth and high fuel prices) and later learns that it faces an entirely different set of conditions (e.g., low load growth and low fuel prices). The utility then adjusts its resource acquisitions to meet the newly perceived conditions.

(Source: Hirst and Schweitzer, 1990).

Scenario analysis appears to be the most favored technique. Utilities often create cases (e.g., Base, High Load Growth, Low Load Growth) and prepare forecasts and plans for each of the cases. Sensitivity analysis involving key variables is also extensively used. Use of probabilistic methods is limited and no case was found where a utility used advanced probabilistic methods, such as discussed in the balance of this report.

Hirst and Schweitzer (1990) also report on how utilities react to uncertainties in IRP. They list five basic strategies: (1) ignore uncertainty, (2) plan very carefully, (3) defer decisions, (4) sell risks to other parties, and (5) adopt flexible strategy that allows for easy and inexpensive changes. In the long-run, ignoring uncertainty and options theory as a matter of course will imperil utilities and their customers. The remaining three strategies, on the other hand, have merit. Future research needs to evaluate how well utilities apply these three strategies.

2.3 COMMENTS

In summary, it is important to point out that utilities are pursuing two complementary approaches to handling uncertainty. The first approach can be labeled technical and deals with how to represent uncertainty quantitatively and manage uncertainty in analytical exercises. Thus, it has been found that utilities make use of probabilities, sensitivity analysis, and worst case scenario analysis.

The second approach is more strategic and process oriented. It relates to attempts to minimize risks associated with uncertainty and results in reducing uncertainty, not necessarily about what might happen in the future, but with respect to the negative consequences of decisions. Thus utilities defer decisions, sell risks to others, and adopt flexible plans.

In general, the utility industry has begun to employ rational and effective techniques for handling uncertainty in integrated resource planning. However, we see several areas where the utility industry could improve its efforts.

(1) Quantitative methods for representing and manipulating uncertainty need wider use. Few of the IRPs make use of probabilistic methods and none use advanced probabilitistic methods. Quantification of uncertainty has two major benefits. First,

the process of quantification requires considerable introspection and analysis, which will help ensure that uncertainties receive honest and rigorous consideration. Second, numerous, powerful methods for manipulating uncertainties and making decisions require quantitative uncertainties. This is not to say that qualitative reasoning is unimportant; the force of qualitative reasoning can be enhanced through the use of quantitative methods.

(2) IRPs need to be set within the larger utility/PUC decisionmaking context. Should IRPs encompass long term strategic directions and strategic decisions in addition to actions that can be taken in the near-term? Should IRPs actually document strategic decisions which in turn effect the scope of action plans? We cannot answer these questions in this report but do argue that all planning and decisionmaking activities within a utility need to be coordinated in an effective fashion.

(3) The plans themselves could incorporate more strategies to reduce uncertainties. For example, consideration should be given to producing robust plans, which are plans that will not collapse if one or two aspects fail to materialize.

(4) The way that uncertainty is communicated in the plans needs to be improved. Figures and tables are often confusing to interpret. It is difficult enough to communicate sophisticated technical information to time- and attention-constrained utility executives and PUC commissioners in the best of conditions. In addition, care needs to be taken to clearly communicate information to a public, that cannot be expected to be technically literate.

This report focuses solely on the first issue.

3. REPRESENTING UNCERTAINTY IN QUANTITATIVE ESTIMATES

3.1 A BRIEF HISTORY OF PROBABILITY

"Probability" entails more than it seems to most people. It is a concept with a long history, controversial interpretation, and significant importance to people who need to represent uncertainty in quantitative estimates. Briefly summarizing the history of probability is a useful exercise in a report about IRP for two major reasons. (1) The discussion will help build a level of comfort with the topic for those who come to this discussion with a limited background in probability. (2) The ideas espoused in this report, including the use of imprecise probabilities, are best appreciated within the broader historical and evolutionary context of the concept. Within any field of endeavor, inertia supports the most familiar theories and formulations. In this case, inertia supports classical probability. However, moving to a more general notion of uncertainty, namely imprecise probability, is actually not a radical step at all if seen in the long-run and when one understands that the original ideas about quantitative probability were more similar to imprecise probabilities than classical probability.

To begin the story, it is interesting to note that prior to the 1660s, the concepts of chance and probability were unquestionably distinct. According to Hacking (1975), probability was associated with opinion and was not mathematical. For example, an esteemed religious authority could argue that a proposition that is "probable is impossible," meaning that the proposition has a favored opinion but cannot be true. Chance, on the other hand, had mathematical qualities because it was associated with games of chance. Thus, historically, the word "Probability" had a much different meaning than it has today and the concepts of chance and probability were not interchangeable.

How then, did the two concepts become synonymous? It took many centuries for humans to understand that: opinion could be founded, in part, on knowledge; that knowledge is not always certain; and that an analogy could be made between one's uncertain knowledge and playing a game of chance. These observations arose from the historical division between the high sciences and low sciences (Hacking 1975), the former encompassing mathematics and astronomy (and presumably no uncertainty) and the latter alchemy, geology, and medicine (and presumably much uncertainty).

To practice a low science, one used signs from nature. For example, physicians used signs to diagnose patients. Unfortunately, for the ancient patients, ancient physicians based diagnoses on signs that arose from the cultural milieu, not on physical symptoms that could be "statistically" associated with certain diagnoses and appropriate treatments.

Hacking finds the first written references that signs are derivable from nature in the 1600s. The earliest reference is from Hobbe's *Humane Nature*, published in 1650, where he says that "if the signs hit 20 times for one missing, a man may lay a wager of twenty to one of the event; but may not conclude it for a truth." The book *Port Royal Logic* (1662) contains the first use of the word "probability" to represent what we might label epistemic or quantitative probability. Pascal made the first link between games of chance and quantitative notions of probability with his wager about the existence of God. Thus began the co-mingling of the concepts of chance and probability that continues to this day.

Shafer (1978) attributes the most important role in the mathemization of probability to Jacob Bernoulli. In his 1713 manuscript, *Ars Conjectandi*, Bernoulli was concerned with different types of arguments to support or reject a proposition. A pure argument provides support for the proposition but does not provide support for competing propositions. A mixed argument could provide support for several competitive propositions.

Examples of pure and mixed arguments can be made with respect to the following proposition, S which is important in IRP:

> S: **Electricity demand in the service area will increase faster than economic growth for the next twenty years.**

The following is a pure argument, A^P, in favor of this proposition:

> A^P: **The use of electricity-intensive information terminology will grow at a faster pace than overall economic growth for the next twenty years.**

The following is a mixed argument, A^M, in favor of this proposition:

> A^M: **Environmental concerns will continue to increase over the next twenty years.**

A^P is a pure argument because at least one component of electricity demand, that related to the use of information technology, will grown faster than the overall rate of economic growth and there are no conditions associated with this argument that would lead one to other conclusions. A^M is a mixed argument because environmental concerns could lead to increases and decreases in the rate of change in electricity demand. For example, increasing concerns could lead to more environmentally benign manufacturing technologies which could be more electricity intensive than previous technologies. On the other hand, increasing environmental concerns could lead to an extreme conservation ethic which would involve high levels of energy, and therefore electricity conservation.

In addition to presenting the outlines for these types of arguments, Bernoulli presented mathematical formulas to implement quantitatively each kind of argument. For example, the probability associated with a pure argument for proposition S is the number of cases it proves correct, a, divided by the total number of cases, $n : P(S) = a/n$. This is the familiar, frequency-based definition of probability.

Because a pure argument does not specify that unallocated probability mass be assigned to the complement of $S, P(\overline{S})$, with respect to a pure argument, $P(S) \leq 1.0$, where a probability of 1.0 ($i.e., P(S) = 1.0$) indicates complete certainty about the truth of a proposition.

This avenue of thought, that is, the frequency-based notion of probability, was pursued by many others, including DeMoivre (1718) and Bayes (1763), who were interested in the calculation of annuities and actuarial tables. This work lead to the development of classical probability (discussed in Section 3.2) and modern statistics.

It should be noted, however, that Bernoulli also pursued another avenue of thought in *Ars Conjectani*, related to combining probabilities of various types of arguments, which might be necessary with respect to synthesizing pieces of evidence in a court of law or to synthesizing various signs to render a medical diagnosis. For example, to combine Z pure arguments related to the truth of S, Bernoulli proposed the following:

$$P(S) = 1 - \{[1 - P_1(S/A_1^P)][1 - P_2(S/A_2^P)] \ldots [1 - P_z(S/A_3^P)]\}. \qquad (3.1)$$

Bernoulli also proposed equations to combine numerous mixed arguments, and one pure and one mixed argument, which is:

$$P(S) = P(S/A^P) + [1 - P(S/A^P)]P(S/A^M). \qquad (3.2)$$

According to Shafer (1978), Lambert (1764) was the only historical figure to extend Bernoulli's work regarding the combination of arguments. For example, Lambert found fault with Equation 3.2 because it does not adequately incorporate arguments against S. He proposed the following more general equation:

$$P(S) = \frac{P_1(S) + P_2(S) - P_1(S)P_2(S) - P_1(S)Q_2(S) - P_2(S)Q_1(\overline{S})}{1 - P_1(S)Q_2(\overline{S}) - P_2(S)Q_1(\overline{S})}, \qquad (3.3)$$

where $P_i(S)$ is the probability of S being true for argument i, $Q_i(\overline{S})$ is the probability of $\overline{S}$, and $P_i(S) + Q_i(\overline{S}) \leq 1$.

It is unclear why this avenue of thought died out in the 1700s. However, for our discussion, it is interesting to note that these ideas resurfaced in the 1960s and 1970s as people became interested again in combining arguments (e.g., in expert systems). For example, Equation 3.3 is a simple version of Dempster's (1967) method of combining upper and lower probabilities and Shafer's (1976) method of combining belief functions, which are a class of imprecise probabilities. Shortliffe (1976) rediscovered Equation 3.2 and made it the cornerstone of his certainty factor theory, which today is a popular method of managing uncertainty in expert systems.

A theme that runs through most of the research recently, and which is found in Bernoulli's and Lambert's ideas, with respect to combining probabilities associated with arguments is that the probabilities need not be additive. That is, $P(S) + P(\overline{S}) \leq 1$, which is known as "nonadditivity," is an acceptable constraint. This is in contrast to classical probability, where it is assumed that $P(S) + P(\overline{S}) = 1$, which is known as "additivity."

What does this seemingly arcane point have to do with IRP? The answer is rather complex. To begin with, IRP encompasses both of Bernoulli's avenues of thought about probability. On one hand, much use is made of databases and statistics calculated within the formal paradigm of classical probability. On the other hand, much of IRP is an argumentative process. The development of propositions and pure and mixed arguments is a natural part of the IRP process. To quantitatively represent and combine probabilities associated with propositions, based on the above presentation, we argue that a less restrictive view of probability, (i.e. non-additivity), is required.

It will not do, however, to advocate the use of two probability paradigms in IRP, one for frequentistic database applications, and one for more subjective, combination of argument applications. This is why we argue for the use of imprecise probability, which Section 3.3 points out, is a generalization of classical probability. Within the imprecise probability domain, one can still maintain the additivity constraints with respect to statistical applications, although authors such as Walley (1991) argue this is not necessary or even prudent. One can also have flexibility in representing subjective knowledge that accompanies nonadditive probabilities. These points will become clearer after the discussions on classical probability, upper and lower probabilities, and upper and lower distribution functions, in the next three subsections, respectively, and through the presentation of examples (Section 3.6).

3.2 CLASSICAL PROBABILITY

This section lays out the mathematical underpinnings of classical probability which, as discussed in Section 3.1, have their historical roots in the work of Bernoulli.

If Ω is a set of possible states of the world ($e.a., w_1, w_2 \ldots w_n$), uncertainty about which state $\omega \in \Omega$ is the true state is often modeled by a *probability measure* P defined on some class of subsets (called *events*) of Ω. The number $P(A)$ assigned to the event A represents the probability that the true state of affairs belongs to the set A. The probability measure P is said to be *objective* if $P(A)$ represents, in some sense, the relative frequency with which the true state belongs to A. If, on the other hand, $P(A)$ reflects the odds that one would consider fair (either as bettor or bookmaker) for a bet that the "the truth lies in A," then P is said to be *subjective*. The latter types of probabilities can also be elicited directly from people in a variety of ways (Wallsten 1983).

Axiomatic accounts of probability theory always postulate that $0 \leq P(A) \leq 1$ for all events A, with $P(\Omega) = 1$. In addition, *additivity* of $P(A \cap B = \emptyset \Rightarrow P(A \cup B) = P(A) + P(B))$ is always postulated, and *countable additivity* $(P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$, for every infinite sequence $A_1, A_2, \ldots$ of pairwise disjoint events) is often postulated (always, among mathematicians). As a consequence of these postulates, one always has $P(A) + P(\overline{A}) = 1$ where $\overline{A} := \{\omega \in \Omega : \ w \notin A\}$, and so $P(\emptyset) = 0$.

It is clear why one might demand additivity of an objective probability. Such probabilities model relative frequencies, and relative frequencies are additive as a matter of simple arithmetic. As for requiring additivity of subjective probabilities, there are arguments, which we shall not pursue, that non-additive subjective probabilities commit one to certain incoherent betting behavior (Skyrms, 1975).

The application of classical probability theory requires that one first complete a demanding assessment exercise: each event must be assigned a single precise probability. But it is clear that the evidence is very often insufficient to ground such an assessment. For example, with respect to objective probabilities, Fine (1973) points out that there are a number of subjective judgments required to estimate an "objective" probability, (e.g., defining Ω and choosing the sample population). As discussed in Section 6.1, there are other factors which could produce uncertainty in an estimate beyond the variability in the data used to calculate it, (e.g., suspect theory, low quality data, lack of data, etc).

With respect to subjective probability, an important problem is the expression of strength of evidence concerning an estimate. According to Keynes (1921), as new evidence about a proposition accumulates, "The magnitude of the probability of the argument may either decrease or increase, but *something* seems to have increased in either case—we have a more substantial basis upon which to rest our conclusion

(Keynes P.71)." It is not possible to represent both a probability and strength of evidence with one number.

Popper (1974) provides an example related to tossing a coin. Prior to any toss, a reasonable subjective probability of heads arising would be 0.50. Now, assume that this coin is tossed a thousand times, and the statistically derived probability is 0.50. Popper states that using subjective probability, there is no way to indicate the accumulated evidence. The next section explores one way of overcoming this problem.

3.3 UPPER AND LOWER PROBABILITIES

A natural way to relax the demand for a single number expressing the probability of an event A is to allow assessment of uncertainty by an *interval* $[\underline{P}(A), \overline{P}(A)]$, where $0 \leq \underline{P}(A) \leq \overline{P}(A) \leq 1$. The numbers $\underline{P}(A)$ and $\overline{P}(A)$, called respectively, the *lower* and *upper probabilities of* A, are chosen so that one is, given present evidence, confident that the probability of A is neither less that $\underline{P}(A)$ nor greater than $\overline{P}(A)$. In a state of complete ignorance, it is entirely appropriate to set $\underline{P}(A) = 0$ and $\overline{P}(A) = 1$. At the other extreme, where one *knows* the objective probability $P(A)$ of A, as in the case of sampling from a known population, it may be appropriate to set $\underline{P}(A) = P(A) = \overline{P}(A)$. This formulation avoids the problem created by the principle of insufficient reason, and noted by Popper, where under complete ignorance, one assumes all events in Ω have the same probability. Once one accumulates evidence, such probabilities may commence from the data, but it is unreasonable to assume so at the outset. We define imprecise probability as containing a family of axiomatic generalizations of classical probability based on the concepts of lower and upper probability.

What properties, in addition to those indicated above, might lower and upper probabilities possess? Suppose that $\underline{P}(A)$ and $\overline{P}(A)$ are construed objectively, (i.e., as lower and upper bounds on the unknown relative frequency of A). Then, at the very least, we should have

$$\underline{P}(\emptyset) = \overline{P}(\emptyset) = 0 \text{ and } \underline{P}(\Omega) = \overline{P}(\Omega) = 1, \tag{3.4}$$

as well as *monotonicity* of $\underline{P}$ and $\overline{P}$, i.e.,

$$A_1 \subseteq A_2 \Rightarrow \underline{P}(A_1) \leq \underline{P}(A_2) \text{ and } \overline{P}(A_1) \leq \overline{P}(A_2) \tag{3.5}$$

And if the bounding functions $\underline{P}$ and $\overline{P}$ are to be useful, there ought, of course, to exist at least one probability measure P satisfying

$$\underline{P}(A) \leq P(A) \leq \overline{P}(A) \quad \text{for all events } A. \tag{3.6}$$

When Ω is finite, checking that (3.6) holds for some P amounts to checking that certain linear inequalities have a solution, which is easily done by linear programming if Ω is not too large.

Interpretations of $\underline{P}$ and $\overline{P}$ in terms of upper and lower odds make it reasonable to demand that (3.4), (3.5), and (3.6) be satisfied in subjective contexts as well. In particular, failure to satisfy (3.6) guarantees that one will suffer a sure loss (Walley 1991).

In what follows, therefore, we shall call functions $\underline{P}$ and $\overline{P}$ a *pair of lower and upper probability measures* if they satisfy (3.4), (3.5), and (3.6). It should be noted that some authors are more stringent in their use of these terms, requiring in addition the properties of *complementarity*, i.e.,

$$\underline{P}(A) + \overline{P}(\overline{A}) = 1 \quad \text{for all events } A, \tag{3.7}$$

superadditivity of $\underline{P}$, i.e.,

$$A_1 \cap A_2 = \emptyset \;\Rightarrow\; \underline{P}(A_1 \cup A_2) \geq \underline{P}(A_1) + \underline{P}(A_2) \tag{3.8}$$

and *subadditivity* of $\overline{P}$, i.e.

$$A_1 \cap A_2 = \emptyset \;\Rightarrow\; \overline{P}(A_1 \cup A_2) \leq \overline{P}(A_1) + \overline{P}(A_2). \tag{3.9}$$

But the above three properties, while clearly desirable - they are properties always possessed by the "tightest possible" lower and upper bounds defining the same class of probability measures P as (3.6) - are not essential. If for some reason it is important to satisfy (3.7)-(3.9), one can, at least for "small" finite sets Ω, upgrade a pair of upper and lower probability measures $\underline{P}$ and $\overline{P}$ to a pair $\underline{P}^{\#}$, $\overline{P}^{\#}$ satisfying (3.4)-(3.9), with $\underline{P}(E) \leq \underline{P}^{\#}(E) \leq \overline{P}^{\#}(E) \leq \overline{P}(E)$ for all events E. Using standard linear programming, one simply computes

$$\underline{P}^{\#}(E) = min\{P(E): \quad P \in \mathcal{P}(\underline{P}, \overline{P})\}, \quad \text{and} \tag{3.10}$$

$$\overline{P}^{\#}(E) = max\{P(E): \quad P \in \mathcal{P}(\underline{P}, \overline{P})\}, \tag{3.11}$$

where $\mathcal{P}(\underline{P}, \overline{P})$ is the closed, convex polyhedral set given by

$$\begin{aligned} \mathcal{P}(\underline{P}, \overline{P}) := \{P: \quad & P \text{ is a probability measure and} \\ & \underline{P}(A) \leq P(A) \leq \overline{P}(A) \text{ for all events } A\}. \end{aligned} \tag{3.12}$$

It is perhaps worth mentioning that one can avoid assessing values of both $\underline{P}$ and $\overline{P}$. For example, one might only assess all lower probabilities $\underline{P}(A)$, with $\underline{P}(\emptyset) = 0$, $\underline{P}(\Omega) = 1$, and $\underline{P}$ monotone (3.5), and such that there is *some* probability measure P for which $\underline{P}(A) \leq P(A)$ for all events A (again, check by linear programming if Ω is small enough). One then simply *defines* $\overline{P}(A) = 1 - \underline{P}(\overline{A})$ for all events A. It follows that $\underline{P}$ and $\overline{P}$ satisfy (3.4)-(3.6), and (3.7) as well.

The idea of upper and lower probabilities is not new, as we have seen. In addition to the early 18th century writings the ideas appear in the work of Mill (1843) and Boole (1854). In our own century Keynes (1921) and Koopman (1941), as well as a number of other scholars, have pursued the idea of representing uncertainty by imprecise probabilities. In recent years, this idea has attracted substantial interest (especially in the disciplines of artificial intelligence, economics, and statistics), as the following brief recent history indicates.

(1) *Artificial intelligence.* Students of artificial intelligence, particularly those concerned with expert systems, were the first to endorse the use of imprecise probabilities in substantial numbers, influenced strongly by Shafer's pathbreaking book, *A Mathematical Theory of Evidence* (1976). Shafer's Belief Functions [See section (4.5) and (4.7)], an abstraction of a highly structured class of lower probabilities first studied by Strassen (1964), provided a substantial generalization of classical probability. It has become clear, however, that one often needs an even more general class of uncertainty measures to honestly represent the evidence at hand (e.g., upper and lower probabilities).

(2) *Economics.* Savage's (1972) axiomatic treatment of decisionmaking under uncertainty, with its rationalization of preference based on expected utility, is justly famous among decision theorists. But almost from its appearance, criticisms have been directed at the stringency of some axioms, particularly the so-called "sure-thing principle." In the mid-1980's, Schmeidler (1986) constructed an account of decisionmaking under uncertainty using rather weak axioms. In Schmeidler's theory, preference is based on expected utilities calculated with respect to lower and/or upper probabilities using the "Choquet integral," [see section (3.5)] which does not require additivity of the measure in question. The best account of this penetration of imprecise probabilities into the realm of decision theory is Fishburn's *Nonlinear Preference and Utility Theory* (1988).

(3) *Statistics.* Shafer's *A Mathematical Theory of Evidence*, mentioned above, was actually addressed to statisticians, despite having found its most appreciative

audience in the AI community. With the publication of Walley's (1991) magisterial treatise, *Statistical Reasoning with Imprecise Probabilities*, it is likely that imprecise probabilities will play an increasing role in statistical inference. Interestingly, the updating of imprecise probabilities in the light of new evidence admits of a number of different methods, which generalize ordinary conditionalization of precise probabilities in various ways.

In summary, the use of imprecise probabilities is no longer the untested, *avant-garde* idea that it was several decades ago. At the same time, it does not replace classical probability where evidence supports precise assessment of uncertainty. *The theory of imprecise probabilities is not a competitor to classical probability, but rather a generalization of classical probability, reducing to the latter when upper and lower probabilities coincide.*

3.4 UPPER AND LOWER DISTRIBUTION FUNCTIONS

Much of the similar work on imprecise probabilities assumes Ω is discrete. This is understandable given that much of the effort focused on expert systems, where Ω is composed of clearly discrete diagnoses (e.g., medical diagnosis). However, with respect to IRP, there are numerous instances where Ω is continuous (e.g., future oil prices). Therefore, this section addresses upper and lower distribution functions.

Let Ω be equipped with a probability measure P. A *random variable on* Ω is a numerical labeling of the outcomes in Ω, i.e., a function $X: \Omega \to R$, the real number system. Under certain mild restrictions, which need not concern us here, a random variable X possesses a *cumulative distribution function* (cdf) $F: R \to [0,1]$ *with respect to* P, where

$$F(x) := P(\{\omega \in \Omega : X(w) \leq x\}) = \text{“}P(X \leq x)\text{”}, \quad \text{for all } x \in R. \tag{3.13}$$

We recall that $x_1 \leq x_2 \rightarrow F(x_1) \leq F(x_2)$, that $F(x) \rightarrow 0$ as $x \rightarrow -\infty$ and $F(x) \rightarrow 1$ as $x \rightarrow \infty$, and that F is "right continuous," i.e., $F(x+h) \rightarrow F(x)$ as $h \rightarrow 0$ through *positive* values.

The generalization of this idea to imprecise probabilities is straightforward. If $\underline{P}$ and $\overline{P}$ are a pair of lower and upper probability measures, we define

$$\underline{F}(x) = \underline{P}(\{\omega \in \Omega : \quad X(w) \leq x\}) = \text{“}\underline{P}(X \leq x)\text{”} \tag{3.14}$$

and

$$\overline{F}(x) = \overline{P}(\{\omega \in \Omega : \quad X(w) \leq x\}) = \text{“}\overline{P}(X \leq x)\text{”} \tag{3.15}$$

and call $\underline{F}$ and $\overline{F}$, respectively, the *lower* and *upper cumulative distribution functions* (lcdf and ucdf) *of* X. Clearly, if $P \in \mathcal{P}(\underline{P}, \overline{P})$ and F is the cdf of X with respect to P, then $\underline{F}(x) \leq F(x) \leq \overline{F}(x)$ for all $x \in R$.

The notion of random variables and their cdfs described above represents a comprehensive, mathematical formulation. In much applied work, $\Omega = R$, and $X(x) = x$. In such cases the cdf F is usually directly assessed, often in the form of a density function f where $F(x) = \int_{-\infty}^{x} f(t)dt$. One can obviously also directly assess lcdfs and ucdfs on R, taking care simply to ensure that $0 \leq \underline{F}(x) \leq \overline{F}(x) \leq 1$ for all $x \in R$, that $\underline{F}(x)$ and $\overline{F}(x) \rightarrow 0$ as $x \rightarrow -\infty$, and $\underline{F}(x)$ and $\overline{F}(x) \rightarrow 1$ as $x \rightarrow \infty$, and that $\underline{F}$ and $\overline{F}$ are right continuous.

3.5 CHOQUET EXPECTED VALUES

An extremely important task in IRP is the calculation of expected values. This section presents for review how to calculate expected values using classical probabilities and then presents the method to calculate expected values using lower and upper probabilities.

Suppose that the random variable $X: \quad \Omega \rightarrow R$ takes on only a finite set of values, say, $x_1 < x_2 < \ldots < x_n$. If P is a probability measure on Ω, then the *expected value of* X *with respect to* P, denoted $\mathcal{E}_{\mathcal{P}}(\mathcal{X})$, is given by the familiar formula

$$\mathcal{E}_P(X) = \sum_{i=1}^{n} x_i \; P(X = x_i), \tag{3.16}$$

where $P(X = x_i)$ is an abbreviation for $P(\{\omega \in \Omega : \quad X(w) = x_i\})$.

Now, by additivity of P we have, for $1 \leq i \leq n-1$, that $P(X = x_i) = P(X \geq x_i) - P(X \geq x_{i+1})$, so an equivalent (though slightly odd looking) formula for $\mathcal{E}_P(X)$ is given by

$$\begin{aligned}\mathcal{E}_P(X) &= \sum_{i=1}^{n-1} x_i\{P(X \geq x_i) - P(X \geq x_{i+1})\} + x_n P(X = x_n) \\ &= x_1 + \sum_{i=2}^{n}(x_i - x_{i-1})P(X \geq x_i). \end{aligned} \tag{3.17}$$

Now if $\underline{P}$ and $\overline{P}$ are a pair of lower and upper probability measures on Ω, then, motivated by (3.17), we *define* $\mathcal{E}_{\underline{P}}(X)$ and $\mathcal{E}_{\overline{P}}(X)$ by the formulas

$$\mathcal{E}_{\underline{P}}(X) := x_1 + \sum_{i=2}^{n}(x_i - x_{i-1})\underline{P}(X \geq x_i) \tag{3.18}$$

and

$$\mathcal{E}_{\overline{P}}(X) := x_1 + \sum_{i=2}^{n}(x_i - x_{i-1})\overline{P}(X \geq x_i). \tag{3.19}$$

We call $\mathcal{E}_{\underline{P}}(X)$ and $\mathcal{E}_{\overline{P}}(X)$ the *Choquet expected values of* X *with respect to* $\underline{P}$ and $\overline{P}$. The above formulas are simply special cases of the general formula, valid for every random variable X,

$$E_\alpha(X) = \int_0^\infty \alpha(X \geq x)dx - \int_{-\infty}^{0} [1 - \alpha(X \geq x)]dx,$$

where $\alpha = \underline{P}, \overline{P}, P$(Fishburn, 1988, p. 189).

Since the quantities $x_i - x_{i-1}$ appearing in (3.18) and (3.19) are positive, it follows immediately that

$$P \in \mathcal{P}(\underline{P}, \overline{P}) \;\Rightarrow\; \mathcal{E}_{\overline{P}}(X) \leq \mathcal{E}_P(X) \leq \mathcal{E}_{\overline{P}}(X), \tag{3.20}$$

and so, with

$$\underline{\mathcal{E}}(X) := min\{E_P(X): \quad P \in \mathcal{P}(\underline{P}, \overline{P})\}, \quad \text{and}$$

$$\overline{\mathcal{E}}(X) := max\{E_P(X): \quad P \in \mathcal{P}(\underline{P}, \overline{P})\}, \tag{3.21}$$

it follows that

$$\mathcal{E}_{\underline{P}}(X) \le \underline{\mathcal{E}}(X) \le \overline{\mathcal{E}}(X) \le \mathcal{E}_{\overline{P}}(X). \tag{3.22}$$

So the crucial quantities $\underline{\mathcal{E}}(X)$ and $\overline{\mathcal{E}}(X)$ may be conservatively approximated, respectively, by the easily computable quantities $\mathcal{E}_{\underline{P}}(X)$ and $\mathcal{E}_{\overline{P}}(X)$. Indeed, in certain cases, we are guaranteed to have $\mathcal{E}_{\underline{P}}(X) = \underline{\mathcal{E}}(X)$ and $\mathcal{E}_{\overline{P}}(X) = \overline{\mathcal{E}}(X)$. This always happens, for example, if the pair $\underline{P}$, $\overline{P}$ satisfy (3.4)-(3.7) and $\underline{P}$ satisfies the following stronger version of superadditivity, called 2-*monotonicity*:

$$\underline{P}(A_1 \cup A_2) \ge \underline{P}(A_1) + \underline{P}(A_2) - \underline{P}(A_1 \cap A_2). \tag{3.23}$$

(See Chateauneuf and Jaffray, 1989; and Thorp, McClure, and Fine, 1982.) Several common constructions of imprecise probabilities yield lower probability measures satisfying (3.23), as we show in Sections 3.6 and 4.3.

3.6 EXAMPLES

3.6.1 Example 1. *Acid Rain Regulations*

The context is a major electric utility located in the Eastern United States. The utility is investing in environment controls to reduce the emissions of SOx and NOx to reduce acid rain. Title IV of the 1990 amendments to the Clean Air Act (CAA) requires controls to be in place by 1996 or 2000. The National Acid Precipitation Assessment Program (NAPAP) will report to Congress in 1996 on the societal costs and benefits of Title IV, and every four years thereafter. An important question facing the utility, and one which significantly affects the next IRP, is whether Congress will again change the acid rain provisions of the Act.

Assume that the solution set, Ω, is discrete and has three members, {stricter provisions (sp), no change (nc), less strict provisions (lsp)}. Hard evidence supporting any of these possibilities is non-existent. There are no published statements from any member of Congress on future legislative intentions with regard to Title IV. In addition, the content of environmental legislation is highly dependent on the party occupying the White House, another unknown. Due to the large uncertainties and the subjective nature of the required judgments, lower and upper probabilities are an appropriate method to represent uncertainty about Ω.

There are several approaches to constructing imprecise probabilities. The approach chosen for this example is based upon eliciting lower probabilities over the power set of Ω, which as Table 3.1 indicates, has seven members. The task for the analyst is to assign lower probabilities to each set, related to the lower probability that the true outcome is in the set. Ω is always assigned a lower probability of 1.0 because, by definition, the truth must reside in this set.

Assume an analyst supplied the lower probabilities ($\underline{P}$) found in the second column of Table 3.1. What do they tell us? Overall, the analyst is confident that the truth is contained in the fourth set, that Congress will issue stricter provisions or not change the current provisions. The analyst was clearly uncomfortable with assigning substantial lower probabilities to the sets with only one member. A small lower probability was assigned to set 2, {no change}, if only because legislature inertia hinders change of any sort. The analyst doesn't give much credence to Congress' lessening the provisions of Title IV.

The third column of Table 3.1 provides the upper probabilities for the power set of Ω, as calculated by (3.7). A quick review of Table 3.1 indicates the following. First, the analyst did not specify a classical probability function, for example, because $\underline{P}\{sp\} \neq \overline{P}\{sp\}$. Second, the function is monotonic because in every case

Table 3.1. Lower and upper probabilities assigned by an analyst in the acid rain example

Subset A of Ω *	$\underline{P}(A)$	$\overline{P}(A)$
1. {sp}	0.2	0.06
2. {nc}	0.4	0.4
3. {lsp}	0.0	0.2
4. {sp, nc}	0.8	1.0
5. {sp, lsp}	0.6	0.6
6. {nc, lsp}	0.4	0.8
7. {sp, nc, lsp}	1.0	1.0

*sp= stricter provisions; nc = no change; lsp = less strict provisions

(3.5) holds. Third, the function is also superadditive, (3.8). Fourth the function is not subadditive, (3.9) because $\overline{P}\{nc, lsp\} \geq \overline{P}\{nc\} + \overline{P}\{lsp\}$.

3.6.2 Example 2. Energy savings attributable to a residential weatherization program

In this example, an imprecise probability function is constructed using a less direct assessment method, which we refer to as compatibility mapping. This method was first introduced by Strassen (1964) and was further developed by Dempster (1967). The method is applied to a problem related to calculating the lower and upper expected values (using Choquet expected values) of energy savings attributable to a residential weatherization program.

Compatibility mapping involves: (1) assessing a probability measure Q on a related set Θ of possible states of the world; and (2) relating Θ to a set Ω of outcomes which are relevant to the problem at hand. The relation between Θ and Ω is given by the "compatibility mapping,"

$$\Gamma : \Theta \rightarrow \{A : A \subseteq \Omega \text{ and } A \neq \phi\} , \tag{3.24}$$

Where $\Gamma(\Theta)$ is the set of all $\omega \epsilon \Omega$ compatible with θ, for each element $\theta \epsilon \omega$.

As is illustrated below, Q and Γ induce lower and upper probabilities $\overline{P}$ and $\underline{P}$ or Ω by the formulas

$$\underline{P}(A) = Q(\{\theta \epsilon \Theta : \Gamma(\theta) \subseteq A\}) \tag{3.25}$$

and

$$\underline{P}(A) = Q(\{\theta \epsilon \Theta : \Gamma(\theta) \cap A = \phi\}). \tag{3.26}$$

In this example, let $\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$, where each θ_i pertains to a particular pattern of household electricity use. For the purposes of this example, let θ_1 represent households with relatively low electricity use and no daily use peaks, θ_2 households with average electricity use and morning and evening peaks, θ_3 households with high electricity use and morning and evening peaks, and θ_4 households with no clear electricity use patterns. Data collected as part of an extensive submetering project indicates that the proportion of households exhibiting these patterns is .10, .25, .35, and .20, for $\theta_1, \theta_2, \theta_3$, and θ_4, respectively.

Assume the utility has been running a residential weatherization program for a number of years and that the program represents one DSM resource that is being considered for inclusion in the next IRP. Before a benefit/cost ratio can be calculated for this program, its energy savings on a per participating household basis needs to be estimated. Analysis of weather corrected electricity bills indicate that households participating in the program reduced their annual electricity use by 2500 kWh.

The problem for the analyst is determining what percentage of this reduction can be attributed to the program as permanent savings. This is a problem because households change in various ways over time. For example, households could purchase/acquire different end-use technologies, and change preferences and behavior with respect to electricity use. Only a few studies have probed this problem. Assume the analyst has anecdotal evidence from the field.

In this example, we have chosen to structure the problem using a compatibility mapping. As mentioned above, we have a probability measure Q on the set Θ over the 4 classifications of household electricity use. We shall define the outcome set, Ω, as also containing four members, where $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and ω_1 = 30-70% of electricity savings can be attributed to the residential weatherization program, ω_2 = 60-80%, ω_3 = 80-90%, and ω_4 = 90-100%.

Table 3.2 represents a compatibility mapping between Θ and Ω, based on sparse studies and anecdotal evidence. A^* in the table indicates that θ_i is compatible with outcome ω_i. For example, θ_1, is compatible with ω_4, which indicates the program is significantly responsible for the savings, because it could be argued that few factors other than the installation of conservation measures could account for a reduction in electricity use in households that use a relatively small amount of electricity to begin with.

A - in the table indicates that θ_i is not compatible with outcome ω_i. For example, θ_4 is judged incompatible with ω_4 because one can imagine numerous factors unrelated to the installation of measures that could underlie irregular patterns of electricity use. Thus, it seems more appropriate that θ_4 households are compatible with ω_1 and ω_2 outcomes.

Applying (3.25) and (3.26) to the contingency table in Table 3.2 yields the lower and upper probabilities contained in Table 3.3. For example, $\underline{P}(\omega_1) = 0$ because no mapping between Θ and Ω yields a subset which is equal to or is subsumed by ω_1. $\underline{P}(\omega_1\omega_2) = .20$ because the mapping between Θ_4 and Ω yields a subset which is equal to $(\omega_1\omega_2)$. $\overline{P}(\omega_1) = .55$ because the mapping between Θ_3 and Θ_4 indicate a non-null intersection, namely ω_1, and $P(\Theta_3) = .35$ and $P(\Theta_4) = .20$, which when added equal .55. $\overline{P}(\omega_1\omega_3) = 1.0$ because $(\omega_1\omega_2)$ has a non-null intersection with every set in Θ.

Table 3.2. Compatibility mappings for residential weatherization example

		Types of Electricity Use Patterns			
		Θ_1 (low, no peaks) .10	Θ_2 (moderate, peaks) .25	Θ_3 high, peaks) .35	Θ_4 ir-regular .20
% Energy savings attributable to installation of measures	$\omega_1(30-70\%)$	-	-	*	*
	$\omega_2(60-80\%)$	-	*	*	*
	$\omega_3(80-90\%)$	*	*	*	-
	$\omega_4(90-100\%)$	*	*	-	-

*Compatible; - Incomplete

(3.18) and (3.19) can be used to calculate lower and upper expected values for savings attributable to the weatherization program. To simplify this example, let's assume that $\omega_1 = .5, \omega_2 = .7, \omega_3 = .85$, and $\omega_4 = .95$. Then $X_1 = 1250$ kWh, $X_2 = 1750$ kWh, $X_3 = 2125$ kWh and $X_4 = 2375$ kWh. Using (3.18), $E_{\underline{P}}\otimes = 1250 + (1750 - 1250) * .35 + (2175 - 1750) * .10 + (2375 - 2125) * 0 = 1462.5$ kWh. Using (3.19), $E_{\overline{P}}(X) = 1250 + (1750 - 1250) * 1.0 + (2125 - 1750) * .80 + (2375 - 2125) * .35 = 2137.5$ kWh.

In conclusion, it should be noted that had the analyst had better information, the contingency table could have been completed with conditional probabilities and the expected value calculations could have been computed the classical way using (3.16). However, as the example indicates, and we argue above, uncertainties plague these types of problems and it is unlikely indeed that in many instances the analyst will have the information needed to complete these types of contingency tables.

3.6.3 Example 3. Oil Price Forecasting

Example 1. features the direct elicitation of upper and lower probabilities. Example 2 features the construction of upper and lower probabilities from an

Table 3.3. Upper and lower probabilities for residential weatherization example

SET	$\underline{P}$	$\overline{P}$	SET	$\underline{P}$	$\overline{P}$
ω_1	0	0.55	$\omega_2\omega_4$	0	1.0
ω_2	0	0.90	$\omega_3\omega_4$	0.10	0.80
ω_3	0	0.80	$\omega_1\omega_2\omega_3$	0.55	1.0
ω_4	0	0.35	$\omega_1\omega_2\omega_4$	0.20	1.0
$\omega_1\omega_2$	0.20	0.90	$\omega_1\omega_3\omega_4$	0.10	1.0
$\omega_1\omega_3$	0	1.0	$\omega_2\omega_3\omega_4$	0.35	1.0
$\omega_1\omega_4$	0	1.0	$\omega_1\omega_2\omega_3\omega_4$	1.0	1.0
$\omega_2\omega_3$	0	1.0			

incomplete contingency table. This example features the construction of upper and lower probability cumulative distribution functions using a betting paradigm. The topic chosen for this example is oil price forecasting.

Assume the utility currently possesses a supply resource base that is dependent upon oil. The future price of oil, then, would be of considerable importance to the utility. Assume that the utility has access to an "expert" on the world oil market and that the goal is to elicit from the expert the expected value of the price for a barrel of oil in the year 2000.

In addition to the two methods for eliciting and constructing upper and lower probability functions discussed in the first two examples, a third method relies on betting behavior to elicit subjective probabilities. Based on the work of Ramsey (1931), De Finetti (1964), and others, the approach assumes that people make bets or accept bets involving loses and gains according to personal assessments of the likelihood of the events associated with the loses and gains. For example, according to De Finetti (1964), a person would be indifferent in making or taking a bet when the gain (or loss) a person would certainly receive (S) is equal to the gain (or loss) a person would expect to receive (S^1) contingent upon the event of the bet coming true. The subjective probability of the event would be $P(\epsilon) = S/S^1$.

The betting method will yield a classical probability function in those instances where there is one probability where the person is indifferent between making or taking a bet. However, as observed by Walley (1991), in real life, people rarely would make the same bet that they would take, or take the same bet that they would make. In other words, from the betting perspective, a person would want to receive more for winning the bet than the person would want to pay out for losing the bet. We attribute this observation to the fact that people intuitively fashion upper and lower probabilities about the world because the real world is so uncertain.

Thus, we argue, the betting paradigm should be generalized for application to uncertain real life situations such as IRP. With respect to oil price forecasting, the following could be pursued.

Assume the goal is to elicit from the expert upper and lower cumulative distribution function (LCDF) of the price of a barrel of oil in the year 2000. The LCDF could be elicited by assigning the expert to assume the position of a bettor (as opposed to bookie) and posing the following general question: What is the minimum payout (Y), you would expect for making a bet that the oil price in the year 2000 would be greater than or equal to X given that winning the bet would incur a sure gain of \$1,000,000 (or some such amount)? The question can be posed for numerous values for a barrel of oil to create, more or less, an LCDF, as shown in Fig. 3.1, using the formula, $\underline{P}(Y) = \$1,000,000/Y$.

Similarly, the upper cumulating distribution function (UCDF) can be elicited by posing the following general question for numerous oil prices: What is the maximum payout, (Z), you would provide for taking a bet that the oil price in the year 2000 would be greater than or equal to X given that losing the bet would incur a sure loss

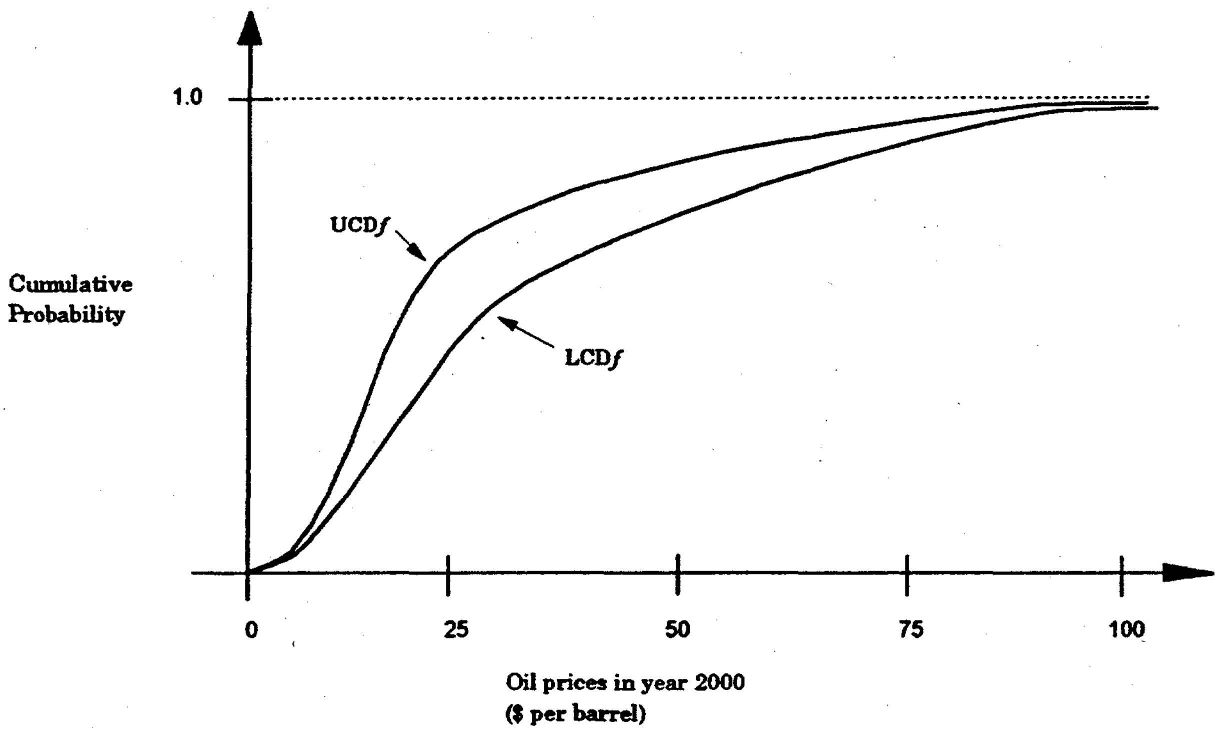

Fig. 3.1. Upper and lower cumulative distribution functions for the price of oil in the year 2000.

of \$1,000,000 (or some such amount)? An UCDF is shown in Figure 3.1, calculated using the formula $\overline{P}(X) = \$1,000,000/Z$. Formulas found in Section 3.5 can be used to calculate the expected values of the LCDF and UCDF.

We need to conclude the discussion of this example on a cautionary note. Examples 1-3 were purposely designed to illustrate different approaches to constructing upper and lower probability functions. In examples 1 and 3, it is assumed that experts could be interviewed to supply the probabilities. Likewise, in Example 2, it was assumed that an expert could be interviewed to supply compatibility judgments. The examples do not indicate, unfortunately, the very real problems associated with working with human experts to elicit/construct these types of estimates of judgments in a reliable and valid fashion.

3.6.4 Example 4. Choosing resource supply options

An important goal of IRP is choosing resource supply options to meet forecasted electricity demand. Lists of supply options typically include new power plants, repowering of existing plants, and demand-side management programs. The challenge is deciding which combination of options to choose to meet the forecasted requirements.

The most basic approach is to rank order the options according to some criterion or criteria and then select as many options as needed to meet the forecast requirements. For example, in the 1991 Northwest Conservation and Electric Power Plan, the options are rank ordered by levelized nominal cost and levelized real cost. In the 1991 Niagara Mohawk Integrated Electric Resource Plan, options are rank ordered according to benefit/cost ratios.

The goal of this example is to illustrate how upper and lower expected values can be used to choose resource supply options. Table 3.4 presents hypothetical data on thirteen resource options.[1] The Table includes, from left to right, lower expected benefits (in present value), upper expected benefits, lower expected costs, upper expected costs, lower expected resources supplied in MW, upper expected resources supplied, a benefit/cost ratio defined as columns A/D, and a benefit/cost ratio defined as columns B/C. The expected values are assumed to have been constructed using methods such as those discussed in examples 1-3.

[1] The options and data are conceptually based upon real analyses contained in the 1991 Niagara Mohawk Integrated Electric Resource Plan. We stress, however, that the values in Table 3.4 are purely hypothetical.

Table 3.4. Summary data on example resource options *

		A	B	C	D	E	F	G	H
	Option description	$\underline{E}(B)$	$\overline{E}(B)$	$\underline{E}(C)$	$\overline{E}(C)$	$\underline{E}(MW)$	$\overline{E}(MW)$	A/D	B/C
1.	New combined cycle unit	550	610	520	600	220	240	0.92	1.17
2.	New simple cycle gas turbine	12	15	18	20	20	25	0.60	0.83
3.	Combined cycle repowering using existing turbines at plant X	950	1000	1200	1350	900	940	0.70	0.83
4.	New combined cycle power plant	1200	1300	1000	1240	920	940	0.97	1.3
5.	Life extension of plant Y	20	35	200	300	320	430	0.07	0.18
6.	800MW phased IGCC at plant Z	2300	2400	1800	2260	790	810	1.02	1.33
7.	Life extension of plant A	100	140	200	300	150	200	0.33	0.70
8.	Two new 300MW pulverized coal units	1700	1940	1800	2110	630	650	0.81	1.08
9.	New 600MW IGCC	2000	2100	1800	1900	590	610	1.05	1.17
10.	New 25MW natural gas fuel cell	115	120	105	115	25	25	1.0	1.14
11.	C&I audit program	20	40	10	20	30	60	1.0	4.0
12.	Residential weatherization program	30	40	15	30	50	75	1.0	2.67
13.	Heat pump water heater program	6	8	6	8	5	10	0.75	1.33

*A - F are in millions of dollars

There are numerous ways to choose resource supply options from the thirteen presented in Table 3.4. Let's assume that the lower expected resource need is 3000 MW and that the upper expected resource need is 3500 MW. The most conservative approach would be to: (1) use the most risk averse benefit/cost ratio, which is the one in Column G, which is calculated by dividing the lower expected benefits by the upper expected costs; (2) rank order the options using this ratio; and (3) choosing options such that the sum of the lower expected resources supplied just exceeds the upper expected resource need. If this approach were chosen, the following options would be chosen —9, 6, 10, 11, 12, 4, 1, 8, 13, and 3. The lower expected resource supply would be 4,160 MW.

To balance this conservative approach, the most optimistic approach could also be explored, which would be to use the benefit/cost ratio in Column H and choose options to just exceed the lower expected resource need. If this approach were chosen, the following option would be chosen—11, 12, 13, 6, 4, 9, 1, 10, and 8. The upper expected resource supply would be 3420.

An interesting observation in this example is that the two approaches yield a nearly identical set of options. The only difference is that option 3 is left out of the optimistic approach. Basically, then, the utility is left with the decision about whether or not to additionally pursue option 3, or substitute option 3 with a smaller, but less financially attractive resource, such as option 2.

4. MANIPULATING UNCERTAIN QUANTITATIVE ESTIMATES

Section 3 addresses only one aspect of representing uncertainty about quantitative estimates; namely, the elicitation/construction of individual upper and lower probability functions. This section addresses what to do if one has two or more functions that could be usefully synthesized in some fashion to provide insights into a problem or question. As it happens, there are numerous quantitative methods available to synthesize uncertainty functions. To provide some guidance about what methods to use, Section 4.1 sets out a theoretical but practical framework within which to understand relationships between pieces of evidence to be brought to bear on a problem.

Four general methods for synthesizing evidence are encompassed within the framework: consensus, combination, updating conditionalization, and diagnostic conditionalization. Mathematical definitions for these methods and illustrations are presented in Sections 4.2 to 4.5 respectively.

4.1 SCHEMA FOR EVIDENTIAL REASONING

Evidential reasoning is defined here to represent the process of assembling and synthesizing pieces of evidence to be brought to bear on a problem. For our purposes, it is assumed that a piece of evidence will take the form of an imprecise probability function over an outcome set Ω. The schema for evidential reasoning presented in this section addresses different relationships between pieces of evidence and why different mathematical methods are needed to synthesize evidence given different relationships.

To begin this discussion, it is important to make the distinction between diagnosis and decisionmaking, because only the former is related to evidential reasoning. Diagnosis is concerned with ascertaining the state of the world, past, present, or future. Decisionmaking is concerned with shaping the future state of the world given relevant diagnosis.

The diagnosis—decisionmaking distinction is common in numerous areas of human endeavor. A physician renders a diagnosis about what malady affects a patient and then, based on the diagnosis, decides what treatment to administer. In our legal system, the first step is to determine the innocence or guilt of the defendant (a diagnosis). Based on the determination, a decision is made on the appropriate punishment. Even economists follow this model when their macroeconomic recommendations are based on whether the economy is determined to be in recession or not.

This model is very applicable to the IRP context. Basically, diagnosis pertains to the establishment of inputs for use in the resource option decisionmaking process. Some of these inputs represent the past (e.g., effectiveness of a DSM program) and some represent the present (e.g., current total plant capacity). What makes IRP particularly challenging is the preponderance of future diagnoses (e.g., electricity demand, oil prices, environmental regulations). Thus, IRP entails identifying the required inputs to the decisionmaking process, specifying the inputs quantatively, representing uncertainty about inputs using imprecise probabilities, and applying an appropriate decision heuristic.

Fig 4.1 is provided to help explain the diagnostic process and the associated schema for evidential reasoning. Let's focus on the top half of the figure and work our way from left to right.

Because the real world is so complex, humans tend to simplify things by reducing the real world into a collection of interrelated systems, such as the oil market, the utility service area, a power plant, and a transmission system. Typically, a diagnosis is related to the state of one such system, past, present, or future. Also, the systems themselves are still complicated, such that several descriptors of the system are needed to render a diagnosis about the system. Thus, as indicated in Figure 4.1, multiple data channels describing the system may need to be tapped to form multiple pieces of evidence about the system.

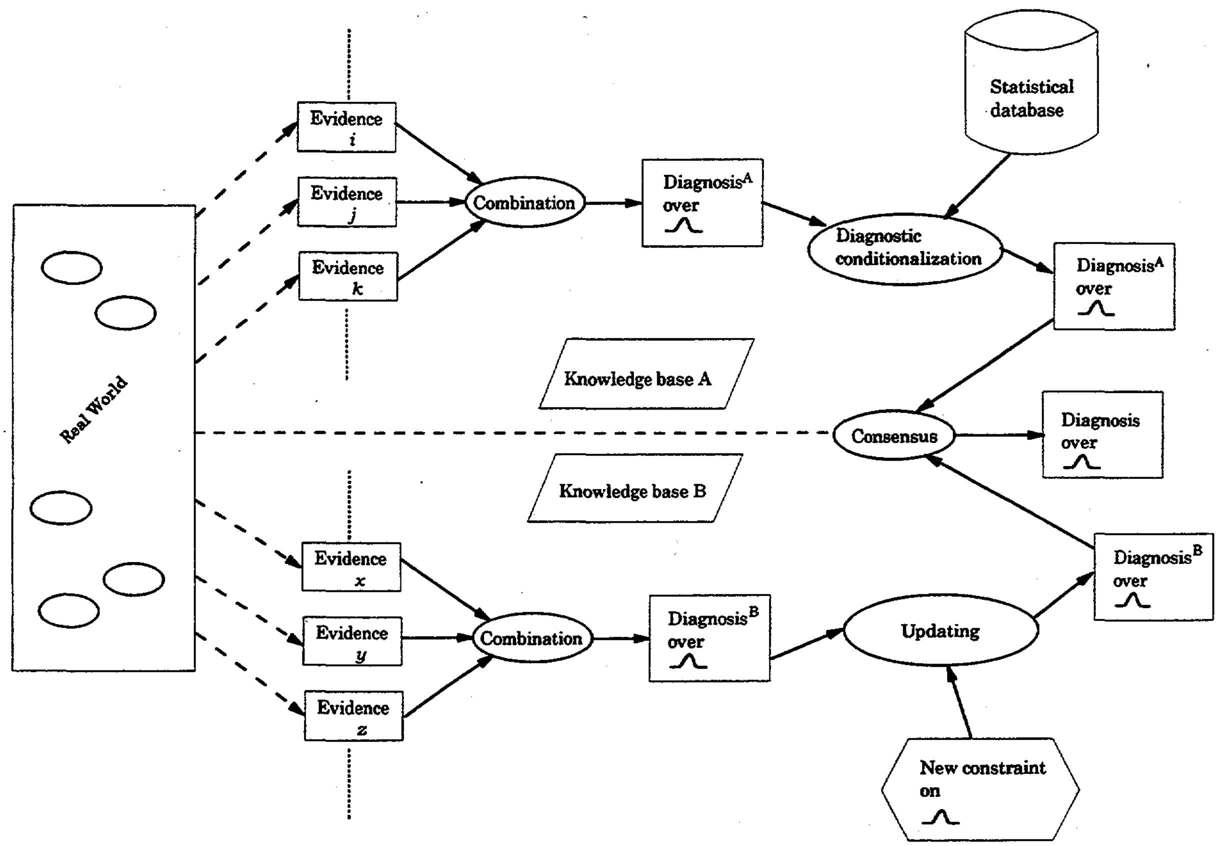

Fig. 4.1. Schema for evidential reasoning.

For example, assume that the utility service area can be considered as an electricity demand system and we want to diagnosis the area as increasing, decreasing, or remaining stable with respect to electricity demand. To determine the current status of the system, data could be collected from various parts of the system (e.g., by sector). To determine changes in the system, data should be collected over time. *A combination method* of some sort is needed to synthesize the data to render a diagnosis about the system [e.g., is it growing, shrinking; See Section (4.3.1) for an example of combination].

In our exposition, we assume that the current real world will first be consulted before rendering a diagnosis. Sometimes, though, a statistical database will exist that contains a history of past diagnosis. This database could be extremely valuable in situations where the evidence from the real world is deficient in some way. The general process of using a statistical database to improve a diagnosis based on current evidence is called *diagnostic conditionalization* [See Section (4.5.1) for an example]. After diagnostic conditionalization, a second diagnosis is rendered.

Notice that diagnosis A is linked to knowledge base A. This means that an identifiable base of expertise or methodology was used to: conceptualize the real world; build pieces of evidence; render a diagnosis; and generally manage the entire diagnostic process. *Consensus methods* are used to synthesize diagnosis rendered from different knowledge bases about the same outcome set Ω [See Section (4.2.1) for an example]. Thus, the right side of Figure 4.1 illustrates consensus between the diagnosis rendered by knowledge bases A and B, respectively, over Ω. One can think of separate knowledge bases as being different experts or models or paradigms, etc.

Sometimes information will become available which places a firm constraint on where the truth lies in Ω. In other words, it is determined that some member(s) of Ω cannot be true. Synthesizing this constraint into a current diagnosis is simply known as *updating* [See Section (4.4.1) for example]. Similar to diagnostic conditionalization, a new diagnosis, Ω^*, is rendered, after updating.

Thus, we have four methods for manipulating imprecise probability functions: consensus, combination, updating, and diagnostic conditionalization. Not all are necessarily used to solve all problems. They can be used in various combinations, according to an evidential reasoning design. The following four sections say more about each method, about what is known about how to actually implement each method, and present examples.

4.2 CONSENSUS

Consensus is probably the best known and researched of the four methods. We sometimes refer to it as a whole evidence scheme because each imprecise probability being synthesized represents an entire diagnosis about a state of the world. Each can stand on its own.

Suppose that N knowledge bases were used to appraise where the truth lies in Ω, resulting in N lower probability functions, $\underline{P}_1, \underline{P}_2 \ldots \underline{P}_N$, over Ω. An appropriate way to construct a single lower probability, $\underline{P}^*$, from $\underline{P}_1, \underline{P}_2, \ldots \underline{P}_N$ is to form a weighted arithmetic mean

$$\underline{P}(A) = \sum_{i}^{N} \omega_i \underline{P}_i(A) \quad , \qquad 4.1$$

where $A \subseteq \Omega$ for all subsets of Ω and ω_i are weights that are nonnegative and sum to one. Consensus methods for imprecise probabilities has been explored axiomatically by Wagner (1989).

4.2.1 Example 5: Consensus of economic growth

Suppose three eminent economists are polled concerning their predictions of economic growth in the utility's service area. Let $\Omega = \{$ high growth (hg), medium growth (mg), recession (r) $\}$. Let's also assume that the three experts provided lower probability functions over Ω that satisfy the conditions set forth in Section 3.3, as shown in Table 4.1. Using Table 4.1, assuming equal weights of .33, the consensus opinion of the three experts is found in the fourth row of the table. Table 3.7 can be used to calculate upper probabilities, if desired.

4.3 COMBINATION

Combination is probably the most used of the four methods, if one considers that people implicitly if not unconsciously use combination rules to synthesize pieces

Table 4.1. Consensus of lower probabilities *

	$\underline{P}(hg)$	$\underline{P}(mg)$	$\underline{P}(r)$	$\underline{P}(hg,mg)$	$\underline{P}(hg,r)$	$\underline{P}(mg,r)$	$\underline{P}(\Omega)$
Expert 1	0	0.2	0.6	0.2	0.6	0.8	1.0
Expert 2	0.4	0.4	0	1.0	0.4	0.4	1.0
Expert 3	0	0	0	0	0	0.5	1.0
Consensus	0.133	0.20	0.20	0.40	0.33	0.57	1.0

*hg-High growth; mg-Medium growth; r-Recession

of evidence in many endeavors, from the professions (e.g., medicine and law) to everyday life (e.g., What will traffic be like today?, What mood is the boss in?). Unfortunately, developing explicit methods to combine imprecise probabilities as proven illusive and represents a definite area for future research.

The term "combination" was coined by Shafer (1976). He uses the term in the similar manner as we do. Specifically, combination can be seen as a partial evidence scheme, where each piece of evidence has equal standing with respect to each other but each alone is only partially definitive. In a partial evidence scheme, one piece of evidence can support a diagnosis that another piece of evidence doesn't support. The challenge of a partial evidence scheme is to develop pieces of evidence which are comprehensive in scope, and have minimum overlap and repetition.

The key assumption about a combination rule, then, is that the pieces of evidence describe different aspects of the same phenomenon, as shown in Figure 4.1. In the mathematical and statistical sense, the pieces of evidence are not independent, because they flow from the same source, albeit in different ways. Unfortunately, Shafer's combination rule has proven controversial in application to dependent pieces of evidence. Indeed, we have found it to be inapplicable in cases when pieces of evidence are completely contradictory, and thus cannot recommend it for uses in IRP.

Thus, at the present time, we are left without a combination rule that we can recommend without hesitation. On the other hand, we have made some progress on a combination rule, which is summarized in Appendix A. This rule is used in the following example.

4.3.1 Example 6. The large industrial customer

The industrial demand for electricity in the utility service area is dominated by one very large customer, Acme Aluminum Company. This company has been a customer for over 50 years. Unfortunately, times have changed and the utility cannot count on Acme's business in the future. Through discussions with the company, it is now known that within the year, Acme will choose among these four options, $\Omega =$ shutdown the plant (sd), continue to buy power from your utility (sq)—for status quo, buying power from an adjacent utility (bp), build its own cogeneration facility (cg).

Because Acme is such an important customer, its decision will significantly affect IRP for the utility. The question is how to get a handle on what Acme may do. Over the past several weeks, four pieces of evidence have surfaced which provide clues concerning Acme's decision.

Evidence 1. (E_1). The chairman of Worldwide Aluminum International, the parent company of Acme Aluminum Company, has publically stated that one of its four North American plants will need to be shut down due to a general downturn in the demand for aluminum. Acme is the oldest of the four plants but enjoys transportation and labor cost advantages over its competitors.

Evidence 2. (E_2) The neighboring utility with whom Acme is said to be negotiating a power purchase contract has little excess capacity. It appears that it would have to purchase power from other utilities to meet its commitments. It is unlikely that Acme could get a better rate by switching suppliers.

Evidence 3. (E_3) The Acme site appears uncondusive for a large co-generation facility. The site has little free space and a lack of water. However, newer technologies might be able to overcome these constraints.

Evidence 4. (E_4) The utility has never interrupted power to Acme over the past 50 years. Negotiations over rates and scheduling have always gone smoothly. Acme has never complained about the service.

It is decided to use the pieces of evidence to construct a lower probability function over Ω. The first step is to create/construct/elicit lower probability functions for each piece of evidence, using methods such as those suggested in Section 3. Table 4.2 presents $\underline{P}$s for each piece of evidence that, hypothetically, could have resulted from such an exercise. Basically, E_1 supports {sd} and none of the other options. E_2 weakly supports {bp} but is seen to more strongly force the utility to favor any of the other options {sd, sq, cg}. E_3 is written such that {cg} cannot be totally dismissed. However, the poor site would lend support for {sd}, and some support for the utility purchase options {sq, bp}. E_4 heavily favors {sq} and also the two utility options together {sq, bp}.

Using the methodology presented in Appendix A, the combination of these four pieces of evidence indicates that the most likely options are shutdown and status quo and the least likely is buying power from the neighboring utility [See $\underline{P}^*$ and $\overline{P}^*$ in Table (4.2)].

4.4 UPDATING

Updating is very useful in situations where new evidence comes to light that provides insight on which members of Ω cannot be true for a particular diagnosis and where it is impractical or illogical to reformulate existing pieces of evidence to a constrained Ω. Because the new piece of evidence cannot stand on its own and

Table 4.2. $\underline{P}$ for the large industrial customer examples *

	$\underline{P}(E_1)$	$\underline{P}(E_2)$	$\underline{P}(E_3)$	$\underline{P}(E_4)$	$\underline{P}^*$	$\overline{P}^*$
sd	0.5	0	0.3	0	0.17	0.51
sq	0	0	0	0.5	0.13	0.68
bp	0	0.1	0	0.1	0.01	0.32
cg	0	0	0.3	0	0.11	0.39
sd sq	0.5	0	0.3	0.5	0.39	0.88
sd bp	0.5	0.1	0.3	0.1	0.19	0.70
sd cg	0.5	0	0.6	0	0.30	0.70
sq bp	0	0.1	0.3	0.8	0.30	0.70
sq cg	0	0	0.3	0.5	0.30	0.81
bp cg	0	0.1	0.3	0.1	0.12	0.61
sd sq bp	0.5	0.1	0.6	0.8	0.61	0.89
sd sq cg	0.5	0.7	0.6	0.5	0.68	0.99
sd bp cg	0.5	0.1	0.6	0.1	0.32	0.87
sq bp cg	0	0.1	0.6	0.8	0.49	0.83
Ω	1.0	1.0	1.0	1.0	1.0	1.0

*sd-shut down; sq-status quo; bp- buy power; cg-cogeneration

says nothing about the remaining options in Ω, we sometimes call the new evidence supplemental evidence.

Generally, suppose that $\underline{P}$ and $\overline{P}$ are a pair of lower and upper probability measures on Ω. The probability measures P compatible with $\underline{P}$ and $\overline{P}$ are, we recall, just those $P \in \mathcal{P}(\underline{P}, \overline{P}) = \{P : \underline{P}(A) \leq P(A) \leq \overline{P}(A)$ for all events A$\}$. Now if some probability measure P is our model of "how uncertainties lie"in Ω and we are apprised of additional information which renders it certain that "the truth lies in E," for some subset $E \subseteq \Omega$ with $P(E) > 0$, it is customary to revise P by "conditionalization"(i.e., updating) to a new probability measure $P(\cdot|E)$, where $P(A|E) = P(A \cap E)/P(E)$ for all events A.

If, instead, we have P delineated only by $\underline{P}$ and $\overline{P}$, and we discover that E is certain, we have the problem of updating $\underline{P}$ and $\overline{P}$ (assume $\underline{P}(E) > 0$). A natural way to do this would be by the formulas

$$\underline{P}(A|E) = min\{P(A|E): \quad P \in \mathcal{P}(\underline{P}, \overline{P})\} \qquad 4.2$$

and

$$\overline{P}(A|E) = max\{P(A|E): \quad P \in \mathcal{P}(\underline{P}, \overline{P})\}. \qquad 4.3$$

That is, $\underline{P}(\cdot|E)$ and $\overline{P}(\cdot|E)$ are just the lower and upper envelopes of the family of all conditionalized probability measures $P(\cdot|E)$ as P runs through the set of all probability measures compatible with $\underline{P}$ and $\overline{P}$. As one would expect, it is in many cases impossible to compute $\underline{P}(A|E)$ and $\overline{P}(A|E)$ exactly. The difficulty is the very one encountered with respect to $\underline{\mathcal{E}}(X)$ and $\overline{\mathcal{E}}(X)$ in Section 3.5. In fact, the situation here is completely analogous to that of Section 3.5, for here we can find conservative approximations to $\underline{P}(A|E)$ and $\overline{P}(A|E)$ that are exact when $\underline{P}$ is two-monotone.

The approximations are easily derived. Let $P \in \mathcal{P}(\underline{P}, \overline{P})$. Since $P(E) = P(A \cap E) + P(\overline{A} \cap E)$ for every event A, one has

$$\begin{aligned} P(A|E) = \frac{P(A \cap E)}{P(A \cap E) + P(\overline{A} \cap E)} &\geq \frac{\underline{P}(A \cap E)}{\underline{P}(A \cap E) + P(\overline{A} \cap E)} \\ &\geq \frac{\underline{P}(A \cap E)}{\underline{P}(A \cap E) + \overline{P}(\overline{A} \cap E)} \quad , \end{aligned} \qquad (4.4)$$

where the first inequality holds because for fixed $c > 0$, $x/x + c$ is an increasing function of x for $x > 0$, and the second inequality holds because $P(\overline{A} \cap E)$ is replaced by the value $\overline{P}(\overline{A} \cap E) \geq P(\overline{A} \cap E)$. Similarly, one can show that

$$P(A|E) \leq \frac{\overline{P}(A \cap E)}{\overline{P}(A \cap E) + \underline{P}(\overline{A} \cap E)}. \qquad (4.5)$$

From (4.2) - (4.5), it follows that

$$\frac{\underline{P}(A \cap E)}{\underline{P}(A \cap E) + \overline{P}(\overline{A} \cap E)} \leq \underline{P}(A|E) \leq \overline{P}(A|E) \leq \frac{\overline{P}(A \cap E)}{\overline{P}(A \cap E) + \underline{P}(\overline{A} \cap E)} \qquad (4.6)$$

Moreover, if $\underline{P}$ and $\overline{P}$ satisfy, in addition to the defining properties (3.4) - (3.6), the complementarity property (3.7), and if $\underline{P}$ is two-monotone (3.23), then the first and third inequalities in (4.6) are actually equalities. And in such a case, $\underline{P}(A|E) + \overline{P}(\overline{A}|E) = 1$, and $\underline{P}(\cdot|E)$ remains two-monotone (Sundberg and Wagner, 1992b).

4.4.1 Example 7. No shutdown of Acme Aluminum

Today the chairman of Worldwide Aluminum International announced that Acme Aluminum Company will remain open for business. However, nothing else new is known about Acme's decision on whether to continue doing business with the utility. It is decided to update $\underline{P}^*, \overline{P}^*$ [see Table (4.2) in Example 6] with the knowledge that SD is no longer in consideration.

To accomplish this, (4.6) is applied to $\underline{P}^*$ in Table 4.2. The results are in Table 4.3. As can be seen, $\underline{P}^*(sd)$ and $\overline{P}^*(sd)$ are now both 0.0. Also, $\underline{P}^*$ (sq, bp, cg) and $\underline{P}^*$(sq, bp, cg) both equal to 1.0, indicating that the truth will lie in the subset that contains the remaining options. Overall, the weight of evidence has shifted to the status quo and co-generation options, with the former carrying the most weight.

4.5 DIAGNOSTIC CONDITIONALIZATION

As mentioned in Section 4.1, diagnostic conditionalization relates to a situation where one has past statistical data that could be used to improve a diagnosis rendered using data directly taken from the system under study. Like updating, diagnostic conditionalization is a supplemental evidence design because, in almost all circumstances, the past statistical data could not stand alone to support a diagnosis. Similar to the combination rule case, methods for diagnostic conditionalization have not been developed for the most general cases. However, we can propose a formula to condition a special kind of imprecise probability, known as a belief function, with

Table 4.3. Updating after the no-shutdown decision *

A	$\underline{P}^*$	$\overline{P}^*$
sd	0	0
sq	0.18	0.85
bp	0.01	0.52
cg	0.14	0.57
sd sq	0.18	0.85
sd bp	0.01	0.52
sd cg	0.14	0.57
sq bp	0.43	0.86
sq cg	0.48	0.99
bp cg	0.15	0.82
sd sq bp	0.43	0.86
sd sq cg	0.48	0.99
sd bp cg	0.15	0.82
sq bp cg	1.0	1.0
Ω	1.0	1.0

*sd-shutdown; sq-status quo;
bp-buy power; cg-cogeneration

past statistical data that can be represented as a classical probability function over the frequency of outcomes in Ω.

To begin, in addition to meeting conditions set out in Equations (3.4, 3.5, 3.7 - 3.9, and 3.23), a lower probability that is also a belief function must meet the additional condition:

$$\underline{P}(A_1 \cup \ldots \cup A_r) \geq \sum_{\substack{I \subseteq (1,\ldots r) \\ I \neq \emptyset}} (-1)^{|I|-1} \underline{P}(\bigcap_{i \in I} A_i) \text{ for all } r \geq 2. \qquad (4.7)$$

This condition is set out and explained in detail by Shafer (1976). The condition is often referred to as r-monotonicity, which is obviously more constrained than monotonicity, (3.5) and 2-monotonicity (3.23), respectively.

As is often required in mathematical reasoning, it is often convenient to transform a formula to find representations that are easier to work with. For example, a transformation was used to explain the derivation of Choquet expected values in Section 3.5. With respect to belief functions, it is useful to transform them using the Möbius Transform:

$$M(A) = \sum_{E \subseteq A} (-1)^{|A-E|} \underline{P}(E). \tag{4.8}$$

Shafer (1976) refers to the M(A) values as basic probability assignments, where $M(\emptyset) = 0$, and $\sum M(A) = 1.0$ for all $A \leq \Omega$. For completeness, a belief function can be induced from a basic probability assignment by:

$$\underline{P}(A) = \sum_{E \subseteq A} M(E). \tag{4.9}$$

Assume a lower probability function, $\underline{P}$, meeting the conditions for representing a belief function has been developed over Ω, and has been transformed to $M(E)$ using (4.8). Also assume, a statistical database is available from which a probability function, P^H, over Ω can be established. Then for each member, A, in $\Omega, \underline{P}$ can be conditioned by P^H by the following:

$$P(A) = \sum_{E \subseteq \Omega} M(E) P^H(A/E). \tag{4.10}$$

(4.10) has several interesting properties. First, it yields a classical probability function over Ω, not an imprecise probability. Second, the formula reduces to the famous formula known as Jeffrey Conditionalization, due to Jeffrey (1983), when the subsets of Ω that have positive M values are pairwise disjoint. Wagner and Tonn (1990) and Sundberg and Wagner (1992b) explore (4.10) in additional depth. As mentioned above, (4.10) needs to be generalized to handle a broader range of imprecise probability functions, specifically imprecise past statistical probabilities, and imprecise probabilities as outcomes.

4.5.1 Example 8. The Hazardous Waste Site

Suppose that the utility is involved in some fashion with an abandoned hazardous waste site (e.g., as a potentially responsible party [PRP] under Superfund). To develop rough cost estimates for remediating the site, the amount and proportion of hazardous wastes at the site must be known. From fragmentary records, it can be determined that the dump contains three types of hazardous wastes, $\Omega = \{T_1, T_2, T_3\}$. It is also known that the site received at least three kinds of shipments, S_1, S_2, and S_3, that accounted for 10%, 20%, and 50% of the mass of hazardous waste at the site. It is known that S_1 and S_2 contained no hazardous wastes of type T_1, and S_3 contains no wastes of type T_3. Nothing is known about the remaining 20% of the shipments, referred to as S_4.

This information can be used to create a lower probability function, $\underline{P}$, over the proportion of mass of each type of hazardous waste at the site (see the first column in Table 4.4). Using (4.8), the Möbius Transform of $\underline{P}$ can be calculated (see the second column in Table 4.4).

Suppose a database maintained by the Regional Planning Agency indicates that factories in the area are known to have produced wastes in Ω, according to the following proportion, $P^H(T_1) = 0.4, P^H(T_2) = 0.3$, and $P^H(T_3) = 0.3$. Using (4.10), we can condition $M(\Omega)$ by $P^H(\Omega)$ to yield $P(\Omega)$ (see the fourth column of Table 4.4). For example, $P(T_1)$ is calculated as follows:

$$\begin{aligned} P(T_1) = M(T_1T_2)&\frac{P^H(T_1)}{P^H(T_1)+P^H(T_2)} + M(T_1,T_2,T_3) \\ &\frac{P^H(T_1)}{P^H(T_1)+P^H(T_2)+P^H(T_3)} = 0.5 \\ &\frac{0.4}{0.4+0.3} + 0.2\frac{0.4}{0.4+0.3+0.3} = .37 \end{aligned} \tag{4.11}$$

Table 4.4. Hazardous waste dump *

	$\underline{P}(\Omega)$	$M(\Omega)$	$P^H(\Omega)$	$P(\Omega)$
T_1	0	0	0.4	0.37
T_2	0	0	0.3	0.42
T_3	0	0	0.3	0.21
T_1, T_2	0.5	0.5		
T_1, T_3	0	0		
T_2, T_3	0.3	0.3		
Ω	1.	0.2		

4.6 SUMMARY REMARKS ON EVIDENTIAL REASONING

This section introduces a schema for evidential reasoning that encompasses four methods for synthesizing imprecise probabilities: consensus, combination, updating, and diagnostic conditionalization. In some cases, such as consensus, the methods have been well explored. In others, the methods need additional development (e.g., combination). In general, we hope this section serves well the purpose of presenting a paradigm of handling uncertainty that relies on constructing imprecise probabilities over important outcome sets given what is known about the problem at hand.

In some sense, Figure 4.1 and the examples present an idealization of what it takes to develop "designs" to synthesize pieces of evidence. In practice, analysts will face numerous situations where the information at hand does not conform to our schema or exactly track the examples. We can think of several examples where this might be true. For instance, the problem of synthesizing two imprecise probabilities specified over two different outcome sets, Ω and Ω^*, to create one imprecise probability over the cartesian product of Ω and Ω^* is not addressed. It turns out that this is a difficult problem even within the classical probability framework and future reasoning should address the more general imprecise probability case.

We have also not explicitly addressed issues surrounding the use of Bayes Theorem, for updating prior uncertainties given new but not conclusive evidence, which is often in the form of new statistical evidence. This approach to evidential reasoning has application to IRP but its proper use, in our minds, is still under consideration. Also, generalizing Bayesion updating using classical probabilities to use imprecise probabilities is yet another open research question. Walley (1991) does address this problem and his treatment deserves consideration in future research.

In summary, though, the ideas and methods contained in this section are more than sufficient to assist utilities in using imprecise probabilities to solve real world problems. Future research should focus on improving and extending the tool kit of methods available to utilities. The next section addresses yet another aspect of handling uncertainty, that of decision trees.

5. IMPRECISE PROBABILITIES AND DECISION TREES

This section addresses the topic of using imprecise probabilities to accumulate expected values at the end points of decision trees. We are using the term "decision trees" in a general sense to represent relationships between random variables and/or decisions that can be represented as a directed graph composed of one root node, and an arbitrary number of variables and end nodes. Figure 5.1 presents the decision tree used in Example 9, discussed below, for consideration.

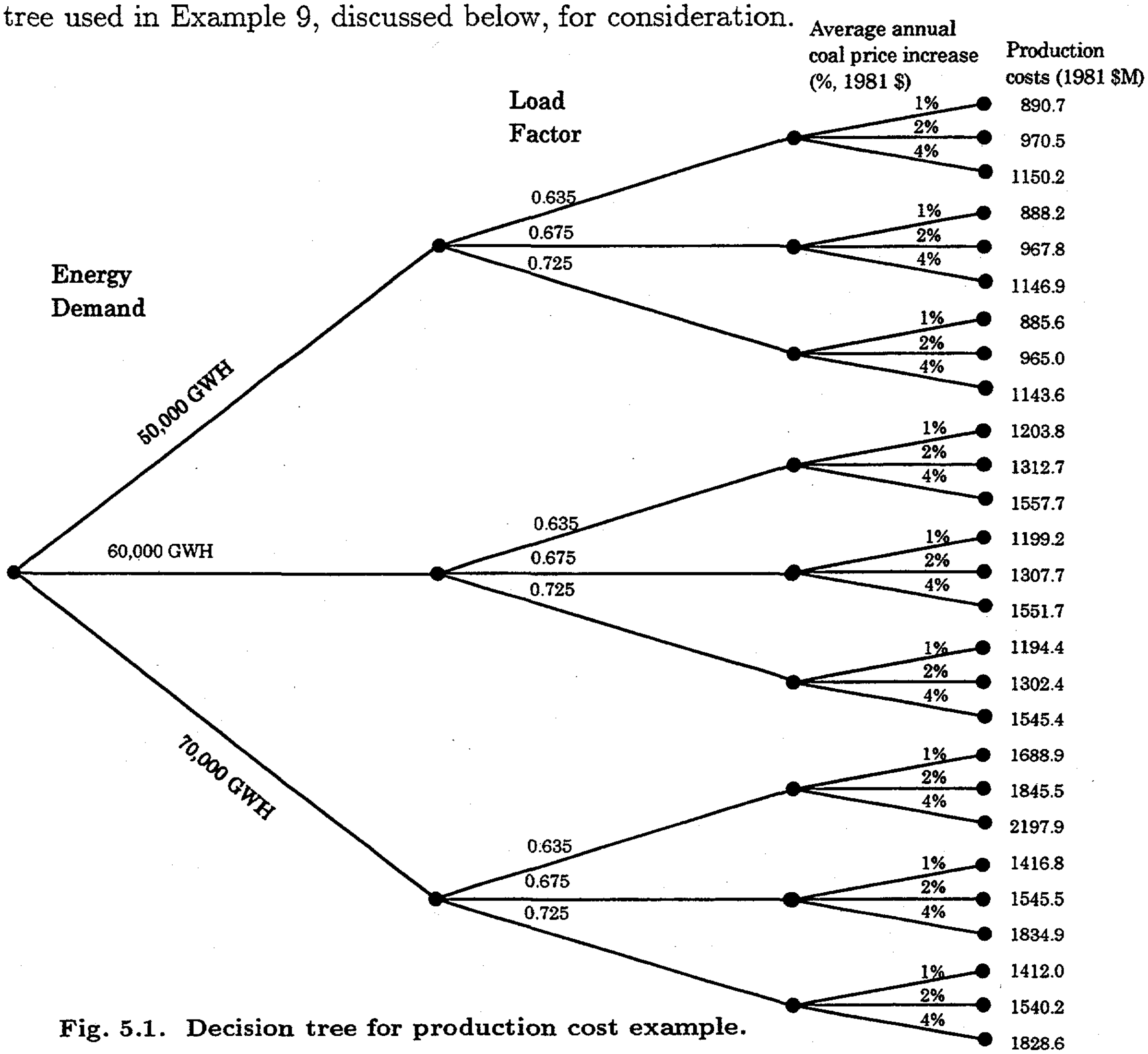

Fig. 5.1. Decision tree for production cost example.

The challenge arising from decision trees is how to handle probabilistic relationships among the variables in a decision tree. This is an important problem to address because decision trees have many applications in IRP. For instance, Example 9 considers the case where three random variables—energy demand, load factor, and coal price increases—are input into a costing model. The goal is to calculate expected values for the costs at the 27 end nodes in Figure 5.1.

Numerous resource problems can be "designed" as decision trees. For example, deciding whether to run, repower, or shut down a power plant can be constructed within the decision tree methodology. Additional examples include: strategic decisionmaking, DSM program implementation, and environmental compliance.

Breaking from the pattern established in Sections 3 and 4, in this section the discussion of the general methodology proposed to propagate imprecise probability through a decision tree is intertwined with the example. This approach was chosen because the example is based on previous research by Thorp, McClure, and Fine (1982) that uses imprecise probabilities in a utility context. In this study, imprecise probabilities were used to calculate expected values of production costs for a utility. To contrast our method with theirs, their method and results are presented first, followed by our method and results. The entire discussion is made possible because the authors of the first study kindly made available their original data.

As a final word, the method proposed herein, [see (5.2)], is based on nonlinear optimization of an expected value calculation. Basically, the goal is to search for those combinations of upper and lower probabilities within upper and lower bounds on the random variables that produce maximum and minimum expected values of the outcome variable under consideration. The approach presented below to accomplish this can be considered a novel result of this research project.

5.0.1 Example 9. FORECASTING PRODUCTION COSTS FOR AN ELECTRIC UTILITY

In a study of Thorp, McClure, and Fine (1982), which is the basis for Example 9, the 1990 production cost, C, of an actual, but unidentified, electric utility depends on the values of the uncertain quantities (i.e., random variables) E = energy demand (GWH), F = load factor, and R = average year to year coal price increase from 1981 to 1990 (%). Given specific values $E = e$, $F = f$, and $R = r$ of these random variables, a standard production costing algorithm can be used to calculate the corresponding production cost $C = C(e, f, r)$.

In this study, the possible values of E, F, and R are given, respectively, by the sets $\Omega_E = \{$ 50,000 GWH, 60,000 GWH, 70,000 GWH$\}$, $\Omega_F = \{0.635, 0.675, 0.725\}$ and $\Omega_R = \{1\%, 2\%, 4\%\}$. To complete the construction of the conceptual apparatus of Section 3.5, the 27-element set Ω is defined by

$$\Omega = \Omega_E \times \Omega_F \times \Omega_R := \{(e, f, r): \quad e \in \Omega_E, \ f \in \Omega_F, \quad \text{and } r \in \Omega_R\}. \qquad (5.1)$$

The 1990 production cost is then a random variable $C: \quad \Omega \to [0, \infty)$.

Figure 5.1 presents a decision tree which graphically depicts this information. The cost values at the end-nodes were produced by a costing algorithm using values of the three random variables as inputs. To facilitate the calculation of Choquet expected values, the values of C are calculated for each of the 27 triples $(e, f, r) \in \Omega$ and arranged in increasing order, $c_1 < c_2 < \ldots < c_{27}$. Then, for each $i = 1, \ldots, 27$, the triples comprising each of the 27 events

$$A_i := \quad \text{“}C \geq c_i\text{”} = \{(e, f, g) \in \Omega: \quad C(e, f, g) \geq c_i\} \qquad (5.2)$$

are identified.

At this point, upper and lower probabilities $\underline{P}(A_i)$ and $\overline{P}(A_i)$ are assessed for $A_1, \ldots, A_{27}$ (never mind how, for the moment; but note that

$\Omega = A_1 \supseteq A_2 \supseteq \ldots \supseteq A_{27}$, so that $1 = \underline{P}(A_1) \geq \underline{P}(A_2) \geq \ldots \geq \underline{P}(A_{27})$ and $1 = \overline{P}(A_1) \geq \overline{P}(A_2) \geq \ldots \geq \overline{P}(A_{27})$). Then the Choquet expected values of C with respect to $\underline{P}$ and $\overline{P}$ are calculated by formulas (3.18) and (3.19), yielding in this case

$$\begin{aligned} \mathcal{E}_{\underline{P}}(C) &= c_1 + \sum_{i=2}^{27}(c_i - c_{i-1})\underline{P}(A_i) \\ &= \$1.159\ B, \end{aligned} \tag{5.3}$$

and

$$\begin{aligned} \mathcal{E}_{\overline{P}}(C) &= c_1 + \sum_{i=2}^{27}(c_i - c_{i-1})\overline{P}(A_i) \\ &= \$1.438\ B. \end{aligned} \tag{5.4}$$

It follows from (3.22) that

$$\$1.159\ B \ \leq \underline{\mathcal{E}}(C) \leq \overline{\mathcal{E}}(C) \leq \$1.438\ B, \tag{5.5}$$

where the crucial quantities of interest, $\underline{\mathcal{E}}(C)$ and $\overline{\mathcal{E}}(C)$, are defined by

$$\underline{\mathcal{E}}(C) = min\{\mathcal{E}_P(C) : \quad P \in \mathcal{P}(\underline{P}, \overline{P})\} \tag{5.6}$$

and

$$\overline{\mathcal{E}}(C) = max\{\mathcal{E}_P(C) : \quad P \in \mathcal{P}(\underline{P}, \overline{P})\}. \tag{5.7}$$

As a consequence of the way in which $\underline{P}$ and $\overline{P}$ are assessed in this particular problem, one can actually place upper bounds on the approximation errors $\underline{\mathcal{E}}(C) -$ \$1.159 B and \$1.438 $B - \overline{\mathcal{E}}(C)$. We shall discuss this issue shortly. At this juncture, however, we need to consider a more basic question: *If one is interested in the numbers $\underline{\mathcal{E}}(C)$ and $\overline{\mathcal{E}}(C)$, why not calculate them, instead of approximating them by $\mathcal{E}_{\underline{P}}(C)$ and $\mathcal{E}_{\overline{P}}(C)$?* After all, (5.6) and (5.7) amount to optimizing a linear function with merely 27 variables over the closed, convex polyhedral set $\mathcal{P}(\underline{P}, \overline{P})$.

The problem, however, is that $\mathcal{P}(\underline{P}, \overline{P})$ is defined by the $2^{27} - 2$ pairs of inequalities $\underline{P}(A) \leq P(A) \leq \overline{P}(A)$, one pair for each nonempty subset A of the 27-element set Ω! Assessing all of the quantities $\underline{P}(A)$ and $\overline{P}(A)$ involved in these bounds is out of the question.

In order to set the stage for the discussion in the next section, and also because it is of interest in its own right, we now describe the method by which Thorp, McClure, and Fine assessed the required values of $\underline{P}$ and $\overline{P}$. What they did was to convene a panel of six experts from the planning department of the company. Each of the experts assessed probabilities over the separate sets $\Omega_E = \{50,000;\ 60,000;\ 70,000\}$, $\Omega_F = \{0.635,\ 0.675,\ 0.725\}$, and $\Omega_R = \{1, 2, 4\}$. Assuming independence, probabilities were multiplied to yield probability measures $P_1, \ldots, P_6$ on $\Omega = \Omega_E \times \Omega_F \times \Omega_R$.

The lower and upper probability measures on Ω were then constructed as the so-called *lower and upper envelopes of the family* $\{P_1, \ldots, P_6\}$, namely

$$\underline{P}(A) = min\{P_1(A), \ldots, P_6(A)\}, \quad \text{and}$$

$$\overline{P}(A) = max\{P_1(A), \ldots P_6(A)\}. \tag{5.8}$$

for each $A \subseteq \Omega$. Note that while (5.8) prescribes a method for calculating $\underline{P}(A)$ and $\overline{P}(A)$ for any of the 2^{27} subsets $A \subseteq \Omega$, this method is only *implemented* for $A = A_1, \ldots, A_{27}$. One can show that the upper and lower envelopes of any set of probability measures always comprise a pair of lower and upper probability measures [i.e., satisfy (3.4)-(3.7)] that satisfy the additional desirable properties (3.7)-(3.9).

But now that the prescription (5.8) is displayed, another question arises: *Would one not get adequate bounds on the expected production cost by simply calculating*

$$m_C := min\{\mathcal{E}_{P_1}(C), \ldots, \mathcal{E}_{P_6}(C)\}, \quad \text{and}$$

$$M_C := max\{\mathcal{E}_{P_1}(C), \ldots, \mathcal{E}_{P_6}(C)\}\ . \tag{5.9}$$

Thorp, McClure, and Fine argue that the interval $[m_C, M_C]$ is unjustifiably narrow, even though it includes not only the values $\mathcal{E}_{P_1}(C), \ldots, \mathcal{E}_{P_6}(C)$, but the infinitely many values $\mathcal{E}_P(C)$, where P belongs to the *convex hull of* $P_1, \ldots P_6$, denoted $\mathcal{H}(P_1, \ldots, P_6)$, and defined by

$$\mathcal{H}(P_1, \ldots, P_6) = \{P : \quad P = \lambda_1 P_1 + \ldots + \lambda_6 P_6, \text{ where} \lambda_1, \ldots, \lambda_6 \text{ is a sequence of nonnegative real numbers summing to one}\}. \tag{5.10}$$

Their reason is that, given the opinions $P_1, \ldots, P_6$, the set of probability measures P that are "compatible" with these opinions extends beyond $\mathcal{H}(P_1, \ldots, P_6)$, the family of weighted arithmetic averages of the six opinions, to the larger family $\mathcal{P}(\underline{P}, \overline{P})$ of all probability measures between the upper and lower envelopes $\underline{P}$ and $\overline{P}$ of the six opinions. We have, in short,

$$\mathcal{E}_{\underline{P}}(C) \leq \underline{\mathcal{E}}(C) \leq m_C \leq M_C \leq \overline{\mathcal{E}}(C) \leq \mathcal{E}_{\overline{P}}(C), \tag{5.11}$$

with strict inequalities the usual state of affairs.

Note that the quantities m_C and M_C do have one important use. In virtue of (5.11), the numbers $m_C - \mathcal{E}_{\underline{P}}(C)$ and $\mathcal{E}_{\overline{P}}(C) - M_C$ provide upper bounds on the respective errors $\underline{\mathcal{E}}(C) - \mathcal{E}_{\underline{P}}(C)$ and $\mathcal{E}_{\overline{P}}(C) - \overline{\mathcal{E}}(C)$ that one makes by employing $\mathcal{E}_{\underline{P}}(C)$ as a conservative estimate for $\underline{\mathcal{E}}(C)$ and $\mathcal{E}_{\overline{P}}(C)$ as a conservative estimate for $\overline{\mathcal{E}}(C)$. In the study of Thorp, McClure, and Fine, $m_C = \$1.164\ B$ and $M_C = \$1.420$, and so it must be the case that $\mathcal{E}_{\underline{P}}(C) = \$1.159\ B$ errs as an approximation to $\underline{\mathcal{E}}(C)$ by at most $\$5M$ and $\mathcal{E}_{\overline{P}}(C) = \$1.438\ B$ errs an approximation to $\underline{\mathcal{E}}(C)$ by at most $\$18M$.

In the next section we explore an entirely different approach to bounding expected production cost, based on nonlinear optimization.

5.1 BOUNDING EXPECTED COST BY NONLINEAR OPTIMIZATION

Let $\underline{P}$ and $\overline{P}$ be a pair of lower and upper probability measures on Ω, and let X be a random variable on Ω. If it is clear, as it sometimes is, that the probability measures P compatible with the evidence are precisely those $P \in \mathcal{P}(\underline{P}, \overline{P})$, then the numbers $\underline{\mathcal{E}}(X) = \min\{\mathcal{E}_P(X) \;:\; P \in \mathcal{P}(\underline{P}, \overline{P})\}$ and $\overline{\mathcal{E}}(X) = \max\{CE_P(X) \;:\; P \in \mathcal{P}(\underline{P}, \overline{P})\}$ are obviously the appropriate bounds on the expected value of X. As noted in the preceding two sections, we often need to content ourselves with estimates of $\underline{\mathcal{E}}(X)$ and $\overline{\mathcal{E}}(X)$.

The set of probability measures $\mathcal{P}(\underline{P}, \overline{P})$ plays a crucial role in the above approach. If this set somehow failed to capture all of the probability measures on Ω compatible with the evidence, then no interest would attach to the numbers $\underline{\mathcal{E}}(X)$ and $\overline{\mathcal{E}}(X)$. In what follows, we argue that the set $\mathcal{P}(\underline{P}, \overline{P})$ employed by Thorp, McClure, and Fine in the study described in the previous section fails to capture the set of probability measures compatible with the evidence.

We would have no quarrel with their approach if the experts had each directly assessed probabilities over the 27-element set Ω. But, in fact, the experts were not called upon to assess probabilities in this way. Instead, judging that the variables E, F, and R were independent, Thorp, McClure, and Fine presented each expert with three assessment problems, one for each of the sets $\Omega_E = \{50{,}000;\ 60{,}000;\ 70{,}000\}$, $\Omega_F = \{0.635,\ 0.675,\ 0.725\}$, and $\Omega_R = \{1, 2, 4\}$. They multiplied the appropriate probabilities provided by each expert to construct probability measures $P_1, \ldots, P_6$ on $\Omega = \Omega_E \times \Omega_F \times \Omega_R$, and then constructed $\underline{P}$ and $\overline{P}$ as the lower and upper envelopes of the family $\{P_1, \ldots, P_6\}$.

The crucial question is whether the probability measures P on Ω compatible with all of the evidence are precisely those $P \in P(\underline{P}, \overline{P})$. *But the evidence in this case is manifested not only in the probabilistic assessments of the six experts for* *the three variables E, F, and R, but also in the judgment that these variables are independent.* By their very construction, the probabilities $P_1, \ldots, P_6$ incorporate that judgement.[1] But the vast majority of probability measures $P \in \mathcal{P}(\underline{P}, \overline{P})$ violate that judgement. Moreover, we would argue that there are probability measures P on Ω, compatible with all relevant evidence, and lying outside $\mathcal{P}(\underline{P}, \overline{P})$.[2] So $\mathcal{P}(\underline{P}, \overline{P})$ is in some ways insufficiently restrictive, and in other ways unduly restrictive.

In what follows we describe what we regard as the correct approach to delineating the set of probability measures on Ω compatible with all of the relevant evidence. The following three tables (5.1)-(5.3) record the experts' probability assessments for the variables E, F, and R (first three columns of each table), expanded by us in the obvious way to record the probabilities of all nonempty, proper events, and with column minima and maxima designated:

Table 5.1. Energy Demand (E)

	$\{50K\}$	$\{60K\}$	$\{70K\}$	$\{50K, 60K\}$	$\{50K, 70K\}$	$\{60K, 70K\}$
1	0.05	0.6	0.35	0.65	0.4	0.95
2	0.625**	0.25*	0.125	0.875	0.75 **	0.375**
3	0 *	0.6	0.4**	0.6 *	0.4	1 *
4	0.6	0.4	0 *	1 **	0.6	0.4
5	0.3	0.55	0.15	0.85	0.45	0.7
6	0.02	0.88**	0.1	0.9	0.12 *	0.98

* = column minimum; ** = column maximum

The probability measures on Ω_E, Ω_F, and Ω_R compatible with the evidence, as manifested in Tables (5.1)-(5.3), are easy to delineate. Let us consider, for example, the case $\Omega_E = \{50\text{K}, 60\text{K}, 70\text{K}\}$, writing $50\text{K} = e_1$, $60\text{K} = e_2$, and $70\text{K} = e_3$ for short. Also, let us denote a typical probability measure on Ω_E by ϵ, and write $\epsilon(e_1) = \epsilon_1$, $\epsilon(e_2) = \epsilon_2$, and $\epsilon(e_3) = \epsilon_3$ for short. It seems obvious that the

Table 5.2. Load Factor (F)

	{.635}	{.675}	{.725}	{.635, .675}	{.635, .725}	{.675, .725}
1	0.4	0.55	0.05 *	0.95**	0.45	0.6 *
2	0.25	0.375*	0.375**	0.625*	0.625**	0.75
3	0.1	0.55	0.35	0.65	0.45	0.9
4	0.4**	0.5	0.1	0.9	0.5	0.6
5	0.25	0.5	0.25	0.75	0.5	0.75
6	0.05*	0.8 **	0.15	0.85	0.2 *	0.95**

* = column minimum; ** = column maximum

Table 5.3. Coal Price Increase (R)

	{1}	{2}	{4}	{1, 2}	{1, 4}	{2, 4}
1	0.2	0.7	0.1	0.9	0.3	0.8
2	0.125	0.5*	0.375**	0.625*	0.5**	0.875
3	0.2	0.75**	0.05*	0.95**	0.25*	0.8
4	0.1 *	0.6	0.3	1.7	0.4	0.9**
5	0.15	0.6	0.25	0.75	0.4	0.85
6	0.25**	0.6	0.15	0.85	0.4	0.75*

* = column minimum; ** = column maximum

probability measures on Ω_E compatible with the evidence manifested in Table 5.1 are precisely those ϵ satisfying

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 1$$

$$0 \leq \epsilon_1 \leq 0.625$$

$$0.25 \leq \epsilon_2 \leq 0.88$$

$$0 \leq \epsilon_3 \leq 0.4$$

$$0.6 \leq \epsilon_1 + \epsilon_2 \leq 1$$

$$0.12 \leq \epsilon_1 + \epsilon_3 \leq 0.75$$

$$0.375 \leq \epsilon_2 + \epsilon_3 \leq 1 \tag{5.12}$$

In fact, (5.12) may be considerably simplified, for the lower and upper bounds stipulated there are values of the lower envelope and upper envelope of the experts' probability measures on Ω_E. And it may easily be proved that the values of lower envelopes may always be deduced from values of upper envelopes in (5.12), (e.g., $\epsilon_3 \leq 0.4$ and $\epsilon_1 + \epsilon_2 + \epsilon_3 = 1$ imply that $0.6 \leq \epsilon_1 + \epsilon_2$, etc.). This means that the left-hand inequalities in (5.12) are all redundant. Hence the probability measures on Ω_E compatible with the evidence manifested in Table 5.1 are precisely those ϵ satisfying

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 1$$

$$0 \leq \epsilon_1 \leq 0.625$$

$$0 \leq \epsilon_2 \leq 0.88$$

$$0 \leq \epsilon_3 \leq 0.4$$

$$0 \leq \epsilon_1 + \epsilon_2 \leq 1$$

$$0 \leq \epsilon_1 + \epsilon_3 \leq 0.75$$

$$0 \leq \epsilon_2 + \epsilon_3 \leq 1 \tag{5.13}$$

Similar considerations with respect to $\Omega_F = \{0.135,\ 0.675,\ 0.725\} = \{f_1, f_2, f_3\}$ dictate that the probability measures φ on Ω_F compatible with the evidence manifested in Table 5.2 are precisely those satisfying

$$\varphi_1 + \varphi_2 + \varphi_3 = 1$$

$$0 \leq \varphi_1 \leq 0.4$$

$$0 \leq \varphi_2 \leq 0.8$$

$$0 \leq \varphi_3 \leq 0.375$$

$$0 \leq \varphi_1 + \varphi_2 \leq 0.95$$

$$0 \leq \varphi_1 + \varphi_3 \leq 0.625$$

$$0 \leq \varphi_2 + \varphi_3 \leq 0.95 \tag{5.14}$$

where $\varphi(f_i) = \varphi_i$, $i = 1, 2, 3$.

Finally, the probability measures ρ on $\Omega_R = \{1,2,4\} = \{r_1, r_2, r_3\}$ compatible with the evidence manifested in Table 5.3 are precisely those satisfying

$$\begin{aligned} &\rho_1 + \rho_2 + \rho_3 = 1 \\ &0 \leq \rho_1 \leq 0.25 \\ &0 \leq \rho_2 \leq 0.75 \\ &0 \leq \rho_3 \leq 0.375 \\ &0 \leq \rho_1 + \rho_2 \leq 0.95 \\ &0 \leq \rho_1 + \rho_3 \leq 0.5 \\ &0 \leq \rho_2 + \rho_3 \leq 0.9, \end{aligned} \tag{5.15}$$

where $\rho(r_i) = \rho_i,\ i = 1,2,3.$

What then are the probability measures π on $\Omega = \Omega_E \times \Omega_F \times \Omega_R$ compatible with the evidence manifested in Tables (5.1)-(5.3) and in the judgement that E, F, and R are independent? They are precisely those π constructed by choosing numbers $\epsilon_1,\ \epsilon_2, \epsilon_3$ satisfying (5.13), $\varphi_1,\ \varphi_2,\ \varphi_3$ satisfying (5.14), and $\rho_1,\ \rho_2,\ \rho_3$ satisfying (5.15) and defining

$$\pi(e_i,\ f_j,\ r_k) = \epsilon_i \varphi_j \rho_k \tag{5.16}$$

Now with π specified by (5.16), the expected value of C with respect to π, $\mathcal{E}_\pi(c)$ is given by the standard formula

$$\mathcal{E}_\pi(C) = \sum_{1 \leq i,j,k \leq 3} C(e_i,\ f_j,\ r_k)\ \epsilon_i \varphi_j \rho_k. \tag{5.17}$$

From the standpoint of the foregoing analysis, the appropriate lower and upper bounds on expected cost are given by $\mathcal{E}_*(C)$ and $\mathcal{E}^*(C)$, the respective solutions to the following nonlinear optimization problems:

$$\begin{aligned} \mathcal{E}_*(C) &= \text{MIN} \sum_{1 \leq i,j,k \leq 3} C(e_i, f_j, r_k)\ \epsilon_i \varphi_j \rho_k \\ \mathcal{E}^*(C) &= \text{MAX} \sum_{1 \leq i,j,k \leq 3} C(e_i, f_j, r_k)\ \epsilon_i \varphi_j \rho_k, \end{aligned} \tag{5.18}$$

subject to the constraints (5.13), (5.14), and (5.15), and with the coefficients $C(e_i, f_j, r_k)$, computed by a standard production costing algorithm, given the information in Table (5.1).

The solutions to (5.18) are:

$$\mathcal{E}_*(C) = \$1.081\ B, \qquad \text{attained when}$$

$$\begin{array}{lll} \epsilon_1 = 0.625 & \epsilon_2 = 0.375 & \epsilon_3 = 0.000 \\ \varphi_1 = 0.050 & \varphi_2 = 0.575 & \varphi_3 = 0.375 \\ \rho_1 = 0.250 & \rho_2 = 0.700 & \rho_3 = 0.050 \end{array} \tag{5.19}$$

and

$$\mathcal{E}^*(C) = \$1.542\ B, \qquad \text{attained when}$$

$$\begin{array}{lll} \epsilon_1 = 0.000 & \epsilon_2 = 0.600 & \epsilon_3 = 0.400 \\ \varphi_1 = 0.400 & \varphi_2 = 0.550 & \varphi_3 = 0.050 \\ \rho_1 = 0.100 & \rho_2 = 0.525 & \rho_3 = 0.375 \end{array} \tag{5.20}$$

Note that these bounds on expected production cost comprise a wider interval than $[\mathcal{E}_{\underline{P}}(C), \mathcal{E}_{\overline{P}}(C)] = [\$1.159B, \$1.438B]$, computed by Thorp, McClure and Fine using Choquet expectation.

To see why this happens, consider first (5.19), and, in particular, the event A = "$E = 50$ & $R = 1$". The probability measure π on Ω defined in (5.19) assigns $\pi(A) = (0.625)(0.250) = 0.15625$. But from Tables (5.1) and (5.3), we have $\overline{P}(A) = P_2(A) = (0.625)(0.125) = 0.078125$. So π violates the condition $\pi \le \overline{P}$ posited by Thorp, McClure, and Fine (reasonably so, we hope by now to have convinced the reader). Moreover, it does so by putting substantial probability on the three scenarios associated with the lowest costs.

In the case of (5.20), consider the event A' = "$E = 70$ &$R = 4$". The probability measure π on Ω defined in (5.20) assigns $\pi(A') = (0.400)(0.375) = 0.150$. But from Tables (5.1) and (5.3), we have $\overline{P}(A') = P_2(A') = (0.125)(0.375) = 0.046875$. So this π also violates $\pi \le \overline{P}$, and by putting substantial probability on the three scenarios associated with the first, second, and fourth highest costs.

We remark in conclusion that the method of bounding expected values illustrated above applies in principle to a wide variety of problems. The constituent random variables need not be independent decision nodes could be included, and the constraints on their probability distributions need not take the form of upper and lower probabilities. Any constraints yielding a closed, convex, polyhedral set of distributions for each constituent variable can be accommodated. Despite the nonlinearity of the objective functions arising in these cases, the special nature of these functions appears to admit certain promising computational economies. We intend in the future to subject this class of objective functions (which have apparently not been investigated by operations researchers) to a detailed study.

Notes

1. Here is a simpler example that illustrates the point. There are two random variables, E and F, judged to be independent, and taking their respective values in the sets $\Omega_E = \{e_1, e_2\}$ and $\Omega_F = \{f_1, f_2\}$. Two experts assign probabilities to Ω_E and Ω_F as follows:

	e_1	e_2	f_1	f_2
expert 1	$\frac{1}{8}$	$\frac{7}{8}$	$\frac{2}{3}$	$\frac{1}{3}$
expert 2	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$

Using independence, their assessments are then extended to singleton subsets of $\Omega = \Omega_E \times \Omega_F$ as follows:

	$\omega_1 = (e_1, f_1)$	$\omega_2 = (e_1, f_2)$	$\omega_3 = (e_2, f_1)$	$\omega_4 = (e_2, f_2)$
expert 1	$\frac{2}{24}$	$\frac{1}{24}$	$\frac{14}{24}$	$\frac{7}{24}$
expert 2	$\frac{4}{24}$	$\frac{8}{24}$	$\frac{4}{24}$	$\frac{8}{24}$

and then to arbitrary subsets of Ω by additivity. The lower and upper envelopes, $\underline{P}$ and $\overline{P}$, of these two probability measures are then computed. For example, $\overline{P}(\{\omega_1\}) = 4/24$, $\overline{P}(\{\omega_2\}) = 8/24$, $\overline{P}(\{\omega_3\}) = 14/24$, $\overline{P}(\{\omega_4\}) = 8/24$, $\overline{P}(\{\omega_1, \omega_2\}) = 12/24$, $\overline{P}(\{\omega_1, \omega_3\}) = 16/24$, $\overline{P}(\{\omega_1, \omega_4\}) = 12/24$, $\overline{P}(\{\omega_2, \omega_3\}) = 15/24$, $\overline{P}(\{\omega_2, \omega_4\}) = 16/24$, $\overline{P}(\{\omega_3, \omega_4\}) = 21/24$, $\overline{P}(\{\omega_1, \omega_2, \omega_3\}) = 17/24$, $\overline{P}(\{\omega_1, \omega_2, \omega_4\}) =$

20/24, $\overline{P}(\{\omega_1, \omega_3, \omega_4\}) = 23/24$, and $\overline{P}(\{w_2, w_3, w_4\}) = 22/24$, with $\underline{P}$ computed similarly. Now consider the probability measure P such that $P(\{\omega_1\}) = 4/24$, $P(\{\omega_2\}) = 8/24$, $P(\{\omega_3\}) = 5/24$, and $P(\{\omega_4\}) = 7/24$, and extended to arbitrary subjects of Ω by additivity. One may (laboriously) check that $\underline{P}(A) \leq P(A) \leq \overline{P}(A)$ for all $A = \Omega$, i.e., that $P \in \mathcal{P}(\underline{P}, \overline{P})$. On the other hand, denoting the marginals of P on Ω_E and Ω_F by P_E and P_F, one has, for example, $P(\{(e_1, f_1)\}) = 4/24$ whereas $P_E(\{e_1\}) \times P_F(\{f_1\}) = (4/24 + 8/24) \times (4/24 + 5/24) = 3/16$, violating the judged independence of E and F.

2. In the example of note 1, above, take expert 1's probabilities for Ω_E and expert 2's probabilities for Ω_F and invoke independence of E and F, getting the probability assignment Q, where $Q(\{(e_1, f_1)\}) = 1/8 \times 1/3 = 1/24$, $Q(\{(e_1, f_2)\}) = 1/8 \times 2/3 = 2/24$, $Q(\{(e_2, f_1)\}) = 7/8 \times 1/3 = 7/24$, and $Q(\{(e_2, f_2)\}) = 7/8 \times 2/3 = 14/24$. Since, for example, $Q(\{(e_2, f_2)\}) > \overline{P}(\{(e_2, f_2)\}) = 8/24$, $Q \notin \mathcal{P}(\underline{P}, \overline{P})$. But Q is certainly compatible with all of the evidence, as manifested in the experts' probability assessments and the judged independence of E and F.

6. FACTORS UNDERLYING AND AFFECTING UNCERTAINTY ESTIMATES

Up to this point, our attention has focused almost exclusively on the quantitative aspects of uncertainty. Methods for representing and manipulating uncertainty have been presented as have several methods for creating upper and lower probability functions. The purpose of this section is to take a step back and explore several associated issues.

First, what factors underlie uncertainty about quantitative estimates? How much can be known about an estimate? How does this knowledge, or lack of it, affect the specification of imprecise probabilities and the ranges between upper and lower probabilities? Answers to these questions are probed in Section 6.1 and must be explored for anyone to intelligently apply the methods outlined in the previous three sections.

Second, how does understanding aspects of uncertainty about quantitative estimates relate to the value and cost of information for IRP? The conclusion one may draw from Section 6.1 is that in many instances, there are limits to what one can know about the true value of a variable. Section 6.2 explores this observation in more depth.

Lastly, how can the discussion of Section 6.1 and 6.2 be of practical use to utilities preparing IRPs? Section 6.3 presents an extended example of the use of the framework and its implications for decision making and collecting additional information.

6.1 DESCRIPTORS OF UNCERTAINTY: QUALITATIVE FRAMES

The purpose of this section is to outline a framework to describe uncertainty about an estimate. The framework takes the form of a frame or checklist. Frames consist of a set of descriptors that can be used to describe every instance of a class of objects, situations, concepts, etc. For example, a frame for automobiles would have descriptors such as: color; engine size; number of doors; price; and manufacturer. The challenge is to develop a frame for automobiles that allows one to describe every instance of an automobile as simply and effectively as possible.

Table 6.1 presents a frame that consists of qualitative descriptors, for the most part, about uncertainty about a quantitative estimate. The frame of Fig 6.1 is intended to allow one to describe how uncertainty afflicts most every kind of quantitative estimate imaginable. The benefits of such a frame for IRP are that it could provide: a systematic basis for understanding uncertainty in an estimate; a systematic way for comparing uncertainties among estimates; and insights about what can and cannot be done to reduce uncertainty in an estimate.

The frame presented in Fig. 6.1 draws upon previous work by Funtowicz and Ravetz (1990) and Tonn and Schaffhauser (1992). However, what is presented below is more comprehensive and better tailored to the needs of IRP.

*** Basic Frame**

The entire frame has three major components: the basic frame; the uncertainty protocol; and the use value frame. The basic frame has four descriptors: the name of the variable; the estimate; the unit of measurement; and the estimator class of the estimate. For example, the name of the variable could be the price of oil in June, 1994, the estimate could be $17, the unit would be per barrel, and the estimator class would be mean. Other estimator classes include: expected value; median; mode; standard deviation, etc.

*** Uncertainty Protocol**

The uncertainty protocol is composed of four subframes: the quantitative representation frame; the inherent uncertainty frame; the operational uncertainty frame and the use value frame. The first expresses uncertainty in forms as discussed in

2.0-5.0. The other three frames capture uncertainty about the estimate, regardless of its intended use.

Table 6.1. Qualitative frame about uncertainty about a quantitative estimate

Basic Frame	Name - (N) Estimate - (E) Unit of measurement - (U) Estimation class (e.g., expected value) - (EC)
<u>Uncertainty protocol</u>	
Quantitative representation frame	Uncertainty measure type - (UM) Uncertainty measure specification - (SP) Upper bound (e.g., on expected value) - (UB) Lower bound - (LB) Level of confidence - (LC)
Inherent Uncertainty frame	Fundamental knowledge that can be gained about the estimate - (FN) Predictability of system encompassing the variable - (SYS) Degree to which variable space can be understood - (VS) -
Operational Uncertainty frame	Soundness of underlying theory (TH) Data collected versus data required (DR) Quality of underlying data (DQ) Reasonableness of estimation methods (EM)
Use value frame	Informativeness (I) Time robustness of the estimates (TR) Relationships between the actual variable and variable needed for policy context (CR) Non-generation of actual variable needed for policy context - (GN) Policy relevance (PR)

- *Quantitative Representation Frame*

This frame has five components: uncertainty measure type; uncertainty measure specification; upper bound; lower bound; and level of confidence. For example, one uncertainty measure type might be an imprecise probability. Others might include fuzzy sets, certainty factors, and classical probability. The specification relates to the form of the uncertainty function. Thus, one could have uniform and normal distributions or discrete functions. The upper and lower bounds are applicable for both continuous and discrete specifications. In the case of an upper and lower probability, the level of confidence would be 100%. In the case of a normal distribution, the upper and lower bounds might pertain to a 95% confidence interval, for example.

- *Inherent Uncertainty Frame*

This frame has three components: fundamental knowledge (FN) that can be gained about a variable; predictability of the system encompassing the variable (SYS); and the degree to which the variable space can be understood (VS). Figure 6.1 presents scales for each of these components. Variables described by the left-hand portion of the figure are said to have more inherent uncertainty that those described by the right-hand portion of the figure. It is argued that inherent uncertainty cannot be overcome by more time and effort (as opposed to operational uncertainty described below).

For example, compare a forecast for electricity demand fifteen years from now to determining how much it cost to replace a transformer at a substation last month. In the later case, the goal is to establish a fact, the system under question (i.e., transformer replacement at a substation) is small scale and orderly, and the variable space (i.e., dollars expended) is well understood. There are no inherent reasons why there should be uncertainty about the cost.

Contrast this to the former. A long-range forecast contains high intrinsic uncertainty because the future is not knowable, the socioeconomic system generating electricity demand is chaotic at best, disorderly at worst, and the variable space (i.e., what contributes to electricity demand) fifteen years hence cannot be said to be well understood. Thus, even in the best of circumstances, there will be uncertainty about the forecast.

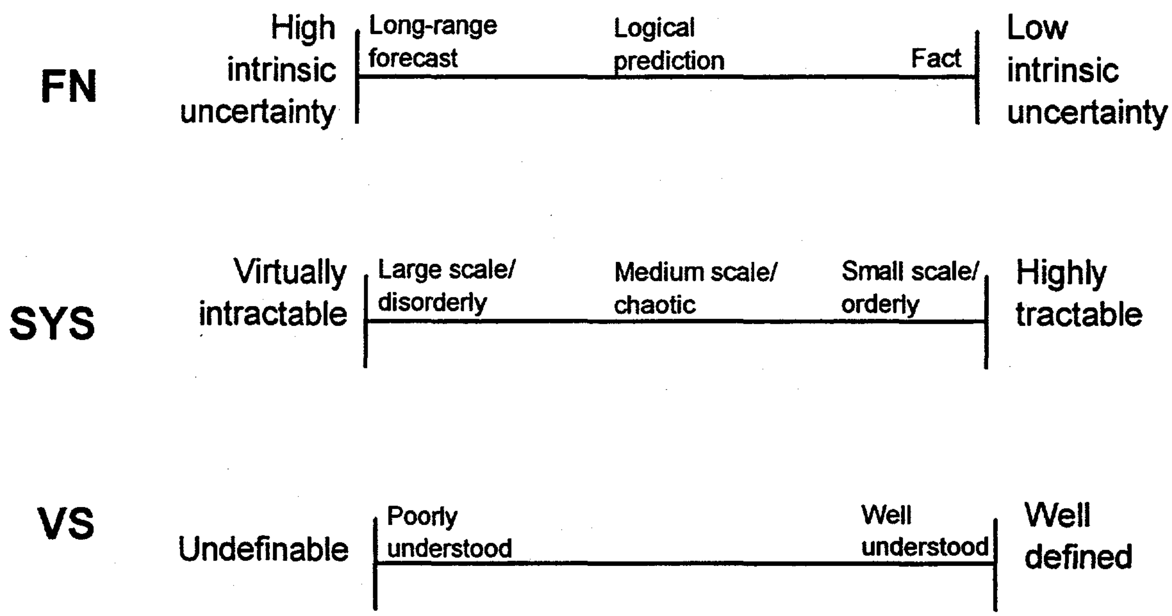

Fig. 6.1. Inherent Uncertainty Frame Scales.

- *Operational Uncertainty Frame*

This frame has four components: soundness of underlying theory (TH); data collected versus data required (DR); quality of underlying data (DQ); and reasonableness of estimation methods (EM). Figure 6.2 presents scales for these components. Variables described by the left-hand portion of the figure are said to have high levels of operational uncertainty, those on the right-hand side low levels of operational uncertainty. It is argued that time and money can be used to overcome operational uncertainty.

To see this, let's continue with the two examples presented above, the fifteen year forecast and the transformer replacement costs. With respect to the later,

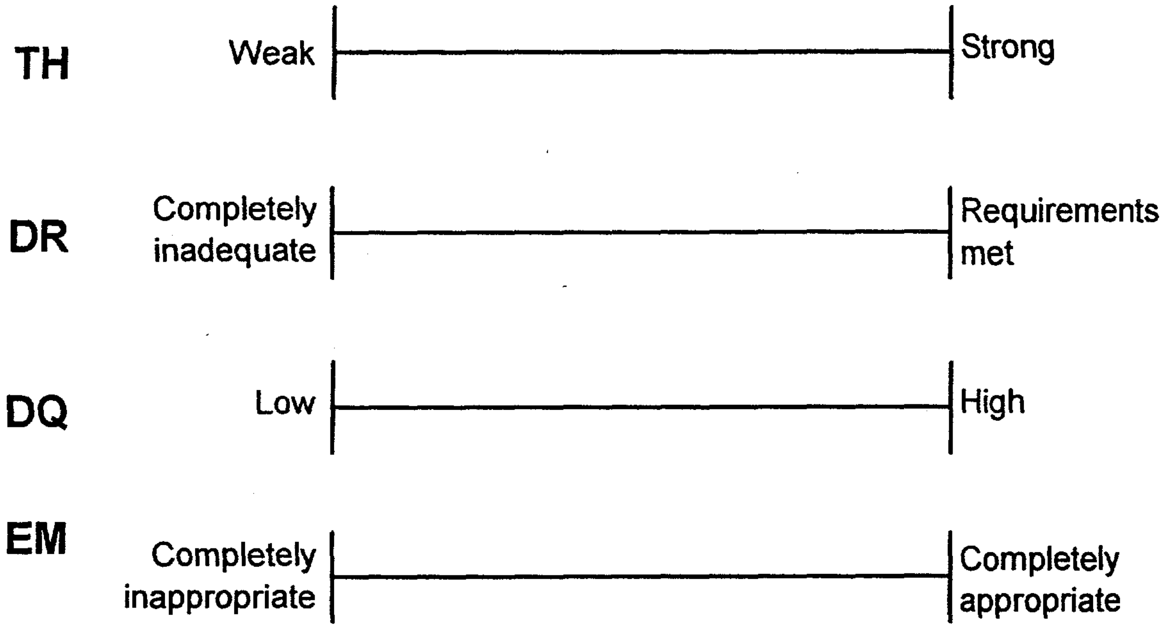

Fig. 6.2. Operational Uncertainty Frame Scales.

theory may not be an issue. One just needs to track the costs. Data to be collected include bills from contractors and vendors and internal costs for labor, materials, overhead, etc. Some uncertainty may arise if vendor bills are late in coming and/or if the bills can be revised within some period of time. The methodology used to arrive at the cost is not a problem in this case either.

The forecast, like many IRP exercises, does entail operational uncertainty. The soundness of economic, demographic, etc. theory underlying the forecast is probably in the middle of the scale. It is rarely the case where a utility has all the data one can imagine on-hand for the analysis. Quality of data, from billing histories to end use metering to measure life-times, is never perfect and can usually be improved in some way. Estimation methods could be excellent (e.g., econometric methods) or ad hoc (e.g., back of the envelop trend lines). Thus, both operational and inherent uncertainty are issues with long-range forecasting.

* Use Value Frame

Estimates have lives independent of their use but in appropriate uses of estimates may cause uncertainty within certain contexts. The Use Value Frame captures this observation in five components: informativeness of the estimate for the policy context (I); time robustness of the estimate (TR); relationships between the actual variable and the variable needed for the policy context (CR); generalization of the actual variable needed for the policy context (GN); and policy relevance (PR). Figure 6.3 provides scales for these components. Variables described by the left-hand side of the figure are essentially worthless for the context, e.g., IRP, where as variables described by the right-hand side are potentially very valuable, depending on the other aspects of uncertainty involved.

Informativeness refers to how useful the range of the estimate is for decision making. An informative range is small enough to rule out many decisions. An uninformative range is so large that anything is still possible. Thus, a forecast with a very large range is not very informative for utility decision makers.

Time robustness refers to the shelf-life of the estimate. If it is only good for a week or a month, it would not be useful. The estimate would also not be useful if it measures something different than what is needed for IRP. For example, an estimate of tons of SO_x emitted into the atmosphere is not a perfect estimate for the potential damage caused by the emissions but may be the best on hand. Similarly, knowing emissions of SO_x from one type of power plant using one type of fuel may or may not be logically generalizable to a utility's entire resource base.

Lastly, not all estimates are equally important to an IRP. It does not make sense to spend more time and money to reduce uncertainty about an unimportant variable. Thus, the policy relevance component was added for completeness.

We understand that it would require a good bit of work to create a qualitative frame for every estimate generated in preparing an IRP. However, we argue that it would be a useful exercise for the most important estimates to at least assist utilities in deciding how to spend their limited data collection and analysis funds. The reasons why are discussed in more detail next.

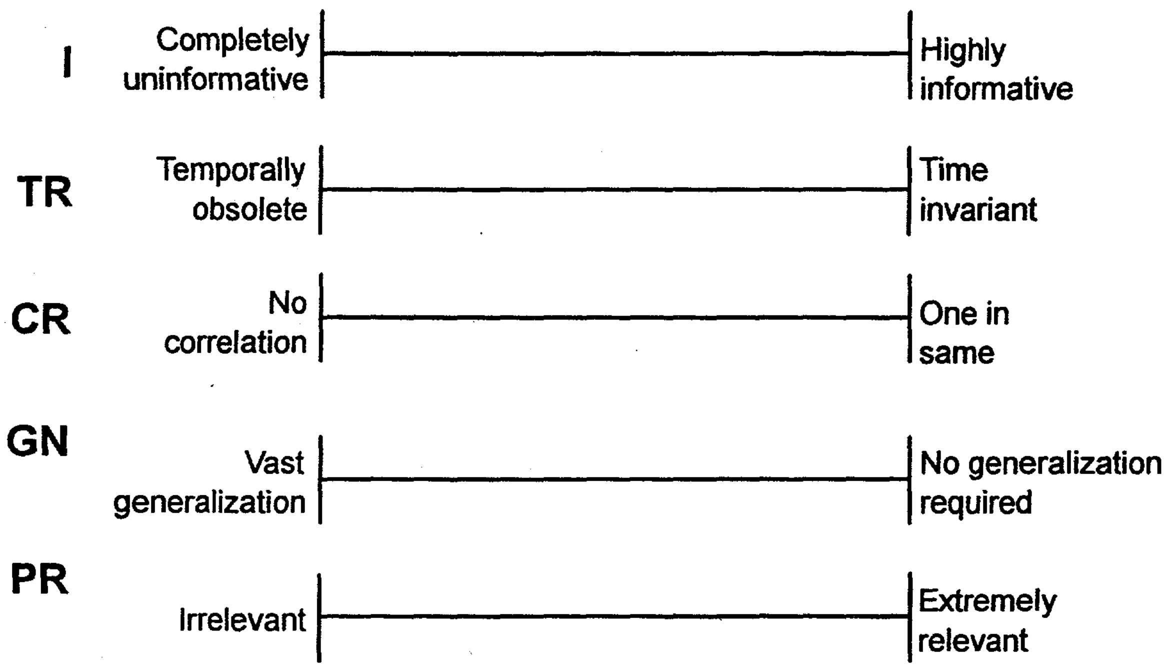

Fig. 6.3. Use value frame scales.

6.2 IMPLICATIONS FOR VALUE AND COST OF INFORMATION

There are at least two important observations to be drawn from the qualitative frame presented above. First, there are some aspects of uncertainty which utilities cannot overcome. Second, there are other aspects of uncertainty that can be overcome, if it makes sense to do so. Let's take these points in order.

Imagine we have an upper and lower probability around a long-range forecast for the price of a barrel of oil. The estimate was made using the opinion of one expert who takes an unconventional approach to understanding the oil markets. The utility was able to elicit from the expert an upper and lower probability for this estimate. However, the utility felt that the range was too narrow given the inherent uncertainty in forecasting oil prices and did not completely trust the judgment of the expert, so the range was broadened to accommodate more inherent uncertainty and increased operational uncertainty.

Figure 6.4 presents the situation faced by the utility. The current information, in terms of upper and lower probabilities around an estimate, is signified by (X,Y). The utility has the option of consulting additional experts, building an econometric model, collecting historical oil price data, etc. The additional information would, we strongly argue, reduce the range between the upper and lower probability around the estimate but could not reduce the difference to zero because of the inherent uncertainty the forecast. Thus, it is virtually inconceivable that an additive, point probability could be established for the estimate.

Figure 6.5 presents this observation from a cost point of view. The figure indicates generally what level of information is currently on hand and what the cost was to collect the information. As is generally assumed, the figure indicates that at some point the added cost of collecting the next piece of additional information increases as one nears what one could theoretically collect. However, different from most value of information models, the figure indicates that the threshold is reached when one overcomes only operational uncertainty. Increased expenditures cannot

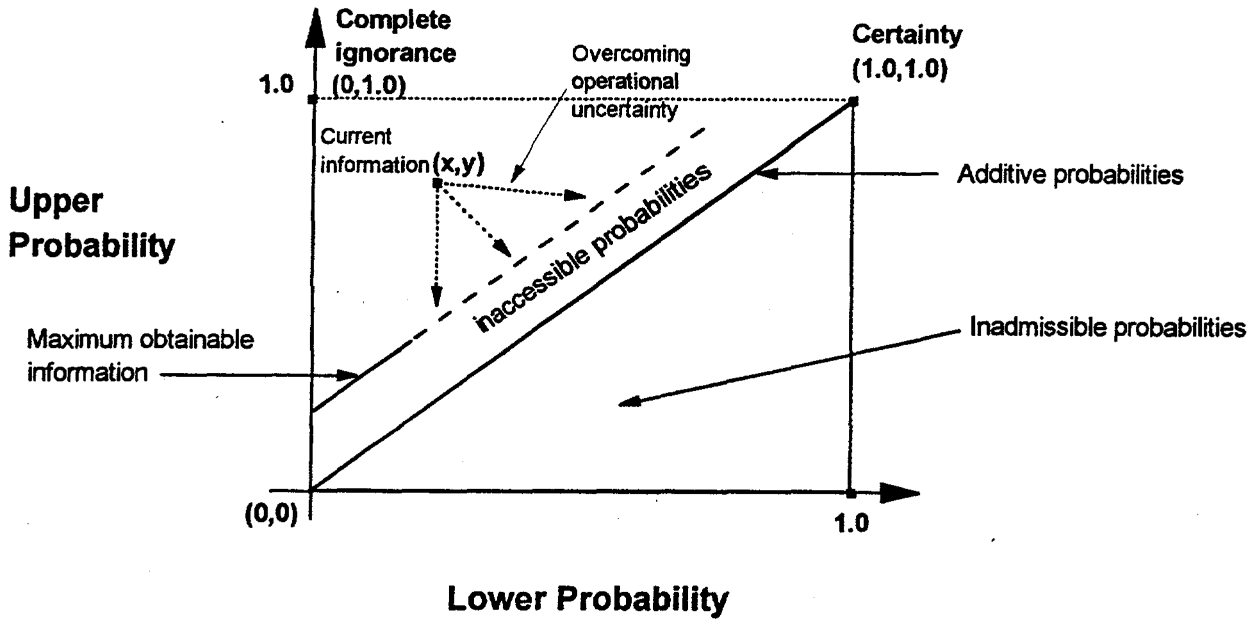

Fig. 6.4. Limits of knowledge.

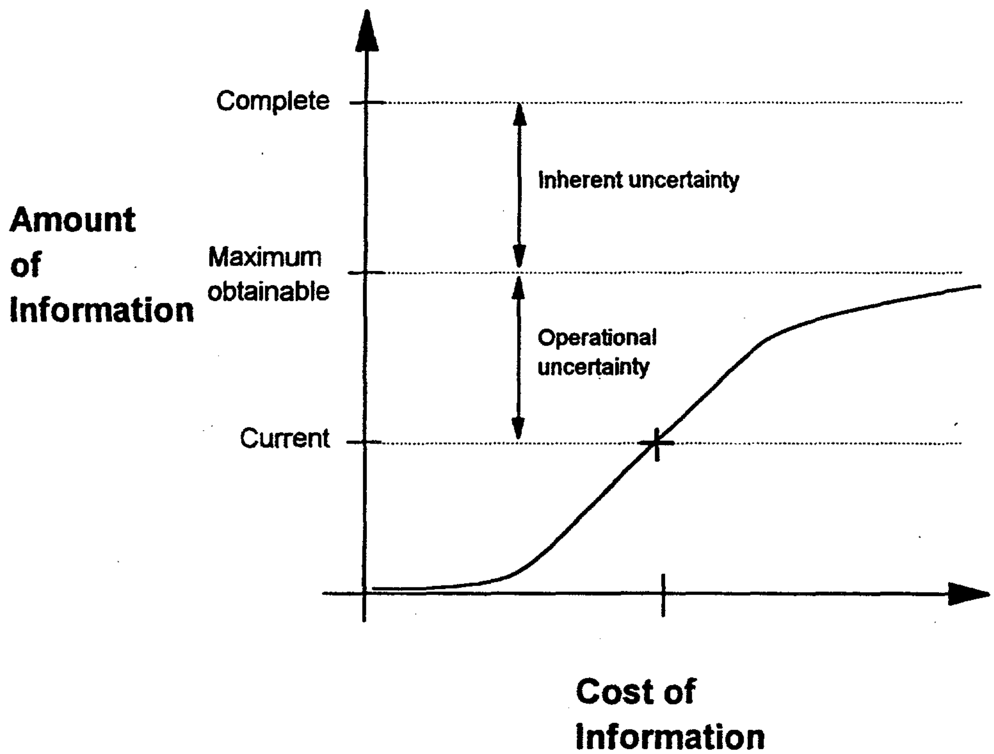

Fig. 6.5. Cost of information.

overcome inherent uncertainty. An important conclusion of this observation is that utilities will always be making decisions that involve uncertainty.

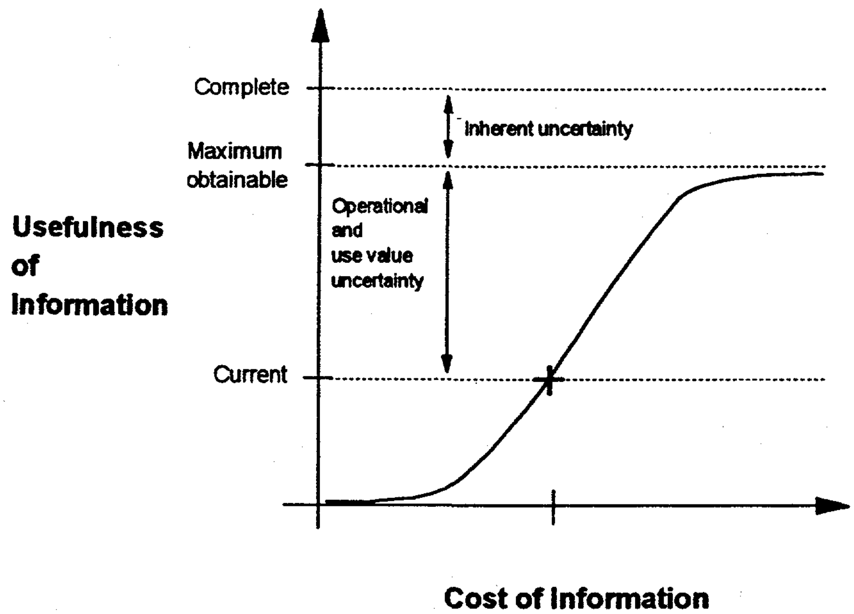

Fig. 6.6. Usefulness of Information.

Figure 6.6 adds the usefulness frame to the picture. In this case, utilities can expend more time and money to overcome operational uncertainty and improve the usefulness of the information at hand. For example, data could be collected for more relevant variables and better methodologies could be employed to reduce the need to generalize findings. It is possible for utilities to have completely useful information, even if uncertainty still plagues the estimates. This could happen if the information is sufficient to provide the necessary guidance to the utility about its current and future decisions. Thus the area between the maximally obtainable useful information and completely useful information could be zero, although it is hard to conceive of such a case in real life.

6.3 EXAMPLE 10: FORECASTING IMPORTANT FACTORS IN IRP

This example takes a broad look at four variables that are important in many IRPs: oil prices; supply costs; electricity demand; and environmental damages. The goal is to forecast each of these variables five years hence. Let's assume the following: oil prices were forecast using a panel of experts; supply costs were generated using a highly accurate model of the utility's physical system; electricity demand was forecast using a time series model; and environmental damages were calculated using information from other utilities and other sources.

Table 6.2 presents hypothetical results of an exercise to create qualitative frames for each of these four variables. The values in the table are meant to be illustrative only and in all likelihood do not conform to any one utility's situation.

Each of the four variables represents a different pattern of uncertainty. The oil price estimate suffers from much inherent uncertainty, and only moderate operational uncertainty. Mainly, the time horizon of the forecast is very long for an oil forecast, which creates a great deal of fundamental uncertainty. The panel of experts convened were the best available and why the oil markets are chaotic is well understood. Possibly better elicitation methods could have been used to improve the quality of the data.

The range of oil prices is not very informative for those engaged in IRP. The bottom line is that while the variable is highly relevant to IRP, there is not much the utility can do to improve its information or the usefulness of the information for IRP. Thus, it might not make sense for the utility to spend additional time or money on oil price forecasts.

Uncertainty about supply costs is quite different although the conclusion is the same. The operation and maintenance of the physical system are quite well known. The system will remain stable over the five year planning horizon. Data collection

Table 6.2. Qualitative frames for important IRP estimates*

N	Oil price	Supply Costs	Electricity demand	Environmental damage
E	N/A	9	N/A	N/A
U	$	cents per KW	MW	$(millions)
EC	average	expected value	expected value	expected value
UM	imprecise probability	probability	imprecise probability	imprecise probability
SP	discrete	weibull	cumulative distribution	discrete
UB	$50	11	600	1000
LB	$8	7	400	10
LC	99.9%	95%	99.9%	99.9%
FN+	L	H	M	M
SYS	L	H	M	VL
VS	H	H	M	VL
TH	H	H	M	VL
DR	VH	VH	L	VL
DQ	M	H	M	L
EM	M	H	M	L
I	L	H	M	VL
TR	L	H	M	H
CR	VH	VH	H	VL
GN	M	VH	H	VL
PR	H	VH	VH	VH

*Assume five years hence
+Scale values are: Very High (VH), High (H), Moderate (M), Low (L), Very Low (VL); Lower levels of scale entail higher levels of uncertianty

E- Estimate
U-Unit
EC - Estimator class
UM - Uncertainty Measures
SP - Uncertainty Measures Specification
UA- Upper Bound
LB - Lower Bound
LC - Level of Confidence
FN - Fundamental Nature
SYS - System
VS - Variable Space
TH - Theory
DR - Data Requirements
DQ - Data Quality
EM - Estimation Methods
I - Informativeness
TR - Time Robustness
CR - Correlation
GN - Generalizability
PR - Policy Relevance

processes are in place to collect all relevant data on costs and the data on whole are very reliable. The forecast is tight, which IRP analysts desire. Overall, there is little uncertainty with respect to supply costs and the estimates are highly informative.

Thus, a conservative conclusion would be for the utility to continue doing what it has been doing with respect to data collection, modeling and analysis.

The electricity demand forecast occupies a middle ground. Five years is a substantial time horizon for a system that is not well-behaved but not so long that demand is expected to change dramatically. Thus, there is only moderate inherent uncertainty in these estimates. Likewise, people have a decent understanding of electricity demand, although knowledge could be much better in the residential and information technology sectors, for example. Data on hand are of decent quality and the times series estimation technique applied to the data was not the most simplistic nor the most sophisticated available. Unfortunately, only sketchy past data were collected with which to estimate the times series model.

Thus, several things could be done to overcome operational uncertainty: collect better data; improve the quality of the data collected; and use a better estimation technique. It might be worthwhile for the utility to do this because the informativeness of the current estimates are only moderate.

The environmental damage estimates in this example exhibit the most uncertainty. The range is extremely large, from virtually inconsequential to very significant. Therefore the informative value of the range is very low. The estimates are plagued with high inherent uncertainty (e.g., the variable space is not well understood) and high inherent uncertainty (e.g., data are sparse and not of high quality). In order to make any estimates at all, proxy variables were used in place of more appropriate variables and other studies were generalized more than they probably should have been.

There is much that can be done to improve the environmental damage estimates, including the collection of better and more comprehensive data, the use of better estimation techniques, and the development of more robust theories about the environment and emissions from power plants. Investing in these activities appears worthwhile, given the very high policy relevance of the variable and the large range of the current estimates.

In making decisions about how to improve the estimates of these four variables, the utility needs to consider the cost effectiveness of its analytical investments. For example, it probably would not be cost effective to spend additional time and money to improve the supply cost estimates because they are already informative and entail little uncertainty.

On the other hand, the oil price and environmental damage estimates are not very informative at this time and the electricity demand forecast is only moderately informative. It is debatable whether any investments could improve the usefulness of oil price forecasts, given high levels of inherent uncertainty. Additionally, investments in environmental analysis may not yield positive payback for many years. The utility needs to assess costs and time horizons with these investments and the sensitivity of decisions to better information when weighing how to allocate scare research and analysis dollars to reduce uncertainty.

6.4 SUMMARY

This section presented a qualitative framework within which to understand causes of uncertainty in quantitative estimates of variables of importance in IRP. The frame has several components, relating to the quantitative representation, inherent and operational aspects of uncertainty, and how the context for using the estimates may cause uncertainty. It was shown at a theoretical level how aspects of uncertainty may impact costs for reducing uncertainty in IRP.

There are several issues associated with the frame that need to be considered. First, it is not clear that utilities will want to develop frames for all variables that are encompassed in IRP. The frame requires a fair amount of information and a number of judgments that may prove difficult to make. Thus, as a first step, it is recommended that the frame be tried on a few of the most important variables. Over time, utilities can develop frames for additional variables. A positive aspect of the frame is that its structure is very amenable for a database application, such that a database about uncertainty about IRP variables could easily be developed, maintained and accessed as needed.

Second, additional use of the frame is needed to evaluate whether its specification as depicted in Fig. 6.1 is best suited for IRP applications. It is possible, for example, that the inherent uncertainty components could be clearer and that some components, such as related to data quality, could be expanded into additional subframes.

Lastly, additional thought is needed to translate information in the frame into decisions about how to most cost effectively reduce uncertainty in IRP. As illustrated in Example 10, it is not always clear that resources should be devoted to reducing uncertainty about the most uncertain and least informative variable if such investments take time, and offer no guarantee of success. Existing methods for conducting value of information analyses are seriously deficient because they assume as a starting point what decisions would be made given perfect information. As we have seen, it is rare that one could even contemplate having complete information, much less information not plagued by operational and use value uncertainties. We believe that if utilities completed frames for the most important IRP variables, qualitative value of information judgments can be confidently made. More rigorous and quantitative value of information analyses await the results of future research.

7. CONCLUSIONS, REFLECTIONS AND RECOMMENDATIONS

This report addresses numerous aspects of the topic of uncertainty. The brief history of the concept indicates that people have been thinking about uncertainty for a very long time and that ideas have changed radically over time. Imprecise probability has its roots in the early 18th century, and, almost paradoxically, can be seen as a generalization of what has become to be known as classical probability. Several examples illustrated how to specify an imprecise probability either directly or indirectly (e.g., using an incomplete contingency table).

Methods for manipulating imprecise probabilities were also presented based on a general framework of evidential reasoning. Methods such as consensus and conditionalization were defined both conceptually and mathematically. Several examples illustrated how different pieces of evidence can be synthesized in various ways to provide more insight into what one knows about a problem.

The important problem of calculating expected values was explored in-depth. One example illustrated how to use a standard formula due to Choquet to calculate expected values involving upper and lower probabilities. The better part of an entire section of the report presented a more sophisticated and theoretically attractive method based on non-linear optimization.

In addition to focusing on techniques, the report also addressed factors that cause uncertainty in quantitative estimates. The qualitative frame offers one means for utilities to keep systematic track of factors that cause uncertainty in important variables in IRP and to facilitate decisions about how to allocate scare resources to reduce uncertainty.

Numerous research issues remain just given the foci of this report. With respect to methods, it was reported that a combination rule has yet to be developed to handle imprecise probabilities, although some progress is reported in the Appendix. In addition, additional research is required on conditionalization methods, methods to synthesize two independent imprecise probability functions, and on extending the non-linear optimization approach for calculating expected values in a decision tree. With respect to the qualitative aspects of uncertainty, the proposed qualitative frame needs to be implemented to test its usefulness and the robustness of its components. Ways to use the frame to make better decisions regarding the cost effective reduction of uncertainty need to also be explored.

There are two major topics that were not addressed in this report that are crucial to utility decision making under uncertainty and IRP. One topic is broadly defined as *psychological aspects of uncertainty.* The other is broadly defined as decision methods. Let's reflect on each in order.

Psychological aspects of uncertainty has two main components: elicitation of uncertainty judgments from experts; and the communication of uncertainty. With respect to the former, a large body of psychological research indicates that at the very least, people, and experts, have difficulties in expressing and thinking in terms of classical probabilities. Experts tend to be overconfident about their diagnoses, which creates a false sense of knowledge. Also, experts have good and bad days, meaning that the reliability and validity of their judgments is not consistent from one day to the next. People in general have a difficult time in conceptualizing low probabilities, over-emphasize certain information, and do not think well probabilistically.

The relationship between these findings and IRP is this: the uncertainty methods presented above are only as good as the uncertainty estimates needed by the methods, and, apparently, it is not a trivial task to elicit good uncertainty estimates from experts. A fair amount of research has been conducted on aids to help experts think about classical probabilities (e.g., probability wheels, sliding bars) but little research has been done with respect to imprecise probabilities. The qualitative frame can be seen as a conceptual aid to experts so that they do not overstate their knowledge by constructing lower and upper probabilities with ranges that are too narrow but no research has been done to tie the frame to uncertainty elicitation. Future research in IRP should explore these issues in more depth.

Communicating uncertainty is crucial to the success of any analytical endeavor. Within a utility, analysts need to communicate to executives. In addition, utilities need to communicate with their shareholders, customers, PUCs, and interest groups. A large body of research falling under the rubric of risk communication indicates that the communication of uncertainty and risk is very difficult, and if done so poorly, can actually lead to unpredictable and unintended consequences. Our review of uncertainty in IRP uncovered many examples of misleading tables and figures, which indicates to us that the communication of uncertainty should be an important topic of future research on IRP and uncertainty.

Of course, utilities are not interested in research for the sake of research. They are interested in making better decisions. Methods for eliciting, representing, manipulating, and communicating uncertainties have to eventually prove their worth in improving decision making. This report only peripherally addressed decision making through Example 4, which focused on choosing a set of resources.

Indeed, the literature on decision making under uncertainty is very robust and should be summarized for use in IRP. Decision methods are important because they provide a second leg for managing uncertainty in IRP. The first leg, addressed in this report, encompasses the explicit representation of what one knows and doesn't know. The second leg provides ways of making decisions to reduce risk and take advantage of opportunities.

For example, one observation we had of the IRPs we reviewed is that they are not set, for the most part, in a decision analytic framework. That is, other than choosing among a set of resource options, one does not find a classical decision analytic model which includes a problem statement, a set of mutually exclusive options, a set of evaluation criteria, weights over the evaluation criteria, assessments about how well each alternative satisfies each of the criteria given different future states of the world, and a method to assimilating this information into an informed choice. Uncertainties play a big part in this model, from representing uncertainty about the future state of the world to representing uncertainty about the outcomes of the different alternatives.

There are other, more sophisticated decision methods that should be examined for use in IRP. Portfolio/options theory is one idea. The irreversibility of decisions is another. Multi-criteria decision making is yet another. Research in decision making needs to be done in conjunction with research on uncertainty.

APPENDIX A
A Combination Rule for Belief Functions

This Appendix presents the results of preliminary research on developing a combination rule for imprecise probabilities. As will be seen, the research has only progressed to developing a fairly complicated procedure for combining Möbius Transforms of an arbitrary number of belief functions.

The Appendix begins with a theoretical discussion on characteristics one might desire a combination rule to have (A.1). Next, the Dempster-Shafer Combination Rule is presented and critiqued (A.2). Our combination rule for the pieces of evidence is presented in A.3. The fourth section, (A.4), presents our approach for combining an arbitrary number of belief functions. A.5 presents a critique of the progress to date.

A.1 CHARACTERISTICS OF A COMBINATION RULE

As discussed in 3.0, the purpose of a combination rule is to synthesize pieces of evidence that have equal standing that bear on the truth of a member in Ω. The pieces of evidence are drawn in some manner from the world. Recall Example 6 where several pieces of evidence where collected and represented as lower probabilities pertaining to the decision of a large industrial customer. In addition, physicians collect pieces of evidence from patients which are synthesized in some manner to render diagnoses and juries are presented pieces of evidence which they synthesize in some manner to render verdicts.

Thus, using another analogy, a combination rule is used to synthesize the work of a detective. How a detective, or physician, or lawyer, determines which pieces of evidence are important is a complicated question. Obviously, the diagnosis or conclusion would be heavily dependent upon what pieces of evidence are included and excluded. For our purposes, let's assume that an inclusionary rule can be established, if only by the use of common sense.

The important consideration with respect to developing a combination rule is that it must appropriately handle any conceivable type and sets of pieces of evidence that are judged to be worthy of inclusion in the analysis. We argue, for example, that a combination rule should handle these three instances in the following manners:

(I) Weight of evidence focusing: This condition addresses what ought to happen when pieces of evidence support each other. Imagine several pieces of evidence that all support {a} in some fashion. Taken separately, no piece of evidence overwhelmingly supports {a}. However, taken together, in case after case, {a} keeps coming up, and in the end, the weight of all the evidence clearly points to {a}.

To state this condition more formally, let's assume that we have a set of evidence, $e = \{e_1, e_2 \ldots e_\infty\}$. Also, let $A \subseteq \Omega$, $B = \{\mathrm{a}\}$, and $\Omega = \{a, b, \ldots\}$. Then, let $\underline{P}_i(A) > 0$ when $A \cap B \neq \phi$, and $\underline{P}_i(A) = o$ when $A \cap B = \phi$, for each e_i. An appropriate combination rule would yield the following: $e_1 \otimes e_2 \otimes e_3 \ldots e\infty = e_*$, where $\underline{P}_*(B) = \overline{P}_*(B) = 1.0$, when the pieces of evidence point to B.

(II) Resolution of inconsistency: Let's assume that two pieces of evidence have been collected, e_1 and e_2, over the set $\Omega = \{a, b, c, d\}$. Let $A = \{a\}, B = \{b\}$,and $C = \{a, b\}$. It turns out that $\underline{P}_1(A) = \overline{P}_1(A) = 1.0$ and $\underline{P}_2(B) = \overline{P}_2(B) = 1.0$. In other words, the two pieces of evidence point conclusively to the different conclusions! While this situation is unlikely to arise very often in real life situations, it is symbolic of the extreme case of a common occurrence where pieces of evidence point to inconsistent conclusions. In cases such as this, the combination rule needs to find the most logical middle ground. In this extreme case, the result should be: $e_1 \otimes e_2 = e_3$, where $\underline{P}_3(C) = \overline{P}_3(C) = 1.0$.

(III) Simple support function identity: This third condition specifies an identity requirement. That is, it specifies under what conditions the combination of two pieces of evidence, E_1 and E_2, will yield either E_1 or E_2. Interestingly enough, there are at least three identity relationships to choose from.

First, one could specify that if $E_1 = E_2$, irregardless of how the probability mass is specified, then $E_1, \otimes E_2 = E_1$. Second, one could specify that $E_1 \otimes E_2 = E_1$, where E_2 is the trivial belief function (i.e., $M(\Omega) = 1.0$), and E_1 is any belief function. Third, one could specify that $E_1 \otimes E_2 = E_1$, where $E_1 = E_2$ and $M_1(A) = M_2(A) = 1.00$. In other words, the identity holds only when each M-function is characterized by all its probability mass resting on the same subset of Ω.

We have chosen the third case for the following reasons. One, the first relationship is incompatible with (I), because it would prevent the focusing of the weight of evidence arbitrarily for arbitrarily defined M-functions. Two, the second relationship presumes that a piece of evidence that supports no diagnosis should have no bearing on the ultimate diagnosis. Our position is this: if the piece of evidence, a priori, is deemed crucial to the diagnosis, it should be included no matter what specifications the M-function takes on. If it is inconclusive, then it should, as a matter of course, lead to higher levels of uncertainty about the diagnosis.

Given these comments, the third specification is a good compromise. Criterion (I) is not violated because there is no possibility of focusing the weight of evidence on any subsets smaller than A. Also, it handles the case where both E_1 and E_2 are trivial belief functions.

It is desirable for the combination rule to have several mathematical properties. These include:

(IV) Commutativity: $E_1 \otimes E_2 = E_2 \otimes E_1$;

(V) Associativity: $(E_1 \otimes E_2) \otimes E_3 = E_1 \otimes (E_2 \otimes E_3)$ (or any order);

(VI) Monotinicity: $E_1 \otimes E_2 = E_3$, where $\underline{P}_3(A \cup B) \geq \underline{P}_3(A)$, and $\underline{P}_3(B)$ and $\overline{P}_3(A \cup B) \leq \overline{P}_3(A)$, and $\overline{P}_3(B)$.

Future research may reveal additional desirable characteristics for a combination rule, or indicate the need for revising I-VI. However, for our purposes, I-VI are quite sufficient to critique an existing rule and propose an alternative.

A.2 DEMPSTER - SHAFER RULE OF COMBINATION

As far as we have been able to determine, the only combination rule for imprecise probabilities proposed in the literature that has been seriously considered is traceable to the work of Dempster (1967) and Shafer (1976). As such, the combination rule is commonly known as the Dempster-Shafer Rule (DSR) within Dempster-Shafer theory, which basically encompasses belief functions and Möbius Transforms of belief functions. Thus, the rule and the theory encompasses only a special, but important, case of imprecise probability.

The DSR is most straightforwardly expressed as the Möbius Transform of a belief function. Recall that a belief function is infinitely monotone (i.e., R-monotone), such that:

$$\underline{P}(A_r \cup \ldots \cup A_r) \geq \sum_{\substack{I \subseteq (1_1, \ldots r) \\ I \neq \emptyset}} (-1)^{|I|-1} \underline{P}(\bigcap_{i \in I} A_i) \qquad (A.1)$$

The Möbius Transform is:

$$M(E) = \sum_{A \subseteq E} (-1)^{|E-H|} \underline{P}(A) \quad , \qquad (A.2)$$

where $E \subseteq \Omega$.

Also, recall that to recreate a belief function from a Möbius Transform, or M-function, one should apply:

$$\sum_{A \subseteq E} M(A) = \underline{P}(E), \text{for all } E \subseteq \Omega. \qquad (A.3)$$

The DSR to combine two pieces of evidence, M_1 and M_2 is:

$$M_3^{(c)} = \frac{1}{1-k} \sum M_1(A) * M_2(B), \text{ for all } A \subseteq \Omega \text{ and } B \subseteq \Omega \qquad (A.4)$$

And where $C = A \cap B$ when $C \neq \emptyset$ and where K is the total value of $M_1(A) * M_2(B)$ where $A \cap B = \emptyset$. The $\frac{1}{1-k}$ term can be viewed as a normalization factor.

Operation of the DSR is best understood through an example presented in tabular form (Table A.1). Let's assume that we have two pieces of evidence defined over $\Omega = \{a, b, c, d\}$. Both E_1 and E_2 are belief functions and have been transformed to be M-functions. Only the focal elements of each M-function are shown in the table (i.e., $M_i(A) > 0$). For comparison purposes, Table A.2 contains the lower probabilities for E_1, E_2 and $E_1 \otimes E_2 = E_3$.

Table A.1. Example of the Dempster-Shafer Rule of combination

M_2	M_1		
	$M_1(ab)$ .3	$M_1(ad)$ .4	M_1(abc) .3
$M_2(a)$.3	{a}=.09	{a}=.12	{a} =.09
$M_2(bc)$.2	{b}=.06	{∅}=.08	{bc}=.06
$M_2(abcd)$.5	{ab}=.15	{ad}=.20	{abc}=.15

$K = .08$
$M_3(a) = (.09 + .12 + .09)/(1 - .08) = .32$
$M_3(b) = (.06)/(1 - .08) = .07$
$M_3(ab) = (.15)/(1 - .08) = .16$
$M_3(ad) = (.20)/(1 - .08) = .22$
$M_3(bc) = (.06)/(1 - .08) = .07$
$M_3(abc) = (.15)/(1 - .08) = .16$

From (A.4) and the example, it can be seen that the DSR satisfies only two of the three major criteria for a rule of combination. As hinted at in the example, DSR is good at weight of evidence focusing. Notice that on balance, smaller subsets of Ω have higher lower probabilities in E_3 than in E_1 or E_2. DSR acts to move

Table A.2. Lower probabilities for DSR example

	E_1	E_2	$E_3 = E_1 \otimes E_2$
a	0.3	0	0.32
b	0	0	0.07
c	0	0	0
d	0	0	0
ab	0.3	0.3	0.55
ac	0.3	0	0.32
ad	0.3	0.4	0.54
bc	0.2	0	0.14
bd	0	0	0.07
cd	0	0	0
abc	0.5	0.6	0.78
abd	0.3	0.7	0.77
acd	0.3	0.4	0.54
bcd	0.2	0	0.14
abcd	1.0	1.0	1.0

probability mass to smaller and smaller subsets of Ω because of the intersection term and because K is, in essence, distributed to the intersections. DSR also satisfies the identity criterion (III), the commutativity criterion (IV), the associativity criterion (V), and the monotinicity criterion (VI). In fact, DSR always yields a belief function if it is combining belief functions. Its behavior is indeterminant if given other kinds of imprecise probabilities.

Unfortunately, DSR is not able to resolve inconsistency criterion (II). Specifically, DSR collapses if given a case of maximal inconsistency. Let $M_1(a) = 1.0$ and $M_2(b) = 1.0$. In this case, $K = 1.0$ and there are no subsets over Ω to normalize over. DSR does not yield the preferred result, $M_3(a, b) = 1.0$. Thus, while DSR meets 5 of 6 criteria, it does not meet the inconsistency resolution criteria, which is especially important for evidential reasoning.

A.3 A FULL EVIDENCE COMBINATION RULE

Searching for a new combination rule is not a trivial task. Conditions I-VI are not specified in such a way that an equation can be logically deduced. Nor are I-VI specified as axioms, such that if, by trial and error, a formula is discovered that it could be proven that it is the only formula, or at least the only family of formulas, to satisfy the criteria. Thus, at the present time, we must be content to explore potential solutions to the problem as best as possible.

Several simple solutions to the problem of combining imprecise probabilities were explored. For example, one could simply average lower probabilities of M-values over N-pieces of evidence.

Such simple solutions failed to meet any of the three major criteria (I-III). Also, at this time, it has proven especially difficult to develop formulas that use lower probabilities directly.

Given these caveats, this section presents an algorithm for combining two M-functions that were transformed from belief functions. The algorithm satisfies all three major criteria, and commutativity and monotonicity. Additions to the algorithm are needed to overcome the fact that it doesn't satisfy the associativity criterion (V). These additions are explained in A.4.

The algorithm takes the same basic conceptual approach as DSR in that each subset of Ω for each piece of evidence is manipulated in some way. The algorithm is:

For every $A \cap B \neq \emptyset, M_3(A \cap B) = M_1(A) * M_2(B) * S$

$$M_3(A) = M_1(A) * M_2(B) * (1 - S) * \frac{M_1(A)}{M_1(A) + M_2(B)}$$

$$M_3(B) = M_1(A) * M_2(B) * (1 - S) * \frac{M_2(A)}{M_1(A) + M_2(B)}$$

$$
\begin{aligned}
\text{For every} A \cap B = \emptyset,\ M_3(A \cup B) &= M_1(A) * M_2(B) * R \\
M_3(A) &= M_1(A) * M_2(B) * (1-R) * \frac{M_1(A)}{M_1(A) + M_2(B)} \\
M_3(B) &= M_1(A) * M_2(B) * (1-R) * \frac{M_2(A)}{M_1(A) + M_2(B)} \\
\text{where } S &= \frac{\| A \cap B \|}{\| A \| + \| B \| - \| A \cap B \|} \\
\text{and } R &= \sum_{A \subseteq \Omega} \frac{| M_1(A) - M_2(A) |}{2} \quad .
\end{aligned}
\tag{A.5}
$$

(A.5) differs from (A.4) in several significant respects. Most noticeable is that the normalization factor K is gone. Instead, (A.5) contains two algorithmic terms, not just one, which are invoked depending upon whether $A \cap B$ is non-null or null. In the non-null case, basic probability mass is focused "downward" onto smaller subsets of Ω.

S can be interrupted as the focusing constant. It regulates focusing based on the cardinality of the sets involved. It prevents basic probability mass on large subsets, which do not carry much information, from focusing, by the intersection term, mass onto very small subsets, which in some sense carry much more information. (A.5) accomplishes this task by retaining mass on the original focal elements based on (1-S) and a proportional term.

In the null case, mass is resolved "upward" to larger subsets. R can be interpreted as a measure of dissimilarity between M_1 and M_2. When $R = 1.0$, the two pieces of evidence are completely dissimilar and each has as a focal element whose value is equal to 1.0. In this case, (A.5) resolves to $M_3(A \cup B) = 1.0$, which satisfies criterion II. As the pieces of evidence become less dissimilar (e.g., $R \rightarrow 0$), then less mass is pushed upward and more mass remains associated with the original focal elements.

(A.5) satisfies criterion III because in this case $S = 1$ and there are no cases where $A \cap B = \emptyset$. A quick inspection will indicate that (A.5) is also commutative and maintains monotinicity (because $\sum_M = 1.0$ in every case and all $M-$values ≥ 0).

Table A.3 presents some results of applying (A.5) to various combinations of two $M-$functions. For example, $M_{2\otimes3}$ illustrates inconsistency resolution. M_1 completely supports {a} and M_2 completely supports {b}. The best one can say in this situation, as output by (A.5), is that $M_{2\otimes3}$ should completely support {a,b}.

$M_{4\otimes4}$ illustrates focusing of weight of evidence given identical pieces of evidence. As indicated, M_4 strongly supports $\{a\}$, weakly supports $\{b\}$, and provides no support to any other diagnosis, either singly or in combination. Assuming two pieces of evidence had been collected exactly like M_4, their combination yields even more weight on $\{a\}$, as criterion (I) would have.

$M_{5\otimes5}$ illustrates the result of the simple support function criterion (III). $M_{1\otimes2}$ illustrates what happens when a trivial belief function, M_1, is combined with a belief function that provides complete support to one prognosis, M_2. As is indicated, $M_{1\otimes2}$ offers less support for $\{a\}$ than does M_2 alone.

A.4 COMBINING N PIECES OF EVIDENCE

The combination rule specified in (A.5) meets five of the six criteria outlined in A.1. An inspection of (A.5) reveals that it is not associative. In other words, for more than two pieces of evidence, the order in which they are combined significantly affects the result. More specifically, the last piece of evidence to be combined has the most impact on the result. This is a potentially fatal flaw with (A.5), even though it is superior to the Dempster-Shafer Rule of Combination because it meets the inconsistency resolution criterion.

Table A.3. Inputs and results of the full evidence combination rule

	Inputs					Results			
	M_1	M_2	M_3	M_4	M_5	M $1 \otimes 2$	M $2 \otimes 3$	M $4 \otimes 4$	M $5 \otimes 5$
a	0	1.0	0	0.95	0	0.625	0	.993	0
b	0	0	1.0	0.05	0	0	0	.007	0
c	0	0	0	0	0	0	0	0	0
d	0	0	0	0	0	0	0	0	0
ab	0	0	0	0	1.0	0	1.0	0	1.0
ac	0	0	0	0	0	0	0	0	0
ad	0	0	0	0	0	0	0	0	0
bc	0	0	0	0	0	0	0	0	0
bd	0	0	0	0	0	0	0	0	0
cd	0	0	0	0	0	0	0	0	0
abc	0	0	0	0	0	0	0	0	0
abd	0	0	0	0	0	0	0	0	0
acd	0	0	0	0	0	0	0	0	0
bcd	0	0	0	0	0	0	0	0	0
abcd	1.0	0	0	0	0	0.375	0	0	0

To overcome this problem, we explored additional criteria to possibly guide the order in which pieces of evidence are combined. Two concepts appear particularly important. First, the pieces of evidence should be combined in an order based on their informativeness. For example, referring to Table A.3, M_2 is maximally informative because it indicates the truth is in a subset of Ω with just one element, {a}. M_1 is minimally informative. M_5 falls in between. Thus, it makes some sense to combine $M_1 \otimes M_5 \otimes M_2$.

Second, pieces of evidence should be combined in some way based on their similarity to each other. In some sense, this criterion argues that there is a meta-physical aspect to combining pieces of evidence in that those that are similar should "band" together to support their shared diagnosis. Thus, pieces of evidence dissimilar to each other should be combined first, followed by pieces of evidence that are more similar.

Conveniently, we already have a measure of dissimilarity, R. Let R' be the measure of similarity and equal $1-R$ for two pieces of evidence. Let $O = \{O_1, O_2 \ldots O_N\}$ indicate the order to combine N pieces of evidence. Then, let the measure of similarity in an ordered array of pieces of evidence be:

$$R' = \sum_{i=2}^{N} (1 - R(E_i, E_{i-1})) * 0_i \ . \qquad (A.6)$$

We would like to find an order which maximized R'.

We have conducted preliminary research on a measure of information (e.g., we also refer to it as a measure of determinantness). Let T be the measure of information in a belief function. T is defined as,

$$T_i = \sum_{A \subseteq \Omega} (2^{M(A)} - 1) \log_2 \frac{|\, \Omega \,|}{|\, E \,|} \ . \qquad (A.7)$$

T_i for piece of evidence i, is equal to 0.0 when E_i is a trivial belief function. T_i is a maximum value when E_i contains a simple support function on a singleton subset of Ω. The maximum value of T is regulated by the number of members in Ω. The higher cardinality of Ω, the more information is needed "determining" the truth in Ω. See Tonn (1993) for an extended discussion of T. To order N pieces of evidence from least to most determinateness, we could maximize:

$$T' = \sum_{i=1}^{N} O_i * \frac{T_i}{T_{max}} \ . \qquad (A.8)$$

To order N pieces of evidence, we maximize:

$$Max \ (T' + R') \ . \qquad (A.9)$$

(A.5)-(A.9) have been coded into software. Specifically, we coded the combination algorithms using C to support an expert system application. Currently, the software is capable of handling Ω of cardinality four and 2-5 pieces of evidence. Table A.4 presents some results for combining more than two pieces of evidence.

Table A.4. Combining three or more pieces of evidence

	Inputs						Results	
	M_1	M_2	M_3	M_4	M_5	M_6	M $1 \otimes 2 \otimes 3$	M $4 \otimes 5 \otimes 6$
a	0.95	0	0	0.3	0.2	0	0.38	0.14
b	0.05	0	0	0.2	0.4	0	0.01	0.26
c	0	0.3	0	0	0.2	0.3	0	0.14
d	0	0.2	0.2	0	0	0.1	0	0
ab	0	0	0	0.3	0	0	0	0.05
ac	0	0	0	0	0	0	0.12	0.04
ad	0	0	0	0	0	0	0.17	0
bc	0	0	0	0	0	0	0.01	0.03
bd	0	0	0	0	0	0	0.01	0.01
cd	0	0.2	0	0	0	0.3	0	0.01
abc	0	0	0	0	0.1	0	0	0.10
abd	0	0	0	0	0	0	0	0.01
acd	0	0	0	0	0	0	0.13	0.03
bcd	0	0	0	0	0	0	0.01	0.03
abcd	0	0.3	0.8	0.2	0.1	0.3	0.16	0.15

First, M-Functions M_1, M_2, and M_3 are combined, which resulted in $M_{1\otimes 2\otimes 3}$. As is evident from visual inspection, M_3 is the least informative of the three vectors, followed by M_2 and then M_1, which is highly informative. With respect to similarity, M_1 supports different conclusions than M_2 and $M_3(\{a,b\}$ vs. $\{c,d\})$. Given these observations, one would expect the algorithm to combine M_3 with M_2 followed by M_1. Indeed, this is what the code does, with the results shown in the seventh column of Table A.4. Because the pieces of evidence are somewhat inconsistent, it can be seen that support for $\{a\}$ decreases from M_1.

In the second case, the similarity strongly affects the combination order. In combining M_4, M_5, and M_6, from visual inspection, M_4 and M_5 appear quite similar. Also, calculations of T indicate that M_5 is the most informative, followed by M_4 and M_6. Given this information, the code combined M_6 with M_4, and then with M_5, with the results shown in the eighth column of Table A.4.

A.5 COMMENTARY

The combination rule presented herein is actually a fairly complex algorithm (A.5). The algorithm has its good points. It satisfies five of six criteria presented in (A.1) that characterize an attractive combination rule. In particular, the algorithm has the capability of both focusing the weight of evidence on smaller subsets of Ω if so warranted or resolving inconsistencies to larger subsets of Ω if so warranted.

The algorithm is not associative. Additional ideas, captured in (A.6)-(A.9), were needed to order pieces of evidence before combination. Ordering based on informative value and similarity has face validity but additional research on this point is recommended. It would be better to develop an algorithm that is associative, but it looks unlikely that such could be accomplished given our current path of algorithm development.

There are several arbitrary specifications within the algorithm. For example, one can imagine a whole family of specifications for R and S, that would amplify or dampen inconsistency resolution and focusing, respectively. Also, the specification for T, T', and R' demand further examination.

In hindsight, it would be preferable to develop an algorithm, or better yet a simple combination rule, that utilizes imprecise probabilities directly, without having to apply the Möbius Transform. Also, the algorithm presented here can only handle belief functions with some confidence. A more general version is needed.

The bottom-line for applied contexts, such as IRP, is that the algorithm appears to yield the desired results. It operates on imprecise probabilities, albeit a restricted case known as belief functions, as wished. The algorithm was fairly straightforward to code.

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