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TESTING TO ESTIMATE THE MUNSON-DAWSON PARAMETERS

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ABSTRACT

Parameter estimation for modern viscoplastic constitutive models often requires data from many tests. Sensitivity coefficients can be used to design an efficient test matrix and reduce testing requirements. The present study derives sensitivity coefficients for each model parameter in the Munson-Dawson constitutive model and evaluates them for several load histories.

INTRODUCTION

Constitutive equations for viscoplastic deformation of salt are highly nonlinear and contain many parameters that are necessary to model the strong rate- and history-dependent behavior. These parameters need to be evaluated accurately so that the constitutive models can be used to make credible predictions of structural deformation and failure. However, the constitutive parameters in these nonlinear models are difficult to evaluate because most parameters do not have an obvious physical meaning such as yield stress or strain-hardening exponent. The traditional approach to parameter estimation can require a large number of tests. Analysis of the data from these tests invokes simplifying assumptions about the material behavior so that model parameters can be evaluated by determining the slope of the plotted data, changes in slope of data, centers of hysteresis loops, etc.

Parameter estimation can be improved by performing precisely controlled tests involving traditional and non-traditional load histories, integrating the constitutive model to simulate the imposed load histories, and using least-squares fitting to evaluate the model parameters (Senseny et al, 1993). This approach can reduce testing requirements if it is coupled with a test matrix that produces information to estimate each parameter, and that imposes load histories in which the material behavior is dominated by phenomenology that is characterized by one

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or only a few parameters. A well-designed matrix allows parameters to be determined efficiently with little uncertainty and with little confounding with other parameters.

This study considers the Munson-Dawson constitutive model (Munson et al, 1989). Parameter sensitivity coefficients, the derivative of the material behavior with respect to the parameter, are derived for each parameter and evaluated for several loading histories. Results show that data from creep tests are very useful for parameter evaluation.

CONSTITUTIVE MODEL

The Munson-Dawson model can be defined by three first-order differential equations. The isothermal scalar form of the equations contains ten parameters:

$$\dot{\sigma} = E(\dot{\varepsilon} - \dot{\varepsilon}^p) \tag{1}$$

$$\dot{\varepsilon}^p = \left\{\exp\left[\left(\alpha + \beta\ln\frac{\sigma}{\mu}\right)\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)^2\right] + \exp\left[-\delta\left\langle\frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3} - 1\right\rangle^2\right] - 1\right\}\times \\ \times\left\{A_1'\left(\frac{\sigma}{\mu}\right)^{5.5} + A_2'\left(\frac{\sigma}{\mu}\right)^{n_2} + B'\sinh\left[\frac{q\langle\sigma - \sigma_o\rangle}{\mu}\right]\right\}\mathrm{sgn}(\sigma) \tag{2}$$

$$\dot{\zeta} = \left\{\exp\left[\left(\alpha + \beta\ln\frac{\sigma}{\mu}\right)\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)^2\right] + \exp\left[-\delta\left\langle\frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3} - 1\right\rangle^2\right] - 2\right\}\times \\ \times\left\{A_1'\left(\frac{\sigma}{\mu}\right)^{5.5} + A_2'\left(\frac{\sigma}{\mu}\right)^{n_2} + B'\sinh\left[\frac{q\langle\sigma - \sigma_o\rangle}{\mu}\right]\right\}\mathrm{sgn}(\sigma) \tag{3}$$

where $\langle x\rangle$ are the MacCauley brackets, which are 0 for $x < 0$, and take on the value of x for $x > 0$; $\dot{\sigma}$, $\dot{\varepsilon}$ and $\dot{\varepsilon}^p$ are the stress rate, total strain rate and plastic strain rate, respectively; E and μ are elastic moduli, 31.0 and 12.4 GPa, respectively; and α, β, δ, K_o', A_1', A_2', B', n_2, q and σ_o are the ten material parameters. The parameters that contain a prime, K_o', A_1', A_2' and B', are the values at 25 C of the corresponding unprimed temperature-dependent expressions in the Munson-Dawson model. Table 1 gives the parameter values for WIPP salt.

TABLE 1
Munson-Dawson Parameter Values

PARAMETER	VALUE	PARAMETER	VALUE
α	-17.37	A_2'	$4.557 \times 10^{5}\ s^{-1}$
β	-7.738	B'	$1.432 \times 10^{-9}\ s^{-1}$
δ	0.58	n_2	5.0
K_o'	9.728×10^{6}	q	5.335×10^{3}
A_1'	$4.040 \times 10^{4}\ s^{-1}$	σ_o	20.57 MPa

SENSITIVITY COEFFICIENTS

Modern testing machines control either stress rate or total strain rate. Generally, the controlled stress rate or strain rate is held constant and is often zero (e.g. creep and stress relaxation tests). Although stress rate and strain rate are controlled, the quantities measured are the stress and strain. Therefore, the response model for a given laboratory test is obtained by specifying the rate of the controlled variable in Eqn 1 and then integrating Eqn 1 in conjunction with Eqns 2 and 3 for the specimen response.

Senseny and Fossum (1995) numerically simulated various loading histories and assessed the importance of each constitutive parameter in producing the specimen behavior for another viscoplastic constitutive model, MATMOD. They used sensitivity coefficients (Bard, 1974) to show the adequacy of the data from a twelve-test test matrix to evaluate MATMOD's nine isothermal parameters.

Sensitivity coefficients are the derivative of the material behavior, stress or strain, with respect to the constitutive parameters. Sensitivity coefficients show the influence of each parameter on the current specimen response as a function of loading history and the parameter values. The expressions for the sensitivity coefficients are obtained by differentiating the expressions for stress rate in strain-rate-control tests, or for strain rate in the stress-rate-control tests with respect to the parameter, P, and integrating with respect to time. The sensitivity coefficient during strain-rate control is obtained by integrating

$$\frac{\partial \dot{\sigma}}{\partial P} = -E\left(\frac{\partial \dot{\epsilon}^p}{\partial P} + \frac{\partial \dot{\epsilon}^p}{\partial \sigma}\frac{\partial \sigma}{\partial P} + \frac{\partial \dot{\epsilon}^p}{\partial \zeta}\frac{\partial \zeta}{\partial P}\right) \tag{4}$$

and for stress-rate control, the sensitivity coefficient is obtained by integrating

$$\frac{\partial \dot{\epsilon}}{\partial P} = \frac{\partial \dot{\epsilon}^p}{\partial P} + \frac{\partial \dot{\epsilon}^p}{\partial \zeta}\frac{\partial \zeta}{\partial P} \tag{5}$$

The expression for the partial derivative of the internal variable ζ is obtained by differentiating the evolutionary equation, Eqn. 3, with respect to the parameter. Interchanging the order of differentiation with respect to the parameter P and time produces an ordinary, first-order differential equation for the desired partial derivative.

Computing the derivatives can be greatly simplified by casting Equations 2 and 3 in the form given by Munson et al (1989)

$$\dot{\epsilon}^p = F\dot{\epsilon}_{ss}\,\mathrm{sgn}(\sigma) \tag{6}$$

$$\dot{\zeta} = (F-1)\dot{\epsilon}_{ss}\,\mathrm{sgn}(\sigma) \tag{7}$$

where

$$F = \exp\left[\left(\alpha + \beta \ln\frac{\sigma}{\mu}\right)\left\langle 1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right\rangle^2\right] + \exp\left[-\delta\left\langle\frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3} - 1\right\rangle^2\right] - 1 \tag{8}$$

$$\dot{\epsilon}_{ss} = A_1'\left(\frac{\sigma}{\mu}\right)^{5.5} + A_2'\left(\frac{\sigma}{\mu}\right)^{n_2} + B'\sinh\left[\frac{q\langle\sigma - \sigma_o\rangle}{\mu}\right] \tag{9}$$

Then what needs to be computed is

$$\frac{\partial \dot{\epsilon}^p}{\partial X} = \left\{\frac{\partial F}{\partial X}\dot{\epsilon}_{ss} + F\frac{\partial \dot{\epsilon}_{ss}}{\partial X}\right\}\mathrm{sgn}(\sigma) \tag{10}$$

$$\frac{\partial \dot{\zeta}}{\partial X} = \left\{\frac{\partial F}{\partial X}\dot{\epsilon}_{ss} + (F-1)\frac{\partial \dot{\epsilon}_{ss}}{\partial X}\right\}\mathrm{sgn}(\sigma) \tag{11}$$

where X is σ, ζ or P. The appendix gives the expressions for the derivatives of F and $\dot{\epsilon}_{SS}$ with respect to σ, ζ and the parameters, P.

RESULTS

Figures 1, 2 and 3 plot specimen response and sensitivity coefficients for three loading histories. The sensitivity coefficients are normalized by dividing by the parameter value to allow ready comparison of the role of the model parameters in producing the material response for the imposed loading history.

Figure 1 shows the results for a stress relaxation test in which the specimen is loaded at a constant strain rate of $10^{-5}s^{-1}$ to a strain of 0.01, which produces a stress of about 23 MPa. The strain rate is reduced to zero and the stress is allowed to decay for 10^3 seconds. This figure shows that stress relaxation tests are most sensitive to the values of α, β and n_2. The highest sensitivity is during the constant-strain rate load-up phase. The sensitivities for these parameters peak very early in the load-up, at about 100 seconds, and the remainder of the test shows little sensitivity to any parameter.

Figure 2 shows the results for a two-stage constant strain-rate test in which the specimen is first loaded at a strain rate of $10^{-7}s^{-1}$ to a strain of 0.01 when the strain rate is suddenly increased to $10^{-5}s^{-1}$. This figure shows that the stress measured is sensitive to A_2 and K_o' as well as the same three parameters as in the stress relaxation test. The sensitivities peak early in the first, low-strain rate stage and then decline slowly. The sensitivities increase sharply following the increase in strain rate, and then decline slowly during the second stage.

Figure 3 shows the results for that the strain measured in a two-stage creep test. The specimen is rapidly loaded to 21 MPa and allowed to creep for 10^7 seconds, when the strain is nearly 0.01. The stress is suddenly reduced to 15 MPa and the specimen is allowed to creep at an accelerating rate for another 10^7 seconds. This figure shows that strain is sensitive to δ as well as to the same five parameters as in the constant strain rate test. The sensitivity to n_2 is extremely large and is not plotted in the figure. Sensitivity to α and β increases rapidly and then becomes relatively constant after the stress is held constant in the first stage. Sensitivities to the other three parameters, n_2, A_2 and K_o', continue to increase in time, which suggests that long-term tests are necessary to obtain accurate estimates of these parameters. As expected, sensitivity to δ increases after the stress is decreased to begin the second stage and recovery contributes to the response.

DISCUSSION

Long-term creep tests are the most efficient tests for evaluating the Munson-Dawson parameters. Individual stress relaxation and constant strain rate tests did not provide information for more or different parameters, and no combination of stress relaxation and constant strain rate tests could match the efficiency of the creep tests.

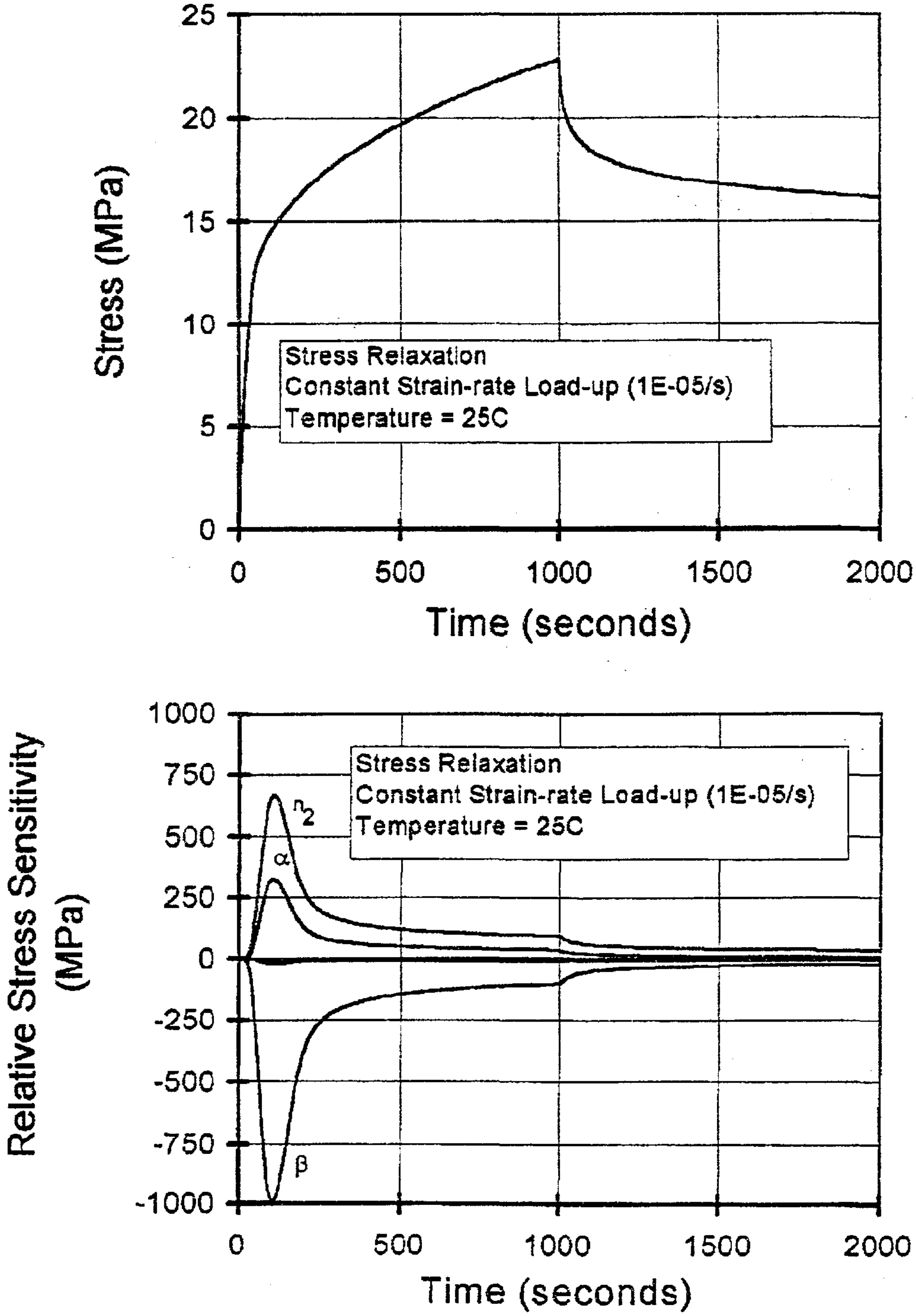

Figure 1. Specimen Response and Sensitivity Coefficients for Stress Relaxation

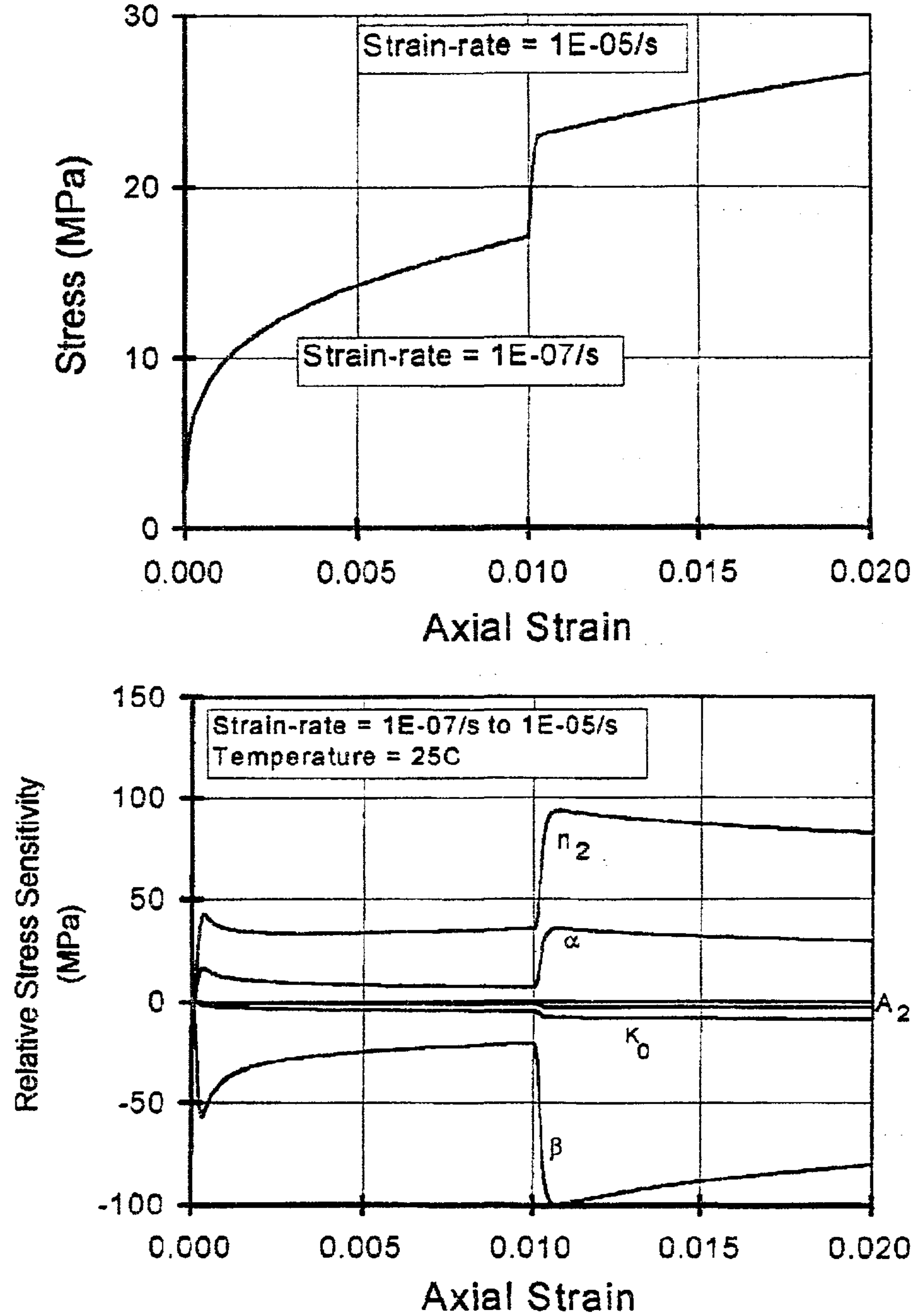

Figure 2. Specimen Response and Sensitivity Coefficients for Constant Strain Rate

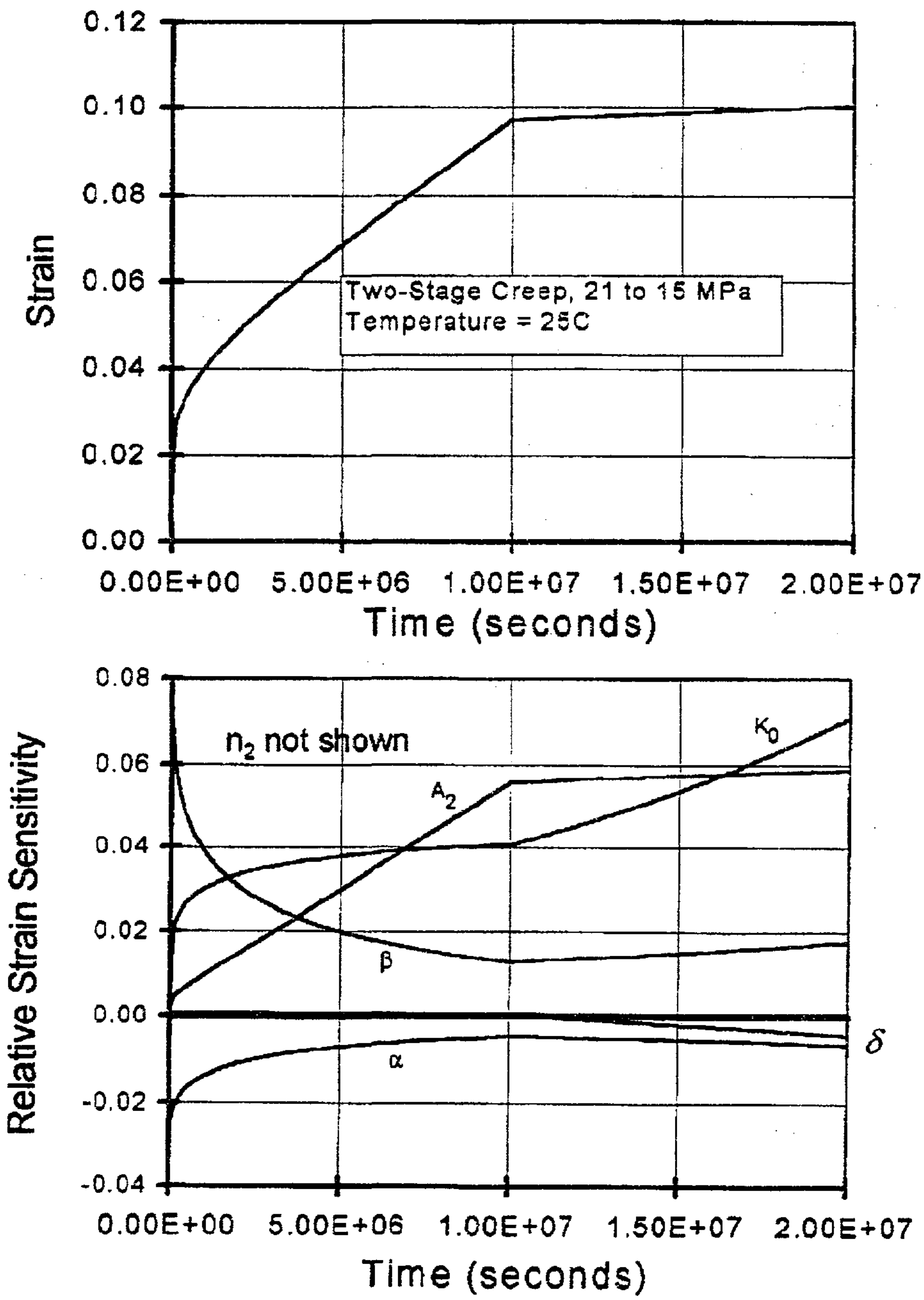

Figure 3. Specimen Response and Sensitivity Coefficients for Creep

Not all model parameters could be evaluated from the low-temperature, low-stress tests that were simulated and analyzed. The sensitivity coefficients for A'_1, B', q and σ_o are nearly zero for the three tests considered. This is a consequence of the explicit incorporation in the model of the micromechanical mechanisms that control the deformation. A fundamental premise of the Munson-Dawson model is that different deformation mechanisms control the rate of inelastic deformation in different stress-temperature regimes. Tests need to be performed at higher stress and temperature to evaluate the remaining model parameters that characterize the deformation resulting from other micromechanical mechanisms.

Extensive use of the Munson-Dawson model to predict the deformation at the WIPP shows that the model parameters A'_1, B', q and σ_o are not important for the low-temperature and low-stress environment at the WIPP. Therefore, low-stress, low-temperature testing is adequate to develop the model for this very important application.

The results showing efficiency of the various tests are different from those obtained previously by Senseny and Fossum (1995) for MATMOD, an internal-variable viscoplastic constitutive model they applied to salt. For that model, long-term tests were not necessary, but loading histories other than creep tests were necessary to evaluate all model parameters. Stress relaxation and constant strain rate tests were needed to obtain large sensitivity coefficients for each parameter. Another difference is that low-stress, low-temperature testing was adequate to evaluate all MATMOD parameters. This is because the MATMOD basis is phenomenological rather than mechanistic. The phenomenological approach uses a single function to approximate the contributions of the individual micromechanical deformation mechanisms over wide ranges in stress and temperature. This compromise in modeling the fundamental deformation processes facilitates evaluation of the MATMOD parameters.

CONCLUSIONS

Sensitivity coefficients have been derived for the parameters in the isothermal form of the Munson-Dawson constitutive model. The coefficients have been evaluated along several loading histories to show the utility of data from such tests in evaluating the constitutive parameters. Data from long-term creep tests are essential to evaluate the Munson-Dawson parameters. This contrasts with results from a previous study with the MATMOD constitutive model which showed that stress relaxation and constant strain rate tests were also important, and that long durations are not required.

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APPENDIX

The equations for the derivatives of F and $\dot{\epsilon}_{ss}$ with respect to σ, ζ and the parameters, P, are

$$\frac{\partial F}{\partial \sigma} = \exp\left[\left(\alpha + \beta \ln\frac{\sigma}{\mu}\right)\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)^2\right]\left\{\frac{\beta}{\sigma}\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)^2 + 2\left(\alpha + \beta \ln\frac{\sigma}{\mu}\right)\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)\frac{3\frac{\zeta}{\sigma}}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right\} + \exp\left[-\delta\left(\frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3} - 1\right)^2\right]\left\{2\delta\left(\frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3} - 1\right)\frac{3\frac{\zeta}{\sigma}}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right\}$$

$$\frac{\partial \dot{\epsilon}_{ss}}{\partial \sigma} = \frac{5.5A_1'}{\mu}\left(\frac{\sigma}{\mu}\right)^{4.5} + \frac{n_2 A_2'}{\mu}\left(\frac{\sigma}{\mu}\right)^{n_2 - 1} + \frac{qB'}{\mu}\cosh\left[\frac{q(\sigma - \sigma_o)}{\mu}\right]H(\sigma - \sigma_o)$$

$$\frac{\partial F}{\partial \zeta} = -\frac{2}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\left\{\exp\left[\left(\alpha + \beta \ln\frac{\sigma}{\mu}\right)\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)^2\right]\left\{\left(\alpha + \beta \ln\frac{\sigma}{\mu}\right)\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)\right\} + \exp\left[-\delta\left(\frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3} - 1\right)^2\right]\delta\left(\frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3} - 1\right)\right\}$$

$$\frac{\partial \dot{\epsilon}_{ss}}{\partial \zeta} = 0$$

$$\frac{\partial F}{\partial \alpha} = \exp\left[\left(\alpha + \beta \ln\frac{\sigma}{\mu}\right)\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)^2\right]\left\{\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)^2\right\}$$

$$\frac{\partial \dot{\epsilon}_{ss}}{\partial \alpha} = 0$$

$$\frac{\partial F}{\partial \beta} = \exp\left[\left(\alpha + \beta \ln\frac{\sigma}{\mu}\right)\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)^2\right]\left\{\ln\frac{\sigma}{\mu}\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)^2\right\}$$

$$\frac{\partial \dot{\epsilon}_{ss}}{\partial \beta} = 0$$

$$\frac{\partial F}{\partial \delta} = -\exp\left[-\delta\left(\frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3} - 1\right)^2\right]\left\{\left(\frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3} - 1\right)^2\right\}$$

$$\frac{\partial \dot{\epsilon}_{ss}}{\partial \delta} = 0$$

$$\frac{\partial F}{\partial K_0'} = \frac{2\zeta}{K_0'^2\left(\frac{\sigma}{\mu}\right)^3}\left\{\exp\left[\left(\alpha + \beta \ln\frac{\sigma}{\mu}\right)\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)^2\right]\left\{\left(\alpha + \beta \ln\frac{\sigma}{\mu}\right)\left(1 - \frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3}\right)\right\} + \exp\left[-\delta\left(\frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3} - 1\right)^2\right]\left\{-\delta\left(\frac{\zeta}{K_o'\left(\frac{\sigma}{\mu}\right)^3} - 1\right)\right\}\right\}$$

$$\frac{\partial \dot{\epsilon}_{ss}}{\partial K_0'} = 0$$

$$\frac{\partial F}{\partial A_1'} = 0$$

$$\frac{\partial \dot{\epsilon}_{ss}}{\partial A_1'} = \left(\frac{\sigma}{\mu}\right)^{5.5}$$

$$\frac{\partial F}{\partial A_2'} = 0$$

$$\frac{\partial \dot{\epsilon}_{ss}}{\partial A_2'} = \left(\frac{\sigma}{\mu}\right)^{n_2}$$

$$\frac{\partial F}{\partial B'} = 0$$

$$\frac{\partial \dot{\epsilon}_{ss}}{\partial B'} = \sinh\left[\frac{q\langle\sigma - \sigma_o\rangle}{\mu}\right]$$

$$\frac{\partial F}{\partial n_2} = 0$$

$$\frac{\partial \dot{\epsilon}_{ss}}{\partial n_2} = A_2'\left(\frac{\sigma}{\mu}\right)^{n_2} \ln\frac{\sigma}{\mu}$$

$$\frac{\partial F}{\partial q} = 0$$

$$\frac{\partial \dot{\epsilon}_{ss}}{\partial q} = B'\cosh\left[\frac{q\langle\sigma - \sigma_o\rangle}{\mu}\right]\frac{\langle\sigma - \sigma_o\rangle}{\mu}$$

$$\frac{\partial F}{\partial \sigma_o} = 0$$

$$\frac{\partial \dot{\epsilon}_{ss}}{\partial \sigma_o} = -\frac{B'q}{\mu}\cosh\left[\frac{q\langle\sigma - \sigma_o\rangle}{\mu}\right]\mathrm{H}(\sigma - \sigma_o)$$