SAND2024-12569C

Uncertainty quantification of single and multi-parameter full-waveform inversion through a variational autoencoder

Abdelrahman Elmeliegy[1*], Mrinal Sen[1], Jennifer Harding[2] and Hongkyu Yoon[2]

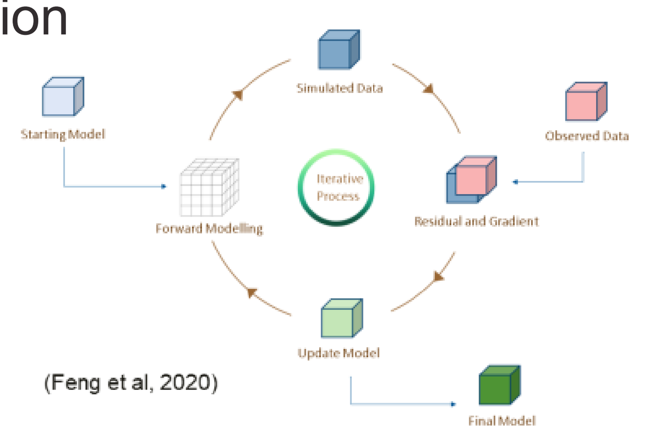
[1] The University of Texas at Austin
[2] Sandia National Laboratories

1. RESEARCH GAP, OBJECTIVE & SCOPE

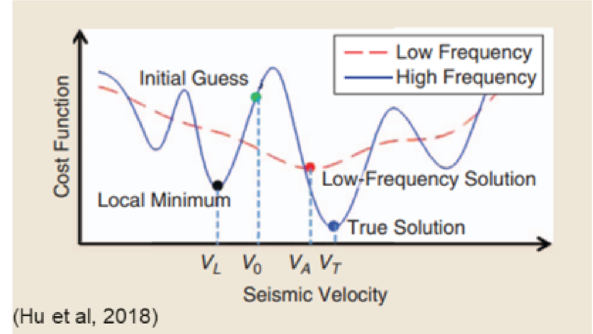
Motivation. Quantify uncertainties of single and multi-parameter full waveform inversion

Full waveform inversion

- Provides high resolution velocity model of subsurface
- Suffer from non-uniqueness, nonlinearity, noise and low-wavenumber content
- Problem: all these hinder the reliability of the FWI results
- Goal: assess the uncertainty of FWI



(Feng et al, 2020)

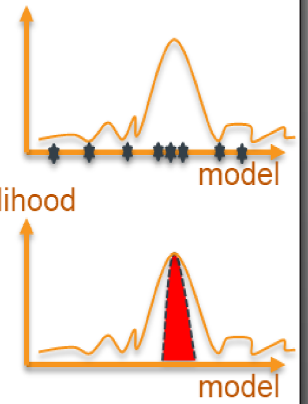


(Hu et al, 2018)

State of the art. Bayesian Inversion

- Sampling methods: e.g., MCMC – robust but computationally expensive
- Variational methods: cheaper but comes with approximations

$$\underbrace{p(\mathbf{m}|\mathbf{d})}_{\text{posterior}} = \frac{p(\mathbf{d}|\mathbf{m})\,p(\mathbf{m})}{p(\mathbf{d})} \propto \underbrace{p(\mathbf{d}|\mathbf{m})}_{\text{likelihood}}\,\underbrace{p(\mathbf{m})}_{\text{prior}}$$



- $\mathbf{m}$ True velocity model
- $p(\mathbf{d})$ Probability of seismic data $\mathbf{d}$ being observed
- $p(\mathbf{m})$ Probability of seismic velocity distribution $\mathbf{m}$
- $p(\mathbf{d}|\mathbf{m})$ Conditional probability of observing $\mathbf{d}$ given seismic model $\mathbf{m}$
- $p(\mathbf{m}|\mathbf{d})$ Conditional probability of model $\mathbf{m}$ when $\mathbf{d}$ is observed

Objective. Provide a computationally affordable framework for the uncertainty quantification of FWI for both acoustic and elastic cases

Scope

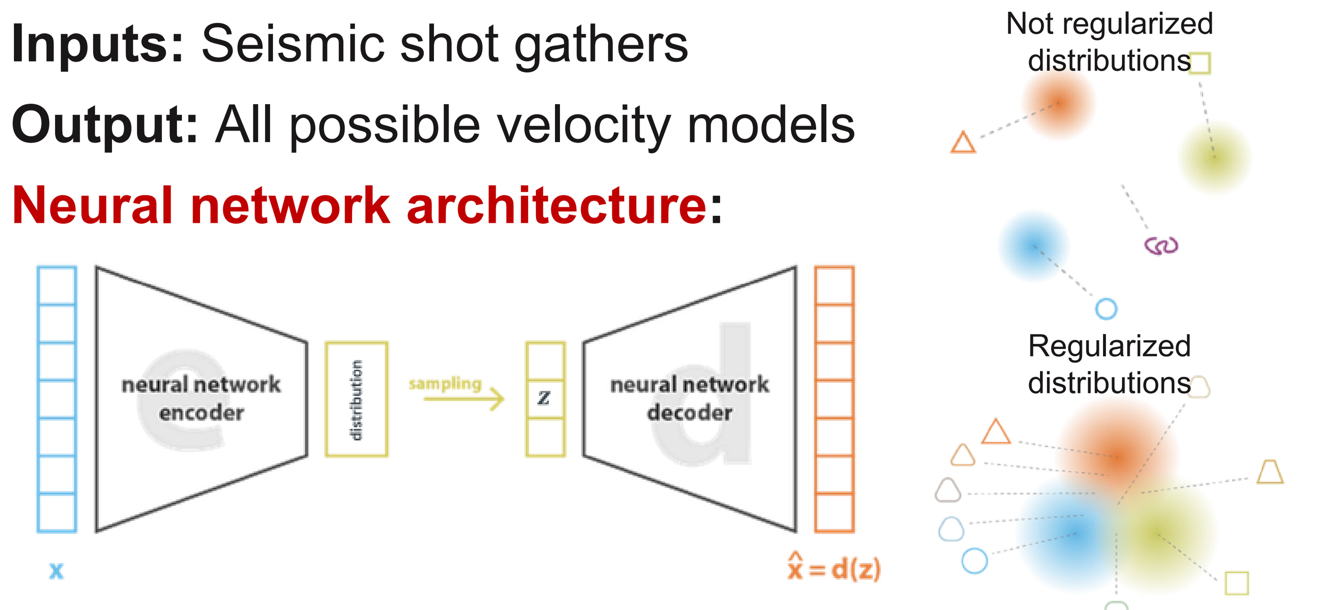
- Deep learning through convolutional Neural Network
- Generative artificial intelligence (Gen AI)

2. VARIATIONAL AUTO-ENCODER

Inputs: Seismic shot gathers

Output: All possible velocity models

Neural network architecture:



1. **Encoder:** Compresses information into a distribution of features defined by some known distribution e.g., Gaussian. Those features are the most important
Decoder: Learns a inverse mapping function that maps the abstracted info such that the reconstructed data/image is close to the input data

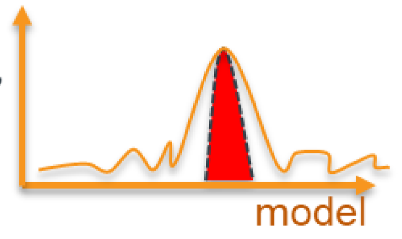
Theory

$$(\hat{g},\hat{h}) = \underset{(g,h)\in G,H}{\operatorname{argmin}}\ \mathcal{D}[q_x(z), p(z|x)],$$

$$(\hat{g}_\theta,\hat{h}_\theta) = \underset{(g,h)\in G,H}{\operatorname{argmin}}\left[\mathbb{E}_{z\sim q_x}\left(\frac{\|x-d_\theta(z)\|^2}{2\sigma^2}\right) + \mathcal{D}(q_x(z),p(z))\right],$$

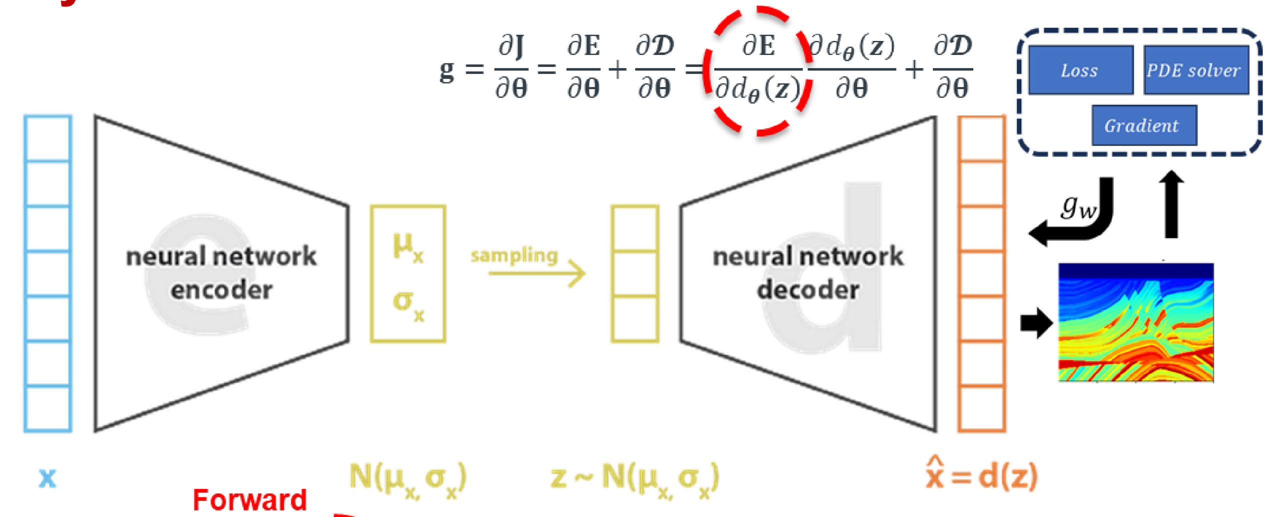
$$(\hat{d}_\theta,\hat{g}_\theta,\hat{h}_\theta) = \underset{(D,g,h)\in D,G,H}{\operatorname{argmin}}\ J(\theta)$$

$$J(\theta) = \left[\mathbb{E}_{z\sim q_x}\left(\frac{\|x-d_\theta(z)\|^2}{2\sigma^2}\right) + \mathcal{D}(q_x(z),p(z))\right].$$



Physics Informed Neural Network

$$g = \frac{\partial J}{\partial \theta} = \frac{\partial E}{\partial \theta} + \frac{\partial \mathcal{D}}{\partial \theta} = \frac{\partial E}{\partial d_\theta(z)}\frac{\partial d_\theta(z)}{\partial \theta} + \frac{\partial \mathcal{D}}{\partial \theta}$$

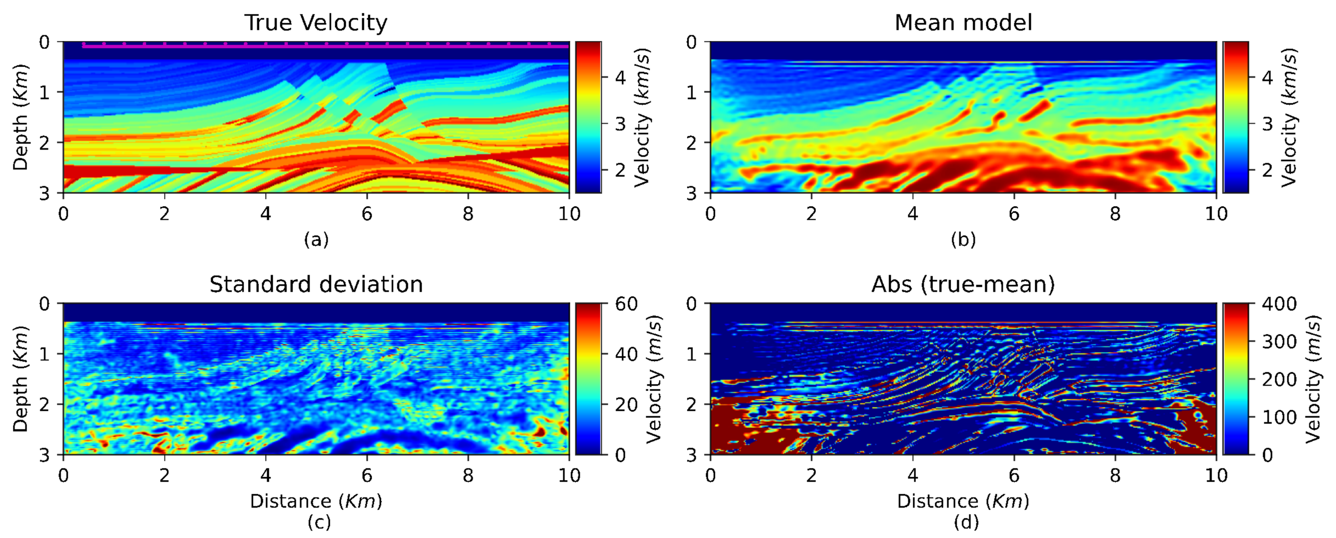


$$J(\theta) = \left[\mathbb{E}_{z\sim q_x}\left(\frac{\|x-F[d_\theta(z)]\|^2}{2\sigma^2}\right) + \mathcal{D}(q_x(z),p(z))\right]$$

Data misfit (promotes accuracy) — Regularization (promotes diversity)

3. ACOSTIC FULL WAVEFORM INVERSION

Example : Marmousi



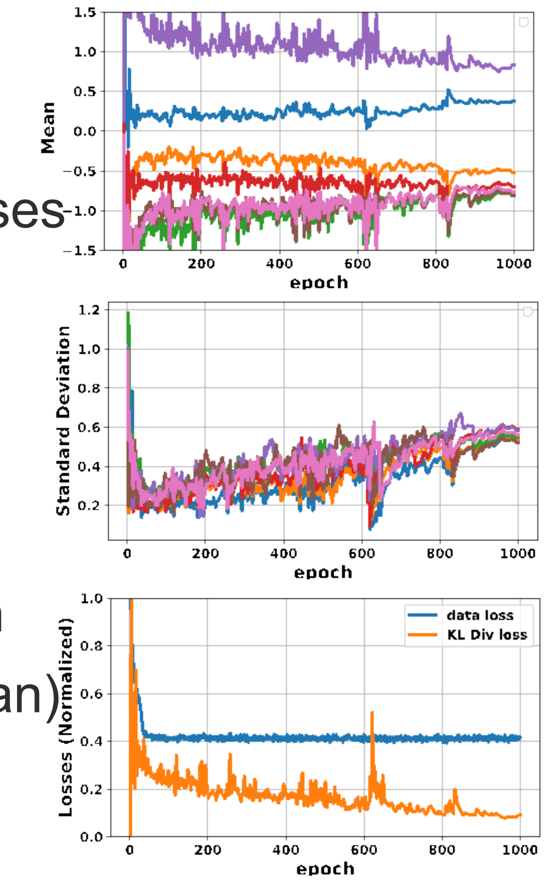
Notes:

1- The KL-divergence losses decrease as inversion progresses

2- There is a tradeoff between the data accuracy and the variability

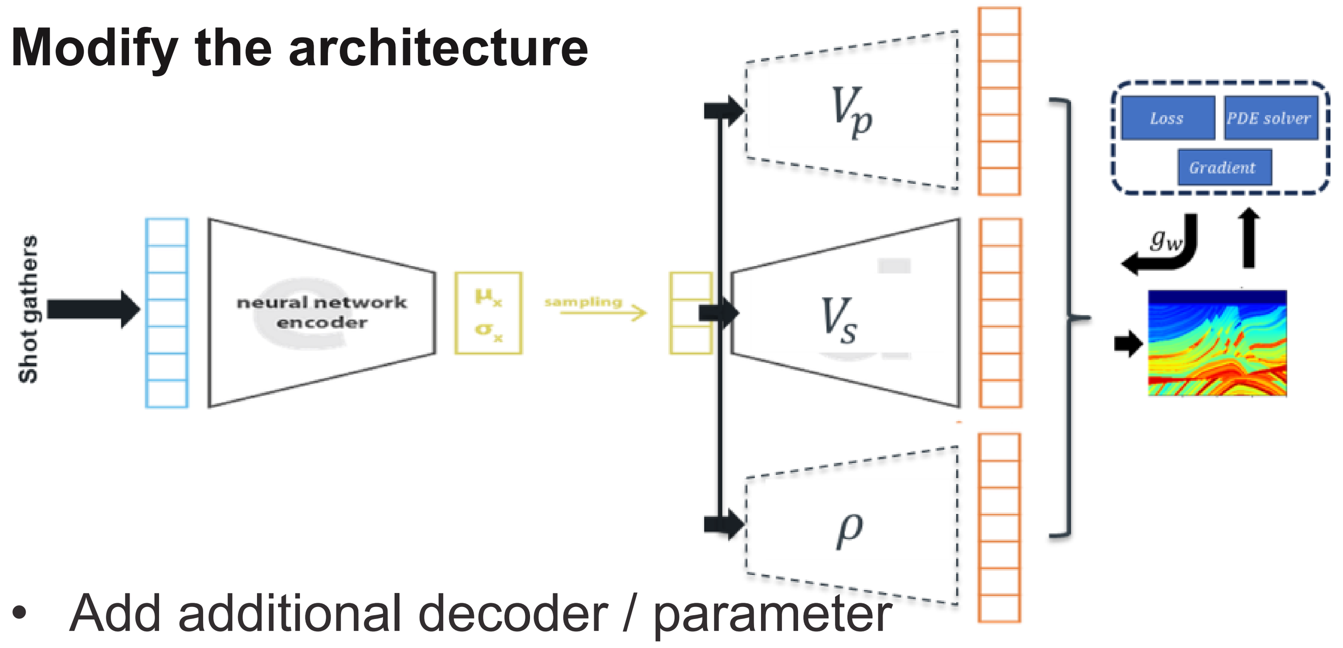
3- Standard deviation of latent variables increases over epoch (to match the standard Gaussian)

4- The mean latent variables are getting clustered as inversion advances



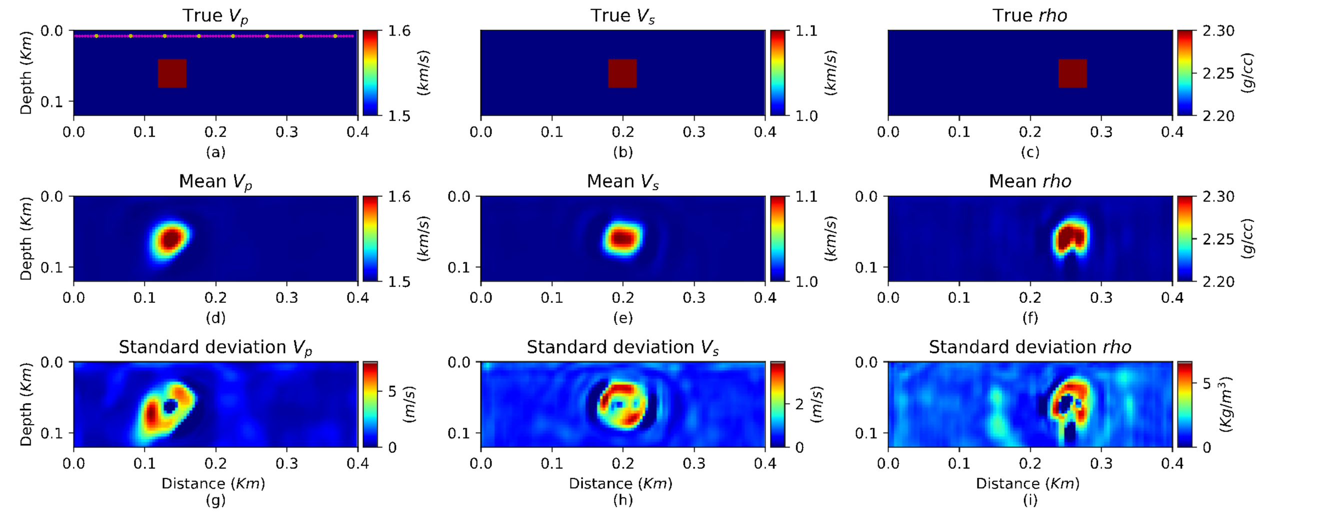
4. ELASTIC FULL WAVEFORM INVERSION

Modify the architecture

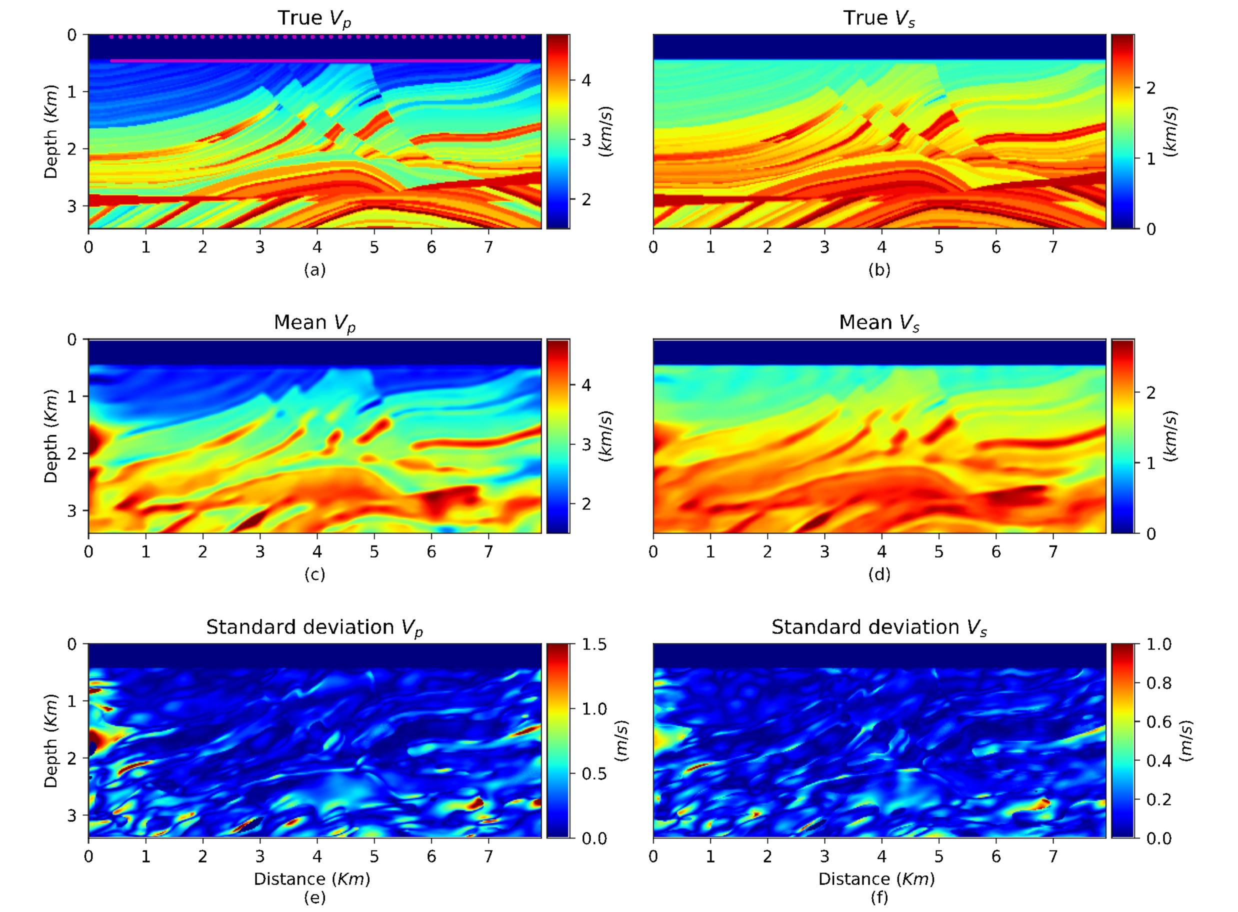


- Add additional decoder / parameter
- This helps in resolving the crosstalk between parameters

Example: Toy model (inclusion)



Example: Marmousi



5. SUMMARY

- VAE framework is a promising approach for quantifying FWI uncertainties.
- Results show that the framework is cheap and can handle single and multi-parameter inversion
- Results should be carefully interpreted especially with the variance since it seems to be underestimated

6. REFERENCES

- Elmeliegy et al, IMAGE Extended Abstract, (2024).
- Dhara and Sen. The Leading Edge 2022 and IEEE Transactions on Geosci. and Remote Sensing 2023.

7. Acknowledgments

This work was supported by the Laboratory Directed Research and Development program at Sandia National Laboratories.

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