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Harmonic balance based methodology for reduced order modeling to predict response of nonlinear structures

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INTRODUCTION

- Detailed FEA capable of accurately simulating structural response of nonlinear structures
  - High computational cost
- ROMs can enable faster simulation
  - Typically still involve coupled ODEs and more equations than modes
- Motivates use of Nonlinear Modal Models (NLMMs)
  - Efficiency of traditional ROMs while capturing nonlinear phenomena of interest
- Application to rapid assessment of nonlinear bolted joint behavior

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Proposal

This work proposes a technique for nonlinear structural response prediction utilizing harmonic balance solutions of a ROM to generate characterization data used to identify nonlinear modal models.

APPROACH

- The approach consists of two main steps: identification and validation
  - Identification consists of using a FEM, HCB ROM, running Harmonic Balance, and identifying nonlinear parameters
  - Validation consists of comparing responses between the HCB model and NLMM model

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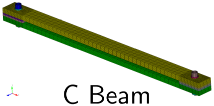
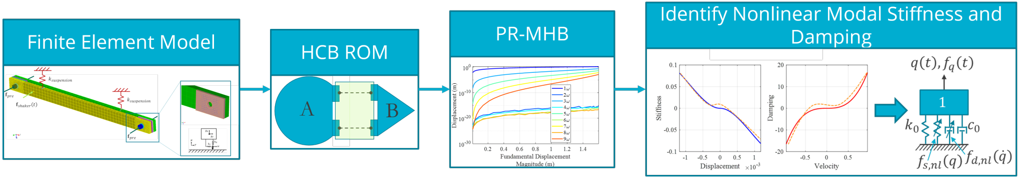
  - Identification consists of using a FEM, HCB ROM, running Harmonic Balance, and identifying nonlinear parameters
  - Validation consists of comparing responses between the HCB model and NLMM model
- The benchmark system used to evaluate the proposed methodology is the C-Beam (S4 Beam)
- Sierra/SD used for the eigen, preload analysis and ROM development
- MATLAB toolset used to generate NL HCB and perform PR-MHB



C Beam

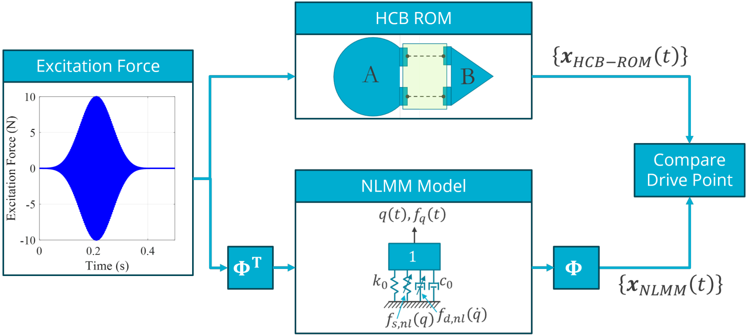
IDENTIFICATION



Identification Process

1. Creation of FE model
2. HCB reduction and physical NL on boundaries
3. PR-MHB characterizes mode of interest; compute resonant frequencies and Fourier Coefficients
4. Use modal space representation and fit NL parameters

VALIDATION

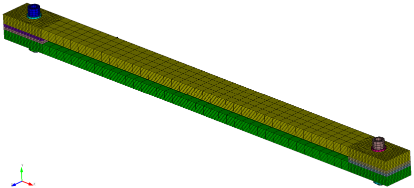


Validation Process

1. Excite mode of interest with windowed sinusoid
2. Run simulations with both HCB ROM and NLMM
3. Transform to physical space and compare drive point responses

MODEL DESCRIPTION

- 3D Sierra FEM of C-Beam (S4) with preloaded bolts
- Mode 2 of interest (bending)
- HCB ROM generated and MATLAB environment used to add NL contact and perform PR-MHB



3D FEM

Mode 2 - 332 Hz

PR-MHB

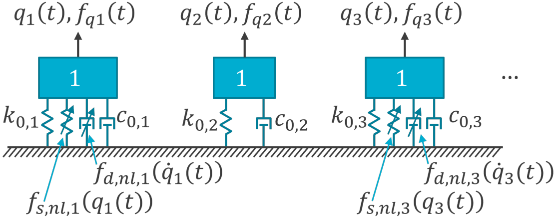
- Compute time-periodic solutions for harmonically forced NL system
- Displacement and input force represented by finite Fourier series
- Obtain periodic solutions using continuation and AFT
- Numerical analogy to force appropriation testing (90 deg phase relation)

$$M\ddot{x} + C\dot{x} + Kx + f_{nl}(x,\dot{x}) = f_{pre} + f_{ext} \tag{1}$$

$$\mathrm{x(t)} = \frac{c_0^x}{\sqrt{2}} + \sum_{k=1}^{N_h}[s_k^x \sin(k\omega t) + c_k^x \cos(k\omega t)] \tag{2}$$

$$\mathrm{f_{ext}(t)} = \frac{c_0^f}{\sqrt{2}} + \sum_{k=1}^{N_f}[s_k^f \sin(k\omega t) + c_k^f \cos(k\omega t)] \tag{3}$$

NLMM



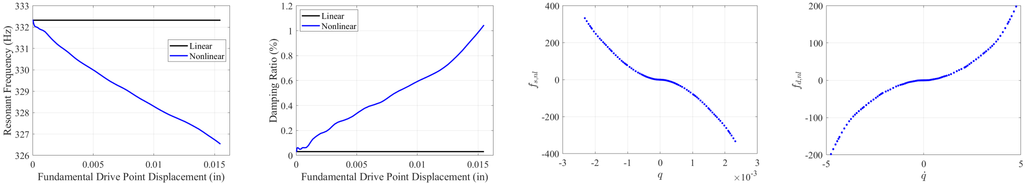
Nonlinear Modal Model

1. Analogous to uncoupled linear modal model, but with NL elements
2. Modal equation of motion now captures NL effects with spring and damper restoring forces per mode:

$$\ddot{q}_j(t) + c_{0,j}\dot{q}_j(t) + f_{d,nl,j}(\dot{q}_j(t)) + k_0 q_j(t) + f_{s,nl,j}(q_j(t)) = f_{q,j}(t) \qquad (4)$$

3. Assume mode shapes do not change with amplitude and modal coupling negligible

IDENTIFICATION RESULTS

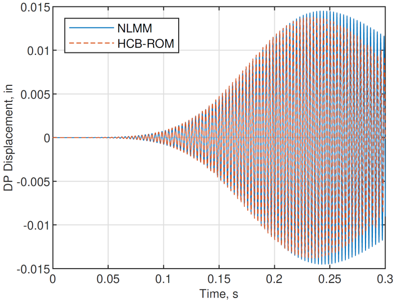


Amplitude-dependent, frequency, damping, and modal stiffness and damping

1. PR-MHB results give amplitude-dependent results. Note changes in frequency and damping (vs drive point displacement)
2. NLMM fit after taking modal space results and creating stiffness vs displacement and damping vs velocity curves
3. NLMM data used as lookup table during simulations

VALIDATION RESULTS

- Drive point displacement response
  - Compare NLMM to original HCB ROM used to derive it
  - Sine beat input
- Preliminary results match well initially working on checking full 1s duration



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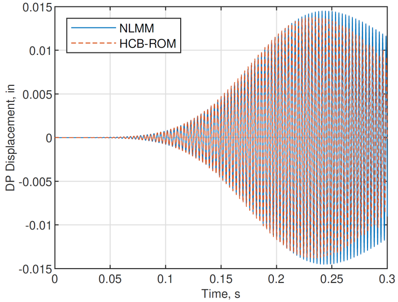


Validation Results

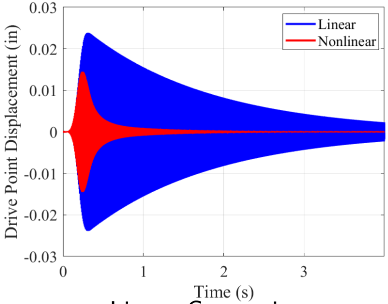
- NLMM is fast
- HCB-ROM / PR-MHB take setup time
  - Over order of magnitude faster than 3D FEM
  - Can quickly re-run for other loading

Time Summary

Method	Sim. Time (s)	Run Time (s)
PR-MHB	-	40000
NLMM	1	50
3D FEM	1	1e5-1e6

NOTES ON LINEARITY

- Linear model highly overpredicts response
- This methodology can be used to quickly get idea of joint nonlinearity
- Better representation of dynamics without cost of full FEA



Linear Comparison

CONCLUSION

- This work demonstrated a technique utilizing harmonic balance solutions of a ROM to generate characterization data used to identify nonlinear modal models
- The nonlinear modal model matched the results from the ROM well and was significantly faster to solve
- Future work will apply to other nonlinear modes of the structure and validate the results against a broadband input
- Predicting joint dissipation has potential to be used in early phase design studies and this methodology can be used to provide valuable information without the need of costly high-fidelity FEA

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ACKNOWLEDGEMENTS

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Questions?

BACKUPS

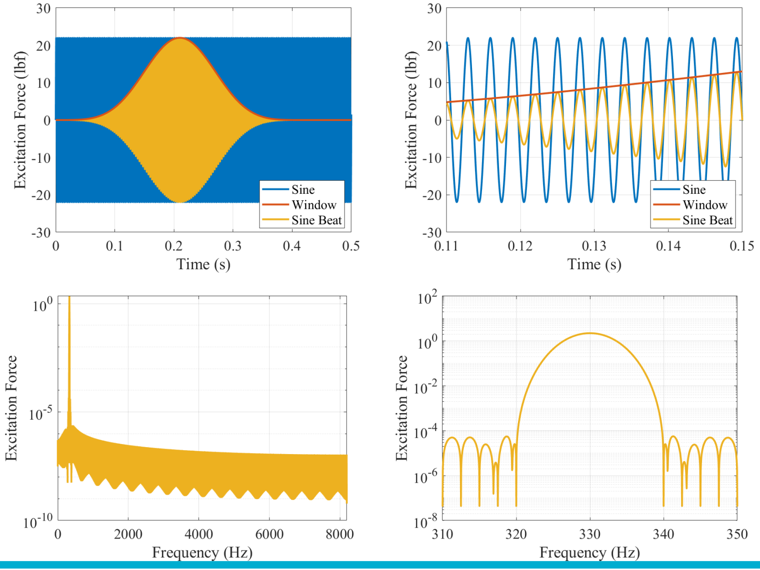
IDENTIFICATION DETAILS

Make lookup tables for $f_{s,nl}$ and $f_{d,nl}$ as functions of q and $\dot{q}$, respectively, which can be used in simulations with the NLMM. Results of the PR-MHB computations on the HCB ROM are cast to modal space $j = 8$ (mode 2). Solve for the NL restoring force, $f_{r,nl}(t)$:

$$f_{d,nl}(\dot{q}) + f_{s,nl}(q) = f_q - \ddot{q} - c_{\mathbf{0}}\dot{q} - k_{\mathbf{0}}q = f_{r,nl}(t) \quad (5)$$

f_q, $\ddot{q}$, $\dot{q}$, and q from the PR-MHB results. $k_{\mathbf{0}}$ and $c_{\mathbf{0}}$ known from linear eigen analysis. Parameters are known or measured. NL stiffness and damping are assumed to be functions of modal disp and vel- determined from the data at time instances when $\dot{q} = 0$ (at time instances $t_{\dot{q}=\mathbf{0}}$) and when $q = 0$ (at time instances $t_{q=\mathbf{0}}$), respectively. For each excitation level (PR-MHB solution), the $q(t_{\dot{q}=\mathbf{0}})$ and $f_{r,nl}(t_{\dot{q}=\mathbf{0}})$ are evaluated for one period of the fundamental frequency and become two abscissa-ordinate pairs $(q(t_{\dot{q}=\mathbf{0}}), f_{r,nl}(t_{\dot{q}=\mathbf{0}}))$ of $f_{s,nl}(q)$. Done for each excitation level, and results combined to form the entire $f_{s,nl}(q)$. Similar for $f_{d,nl}(\dot{q})$ using $\dot{q}(t_{q=\mathbf{0}})$ and $f_{r,nl}(t_{q=\mathbf{0}})$ for each excitation level. Equivalent to taking slices of the restoring force surface (3D space defined by q, $\dot{q}$, $f_{r,nl}(q, \dot{q})$) at the $\dot{q} = 0$ plane for $f_{s,nl}(q)$ and at the $q = 0$ plane for $f_{d,nl}(\dot{q})$. $f_{s,nl}(q)$ and $f_{d,nl}(\dot{q})$ data used as lookup tables during the NLMM validation simulation.

VALIDATION LOAD- SINE BEAT

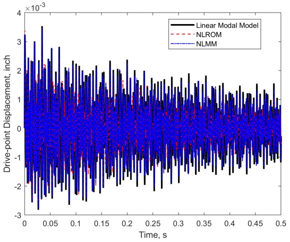


MODELING STEPS

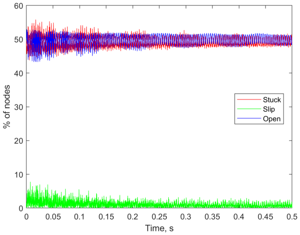
1. Generate 3D mesh and run Sierra modal and static preload
2. Create HCB reduction and obtain ROM matrices
3. Use NL contact on node basis with slider elements to preload structure and do modal analysis
4. Use dynamic matrix info from ROM and preloaded state to feed into MHB code
5. Perform PR-MHB- apply loading, use continuation algorithm for solution
6. Obtain Fourier coefficients from MHB and convert data to modal space
7. Get modal force and acceleration at each loading point and get backbone
8. Fit NLMM from modal information
9. Generate loading for validation
10. Run NLMM and HCB with load and compare drive point response

HAVERSINE RESULTS

- Preliminary results from haversine shock (100 lbf and 200 lbf, 1 ms input)
- Displacement and contact status response
- Note difference vs linear and ability to capture joint slip during loading



Time Response



Joint Contact Status