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Low Frequency Cable Dual Shield Penetration Using Complete Transfer Immittance Model

Larry K. Warne and Alden R. Pack

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico
87185 and Livermore,
California 94550

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Low Frequency Cable Dual Shield Penetration Using Complete Transfer Immittance Model

L. K. Warne, Plasma/EM Software Dept.
and A. R. Pack, EM/Electronics Analyses Dept.
Sandia National Laboratories
P. O. Box 5800
Albuquerque, NM 87185-1152

Abstract

This report discusses low frequency coupling and subsequent penetration of a dual shield cable using a transfer immittance model with reciprocal sources. Configurations include: 1) where the cable is above a ground plane and shorted at both ends, 2) a monopole arrangement where it is shorted at one end and open at the other end, and 3) a monopole arrangement where the open end is loaded by a disc. The open circuit voltage within the cable is estimated for each case.

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1 LOW FREQUENCY SHIELD CURRENT

The shield current at low frequencies is now estimated for short circuit conditions at both ends and for an open circuit at one end and a short circuit at the opposite end. We initially neglect the interior currents compared to the outer shield layer.

1.1 Short-Short Circuit Cable Case

In the case where the cable is taken to be above some ground plane structure and shorted at both ends, we estimate the magnetic flux between the cable and the return (with no current on the cable)

$$\Phi_0 = \ell\phi_0 = L_{loop}I_s^{sc} = \int_S \underline{B} \cdot \underline{n}dS \approx A_{loop}\mu_0H_0 \quad (1)$$

where

$$A_{loop} = O(\ell h_g) \quad (2)$$

and H_0 is the drive field above the ground plane and ϕ_0 is the average flux per unit length between the cable and ground plane. The inductance of the loop is estimated from the wire of radius b , a distance h_g above ground, inductance per unit length

$$L = \frac{\mu_0}{2\pi} \text{arccosh}(h_g/b) = \frac{\mu_0}{2\pi} \ln \sqrt{\frac{h_g + h_e}{h_g - h_e}} \quad (3)$$

where

$$h_e = \sqrt{h_g^2 - b^2} \quad (4)$$

and the total loop inductance is

$$L_{loop} \approx L\ell \quad (5)$$

where ℓ is the wire length. For fixed H_0 we see that the low frequency value of the shield current I_s^{sc} is constant in frequency

$$I_s^{sc} \approx A_{loop}\mu_0H_0/L_{loop} \quad (6)$$

1.2 Open-Short Circuit Cable Case

In the case where the cable is shorted at one end to a substantial structure and open at the other end, it is driven by the incident electric field in the presence of the structure E_0 . the open circuit voltage at the shorted end of the cable is

$$V_s^{oc} = -Q_{cab}/C_{cab} = -h_e^{oc}E_0 \quad (7)$$

where the effective height is related to the monopole height h by using the current distribution for an electrically short transmitting monopole

$$I_t(z) = I_t(0)(h-z)/h \quad (8)$$

where the open circuit voltage is found from

$$V_s^{oc} = \frac{1}{I_t(0)} \int_0^h I_t(z) E_z dz = \frac{E_0}{h} \int_0^h (h-z) dz = -\frac{1}{2} E_0 h \quad (9)$$

and therefore

$$h_e^{oc} \sim h/2 \quad (10)$$

The average charge per unit length q_0 is

$$Q_{cab} \approx h q_0 = C_{cab} E_0 h / 2 \quad (11)$$

where the capacitance is taken as that of a monopole above a ground plane, which from the first order Hallen iteration method is

$$C_{cab} \sim \frac{4\pi\epsilon_0 h}{\Omega_e} = \epsilon_0 2\pi h / \ln\left(\frac{h}{be}\right), \quad \Omega_e = 2 \ln(2h/b) - 2(1 + \ln 2) = 2 \ln\left(\frac{h}{be}\right), \quad h/b \geq 10 \quad (12)$$

The received current at the base of the monopole is

$$I_s^{sc} = \frac{d}{dt} Q_{cab} = -i\omega Q_{cab} \quad (13)$$

and the distribution is

$$I_s(z) = I_s^{sc} (h-z) / h \quad (14)$$

1.2.1 Disc Loaded Monopole

If we load the monopole with a disc having much larger radius $b_d \gg b$, but still with $h > b_d$, we add an additional charge at the tip

$$Q_{disc} = C_{disc} E_0 h \quad (15)$$

where a disc in free space has capacitance

$$C_{disc} = \epsilon_0 8b_d \quad (16)$$

Then using the transmitting current distribution

$$I_t(z) = I_{disc} + I_{cab}(0) (h-z) / h \quad (17)$$

where

$$I_{cab}(0) = -i\omega Q_{cab} = -i\omega \frac{h}{2} E_0 C_{cab} \quad (18)$$

$$I_{disc} = -i\omega Q_{disc} = -i\omega h E_0 C_{disc} \quad (19)$$

we find

$$V_{oc} = -\frac{E_0}{C_{disc} + \frac{h}{2} C_{cab}} \int_0^h \left[C_{disc} + \frac{1}{2} C_{cab} (h-z) / h \right] dz = -\frac{E_0}{C_{disc} + \frac{h}{2} C_{cab}} \left[z C_{disc} - \frac{1}{4} C_{cab} (h-z)^2 / h \right]_0^h \quad (20)$$

so that the effective height in this loaded case is

$$h_e^{oc} = h \frac{C_{disc} + \frac{1}{4}C_{cab}}{C_{disc} + \frac{1}{2}C_{cab}} \quad (21)$$

and again

$$Q_{cab} = hq_0 = C_{cab}E_0h/2 \quad (22)$$

$$C_{cab} \sim \frac{4\pi\epsilon_0h}{\Omega_e} \quad (23)$$

We see that the effective height varies over the range

$$h/2 \leq h_e^{oc} \leq h \quad (24)$$

The current at the base in this case is

$$I_s^{sc} = \frac{d}{dt}(Q_{disc} + Q_{cab}) = -i\omega(Q_{disc} + Q_{cab}) \quad (25)$$

For fixed E_0 we see that the low frequency value of the shield current I_s^{oc} is proportional to frequency in these cases with open circuits at the top end (versus the previous short circuited case driven by the magnetic field).

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2 CABLE SHIELD TRANSFER IMPEDANCE

If the cable shield has two layers, a braid layer and a solid layer, we first discuss models for each and then assemble the total.

2.1 Braid Layer

The Kley model [1] for the braid penetration has transfer impedance

$$Z_T \approx Z_{TR}^{int} - i\omega L_T + (1 - i)\omega L_S \quad (26)$$

where the diffusion contribution is [1]

$$Z_{TR}^{int} = R_{gs} \frac{\gamma d_R}{\sin(\gamma d_R)} = R_{gs} \frac{(1 + i) d_R / \delta}{\sin(d_R / \delta) \cosh(d_R / \delta) + i \cos(d_R / \delta) \sinh(d_R / \delta)} \quad (27)$$

$$R_{gs} = \frac{4}{\sigma m n d^2 \pi \cos \alpha} = \frac{1}{\sigma G_0 \cos \alpha} \frac{2}{\pi^2 D_m d} \quad (28)$$

$$d_R \approx 0.67d / \sqrt{\cos \alpha} \quad (29)$$

$$\gamma = (1 + i) / \delta \quad (30)$$

$$\delta = \sqrt{2 / (\omega \mu \sigma)} \quad (31)$$

The external inductive contribution has hole and porpoising parts [1]

$$L_T = M_L + L_G \quad (32)$$

$$M_L \approx \mu_0 0.875 \frac{\pi (2 - \cos \alpha)}{6m} (1 - G)^3 e^{-\tau_H} \quad (33)$$

$$\tau_H = 0.8\tau_E = 9.6G \sqrt[3]{B^2 d / D_m} \quad (34)$$

$$L_G \approx -\mu_0 \frac{0.11d}{2\pi D_m G_0} \cos(2k_1 \alpha) = -\mu_0 \frac{0.11}{mn} \cos(2k_1 \alpha) \quad (35)$$

$$k_1 = \frac{\pi}{4} \left(\frac{2}{3} G_0 + \frac{\pi}{10} \right)^{-1} \quad (36)$$

and the internal impedance contribution also has hole and porpoising contributions [1]

$$\omega L_S = \frac{1}{\pi \sigma \delta} (1/D_L + 1/D_G) \quad (37)$$

$$D_L^{-1} \approx 10\pi G_0^2 \frac{\cos \alpha}{D_m} (1 - G) e^{-\tau_E} \quad (38)$$

$$D_G^{-1} \approx -\frac{3.3}{2\pi D_m G_0} \cos(2k_2 \alpha) \quad (39)$$

$$k_2 = \frac{\pi}{4} \left(\frac{2}{3}G_0 + \frac{3}{8} \right)^{-1} \quad (40)$$

For the Belden 8240 cable the geometry parameters are

$$m = 16 \quad (41)$$

$$n = 7 \quad (42)$$

$$d = 0.005 \text{ in} \quad (43)$$

$$D_0 \approx 0.116 \text{ in} \quad (44)$$

$$D_m = D_0 + 2d + h \approx D_0 + 2.5d \approx 0.1285 \text{ in} \quad (45)$$

$$s \approx 0.889 \text{ in} \quad (46)$$

$$\alpha = \arctan(\pi D_m/s) = 24.4^\circ \quad (47)$$

$$G_0 = mnd/(2\pi D_m) \approx 0.6936 \quad (48)$$

$$G = G_0/\cos \alpha \approx 0.7616 \quad (49)$$

$$B = G(2 - G) \approx 0.943 \quad (50)$$

where B is the optical coverage. Then the Kley parameters are

$$d_R \approx d(0.70209) \quad (51)$$

$$\sigma \approx 5.8 \times 10^7 \text{ S/m (copper at } 20^\circ\text{C)} \quad (52)$$

$$R_{gs} \approx 0.0133 \text{ ohms/m} \quad (53)$$

$$\tau_H \approx 1.4766 \quad (54)$$

$$M_L \approx \mu_0 (3.902 \times 10^{-5}) \quad (55)$$

$$k_1 \approx 1.0114 \quad (56)$$

$$L_G \approx -\mu_0 (6.3973 \times 10^{-4}) \quad (57)$$

$$D_L \approx 0.76954 \text{ in} \quad (58)$$

$$k_2 \approx 0.9379 \quad (59)$$

$$D_G \approx -0.24328 \text{ in} \quad (60)$$

Consequently we can write the transfer impedance terms as ($\delta = 0.026018 \text{ in}$ at 1 MHz) $d_R/\delta \approx 1.3492436$

$$Z_{TR}^{int} \approx R_{gs} \frac{(1+i) 1.3492436}{2.006695 + i0.395} \approx R_{gs} 0.7747042 (1 + i0.6710657) \text{ , at 1 MHz} \quad (61)$$

$$|Z_{TR}^{int}| \approx R_{gs} (0.9329734) \approx 0.012408 \text{ ohms/m at 1 MHz} \quad (62)$$

$$\omega L_S = \frac{1}{1.2041595 \times 10^4 \text{ S}} (1/0.76954 \text{ in} - 1/0.24328 \text{ in}) \approx -0.00919 \text{ ohms/m at 1 MHz} \quad (63)$$

$$\omega L_T \approx -\omega \mu_0 (6.0071 \times 10^{-4}) \approx -0.004743 \text{ ohms/m at 1 MHz} \quad (64)$$

We see from these that the largest term is the diffusion term and this becomes slightly larger (approaching R_{gs}) as the frequency is reduced, whereas the other terms decrease with frequency ($\omega L_s = O(\sqrt{\omega})$ and $\omega L_T = O(\omega)$).

However, for frequencies where $d/2 \leq \delta$, which occurs at 1.083 MHz, isolated wires behave nearly the same as that of a wire with uniform current density; it is thus somewhat unclear if the higher frequency forms in the internal impedance in Kley are really accurate; although these terms become small at lower frequencies anyway. The external inductance is also of somewhat questionable accuracy, but since the porpoising is dominant for this optical coverage (which depends less on wire current distribution) we will include it and take

$$Z_T \approx R_{gs} - i\omega L_T \text{ below 1 MHz} \quad (65)$$

The self impedance of the braid could be taken as

$$Z_s = Z_R^{int} - i\omega \Delta L \approx Z_R^{int} \quad (66)$$

where ΔL is the correction for the braid structure versus a solid wall, and

$$Z_R^{int} \approx R_{gs} \frac{\gamma d_R}{\tan(\gamma d_R)} = Z_{TR}^{int} \{ \cos(d_R/\delta) \cosh(d_R/\delta) - i \sin(d_R/\delta) \sinh(d_R/\delta) \} \quad (67)$$

2.2 Solid Layer

If the shield also has a solid layer of thickness Δ and mean radius b_2 its transfer impedance is [2]

$$Z_{2TR}^{int} = R_2 \frac{\gamma_2 \Delta}{\sin(\gamma_2 \Delta)} = R_{gs} \frac{(1+i) \Delta/\delta_2}{\sin(\Delta/\delta_2) \cosh(\Delta/\delta_2) + i \cos(\Delta/\delta_2) \sinh(\Delta/\delta_2)} \quad (68)$$

and the self impedance is

$$Z_{2R}^{int} = R_2 \frac{\gamma_2 \Delta}{\tan(\gamma_2 \Delta)} = Z_{2TR}^{int} \cos(\gamma_2 \Delta) = Z_{2TR}^{int} \{ \cos(\Delta/\delta_2) \cosh(\Delta/\delta_2) - i \sin(\Delta/\delta_2) \sinh(\Delta/\delta_2) \} \quad (69)$$

$$R_2 = \frac{1}{2\pi b_2 \Delta \sigma_2} \quad (70)$$

$$\gamma_2 = (1 + i) / \delta_2 \quad (71)$$

$$\delta_2 = \sqrt{2 / (\omega \mu \sigma_2)} \quad (72)$$

3 COMPLETE TRANSFER IMMITTANCE MODEL

Usually the interior cable current I is small compared to the exterior cable shield current $I \ll I_s$, and the reciprocal sources on the cable exterior are neglected; this typically results when the system is driven from the exterior and there are loads on the interior (these loads can result from the gap inductance between conductors if shorted or actual loads). Under conditions where the interior current is comparable to the exterior current, the complete transfer immittance model can be used [2]. The full transfer immittance cable model has the form shown in Figure 1. The system of equations are then

$$\frac{dV}{dz} + ZI = Z_T I_s \quad (73)$$

$$\frac{dI}{dz} + YV = -Y_T V_s \quad (74)$$

$$\frac{dV_s}{dz} + Z_s I_s = Z_T I + i\omega\phi_0 \quad (75)$$

$$\frac{dI_s}{dz} + Y_s V_s = -Y_T V + i\omega q_0 \quad (76)$$

where ϕ_0 is the magnetic flux per unit length linking the loop, q_0 is the electric charge per unit length deposited by the electric field, and

$$Z = Z^{int} - i\omega L^{ext} \quad (77)$$

$$Z_s = Z_s^{int} - i\omega L_s \quad (78)$$

$$Z_T = Z_T^{int} - i\omega L_T^{ext} \quad (79)$$

$$Y = G - i\omega C \quad (80)$$

$$Y_s = G_s - i\omega C_s \quad (81)$$

the superscript *int* refers to the internal impedance (resulting from finite metal conductivity) of the conductors and *ext* refers to the external inductance (if the conductors are perfect); self and transfer diffusion solutions can be inserted for the *int* terms, which will involve the shield thickness. The part of the internal impedances due to the diffusion of an equivalent layer of thickness d_R (see semi-empirical models in Kley) could be taken as

$$Z_{TR}^{int} \approx R_{gs} \frac{\gamma d_R}{\sin(\gamma d_R)} \quad (82)$$

$$Z_R^{int} \approx R_{gs} \frac{\gamma d_R}{\tan(\gamma d_R)} \quad (83)$$

where R_{gs} is the DC resistance per unit length (for an actual layer of radius b , $1/(2\pi b\sigma d_R)$), and the propagation constant and skin depth are

$$\gamma = (1 + i)/\delta \quad (84)$$

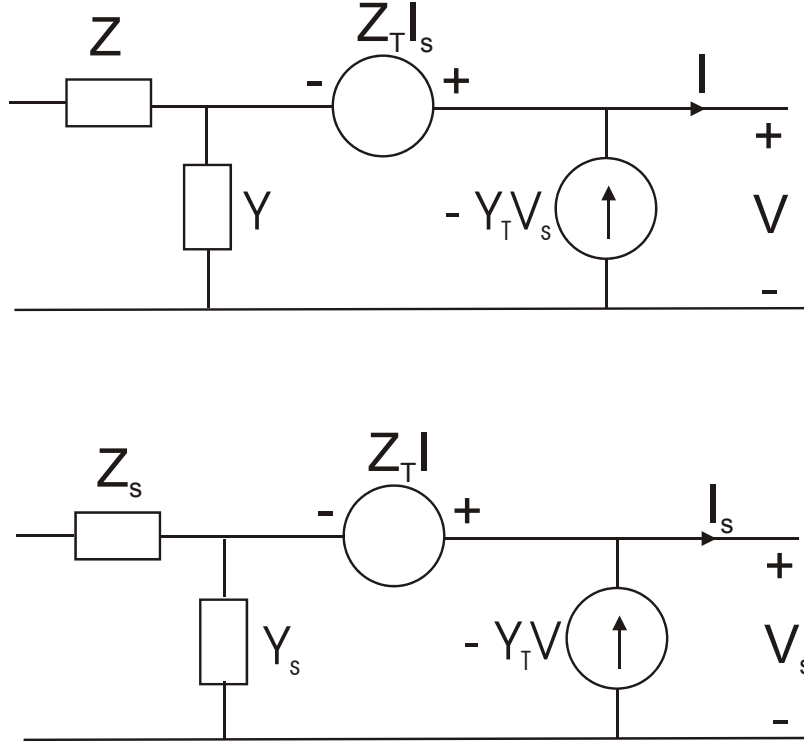


Figure 1: Complete transfer immittance cable model.

$$\delta = \sqrt{2/(\omega\mu\sigma)} \quad (85)$$

3.0.1 Extremely Low Frequency Limits

To illustrate the behavior at extremely low frequencies (taking the skin depth to be large compared to the thickness of the metallic conductors) we set

$$Z \sim Z^{int} \sim R_s + R_c \rightarrow R_s \quad (86)$$

$$Z_s \sim Z_s^{int} \sim R_s + R_r \rightarrow R_s \quad (87)$$

$$Z_T \sim Z_T^{int} \sim R_s \quad (88)$$

where here R_s is the shield resistance per unit length, the outer chassis return resistance per unit length is for convenience taken as perfectly conducting $R_r = 0$, and the center conductor resistance per unit length is also for convenience taken as perfectly conducting $R_c = 0$. The extremely low frequency equations then become

$$\frac{dV}{dz} + R_s I \approx R_s I_s \rightarrow \frac{dV}{dz} \approx R_s (I_s - I) \quad (89)$$

$$\frac{dI}{dz} = 0 \quad (90)$$

$$\frac{dV_s}{dz} + R_s I_s \approx R_s I \rightarrow \frac{dV_s}{dz} \approx R_s (I - I_s) \quad (91)$$

$$\frac{dI_s}{dz} = 0 \quad (92)$$

Defining the total shield current as

$$I_s^{tot} = I_s - I \quad (93)$$

the voltage equations become

$$\frac{dV}{dz} \approx R_s I_s^{tot} \quad (94)$$

$$\frac{dV_s}{dz} \approx -R_s I_s^{tot} \quad (95)$$

which obviously make sense at the low frequencies (noting that V is the center conductor-to-shield voltage and V_s is the shield-to-return chassis voltage).

If we have conductive material for the dielectric and a perforated perfectly conducting shield with $R_s = 0$

$$Y = G \quad (96)$$

$$Y_s = G_s \quad (97)$$

$$Y_T = G_T \quad (98)$$

$$\frac{dI}{dz} = -(G_T V_s + G V) \quad (99)$$

$$\frac{dI_s}{dz} = -(G_T V + G_s V_s) \quad (100)$$

$$\frac{dI_s^{tot}}{dz} = (G_T - G_s) V_s + (G_T - G) (-V) \quad (101)$$

where V_s is the voltage from the shield to the chassis and $-V$ is the voltage from the shield to the center conductor. The symmetrical form of the conductance elements makes sense here.

3.1 Application To Short-Short Cable Case

In the case where both exterior ends of the cable are shorted, let us ignore the current sources due to Y_T , drop Y_2 and Y_s , as well as labeling the solid shield current as I_2 and the solid shield voltage V_2

$$\frac{dV_2}{dz} + Z_2 I_2 = Z_T I_s \quad (102)$$

$$\frac{dI_2}{dz} = 0 \quad (103)$$

$$\frac{dV_s}{dz} + Z_s I_s = Z_T I_2 + i\omega\phi_0 \quad (104)$$

$$\frac{dI_s}{dz} = 0 \quad (105)$$

We need to keep in mind that Z_2 includes not only the inductance per unit length between braided and solid shields, but also the sums of the impedances per unit length of both shields (in a typical transmission line the losses on the walls for current on the center conductor and on the return are summed); similarly, in principal Z_s includes the external inductance per unit length to the cable, but also the sum of the braid and chassis return impedances per unit length.

Now with $V_2(0) = 0$

$$V_2(z) = z(Z_T I_s - Z_2 I_2) \quad (106)$$

Enforcing $V_2(\ell) = 0$

$$Z_T I_s = Z_2 I_2 \quad (107)$$

Also taking $\phi_0 = \Phi_0/\ell$ with $V_s(0) = 0$

$$V_s(z) = z(Z_T I_2 - Z_s I_s) + i\omega z \Phi_0/\ell \quad (108)$$

Enforcing $V_s(\ell) = 0$

$$(Z_T I_2 - Z_s I_s) \ell + i\omega \Phi_0 = 0 \quad (109)$$

Now using the preceding result to eliminate I_2

$$(Z_T^2/Z_2 - Z_s) I_s + i\omega \Phi_0/\ell = 0 \quad (110)$$

giving

$$I_s = \frac{i\omega \Phi_0/\ell}{Z_s - Z_T^2/Z_2} = (i\omega \Phi_0/\ell) \frac{Z_2}{Z_s Z_2 - Z_T^2} \quad (111)$$

$$I_2 = \frac{(Z_T/Z_s) i\omega \Phi_0/\ell}{Z_2 - Z_T^2/Z_s} = (i\omega \Phi_0/\ell) \frac{Z_T}{Z_2 Z_s - Z_T^2} \quad (112)$$

Finally we find

$$V_{oc} = I_2 Z_{2TR}^{int} = (i\omega \Phi_0/\ell) \frac{Z_T Z_{2TR}^{int}}{Z_2 Z_s - Z_T^2} \quad (113)$$

$$Z_{2TR}^{int} = R_2 \frac{\gamma_2 \Delta}{\sin(\gamma_2 \Delta)} \quad (114)$$

Then inserting the parameters

$$Z_T \approx Z_{TR}^{int} - i\omega L_T + (1-i)\omega L_S \text{ or } Z_T \approx R_{gs} - i\omega L_T \text{ at the lower frequencies} \quad (115)$$

$$Z_s = Z_R^{int} - i\omega L \quad (116)$$

$$Z_2 = Z_{2R}^{int} + Z_R^{int} - i\omega L_2 \approx Z_{2R}^{int} + Z_R^{int} \quad (117)$$

$$Z_{2R}^{int} = R_2 \frac{\gamma_2 \Delta}{\tan(\gamma_2 \Delta)} \quad (118)$$

$$L_2 \approx \frac{\mu_0}{2\pi} \left(\frac{D_0 - 2b_2 - \Delta}{D_0 + 2b_2 + \Delta} \right) \rightarrow 0 \quad (119)$$

Finally we find

$$V_{oc} = I_2 Z_{2TR}^{int} = (i\omega \Phi_0 / \ell) \frac{Z_T Z_{2TR}^{int}}{(Z_{2R}^{int} + Z_R^{int})(Z_R^{int} - i\omega L) - Z_T^2} \quad (120)$$

$$I_2 = (i\omega \Phi_0 / \ell) \frac{Z_T}{(Z_{2R}^{int} + Z_R^{int})(Z_R^{int} - i\omega L) - Z_T^2} \quad (121)$$

$$I_s = (i\omega \Phi_0 / \ell) \frac{Z_{2R}^{int} + Z_R^{int}}{(Z_R^{int} - i\omega L)(Z_{2R}^{int} + Z_R^{int}) - Z_T^2} \quad (122)$$

3.1.1 Low Frequency Form

Suppose we take

$$Z_{2TR}^{int} \rightarrow R_2 \quad (123)$$

$$Z_{2R}^{int} \rightarrow R_2 \quad (124)$$

$$Z_R^{int} \rightarrow R_{gs} \quad (125)$$

$$I_s = (i\omega \Phi_0 / \ell) \frac{R_2 + R_{gs}}{(R_{gs} - i\omega L)(R_2 + R_{gs}) - (R_{gs} - i\omega L_T)^2} \quad (126)$$

$$I_2 = (i\omega \Phi_0 / \ell) \frac{R_{gs} - i\omega L_T}{(R_2 + R_{gs})(R_{gs} - i\omega L) - (R_{gs} - i\omega L_T)^2} \quad (127)$$

$$I_s^{tot} = I_s - I_2 = (i\omega \Phi_0 / \ell) \frac{R_2 + i\omega L_T}{(R_{gs} - i\omega L)(R_2 + R_{gs}) - (R_{gs} - i\omega L_T)^2} \quad (128)$$

$$V_{oc} \rightarrow (i\omega \Phi_0 / \ell) \frac{(R_{gs} - i\omega L_T) R_2}{(R_2 + R_{gs})(R_{gs} - i\omega L) - (R_{gs} - i\omega L_T)^2} \quad (129)$$

Very Low Frequency Limit Check If the very low frequency limit of these is taken

$$I_s = \frac{i\omega\Phi_0/\ell}{R_{gs} - R_{gs}^2/(R_{gs} + R_2)} = \frac{i\omega\Phi_0/\ell (R_{gs} + R_2)}{R_{gs} R_2} \quad (130)$$

$$I_s^{tot} = I_s - I_2 = \frac{i\omega\Phi_0/\ell}{R_{gs}} \quad (131)$$

$$I_2 = \frac{(R_{gs}/R_{gs}) i\omega\Phi_0/\ell}{R_{gs} + R_2 - R_{gs}^2/R_{gs}} = \frac{i\omega\Phi_0/\ell}{R_2} \quad (132)$$

This second set is simply the parallel combination of the two resistances

$$I_s^{tot} + I_2 = I_s = (i\omega\Phi_0/\ell) (1/R_{gs} + 1/R_2) \quad (133)$$

3.2 Application To Open-Short Cable Case

In the case where we have an open circuit at one end, the shield current is no longer strictly uniform along the length of the cable. Again dropping Y_T , Y_2 , Y_s as well as labeling the solid shield current as I_2 and the solid shield voltage V_2

$$\frac{dV_2}{dz} + Z_2 I_2 = Z_T I_s \quad (134)$$

$$\frac{dI_2}{dz} = 0 \quad (135)$$

$$\frac{dV_s}{dz} + Z_s I_s = Z_T I_2 \quad (136)$$

$$\frac{dI_s}{dz} = i\omega q_0 \quad (137)$$

and thus I_2 is constant and I_s is linear (if q_0 is constant). If there is no disc load at the end of the cable we take $q_0 = Q_{cab}/h$ and with $I_s(h) = 0$

$$V_s^{oc} = Q_{cab}/C_{cab} = h_e^{oc} E_0 = E_0 h/2 \quad (138)$$

$$C_{cab} \sim \frac{4\pi\epsilon_0 h}{\Omega_e} = \epsilon_0 2\pi h / \ln\left(\frac{h}{be}\right), \quad \Omega_e = 2 \ln(2h/b) - 2(1 + \ln 2) = 2 \ln\left(\frac{h}{be}\right), \quad h/b \geq 10 \quad (139)$$

$$I_s(z) = i\omega(z-h) Q_{cab}/h \quad (140)$$

$$\frac{dV_s}{dz} = Z_T I_2 - Z_s i\omega(z-h) Q_{cab}/h \quad (141)$$

$$\frac{dV_2}{dz} = Z_T i\omega(z-h) Q_{cab}/h - Z_2 I_2 \quad (142)$$

Now we enforce $V_s(0) = 0 = V_2(0)$

$$V_s(z) = zZ_T I_2 - \frac{1}{2} Z_s i\omega z (z - 2h) Q_{cab}/h \quad (143)$$

$$V_2(z) = \frac{1}{2} Z_T i\omega z (z - 2h) Q_{cab}/h - zZ_2 I_2 \quad (144)$$

Next we also enforce $V_2(h) = 0$

$$-i\omega Q_{cab} \frac{1}{2} (Z_T/Z_2) = I_2 \quad (145)$$

Then the open circuit voltage on the center conductor is

$$V_{oc} = Z_{2TR}^{int} I_2 = -i\omega Q_{cab} \frac{1}{2} (Z_T/Z_2) R_2 \frac{\gamma_2 \Delta}{\sin(\gamma_2 \Delta)} \quad (146)$$

$$Z_{2TR}^{int} = R_2 \frac{\gamma_2 \Delta}{\sin(\gamma_2 \Delta)} \quad (147)$$

3.2.1 Low Frequency Form

Suppose we take

$$Z_2 = Z_{2R}^{int} + Z_R^{int} - i\omega L_2 \approx Z_{2R}^{int} + Z_R^{int} \quad (148)$$

$$Z_{2TR}^{int} \rightarrow R_2 \quad (149)$$

$$Z_{2R}^{int} \rightarrow R_2 \quad (150)$$

$$Z_R^{int} \rightarrow R_{gs} \quad (151)$$

then

$$V_{oc} \rightarrow I_2 R_2 \approx -i\omega Q_{cab} \frac{1}{2} (R_{gs} - i\omega L_T) \left(\frac{R_2}{R_2 + R_{gs}} \right) = -i\omega Q_{cab} \frac{1}{2} \left(\frac{R_{gs} - i\omega L_T}{R_{gs}} \right) \left(\frac{R_2 R_{gs}}{R_2 + R_{gs}} \right) \quad (152)$$

$$I_2 \rightarrow -i\omega Q_{cab} \frac{1}{2} \left(\frac{R_{gs} - i\omega L_T}{R_2 + R_{gs}} \right) \quad (153)$$

$$I_s^{tot} = I_s(0) - I_2 \rightarrow -i\omega Q_{cab} \frac{1}{2} \left(1 - \frac{R_{gs} - i\omega L_T}{R_2 + R_{gs}} \right) \quad (154)$$

Very Low Frequency Limit Check If the very low frequency limit of these is taken

$$I_s^{tot} = I_s(0) - I_2 \rightarrow -i\omega Q_{cab} \left(1 - \frac{hR_{gs}/2}{hR_2 + hR_{gs}} \right) \quad (155)$$

$$I_2 \rightarrow -i\omega Q_{cab} \left(\frac{R_{gs}/2}{R_2 + R_{gs}} \right) \quad (156)$$

Therefore some current is transferred to the inner solid shield. It is interesting that even when $R_2 = 0$, only half the total current is transferred. This actually makes sense since the deposited charge and resulting current on the outer shield must flow up the outer shield to $z = h$ in order to reach the connection point to the inner shield.

3.3 Application To Open-Short Cable Case With Disc

When we add a large disc to the top of the open circuited cable the current remains more nearly constant along the cable. The charges are found from

$$V_s^{oc} = Q_{cab}/C_{cab} = h_e^{oc} E_0 \approx hE_0 \quad (157)$$

$$Q_{cab} \approx hq_0 = C_{cab} \frac{h}{2} E_0 \quad (158)$$

$$C_{cab} \sim \frac{4\pi\epsilon_0 h}{\Omega_e} \quad (159)$$

$$Q_{disc} = E_0 h C_{disc} \quad (160)$$

$$C_{disc} = \epsilon_0 8b_d \quad (161)$$

Again dropping Y_T , Y_2 , Y_s as well as labeling the solid shield current as I_2 and the solid shield voltage V_2

$$\frac{dV_2}{dz} + Z_2 I_2 = Z_T I_s \quad (162)$$

$$\frac{dI_2}{dz} = 0 \quad (163)$$

$$\frac{dV_s}{dz} + Z_s I_s = Z_T I_2 \quad (164)$$

$$\frac{dI_s}{dz} = i\omega q_0 \quad (165)$$

The second equation says that I_2 is a constant, and from the final equation using the disc constant contribution to the current, we can write

$$I_s(z) = i\omega q_0 (z - h) - i\omega Q_{disc} \quad (166)$$

Then from the voltage equations enforcing $V_s(0) = 0 = V_2(0)$

$$V_2(z) + zZ_2I_2 = Z_T \left[i\omega q_0 \frac{1}{2} z(z - 2h) - i\omega Q_{disc} z \right] \quad (167)$$

$$V_s(z) + Z_s \left[i\omega q_0 \frac{1}{2} z(z - 2h) - i\omega Q_{disc} z \right] = zZ_T I_2 \quad (168)$$

Now also enforcing $V_2(h) = 0$ gives

$$I_2 = -i\omega (Z_T/Z_2) \left(q_0 h \frac{1}{2} + Q_{disc} \right) = -i\omega (Z_T/Z_2) \left(\frac{1}{2} Q_{cab} + Q_{disc} \right) \quad (169)$$

The open circuit voltage on the interior of the cable is then

$$V_{oc} = Z_{2TR}^{int} I_2 = -i\omega \left(\frac{1}{2} Q_{cab} + Q_{disc} \right) (Z_T/Z_2) R_2 \frac{\gamma_2 \Delta}{\sin(\gamma_2 \Delta)} \quad (170)$$

$$Z_{2TR}^{int} = R_2 \frac{\gamma_2 \Delta}{\sin(\gamma_2 \Delta)} \quad (171)$$

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4 INTERIOR OPEN CIRCUIT VOLTAGE AND CURRENT

From our previous results we have the interior cable open circuit voltage which can be used to assess low frequency upset issues. We can also use it to estimate current into a load.

4.1 Upset Issues

At these low frequencies we expect to be below cable resonances. Hence, the preceding open circuit voltages can be compared to a threshold screening level such as for logic circuits

$$V_{th} = 1 \text{ volt} \quad (172)$$

Note that it is possible to attach a lumped capacitance load with a value $\geq 0.1 \mu\text{F}$ to create resonances in this region (see cable inductance below), but we do not expect these to be present. If we believe these to be present the levels estimated here can be increased by the cable quality factor (which also includes losses associated with this lumped load) which can be taken as $O(100)$.

4.2 Interior Inductive Reactance Versus Load Impedance

The interior coaxial-type region has an inductance per unit length between the center conductor and shield. The inductance per unit length of the Belden 8240 cable is

$$L_{coax} = \frac{\mu_0}{2\pi} \ln \left(\frac{D_0}{d_0} \right) \approx 260.19 \text{ nH/m} \quad (173)$$

where d_0 is the interior center conductor diameter. The total interior inductance is then

$$L_{coax}^{tot} = \ell_{coax} L_{coax} \quad (174)$$

The interior inductive reactance for a meter of such cable at 1 MHz is $-i1.635$ ohms. If we put a $R_L = 1$ ohm load at the end of the cable (a typical hot-wire electro-explosive device value) we thus see that the resulting voltage divider will supply a reduced current

$$I_{sc} = \frac{V_{oc}}{R_L - i\omega L_{coax}^{tot}} \leq V_{oc}/R_L \quad (175)$$

Thus, if $V_{oc} < V_{th} = 1$ volt we will not supply sufficient current or power $P_{th} = 1$ W for initiation.

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5 CONCLUSIONS

This report estimates the coupling to a cable at low frequencies and then uses a complete immittance penetration model to compute the interior voltage for a dual shielded cable.

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